# Cascade model for exotic atoms with $Z > 2$

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**Abstract**

A cascade model for exotic atoms with $Z > 2$ is presented. The model includes radiative transitions, internal Auger effect, electron refilling, nuclear absorption, and decay of the exotic particle. The case kaonic nitrogen is treated in detail.

## 1 Introduction

Exotic atoms are formed by stopping negative particles $x^-$ ($x^- = \mu^-, \pi^-, K^-, \bar{p}$) in matter. The particles are captured by the target atoms and occupy initially highly excited states. After the formation, the so-called atomic cascade takes place: the exotic atoms deexcite to lower states via different processes (cascade mechanisms) until the ground state is reached or a nuclear reaction takes place.

In this Note a cascade model for exotic atoms with $Z > 2$ will be presented. In these exotic atoms the deexcitation take place in a competition between the internal Auger effect where an electron is ejected and radiative transitions. This is qualitatively different from the exotic hydrogen atoms ($Z = 1$) which do not contain electrons and deexcite in collisions with hydrogen molecules. The exotic helium atoms ($Z = 2$) are also special among the exotic atoms: they are the only atoms where highly excited metastable states have been observed.

Cascade programs are useful in the interpretation of experimental data. In hadronic atoms, for example, the strong interaction widths can be measured only for the lowest states. Using the experimental X-ray yields and a cascade program, the widths of higher-lying states can be determined indirectly.

## 2 Cascade model

### 2.1 Processes

The cascade model includes radiative transitions, internal Auger effect, electron refilling, nuclear absorption, and decay of the exotic particle.

#### 2.1.1 Radiative transitions

The radiative transitions

$$(x^-X)_{n_il_i} \to (x^-X)_{n_fl_f} + \gamma \tag{1}$$

take place with rates which can be obtained from those of atomic hydrogen [1] by simple scaling

$$\Gamma^{\mathrm{rad}}_{n_il_i\to n_fl_f} = \mu Z^4 \Gamma^{\mathrm{rad}}_{n_il_i\to n_fl_f}(\mathrm{H}) \ . \tag{2}$$

Here $\mu$ is the reduced mass of the exotic atom and $Z$ is the electric charge of the nucleus. The well-known properties of the E1 transitions are as follows: only $\Delta l = \pm 1$ transitions are possible. The rates decrease fast with increasing $n_i$ and the fastest of the radiative transitions is the $2p \to 1s$ deexcitation.

### 2.1.2 Internal Auger effect

The present model includes the internal Auger effect only for the K shell electrons:

$$(x^-X)_{n_il_i}2e^- \to (x^-X)_{n_fl_f}e^- + e^- \ . \tag{3}$$

In the following the $Z_e$ is the effective charge seen by the electrons and $k_e$ is the momentum of the ejected electron. The exotic particle is assumed to orbit well inside the electron K shell and experiences an electric charge $Z$. We use the internal Auger rates calculated by Burbidge and de Borde [2] for two electrons in the K shell:

$$\begin{aligned}\Gamma^{\mathrm{Auger}}_{n,n-1\to n-1,n-2} &= \frac{1}{3}\left(\frac{Z_e}{Z}\right)^2 C^2 \frac{\pi}{\mu^2}\frac{2^{2n+6}n^{2n+2}(n-1)^{2n+4}}{(2n-1)^{4n+2}} \\ &\times \frac{y^2\exp(y(4\tan^{-1}y-\pi))}{(1+y^2)\sinh \pi y} \ ,\end{aligned} \tag{4}$$

where

$$y = \frac{Z_e}{k_e} \ . \tag{5}$$

The factor $C$ is given by

$$\begin{aligned}C &= 1-\left(\frac{Z_e}{Z}\right)^2 \frac{(2n+1)(2n+2)n^2(n-1)^2}{3(2n-1)^2\mu^2} \\ &\times \frac{1+y^2}{y^2\exp(y(4\tan^{-1}y-\pi))}\left(\frac{C_1}{2}-\frac{C_2}{5}\right) \ .\end{aligned} \tag{6}$$

According to Ref. [2], the correction factors $C_1$ and $C_2$ are generally between 0 and 1 in which case we have to a very good approximation

$$C \approx 1 \ . \tag{7}$$

The rates for the non-circular transitions can be obtained from Eq. (4) and the $nl$ dependence

$$\Gamma^{\mathrm{Auger}}_{n_il_i\to n_fl_f} \propto (C^{l_f0}_{l_i010})^2 (R^{n_fl_f}_{n_il_i})^2 \tag{8}$$

where $C^{l_f0}_{l_i010}$ is a Clebsch-Gordan coefficient and $R^{n_fl_f}_{n_il_i}$ is the radial matrix element [1].

If only one electron is present in the K shell, Eq. (4) is multiplied by 1/2. For the effective electric charge we use

$$Z_e = \begin{cases} Z-1 & \text{for one electron} \\ Z-1-5/16 & \text{for two electrons} \end{cases} \tag{9}$$

corresponding to full screening of the nuclear charge by the exotic particle.

### 2.1.3 Electron refilling, nuclear absorption, and decay of the exotic particle

When there are less than two electrons in the K shell of the exotic atom, electron refilling can take place with electrons from the surroundings. Here, the surroundings are the higher shells of the exotic atoms and nearby molecules (important at high densities). In the present cascade model the electron refilling is modeled in the simplest possible way with an adjustable parameter representing the refilling rate for an empty state in the K shell.

Nuclear absorption takes place due to the wave function overlap between the exotic particle and the nucleus. Once the nuclear widths for the circular states are known, the widths of the other states are given by

$$\begin{aligned} \Gamma_{ns}^{\mathrm{nuc}} &= \frac{\Gamma_{1s}^{\mathrm{nuc}}}{n^3} && (10) \\ \Gamma_{np}^{\mathrm{nuc}} &= \frac{32(n^2-1)}{3n^5}\Gamma_{2p}^{\mathrm{nuc}} && (11) \\ \Gamma_{nd}^{\mathrm{nuc}} &= \frac{2187}{40n^3}\left(1-\frac{5}{n^2}+\frac{4}{n^4}\right)\Gamma_{3d}^{\mathrm{nuc}} && (12) \\ \text{etc.} & \end{aligned}$$

or generally [3]

$$\Gamma_{n+1l}^{\mathrm{nuc}}/\Gamma_{nl}^{\mathrm{nuc}} = \left(\frac{n}{n+1}\right)^{2l+4}\frac{n+l+1}{n-l}\,. \qquad (13)$$

Finally, in the case of particles with finite life time, the atomic cascade can be terminated by decay of the exotic particle.

## 2.2 Atomic cascade simulation

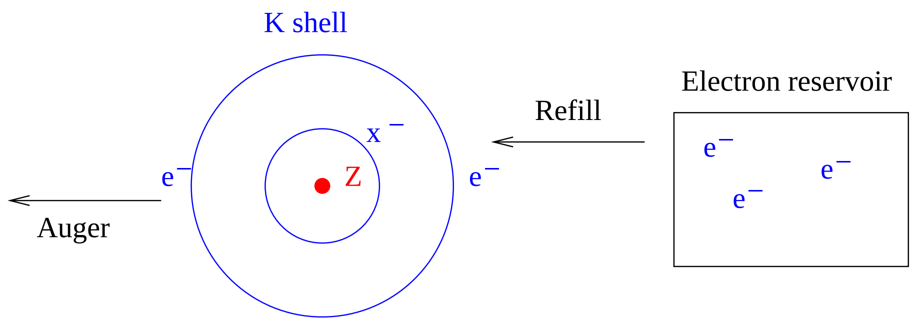


Figure 1: The atomic cascade model.

Using the rates from Sect. 2.1 a Fortran 90 cascade simulation program that calculates X-ray yields and absorption fractions has been written. Figure 1 illustrates the model. The following input must be provided:

1. Initial state: a distribution in quantum numbers $n$ and $l$.

2. Number of electrons in the reservoir.

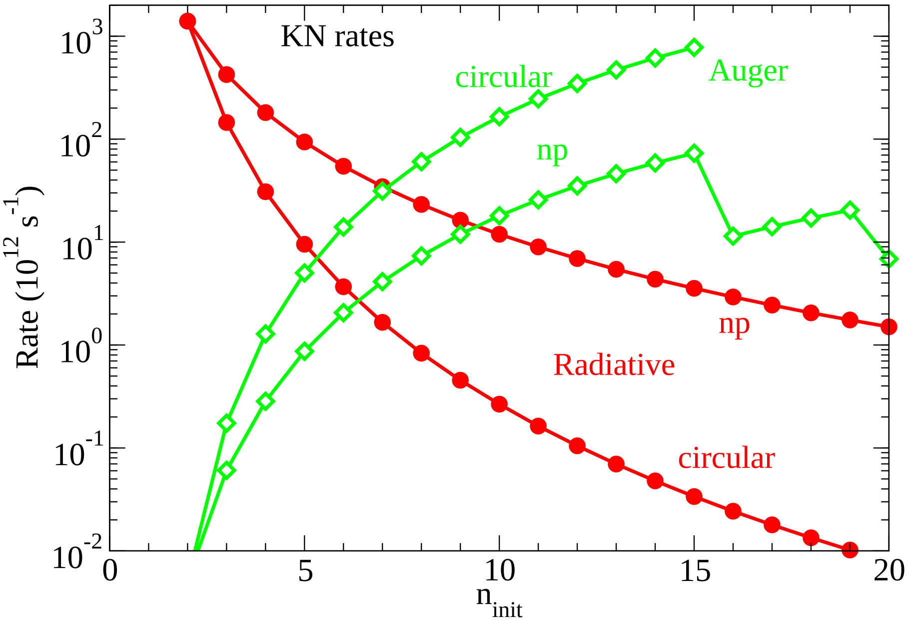


Figure 2: The $n$ dependence of the radiative (filled circles) and internal Auger (diamonds) rates in kaonic nitrogen. The rates are shown for the circular states and the $np$ states.

3. Electron refilling rate.
4. The nuclear widths of the $n, n-1$ states.

The results of the cascade calculations are:

1. X-ray yields for all possible radiative transitions.
2. Cascade times.
3. Nuclear absorption fractions.
4. Weak decay fractions.

# 3 Example: kaonic nitrogen

Figure 2 shows the radiative and internal Auger rates in kaonic nitrogen. The Auger effect dominates the upper part of the cascade if there are electrons available whereas the radiative transitions are most important at low $n$.

The internal Auger rates (4) are calculated with $C = 1$. Varying $C_1$ and $C_2$ between 0 and 1 in (6) in order to determine uncertainties as suggested by Burbidge and de Borde [2] has virtually no effect on the final results. The nuclear absorption rates are all set to zero. The effect of nuclear absorption will be examined in Sect. 3.1.

At high density, one can get a crude estimate of the electron refilling rate: it should be comparable with the electron $2p \to 1s$ radiative rate for an effective charge $Z_e = Z - 1$:

$$\Gamma^{\mathrm{rad}}_{2p\to 1s} = Z_e^4 \alpha^3 \left(\frac{2}{3}\right)^8 \tag{14}$$

| Transition | keV | yield |
|---|---|---|
| $2 \to 1$ | 465.44 | 0.906 |
| $3 \to 1$ | 551.63 | 0.035 |
| $3 \to 2$ | 86.19 | 0.798 |
| $4 \to 2$ | 116.36 | 0.054 |
| $4 \to 3$ | 30.17 | 0.689 |
| $5 \to 3$ | 44.13 | 0.061 |
| $5 \to 4$ | 13.96 | 0.587 |
| $6 \to 4$ | 21.55 | 0.061 |
| $6 \to 5$ | 7.58 | 0.470 |
| $7 \to 5$ | 12.16 | 0.050 |
| $7 \to 6$ | 4.57 | 0.350 |
| $8 \to 6$ | 7.54 | 0.041 |
| $8 \to 7$ | 2.97 | 0.251 |
| $9 \to 7$ | 5.00 | 0.032 |
| $9 \to 8$ | 2.04 | 0.175 |

Table 1: The yields of the $K^-$N X-ray transitions. $n_{\rm init} = 20$. $\Gamma^{\rm refill} = 0.5 \times 10^{12}$ s$^{-1}$.

For exotic nitrogen atoms this gives 0.81 (ps)$^{-1}$. In the following, $\Gamma^{\rm refill} = 0.5$ (ps)$^{-1}$ is used unless indicated otherwise. For the initial state, $n_{\rm init} = 20$ and statistical distribution in $l$ are used. Initially there are two electrons in the K shell and an infinite supply of electrons in the reservoir.

Figure 3 shows the X-ray yields as a function of the electron refilling rate. The yields decrease strongly for increasing refilling rate because the Auger rates dominate the radiative rates. Figure 4 shows the dependence of the yields on the initial state $n_{\rm init}$. The yields increase for higher $n_{\rm init}$ because starting at high n leads to a higher population of the the lower-lying circular states.

The kaonic nitrogen X-ray spectrum is shown in Fig. 5 and some of the yields are given in Table 1. The rather high yield at very low energies is an artifact of the model which only takes the K shell electrons into account. For example, the transition $16, 15 \to 15, 14$ does not release enough energy to eject a K shell electron so the transition must occur radiatively. In reality, however, Auger transitions are possible by ejecting electrons from the L shell.

## 3.1 The effect of nuclear absorption

Based on the experimental data for other kaonic atoms (see [4] and references therein) crude estimates of the nuclear widths can be obtained:

$$\begin{aligned} \Gamma^{\rm nuc}_{1s} &\sim 100 \text{ keV} \\ \Gamma^{\rm nuc}_{2p} &\sim 2 \text{ keV} \\ \Gamma^{\rm nuc}_{3d} &\sim 2 \text{ eV} \end{aligned}$$

Including the widths for $l = 0 - 2$ in the simulation reduces the $4 \to 3$ yield only a little: from 68.9% to 67.3%. The potentially important nuclear width is that of the $4f$ state as illustrated in Fig. 6.

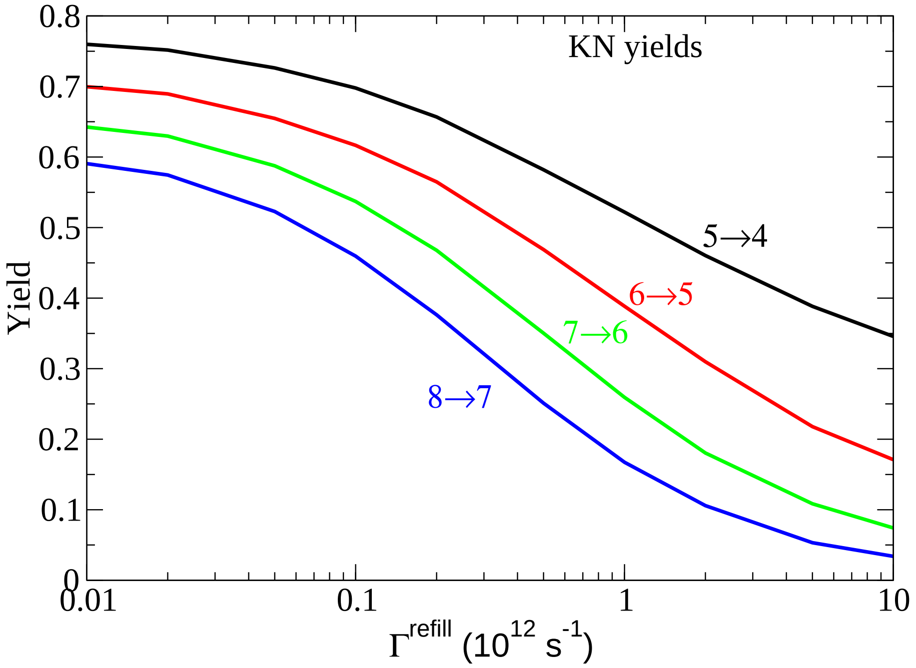


Figure 3: The dependence of the X-ray yields in $K^{-}$N on the electron refilling rate. The initial state is $n_{\rm init} = 20$.

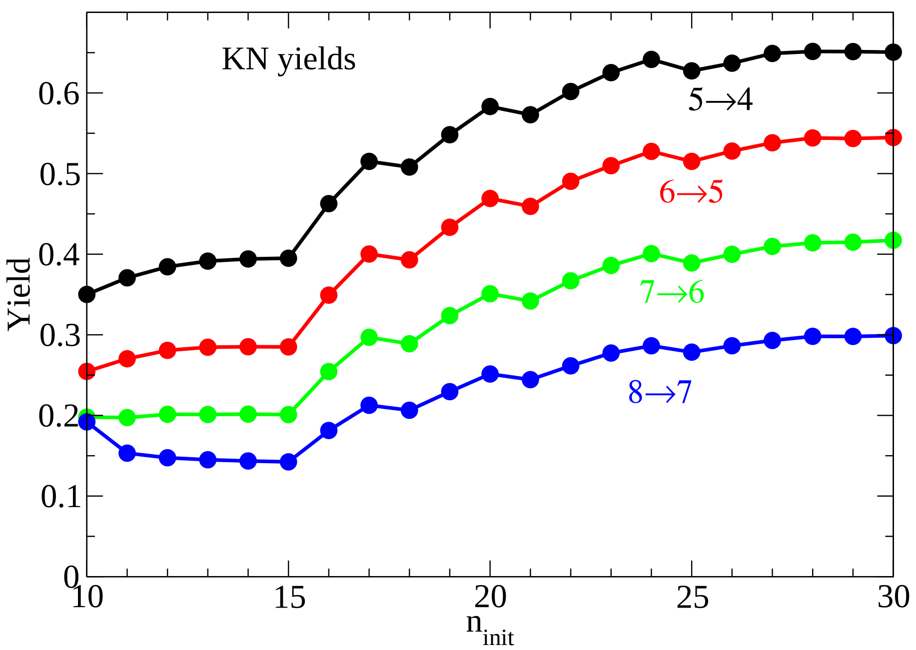


Figure 4: The dependence of the X-ray yields in $K^{-}$N on the initial state $n_{\rm init}$. The electron refilling rate is $\Gamma^{\rm refill} = 0.5 \times 10^{12}$ s$^{-1}$.

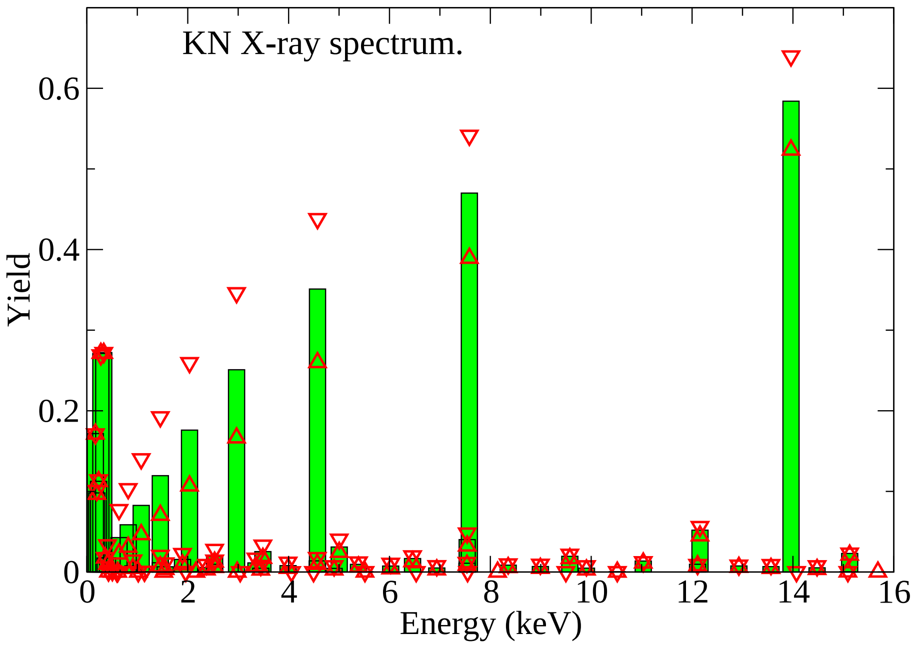


Figure 5: The $K^-$N X-ray spectrum. The initial state is $n_{\rm init} = 20$. The results are shown for electron refilling rates $\Gamma^{\rm refill} = 0.5 \times 10^{12}$ s$^{-1}$ (bars), $0.25 \times 10^{12}$ s$^{-1}$ (triangle down), and $1.0 \times 10^{12}$ s$^{-1}$ (triangle up).

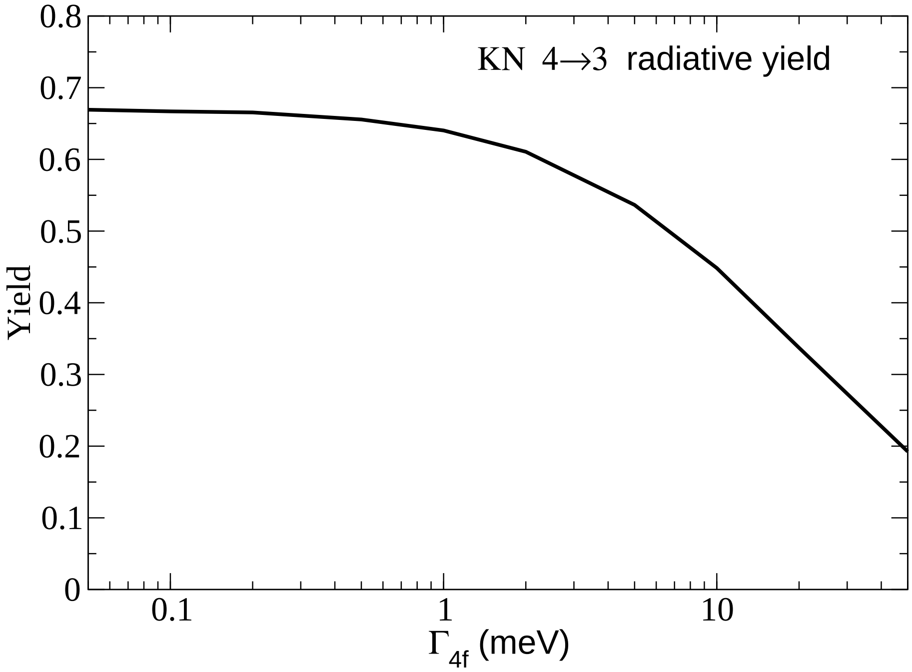


Figure 6: The yield of the $K^-$N $4 \to 3$ X-ray transition as a function of the nuclear width $\Gamma^{\rm nuc}_{4f}$. $n_{\rm init} = 20$. $\Gamma^{\rm refill} = 0.5 \times 10^{12}$ s$^{-1}$.

# 4 Conclusions

A cascade model for exotic atoms with $Z > 2$ has been developed. It includes radiative transitions, internal Auger effect, electron refilling, nuclear absorption, and decay of the exotic particle. A Fortran 90 program is used to calculate the yields of the radiative transitions using input such as the nuclear absorption widths. The program is suitable for analyzing experimental X-ray data from which information on the initial state and the nuclear widths can be obtained.

The present model can be improved by modeling the L shell in addition to the K shell (like in Refs. [5, 6]). This would lead to a better treatment of internal Auger effect and electron refilling.

# References


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