The relation between the long-term detector response measurement and the periodic stability check

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1 Introduction

The material presented below was produced in response to Carlo's question "How many counts do we need in each daily-calibration peak?" By now, I think everybody has come to agree that we need both the long-term response measurement (formerly called the long-term calibration) and the periodic stability check (formerly called the daily calibration)[1] as pointed out in Masa's and Ross's memoranda, and the requirements for the long-term response measurements (eg. the required number of counts in each calibration peak, etc.) are relatively clear. However, the relation between the requirements for the periodic stability check and the accuracy of measurement we are aiming at, and the relation between the periodic stability check and the long-term response measurement are not yet that clear.

I would like to start by reviewing the purpose and the requirements of the long-term response measurement.

2 Long-term Response Measurement

To my understanding, the purpose of the long-term response measurement is two-fold:

1. To provide the absolute energy scale in the final x-ray energy spectra (calibration).

2. To provide a good measure of what the response function of our apparatus (after a run over a long time) is. (The long-term calibration peaks seen in the final x-ray spectra, after all the analyses, are the response function itself apart from the fact that the width has to be scaled according to the energy difference between the calibration peaks and the peak of interest.)

[1]It is true that these are both "calibrations", but in this material I do not use the term "calibration" for either of them to avoid possible confusion.

Roughly corresponding to each of the above, there are requirements that the number of counts in the calibration peaks and the width of the peaks must satisfy:

1. The long-term calibration peak should have enough counts so that

$$\epsilon(p_{\text{calib}}) = \frac{\sigma_{\text{calib}}}{\sqrt{N_{\text{calib}}}} < 3 \text{ eV}, \tag{1}$$

   where σ_{calib} is the width of the calibration peak (=the width of the response function at the energy of the calibration line), N_{calib} is the number of counts in the calibration peak, and $\epsilon(p_{\text{calib}})$ is the accuracy to which the position of the calibration peak is determined. In order to achieve an error of ~3 eV in the determination of the kaonic hydrogen K_α line position, the peak position of each calibration peak should be determined to a similar accuracy so that the systematic error (due to calibration) ~ the statistical error.

2. The long-term calibration peaks should be narrow enough so that

$$\sigma_{\text{calib}} < \frac{\Gamma_{1s}}{2.35}, \tag{2}$$

   and

$$\frac{\sqrt{\sigma_{\text{calib}}{}^2 + (\frac{\Gamma_{1s}}{2.35})^2}}{\sqrt{N_{K_\alpha}}} < 3 \text{ eV}. \tag{3}$$

   Requirement eq.(2) comes from the fact that the width of the response function should be smaller than the natural width of the kaonic hydrogen K_α line in order to determine the K_α width accurately. Requirement eq.(3) is simply that we would like to determine the peak position of the kaonic hydrogen K_α line to an accuracy of 3 eV and that the narrower the response function is, the easier an accurate determination will be.

3 Periodic Stability Check

As pointed out by Masa, the calibration may change over time, due to various causes, and this variation in calibration would appear as an increase in the width of the response function. The purpose of the periodic stability check is to detect these changes and allow an appropriate correction to keep the response function as narrow as possible.

In the next section, I present one possible way of using the periodic stability check data to correct the calibration and give an estimate of how accurate the periodic stability check should be.

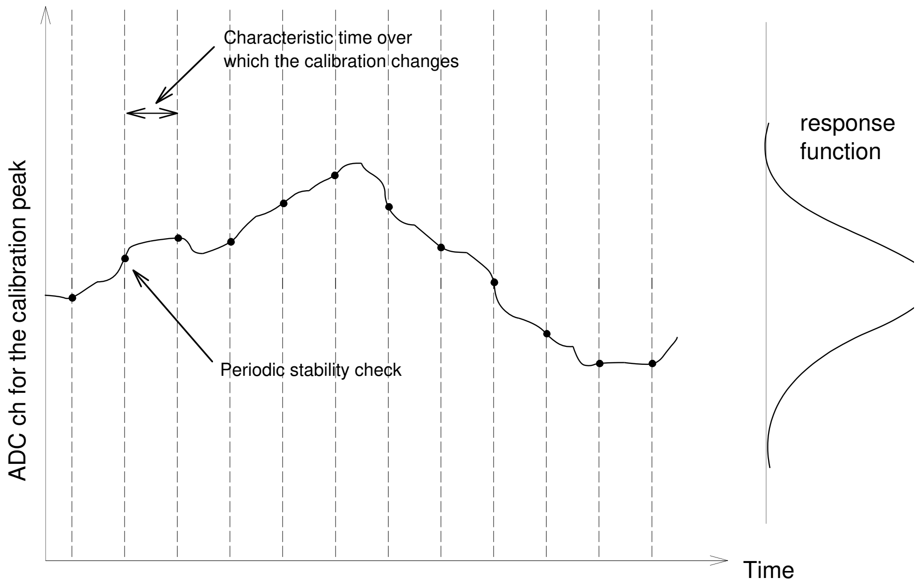

Figure 1: The change in the calibration over time causes a wide response function.

4 The Relation Between the Long-term Response Measurement and the Periodic Stability Check

Figure 1 shows how the calibration might change over time. If there is no correction made for this change, the response function we will see in the x-ray spectrum will be wide, as shown in the figure, and possibly asymmetric. If we correct for the change in the calibration using the data from the periodic stability check (calibration done in a relatively short period of time using a radioactive source or an x-ray tube), then we can make the response function narrower (Fig. 2).

There are three contributions to the width of the final response function: the intrinsic resolution of the CCD, $\sigma_{\rm CCD}$; the error in the periodic stability check, $\sigma_{\rm periodic}$ (although these two are not explicitly shown in the figures); and the change in the calibration over the period between two periodic stability checks, $\sigma_{\rm fluct}$. That is (approximately):

$$\sigma_{\rm calib} = \sqrt{{\sigma_{\rm CCD}}^2 + {\sigma_{\rm periodic}}^2 + {\sigma_{\rm fluct}}^2} \tag{4}$$

Obviously, one cannot gain much by making $\sigma_{\rm periodic}$ much smaller than $\sigma_{\rm fluct}$. So, the accuracy of the periodic stability check should be

$$\sigma_{\rm periodic} \sim \sigma_{\rm fluct}. \tag{5}$$

In other words, the frequency of the periodic stability check should be so chosen that both $\sigma_{\rm periodic}$ and $\sigma_{\rm fluct}$ are much smaller than $\sigma_{\rm CCD}$.

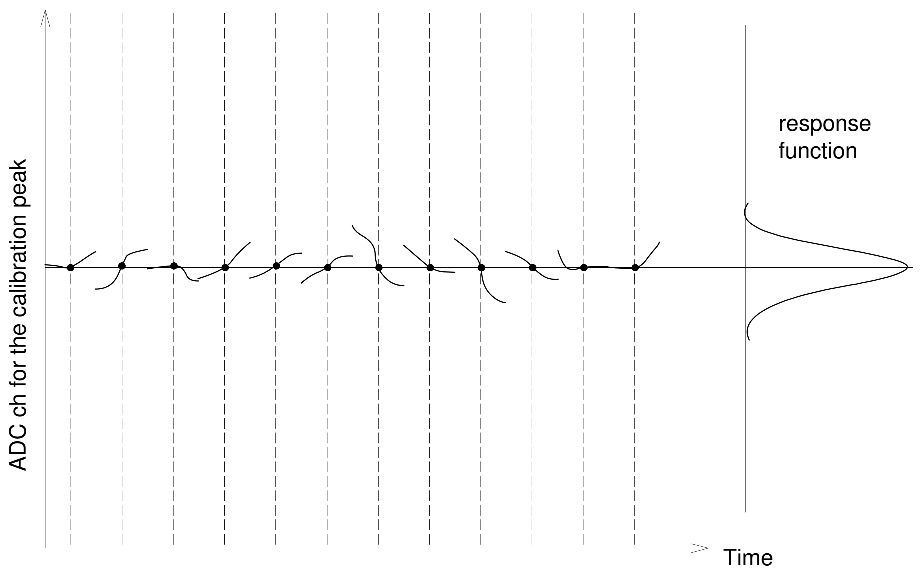

Figure 2: The change in the calibration over time corrected by the periodic stability check can make the response function narrower.

Then, the next question is "what is the characteristic period of the calibration change over time and how big is the change?"

The stability tests done by Aaron in the summer obtained an approximate value of σ_{fluct} of 12 eV over a day[2]. Assuming this value for σ_{fluct} and $\sigma_{\text{CCD}} = 180/2.35$ eV, σ_{calib} is plotted as a function of σ_{periodic} in Fig. 3. Obviously, one cannot gain much by making σ_{periodic} smaller than $\sigma_{\text{fluct}} \sim 12$ eV. From

$$\sigma_{\text{periodic}} = \frac{\sigma_{\text{CCD}}}{\sqrt{N_{\text{periodic}}}}, \tag{6}$$

$\sigma_{\text{periodic}} \sim 12$ eV corresponds to $N_{\text{periodic}} \sim 40$. That is, if the typical variation in the calibration over a day is ~ 12 eV, and if one makes the stability check once a day, one cannot gain much by making N_{periodic} larger than 40. At first sight, this result seems quite surprising and that, if σ_{fluct} is as small as 12 eV, the periodic stability check is unimportant, because σ_{CCD} produces the major contribution to σ_{calib}. However, the periodic stability check is important for the following reasons:

1. Even if σ_{fluct} over a day is as small as 12 eV, it can cause a larger variation in the calibration for a longer period of time: if one simply assumes a random-walk like behavior (although it is too simple an assumption), the variation of

[2]The fluctuation observed was $\sim \pm 1$ ADC channel. Now that the readout boxes are rescaled so that the full range is 17 keV, if the main cause of the fluctuation is in the circuit before the ADC then the fluctuation is still ~ 12 eV, but if it is the ADC itself, then the fluctuation is now ~ 20 eV.

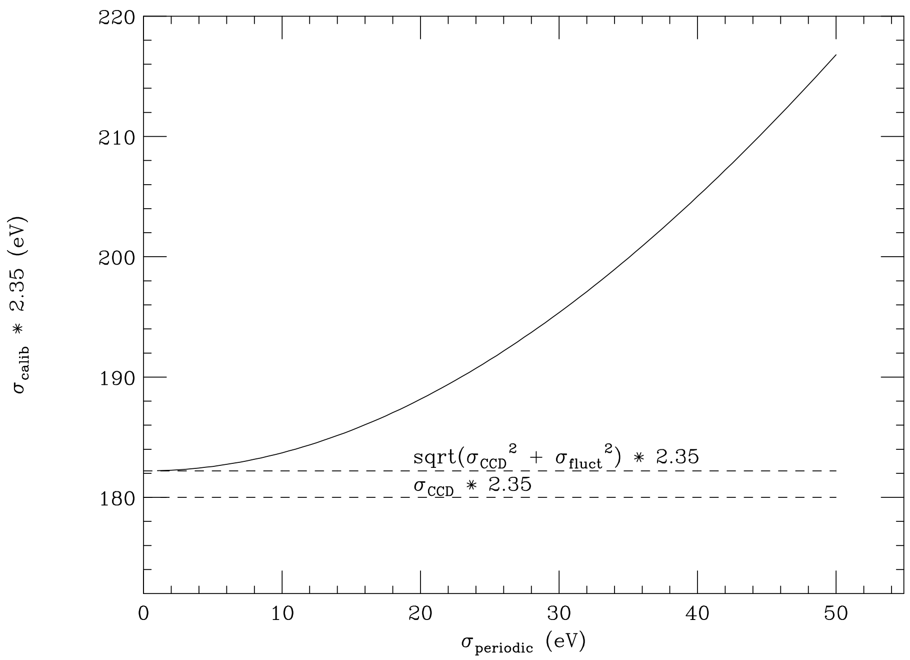

Figure 3: $\sigma_{\rm calib}$ plotted as a function of $\sigma_{\rm periodic}$.

the calibration over N days ($\sigma_{\rm Nth\ day}$) is given by $\sigma_{\rm Nth\ day} = \sqrt{N}\times$ (fluctuation over a day).

2. Also, the value of $\sigma_{\rm fluct} = 12$ eV is a rather optimistic value obtained in the laboratory. We have to perform a stability test of the system under circumstances very similar to those encountered during actual data taking (i.e. at the pit with the machine in operation). There are factors which may cause a considerably different behavior of stability (e.g. the grounding at the pit may not be as clean as the one in the laboratory).

3. Even after such a stability test shows that $\sigma_{\rm fluct}$ is small, $\sigma_{\rm fluct}$ for the real data can only be determined by the periodic stability check. That is, in order to be sure that $\sigma_{\rm fluct}$ is small, we have to make the periodic stability check.

5 Monte Carlo

In order to check the validity of the argument above, I performed a Monte Carlo calculation. The framework used is as follows:

- For each period of time ΔT,

  - events for the long-term calibration are generated: the number of events follows a Possion distribution of mean 3 and the energy of these events follows a Gaussian distribution of mean E_0 and standard deviation $\sigma_{\rm CCD}$.
  - the calibration E_0 is shifted by ΔE, which follows a Gaussian distribution of mean 0 and standard deviation $\sigma_{\rm hour}$

- For each $24 \times \Delta T$, $N_{\rm periodic}$ events for the periodic stability check are generated (hence, ΔT in this Monte Carlo calculation can be regarded as corresponding to one hour and the periodic stability check as being done daily): the energy of these events follows a Gaussian distribution of mean E_0 and standard deviation $\sigma_{\rm CCD}$.

- Sudden jumps in the calibration are also taken into account. For each ΔT, a random number which follows a uniform distribution between 0 and 1 is generated and if this number is smaller than a certain value, then the calibration E_0 is shifted by an amount of energy $\Delta E_{\rm jump}$, which follows a Gaussian distribution of mean 0 and standard deviation $\sigma_{\rm jump}$.

The Monte Carlo calculation was performed for two sets of parameters: (1) $\sigma_{\rm hour}$= 3.5 eV, $\sigma_{\rm jump}$ = 100 eV and (2) $\sigma_{\rm hour}$= 20 eV, $\sigma_{\rm jump}$ = 500 eV. $\sigma_{\rm hour}$= 3.5 eV and $\sigma_{\rm hour}$= 20 eV are expected to roughly correspond to $\sigma_{\rm fluct} = \sqrt{24/2} \times \sigma_{\rm fluct}/\sqrt{2}$ = 8.5 eV and 50 eV. The parameters related to the sudden jump are set such that the average frequency of jumps is once per ten days. The calculation was performed for 1000 days. For each set of parameters, the calculation was performed for each case of $N_{\rm period}$ = 10, 50, 100, 500, and 1000.

Figures 4 and 5 show the change in the calibration over time generated by this Monte Carlo calculation for $\sigma_{\rm hour}$ = 3.5 eV and $\sigma_{\rm hour}$ = 20 eV (for simplicity, the starting values for the calibration were chosen to be zero).

In Figs. 6 and 7, the raw spectra and the corrected spectra (corrected using the periodic stability check in the manner of Fig. 2) from the long-term calibration data for $N_{\rm period}$ = 500 are shown for $\sigma_{\rm hour}$ = 3.5 eV and $\sigma_{\rm hour}$ = 20 eV.

In Fig. 8, the values of $\sigma_{\rm calib}$ obtained after correction using the periodic stability check are shown as a function of $N_{\rm periodic}$ for $\sigma_{\rm hour}$ = 3.5 eV and $\sigma_{\rm hour}$ = 20 eV. Also shown in dashed line are the best fit curves obtained using eqs. (4) and (6) and treating $\sigma_{\rm fluct}$ as a free parameter. The best fit values for $\sigma_{\rm fluct}$ are 15.4 ± 0.5 eV and 52.7 ± 0.2 eV for $\sigma_{\rm hour}$ = 3.5 eV and $\sigma_{\rm hour}$ = 20 eV, respectively. These values are very reasonable when compared to the expected values (8.5 eV and 50 eV) if we take into account the contribution of the sudden jumps.

The general behavior of $\sigma_{\rm CCD}$ is adequately described by eqs. (4) and (6).

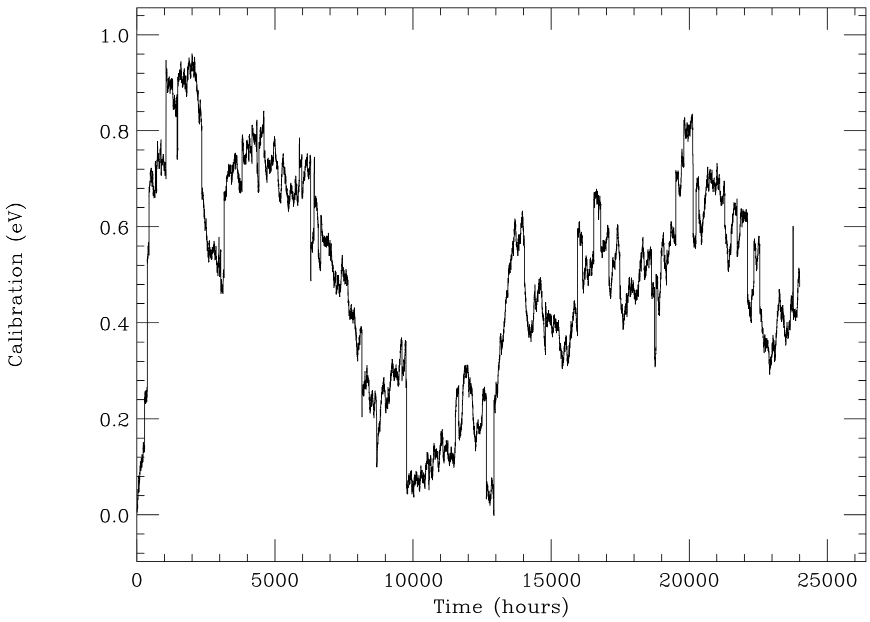

Figure 4: The change in calibration over time generated by a Monte Carlo calculation for $\sigma_{\rm hour} = 3.5$ eV (for simplicity, the starting values for the calibration were chosen to be zero).

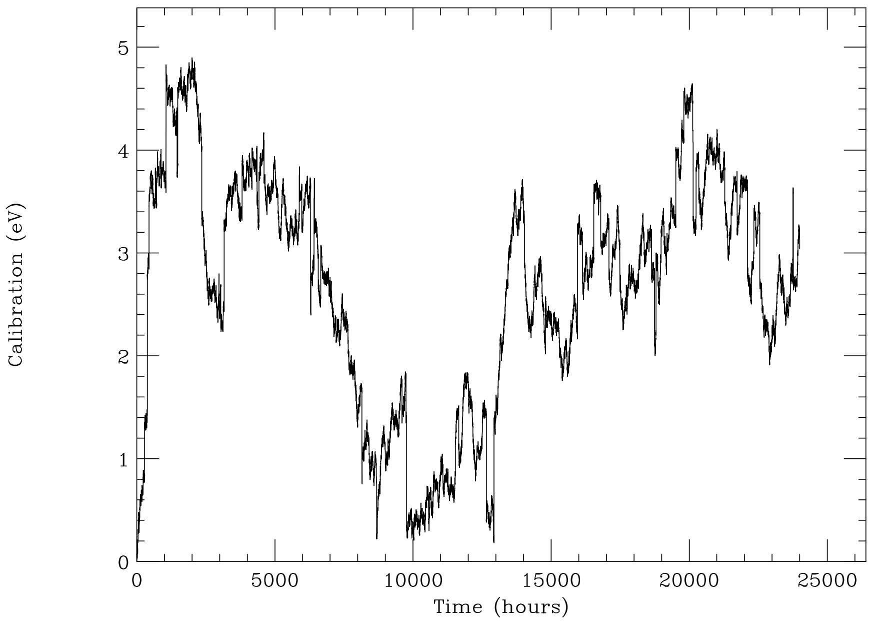

Figure 5: The change in calibration over time generated by a Monte Carlo calculation for $\sigma_{\rm hour} = 20$ eV (for simplicity, the starting values for the calibration were chosen to be zero).

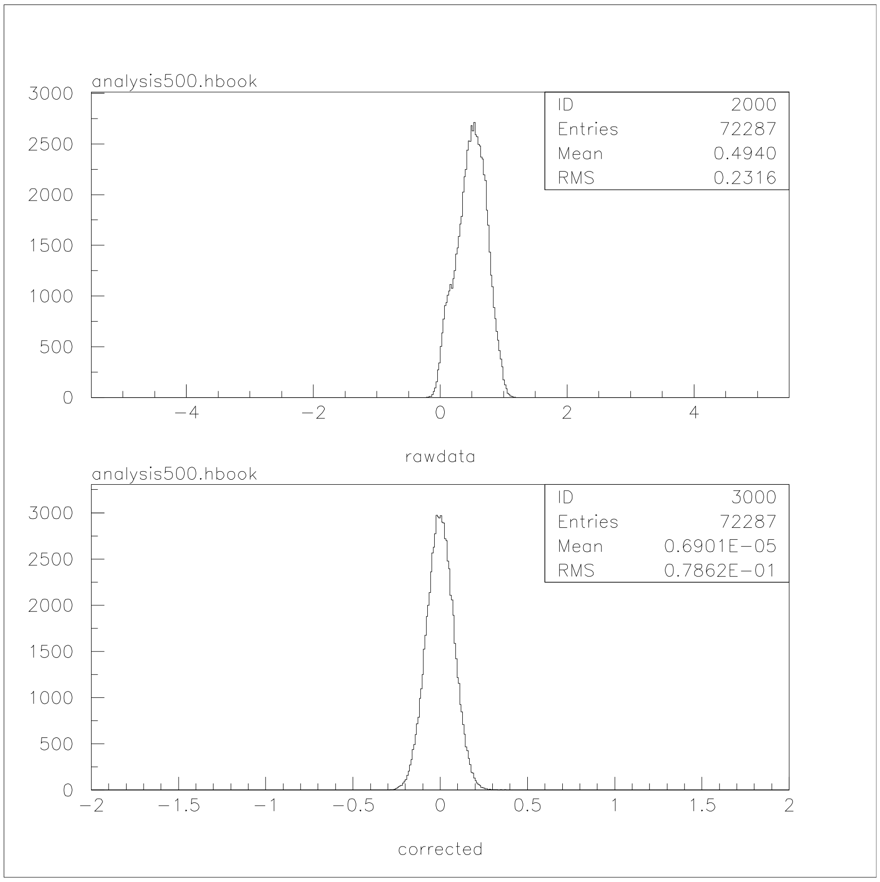

Figure 6: Spectra from the long-term calibration data for $\sigma_{\text{hour}} = 3.5$ eV. The raw spectrum (top) and the spectrum corrected using the periodic stability check data for $N_{\text{periodic}} = 500$ (bottom) are shown.

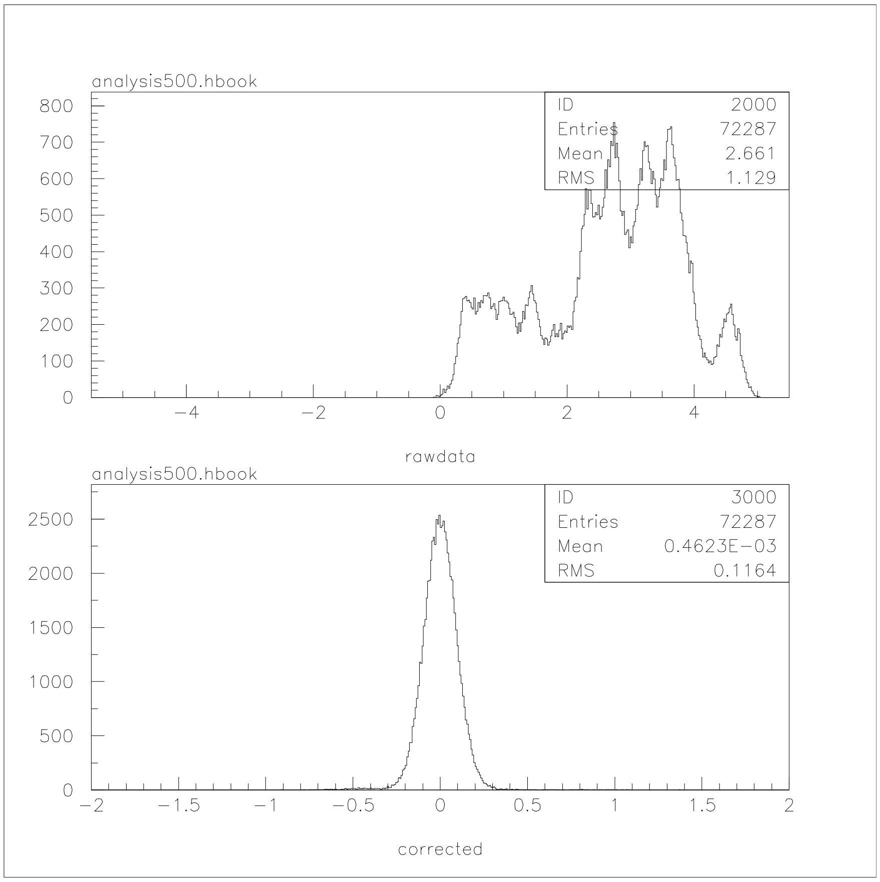

Figure 7: Spectra from the long-term calibration data for $\sigma_{\text{fluct}} = 20$ eV. The raw spectrum (top) and the spectrum corrected using the periodic stability check data for $N_{\text{periodic}} = 500$ (bottom) are shown.

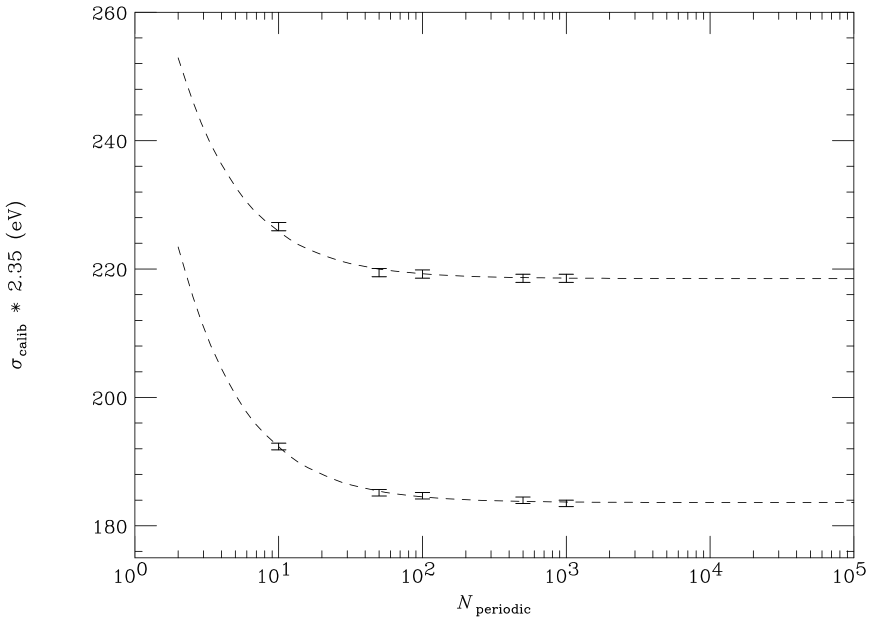

Figure 8: The values of σ_{calib} obtained after correction using the periodic stability check are shown as a function of N_{periodic} for $\sigma_{\mathrm{hour}} = 3.5$ eV and $\sigma_{\mathrm{hour}} = 20$ eV, respectively. Also shown in dashed line are the best fit curves obtained using eqs. (4) and (6) and treating σ_{fluct} as a free parameter. The best fit values for σ_{fluct} are 15.4 ± 0.5 eV and 52.7 ± 0.2 eV for $\sigma_{\mathrm{hour}} = 3.5$ eV and $\sigma_{\mathrm{hour}} = 20$ eV, respectively.

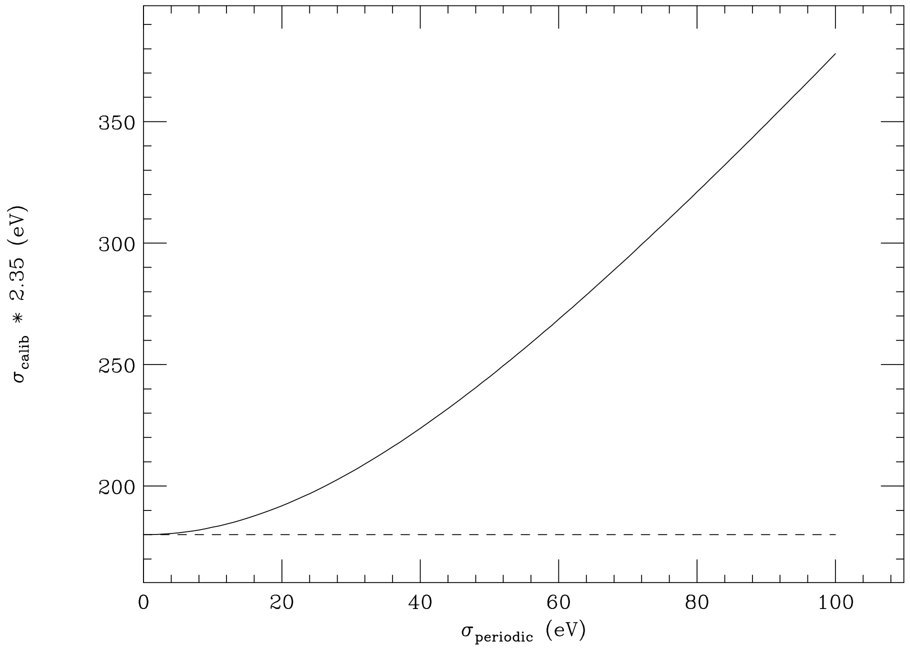

Figure 9: $\sigma_{\rm calib}$ plotted as a function of $\sigma_{\rm periodic}$ assuming $\sigma_{\rm fluct} \sim \sigma_{\rm periodic}$.

6 Conclusion

As seen in section 4, one cannot gain much by making $\sigma_{\rm periodic}$ much smaller than $\sigma_{\rm fluct}$ and it is sufficient to chose $\sigma_{\rm periodic}$ to be

$$\sigma_{\rm periodic} \sim \sigma_{\rm fluct}.$$

Also, the period between two adjacent periodic stability measurement should be so chosen that both $\sigma_{\rm periodic}$ and $\sigma_{\rm fluct}$ are reasonably smaller than $\sigma_{\rm CCD}$. (Again, this can be determined only after a stability test of the system under circumstances very similar to those encountered during actual data taking.) In Fig. 9, $\sigma_{\rm calib}$ is plotted as a function of $\sigma_{\rm periodic}$ assuming eqs. (4) and (5), and $\sigma_{\rm CCD} = 180/2.35$ eV. Also, the relation between the number of counts in each periodic stability check peak ($N_{\rm periodic}$) and $\sigma_{\rm calib}$ is shown in Fig. 10 assuming again eqs. (4), (5), and (6), and $\sigma_{\rm CCD} = 180/2.35$ eV. As can be seen from the figure, little improvement can be achieved beyond $N_{\rm periodic} \approx 10^3$

Acknowledgment

I thank Aaron for the valuable comments he made on the draft of this material, which helped a lot to clarify the argument.

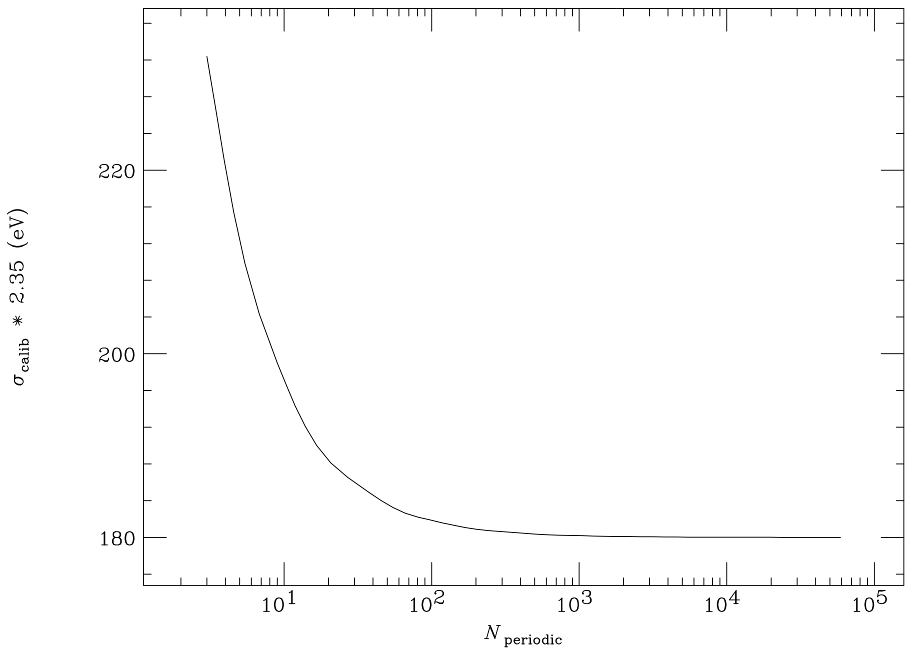

Figure 10: $\sigma_{\rm calib}$ plotted as a function of $N_{\rm periodic}$.