Time-like Baryon Form Factors and Initial State Radiation

S. Pacetti

Perugia University and INFN Perugia, Italy

BESIII/ISR - Form Factors Meeting

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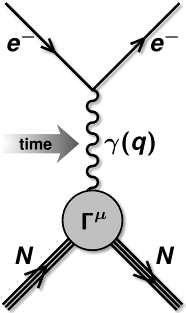
Agenda

- **Form Factors: definitions, formulae and other facts**
- **Form Factor Analysis: study of the ratio G_E^p / G_M^p**
- **The ISR technique**
- **Threshold behaviors in $p\bar{p}$ and neutral channels**
- **J/ψ strong and electromagnetic phase**

Form Factors: definitions, formulae and other facts

Baryon Form Factors definition
Space-like region ($q^2 < 0$)



- **Electromagnetic current** ($q = p' - p$)

$$j^\mu = \langle N(p')|J^\mu(0)|N(p)\rangle = e\overline{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_2(q^2)\right]u(p)$$

- **Dirac and Pauli form factors F_1 and F_2 are real**
- **In the Breit frame**

$$\begin{cases} p = (E, -\vec{q}/2) \\ p' = (E, \vec{q}/2) \\ q = (0, \vec{q}) \end{cases} \qquad \begin{cases} \rho_q = j^0 = e\left[F_1 + \frac{q^2}{4M^2}F_2\right] \\ \vec{j}_q = e\,\overline{u}(p')\vec{\gamma}u(p)\,[F_1 + F_2] \end{cases}$$

- Total charge conservation in the limit $\vec{p'} \to \vec{p}$: $\langle N(p)|J^\mu(0)|N(p)\rangle = e\,F_1(0)$
- Let $\vec{B} = \vec{\nabla}\times\vec{A}$, in the limit $\vec{p'} \to \vec{p}$: $\langle N|\int d^3x\,\vec{S}\cdot\vec{A}|N\rangle = \frac{e}{M}\,[F_1(0)+F_2(0)]\,\vec{S}\cdot\vec{B}$

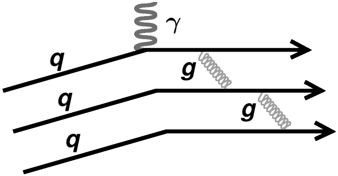
Sachs form factors

$$G_E = F_1 + \frac{q^2}{4M^2}F_2$$
$$G_M = F_1 + F_2$$

Normalizations

$$F_1(0) = Q_N \qquad G_M(0) = \mu_N$$
$$F_2(0) = \kappa_N \qquad G_E(0) = Q_N$$

pQCD asymptotic behavior
Space-like region



- **pQCD**: as $q^2 \to -\infty$, $F_1(q^2)$ and $F_2(q^2)$ must follow counting rules
- Quarks exchange gluons to distribute momentum

Dirac form factor F_1

- Non-spin flip
- Two gluon propagators
- $F_1(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-2}$

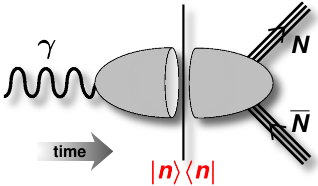
Pauli form factor F_2

- Spin flip
- Two gluon propagators
- $F_2(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-3}$

Sachs form factors G_E and G_M

- $G_{E,M}(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-2}$
- Ratio: $\frac{G_E}{G_M} \underset{q^2 \to -\infty}{\sim}$ constant

Nucleon form factors
Time-like region ($q^2 > 0$)



- **Crossing symmetry**:

$$\langle N(p')|J^\mu|N(p)\rangle \to \langle \overline{N}(p')N(p)|J^\mu|0\rangle$$

- **Form factors are complex functions of** q^2

Cutkosky rule for nucleons

$$\mathrm{Im}\langle \overline{N}(p')N(p)|J^\mu(0)|0\rangle \sim \sum_n \langle \overline{N}(p')N(p)|J^\mu(0)|n\rangle\langle n|J^\mu(0)|0\rangle \Rightarrow \begin{cases} \mathrm{Im}F_{1,2} \neq 0 \\ \text{for } q^2 > 4m_\pi^2 \end{cases}$$

$|n\rangle$ are on-shell intermediate states: 2π, 3π, 4π, ...

Time-like asymptotic behavior

Phragmèn Lindelöf theorem:

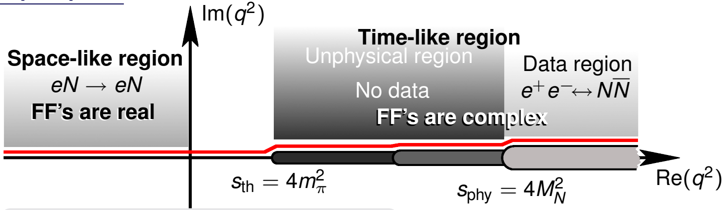
If a function $f(z) \longrightarrow a$ as $z \longrightarrow \infty$ along a straight line, and $f(z) \longrightarrow b$ as $z \longrightarrow \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $a = b$ and $f(z) \longrightarrow a$ uniformly in this angle.

- $$\underbrace{\lim_{q^2 \to -\infty} G_{E,M}(q^2)}_{\text{space-like}} = \underbrace{\lim_{q^2 \to +\infty} G_{E,M}(q^2)}_{\text{time-like}}$$

- $$G_{E,M} \underset{q^2 \to +\infty}{\sim} (q^2)^{-2}$$ **real**

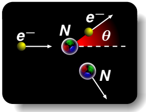
Cross sections and analyticity

q^2-complex plane



Crossing: tot. helicity $= \begin{cases} 1 \Rightarrow G_E \\ 0 \Rightarrow G_M \end{cases}$

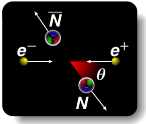
$G_E(4M_N^2) = G_M(4M_N^2)$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E'_e \cos^2\frac{\theta}{2}}{4E_e^3 \sin^4\frac{\theta}{2}} \left[G_E^2 - \tau\left(1 + 2(1-\tau)\tan^2\frac{\theta}{2}\right)G_M^2\right]\frac{1}{1-\tau}$$

$$\tau = \frac{q^2}{4M_N^2}$$



Annihilation

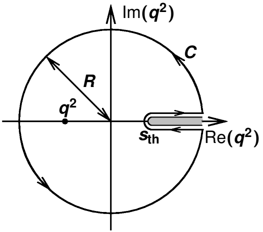
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta |G_E|^2\right]$$

$$\beta = \sqrt{1 - \frac{1}{\tau}}$$
$\mathcal{C}$ = Coulomb factor

Dispersion Relations

A form factor $f(q^2)$ is an analytic function on the q^2 complex plane with the cut: $(s_{th} = 4m_\pi^2, \infty)$

$$f(q^2) = |f(q^2)|e^{i\delta(q^2)}$$



Dispersion relation for the imaginary part

$$f(q^2) = \lim_{R\to\infty} \frac{1}{2\pi i} \oint_C \frac{f(z)dz}{z - q^2} = \frac{1}{\pi} \int_{s_{th}}^{\infty} \frac{\mathrm{Im} f(s)ds}{s - q^2}$$

Dispersion relation for the logarithm

Assuming no zeros on the physical sheet and using the function

$$\Phi(z) = \frac{\ln[f(z)]}{\sqrt{s_{th} - z}}$$

- $q^2 < s_{th}$: $\ln[f(q^2)] = \frac{\sqrt{s_{th} - q^2}}{\pi} \int_{s_{th}}^{\infty} \frac{\ln|f(s)|ds}{(s - q^2)\sqrt{s - s_{th}}}$
- $q^2 \geq s_{th}$: $\delta(q^2) = -\frac{\sqrt{q^2 - s_{th}}}{\pi} \mathrm{Pr}\int_{s_{th}}^{\infty} \frac{\ln|f(s)|ds}{(s - q^2)\sqrt{s - s_{th}}}$

The ratio $R = \mu_p G_E^p / G_M^p$

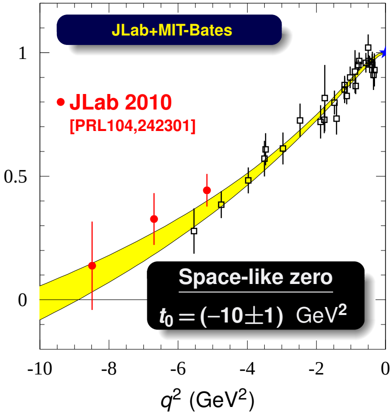
- Dispersion relation for the imaginary part
  - Model-independent approach
    - First time-like $|G_E| - |G_M|$ separation

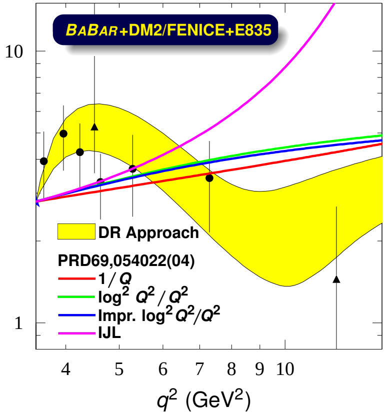
$\Longrightarrow$ Ratio in the whole q^2 complex plane

$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{s_{th}}^{\infty} \frac{\mathrm{Im}R(s)}{s(s-q^2)}\,ds$$

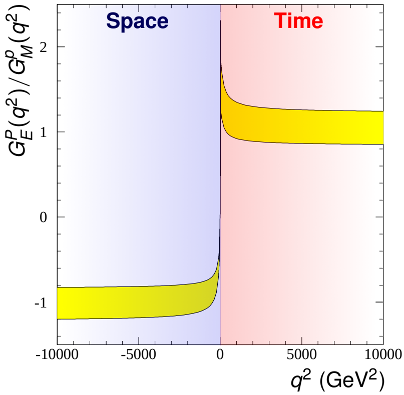
$R(q^2)$ space-like

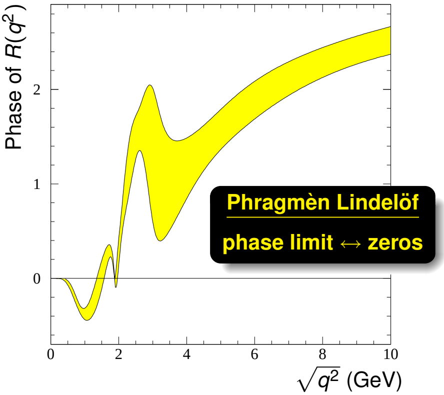
$|R(q^2)|$ time-like





$R(q^2)$: space-like zero and phase





Asymptotic limits

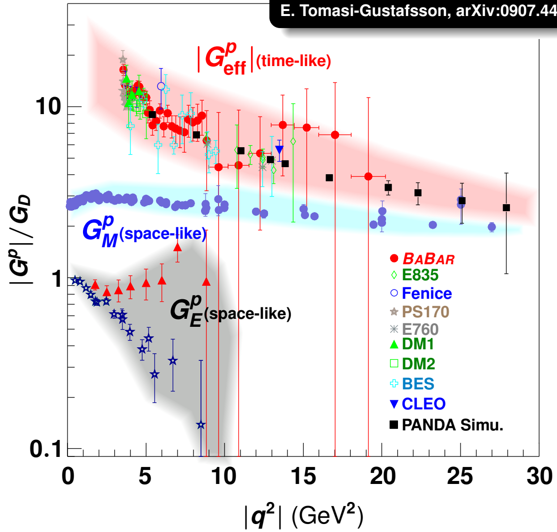
$$\left.\frac{G_E^p}{G_M^p}\right|_{|q^2|\to\infty} \longrightarrow -1.0 \pm 0.2$$

Phase from DR

$$\phi(q^2) = -\frac{\sqrt{q^2 - s_{\rm th}}}{\pi}\,\Pr\int_{s_{\rm th}}^{\infty} \frac{\ln|R(s)|\,ds}{\sqrt{s - s_{\rm th}}\,(s - q^2)}$$

Asymptotic behavior

E. Tomasi-Gustafsson and M. P. Rekalo, PLB504,291
E. Tomasi-Gustafsson, arXiv:0907.4442



pQCD

$$G^p_{\text{eff}}(q^2) \underset{q^2\to\infty}{\sim} G^p_M(q^2)$$

Phragmèn Lindelöf

$$\lim_{q^2\to\infty} \frac{G^p_{\text{eff}}(q^2)}{G^p_M(-q^2)} = 1$$

The ISR technique

ISR: Physics Motivations

- Existing ISR results, obtained by ***BaBar***, show interesting and unexpected behaviors, mainly at thresholds, for

$e^+e^- \to p\bar{p}$ and $e^+e^- \to \Lambda\bar{\Lambda}\,,\Sigma^0\bar{\Sigma}^0\,,\Lambda\bar{\Sigma}^0$

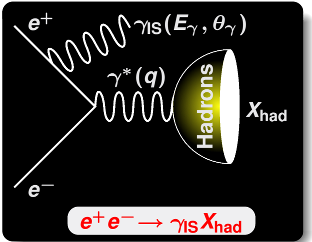
- Only one measurement (**FENICE** with energy scan) for

$e^+e^- \to n\bar{n}$

There are physical limits in reaching the threshold of many of these channels via energy scan (stable hadrons produced at rest can not be detected)

The Initial State Radiation technique provides a unique tool to access threshold regions working at higher resonances

Initial State Radiation



- $$\frac{d^2\sigma}{dE_\gamma \, d\cos\theta_\gamma} = W(E_\gamma, \theta_\gamma)\,\sigma_{e^+e^- \to X_{had}}(s)$$
- $$W(E_\gamma, \theta_\gamma) = \frac{\alpha}{\pi x}\left(\frac{2 - 2x + x^2}{\sin^2\theta_\gamma} - \frac{x^2}{2}\right)$$

- $s = q^2$, q .... X_{had} momentum
- E_γ, θ_γ ..CM γ_{IS} energy, scatt. ang.
- E_{CM} .... CM e^+e^- energy
- $x = E_\gamma / 2E_{CM}$

- **All energies (q^2) at the same time** $\Rightarrow$ **Better control on systematics (greatly reduced point to point)**
- **Detected ISR at large angles** $\Rightarrow$ **full X_{had} angular coverage**
- **CM boost** $\Rightarrow$ **at threshold $\epsilon \neq 0$; energy resolution $\sim$ 1 MeV**

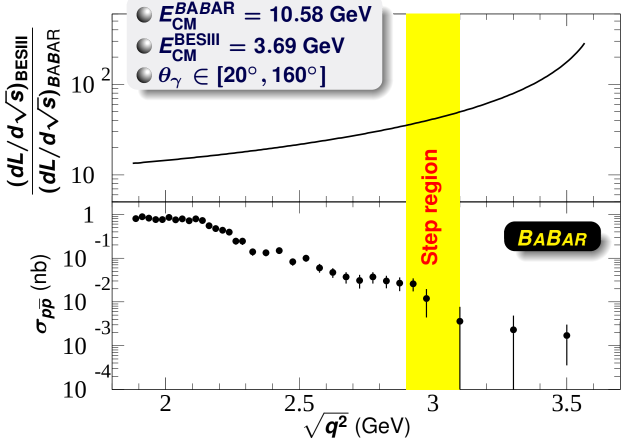
ISR: BESIII vs *BaBar*

$$\frac{d^2 L}{d\cos\theta_\gamma\, d\sqrt{s}} = \frac{2\sqrt{s}\,L_{e^+e^-}}{E^2_{CM}}\,\frac{\alpha}{\pi x}\left(\frac{2-2x+x^2}{\sin^2\theta_\gamma} - \frac{x^2}{2}\right)$$

$L_{e^+e^-}$ = luminosity

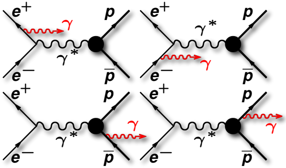
$$x = \frac{2E_\gamma}{E_{CM}} = 1 - \frac{s}{E^2_{CM}}$$

- $E^{BABAR}_{CM} = 10.58$ GeV
- $E^{BESIII}_{CM} = 3.69$ GeV
- $\theta_\gamma \in [20^\circ, 160^\circ]$



ISR and final state radiation

PRD84,017301

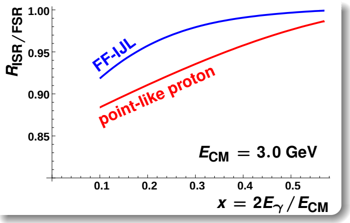


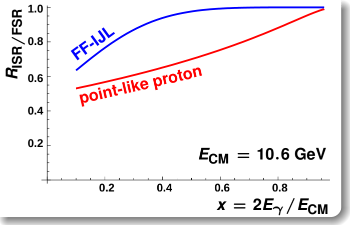
$$\frac{d^2\sigma_{\rm ISR}}{dE_\gamma d\theta_\gamma} = \frac{\alpha^3 E_\gamma}{3E_{\rm CM}^2 s}\left(|G_M^p(s)|^2 + \frac{|G_E^p(s)|^2}{2\tau}\right)\mathcal{W}(E_\gamma,\theta_\gamma)$$

$$\frac{d^2\sigma_{\rm FSR}}{dE_\gamma d\theta_\gamma} = \frac{\alpha^3 E_\gamma}{3E_{\rm CM}^4}\mathcal{F}\left[E_\gamma,\theta_\gamma, G_E^p(E_{\rm CM}^2), G_M^p(E_{\rm CM}^2)\right]$$

No ISR-FSR interference after $d\Phi(p\bar{p})$ integration

$$R_{\rm ISR/FSR} = \frac{d\sigma_{\rm ISR}/dE_\gamma}{d\sigma_{\rm ISR}/dE_\gamma + d\sigma_{\rm FSR}/dE_\gamma}\,[20^\circ \leq \theta_\gamma \leq 160^\circ]$$

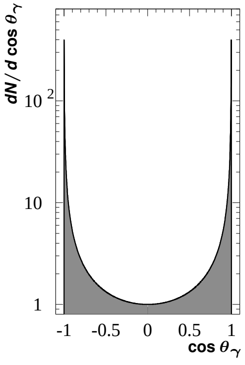




For large values of x or at small angle θ_γ of photon emission the final state radiation is strongly suppressed

ISR angular distribution and zero-degree tagging

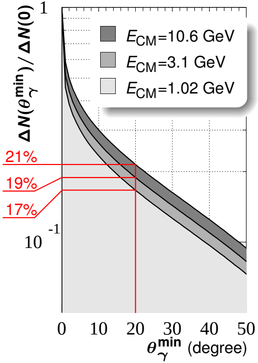
ISR angular distribution peaked at low angles



$$\frac{dN}{d\cos\theta_\gamma} = \frac{1-\cos\theta_\gamma^2}{\left(1-\beta_e^2\cos^2\theta_\gamma\right)^2}$$

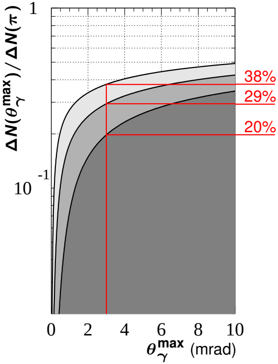
$$\beta_e = \sqrt{1-4m_e^2/E_{CM}^2}$$

$$\Delta N(\theta^{min}) \propto \int_{90^o}^{\theta_\gamma^{min}} d\theta_\gamma \frac{dN}{d\theta_\gamma}$$



With a typical $\theta_\gamma^{min} = 20^o$
$\sim$ 80% of events is lost!

$$\Delta N(\theta^{max}) \propto \int_{0}^{\theta_\gamma^{max}} d\theta_\gamma \frac{dN}{d\theta_\gamma}$$



With $\theta^{max} = 3$ mrad more statistics than at wide angle!

Proposal for a zero-degree detector

- **J/ψ, $\psi(2S)$, $\psi(3770)$ resonances decay with high BR's to final states with π^0 and γ_{FS} (final state)**
- **At BESIII these decay channels represent severe backgrounds for typical ISR final states with γ_{IS} detected at wide angle**

- **π^0 and final γ angular distributions are isotropic**
- **ISR angular distribution is peaked at small angles**

$\Rightarrow$

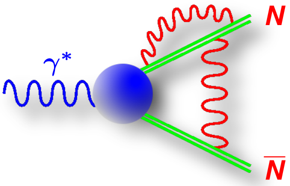
A zero-degree radiative photon tagger will suppress most of these backgrounds

A new zero-degree detector (ZDD) has been installed this summer at BESIII to tag ISR photons as well as to measure the luminosity

(Manica Bertani this morning)

Sommerfeld resummation factor needed?

The Coulomb Factor



$$\sigma_{p\bar{p}} = \frac{4\pi\alpha^2\beta\,\mathcal{C}}{3q^2}\left[|G_M^p(q^2)|^2 + \frac{2M_p^2}{q^2}|G_E^p(q^2)|^2\right]$$

$\mathcal{C}$ describes the $p\bar{p}$ Coulomb interaction as FSI

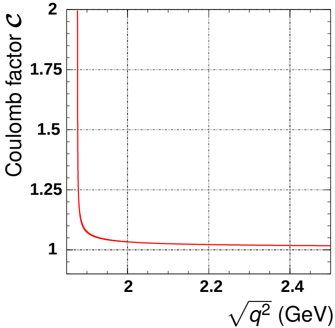
[Sommerfeld, Sakharov, Schwinger, Fadin, Khoze]

Distorted wave approximation

$$\mathcal{C} = |\Psi_{\mathrm{Coul}}(0)|^2$$

- **S-wave:** $\mathcal{C} = \dfrac{\frac{\pi\alpha}{\beta}}{1-\exp\left(-\frac{\pi\alpha}{\beta}\right)} \xrightarrow[\beta\to 0]{} \dfrac{\pi\alpha}{\beta}$
- **D-wave:** $\mathcal{C} = 1$

No Coulomb factor for boson pairs (P-wave)



Sommerfeld Enhancement and Resummation Factors

Coulomb Factor $\mathcal{C}$ for S-wave only:

- **Partial wave FF:** $G_S = \dfrac{2G_M\sqrt{q^2/4M^2} + G_E}{3}$ $\qquad G_D = \dfrac{G_M\sqrt{q^2/4M^2} - G_E}{3}$
- **Cross section:** $\sigma(q^2) = 2\pi\alpha^2\beta\,\dfrac{4M^2}{(q^2)^2}\left[\mathcal{C}\,|G_S(q^2)|^2 + 2|G_D(q^2)|^2\right]$

$$\mathcal{C} = \mathcal{E} \times \mathcal{R}$$

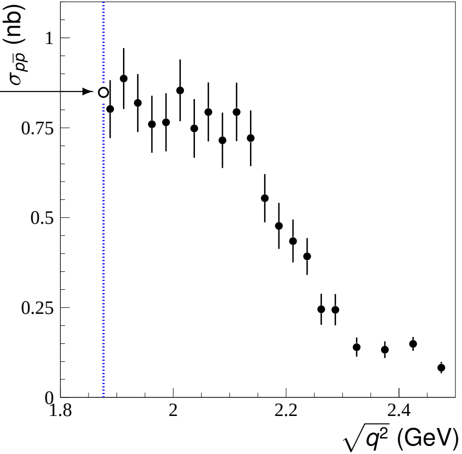
- **Enhancement factor:** $\mathcal{E} = \pi\alpha/\beta$
- **Step at threshold:** $\sigma_{p\bar{p}}(4M_p^2) = \dfrac{\pi^2\alpha^3}{2M^2}\dfrac{\cancel{\beta}}{\cancel{\beta}}|G_S^p(4M_p^2)|^2 = 0.85\,|G_S^p(4M_p^2)|^2$ nb

- **Resummation factor:** $\mathcal{R} = 1/[1 - \exp(-\pi\alpha/\beta)]$
- **Few MeV above threshold:** $\mathcal{C} \simeq 1 \;\Rightarrow\; \sigma_{p\bar{p}}(q^2) \propto \beta\,|G_S^p(q^2)|^2$

EPJA39, 315

Expected cross section with

$\left|G^p(4M_p^2)\right| = 1$



At the threshold

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\frac{\beta_p}{\beta_p}\left|G^p(4M_p^2)\right|^2$$

$$\sigma_{p\bar{p}}(4M_p^2) = 850\left|G^p(4M_p^2)\right|^2 \text{ pb}$$

$\Longrightarrow$

$\left|G^p(4M_p^2)\right| \equiv 1$

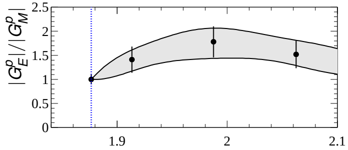
as pointlike fermion pairs!

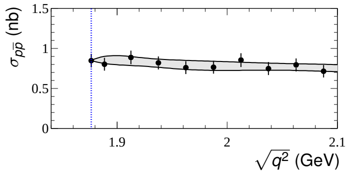
BaBar: $|G_E^p|/|G_M^p|$ and $\sigma(e^+e^- \to p\bar{p})$

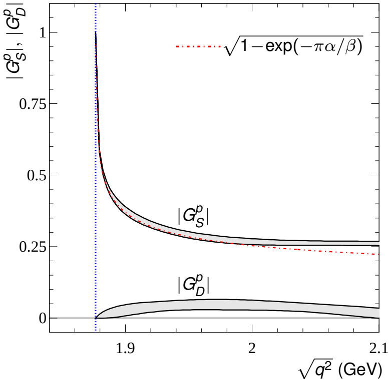
[PRD73, 012005]

Extracting $|G_S^p|$ and $|G_D^p|$ using

- data on $\sigma_{p\bar{p}}$
- data on $|G_E^p|/|G_M^p|$
- G_E^p/G_M^p phase $\phi \simeq 0$







- $|G_S^p| \simeq \sqrt{1-\exp(-\pi\alpha/\beta)}$
- **No need of resummation factor**

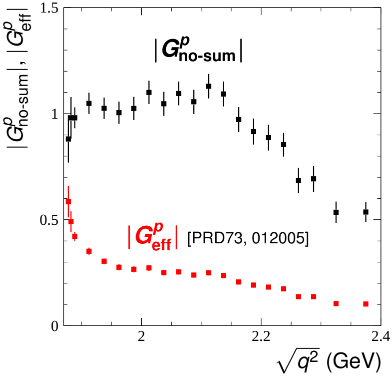
$B\textsc{a}B\textsc{ar}$: integrated Sommerfeld factor and G^p_{eff}

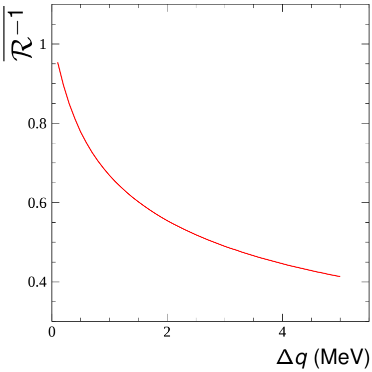
$$|G^p_{\text{eff}}|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\mathcal{C}\,\frac{16\pi\alpha^2}{3}\,\frac{\sqrt{1-1/\tau}}{4q^2}\left(1+\frac{1}{2\tau}\right)}$$

$$|G^p_{\text{no-sum}}|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\mathcal{E}\,\frac{16\pi\alpha^2}{3}\,\frac{\sqrt{1-1/\tau}}{4q^2}\left(1+\frac{1}{2\tau}\right)}$$

$$\overline{\mathcal{R}^{-1}} = \frac{1}{\Delta q}\int_0^{\Delta q}\left[1 - e^{-\frac{\pi\alpha}{\beta}}\right] d\sqrt{q^2}$$

$$\Delta q = \sqrt{q^2} - 2M_p$$

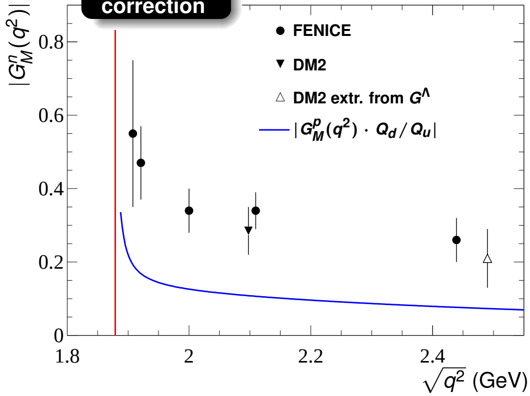




The neutral baryons puzzle

Time-like $|G_M^n|$ measurements

No Coulomb correction

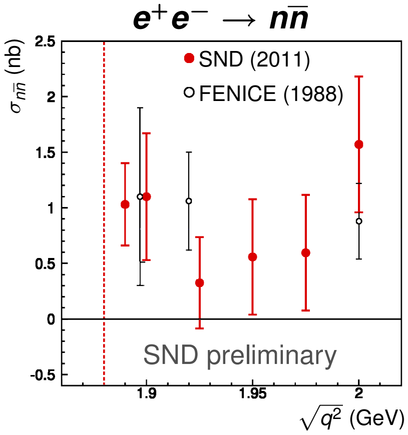


	$\lvert G_M^n / G_M^p \rvert$
Data	$\sim$ **1.5**
Naively	$\sim \lvert Q_d / Q_u \rvert$
pQCD	$<$ **1**
Soliton models	$\sim$ **1**
VMD (Dubnicka)	$\gg$ **1**

BESIII has the unique possibility to measure this cross section

No other experiments at present and in near future will be able to perform such a measurement

$e^+e^- \longrightarrow n\bar{n}$: preliminary result from SND



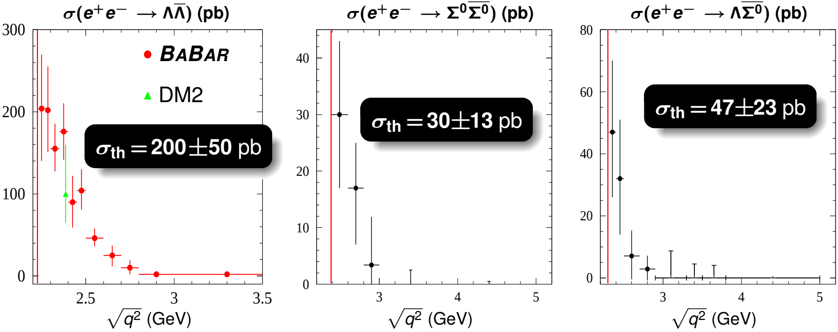
- Scan 2011
- Maximum energy: **2 GeV**
- Efficiency $\sim$ **30%**
- Above $n\bar{n}$ threshold: $\sigma_{n\bar{n}} = 0.8 \pm 0.2$ nb

Neutral Baryons puzzle (*BABAR*)

[PRD76, 092006]

$$\sigma_{N^0\overline{N}^0} = \frac{4\pi\alpha^2\beta C_0}{3q^2}\left[\left|G_M^{N^0}(q^2)\right|^2 + \frac{2M_{N^0}^2}{q^2}\left|G_E^{N^0}(q^2)\right|^2\right] \underset{\sqrt{q^2}\to 2M_{N^0}}{\longrightarrow} \frac{\pi\alpha^2\beta}{2M_{N^0}^2}|G^{N^0}|^2 \to 0$$

No Coulomb correction at hadron level: $C_0 = 1$



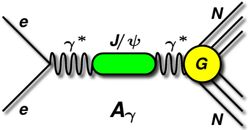
- **BESIII can improve these results**
- **Polarization for free: $\Lambda \to p\pi^-$**

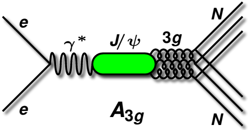
J/ψ strong and electromagnetic phase

Measurement of $J/\psi \longrightarrow p\bar{p}$, $n\bar{n}$

- $p\bar{p}$ amplitude A_γ^p from BABAR data
- $n\bar{n}$ amplitude A_γ^n from FENICE data
- $A_\gamma^p - A_\gamma^n$ relative phase from pQCD

$$B(J/\psi \to n\bar{n}) = \left| \frac{A_{3g} + A_\gamma^n}{A_{3g} + A_\gamma^p} \right|^2 B(J/\psi \to p\bar{p}) = (1.4 \pm 0.2) \times 10^{-3}$$

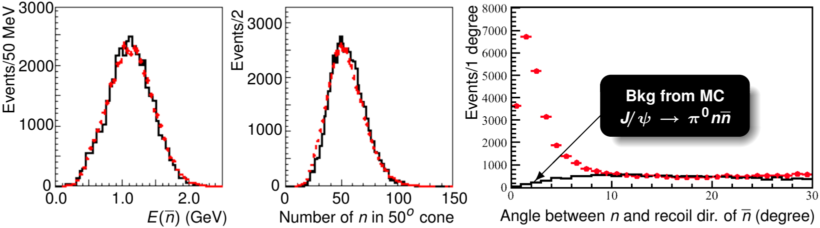




- BESII at BEPC [PLB591,42]: $B(J/\psi \longrightarrow p\bar{p}) = (2.26 \pm 0.01 \pm 0.14) \times 10^{-3}$
- FENICE at ADONE [PLB444,111]: $B(J/\psi \longrightarrow n\bar{n}) = (2.2 \pm 0.4) \times 10^{-3}$

$$B(J/\psi \longrightarrow p\bar{p}) \simeq B(J/\psi \longrightarrow n\bar{n})$$
$$\Downarrow$$
large $A_{3g}^N - A_{3g}^N$ relative phase

$\bar{n}$ identification



BESIII

$$B(J/\psi \to n\bar{n}) = (2.07 \pm 0.01 \pm 0.14) \cdot 10^{-3}$$

$$B(J/\psi \to p\bar{p}) = (2.112 \pm 0.004 \pm 0.027) \cdot 10^{-3}$$

PDG

$$B(J/\psi \to n\bar{n}) = (2.2 \pm 0.4) \cdot 10^{-3}$$

$$B(J/\psi \to p\bar{p}) = (2.17 \pm 0.07) \cdot 10^{-3}$$

$$B(J/\psi \to p\bar{p}) \simeq B(J/\psi \to n\bar{n})$$

suggests a phase $\sim 90^\circ$ between strong and em amplitudes!

Conclusions

- Asymptotic behavior not well understood
- Pointlike behavior not only at threshold
- No Sommerfeld resummation factor
- Neutral baryons puzzle

Perspectives with BESIII

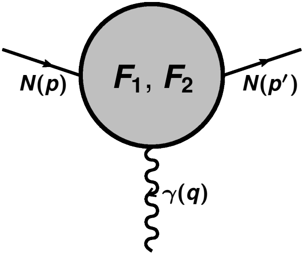
- More precise data on $\sigma_{p\bar{p}}$ above 3 GeV allow:
  - accurate study of the step around 3 GeV
  - precise measurement of the ratio $|G_E^p|/|G_M^p|$
- Unique possibility to measure the $n\bar{n}$ cross section with scan and ISR
- New data on neutral light hyperon cross sections
- Measurement of the relative phase between e.m. and strong amplitudes in $\Psi \to N\bar{N}$ decays

Additional slides

The analytic structure of the matrix element for the vertex $N\gamma N$ at order n of perturbation theory is given by

$$I(q_i) = \int_0^1 \!\!\cdots\! \int_0^1 \frac{\delta(1-\sum_{i=1}^n \alpha_i)\, d\alpha_1 \ldots d\alpha_n}{\left[C(\alpha_j) q^2 - \sum_{i=1}^n m_i^2 \alpha_i + i\epsilon\right]^{n-2k}}$$

- n internal lines of masses m_i, k loops
- C, function of α_i, is positive
- On-shell baryon external lines: $p^2 = p'^2 = M^2$



Nucleon form factors

- F_1 and F_2 depend only on photon q^2
- The integrand has poles in the α_i-domain from q^2_{th} up to ∞

$$q^2_{\text{th}} = [\text{mass of the lightest hadronic state coupled to } \gamma] = (2m_\pi)^2$$

- $F_{1,2}(q^2)$ is analytic in $\text{Im}\,q^2 > 0$ and real for $q^2 \leq q^2_{\text{th}} \Longrightarrow F_{1,2}(q^{2*}) = F^*_{1,2}(q^2)$

Unitarity and Cutkosky rule

Scattering $\boldsymbol{S}$ matrix in terms of transition matrix $\boldsymbol{T}$: $$S = 1 + iT$$

Unitarity ($S^\dagger S = S\, S^\dagger = 1$): $$-i(T - T^\dagger) = T^\dagger T$$

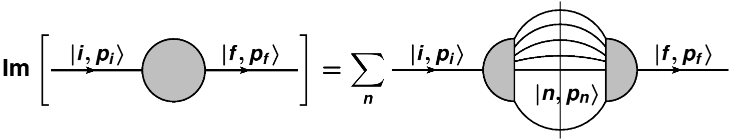
Four-momentum conservation in $|i\rangle \to |f\rangle$ transition:

$$\begin{cases} \langle f|T|i\rangle = (2\pi)^4 \delta^4(p_f - p_i)\mathcal{T}_{fi} \\ \langle f|T^\dagger|i\rangle = (2\pi)^4 \delta^4(p_f - p_i)\mathcal{T}_{if}^* \end{cases}$$

Using $\sum_n |n\rangle\langle n| = 1$:

$$\langle f|T^\dagger T|i\rangle = \sum_n \langle f|T^\dagger|n\rangle\langle n|T|i\rangle$$

$$\langle f|T^\dagger T|i\rangle = (2\pi)^4\delta^4(p_f - p_i)\sum_n (2\pi)^4\delta^4(p_n - p_i)\mathcal{T}_{nf}^*\mathcal{T}_{ni}$$



Cutkosky rule

$$2\,\mathrm{Im}\left[\mathcal{T}_{fi}\right] = -i(\mathcal{T}_{fi} - \mathcal{T}_{if}^*) = \sum_n (2\pi)^4\delta^4(p_n - p_i)\mathcal{T}_{nf}^*\mathcal{T}_{ni}$$

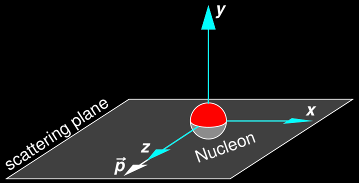
Polarization formulae in the time-like region

[A.Z. Dubnickova, S. Dubnicka, M.P. Rekalo, NCA109,241(96)]

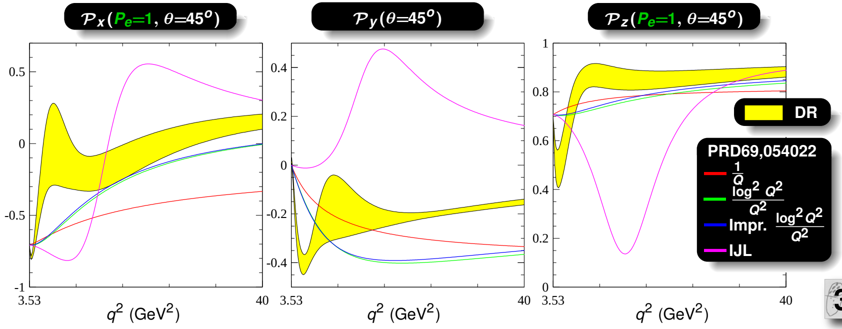
$$\mathcal{P}_y = -\frac{\sin(2\theta)|R|\sin(\phi)}{D\sqrt{\tau}} = \mathcal{A}_y$$

$$\mathcal{P}_x = -P_e \frac{2\sin(2\theta)|R|\cos(\phi)}{D\sqrt{\tau}}$$

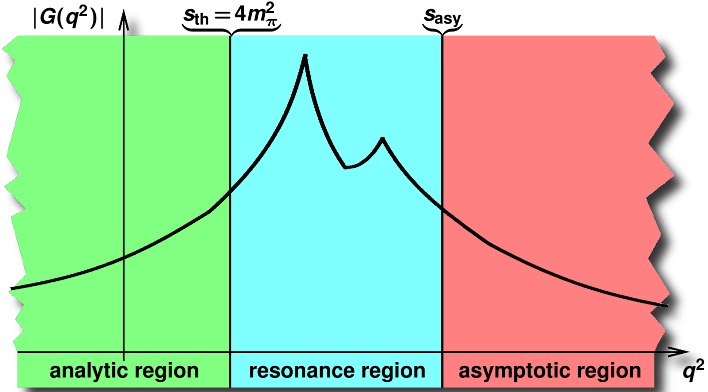
$$\mathcal{P}_z = P_e \frac{2\cos(\theta)}{D}$$



ϕ = phase of R , $D = (1 + \cos^2\theta + |R|^2 \sin^2\theta/\tau)/\mu_p$, P_e = electron polarization



Form factors in three regions

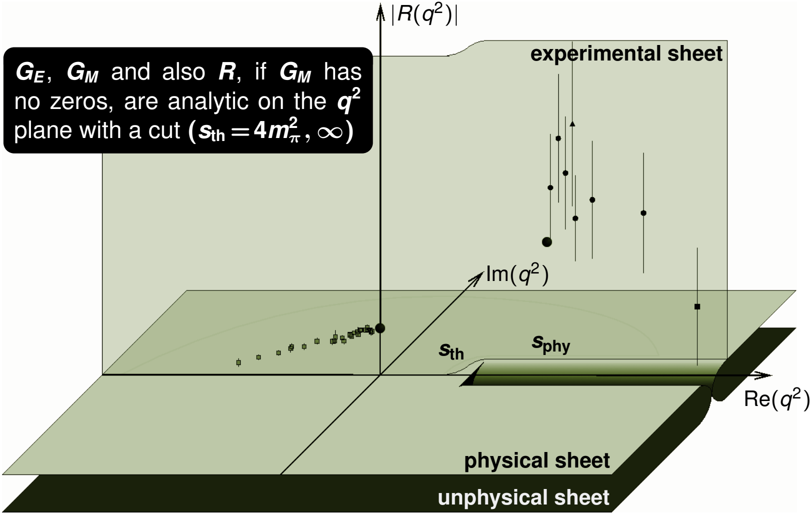


Asymptotic ⇐ q^2-power laws from perturbative QCD

Resonance ⇐ Intermediate vector meson contributions

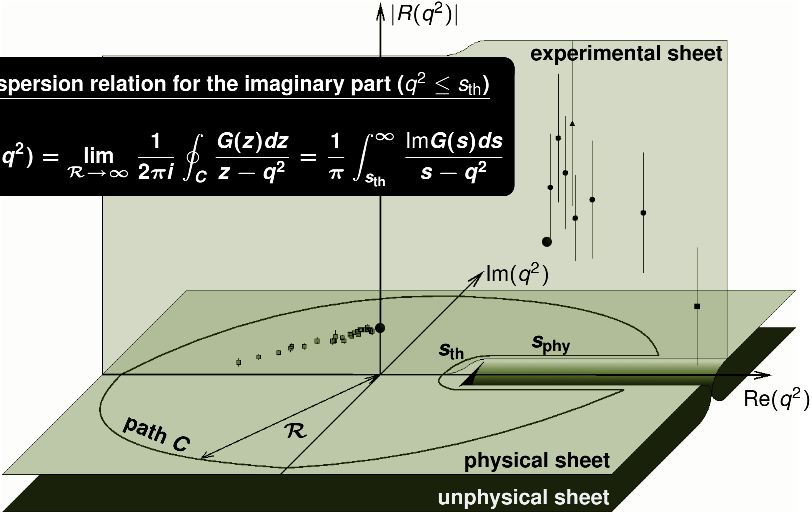
Analytic ⇐ Dispersion relations with pQCD, data and resonances

$R(q^2)$ in the complex plane



G_E, G_M and also R, if G_M has no zeros, are analytic on the q^2 plane with a cut $(s_{th} = 4m_\pi^2, \infty)$

$R(q^2)$ in the complex plane



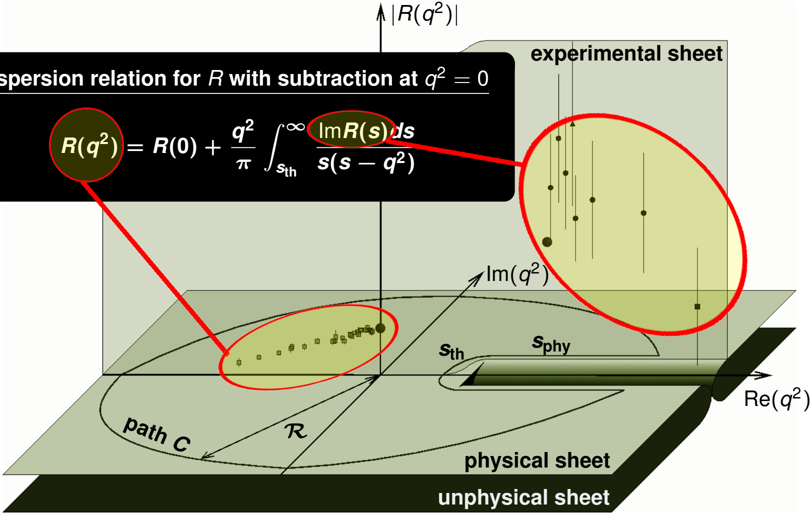
Dispersion relation for the imaginary part ($q^2 \le s_{\mathrm{th}}$)

$$G(q^2) = \lim_{\mathcal{R}\to\infty} \frac{1}{2\pi i} \oint_C \frac{G(z)dz}{z - q^2} = \frac{1}{\pi} \int_{s_{\mathrm{th}}}^{\infty} \frac{\mathrm{Im}G(s)ds}{s - q^2}$$

$R(q^2)$ in the complex plane

Dispersion relation for R with subtraction at $q^2 = 0$

$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{s_{th}}^{\infty} \frac{\mathrm{Im}R(s)\,ds}{s(s-q^2)}$$



S and D waves

$$\begin{cases} P_\gamma = -1 & P_{N\bar{N}} = (-1)^L \times (-1) \quad \Rightarrow \quad L = 0, 2 \\ J_\gamma = 1 & S = 0: \quad L = 1 \text{ forbidden} \longrightarrow S = 1 \end{cases}$$

$$G_E = G_S - 2G_D \qquad G_M = \frac{G_S + G_D}{\sqrt{q^2}/2M}$$

At threshold S wave only: $G_E = G_M$

$$\begin{cases} G_E = F_1 + \dfrac{q^2}{4M^2} F_2 \\ G_M = F_1 + F_2 \end{cases} \Longrightarrow \quad G_E = G_M$$

40

Höhler, Mergell, Meissner, Hammer procedure

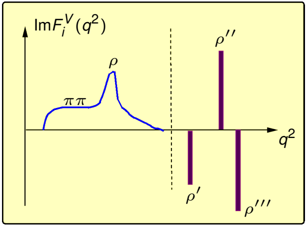
- Optical theorem
  - Dispersion relation for the imaginary part
    - No time-like $|G_E| - |G_M|$ separation

$\Longrightarrow$ $G_E^{p,n}$ and $G_M^{p,n}$ in space and time-like region

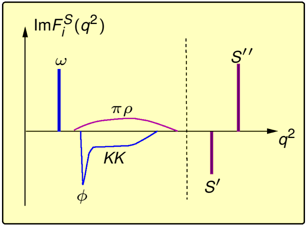
H_2M_2: Höhler, Mergell, Meissner, Hammer,. . . PRC75 035202

Spectral decomposition

$$\text{Im}\langle\overline{N}(p')N(p)|j^\mu|0\rangle \sim \sum_n \langle\overline{N}(p')N(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle \implies \begin{cases} \text{Im}F_{1,2}^{V,S} \neq 0 \\ \text{for } q^2 > 4m_\pi^2 \end{cases}$$



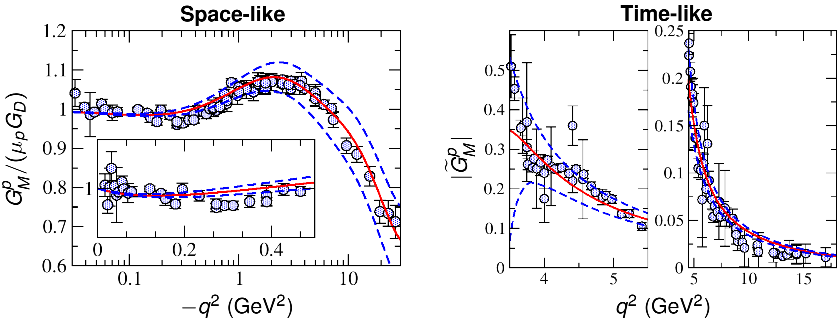
- 2π continuum is known for $q^2 \in [4m_\pi^2, \sim 40m_\pi^2]$
- The singularity on the second Riemann sheet in $\pi N \to \pi N$ amplitude gives the strong shoulder at threshold
- Poles for higher mass states



- *KK* continuum from analytic continuation of *KN* scattering amplitude
- Further contribution in the ϕ-region is due to $\pi\rho$ exchange
- Anomalous threshold behavior is masked because the pole in the second Riemann sheet is not close to $(3m_\pi)^2$
- Poles for higher mass states

H_2M_2: Theoretical constraints and result for G_M^p

- Asymptotic behaviors from perturbative QCD
- Superconvergence relations: $\int_{4m_\pi^2}^{\infty} \mathrm{Im}\, F_{1,2}(q^2)dq^2 = \int_{4m_\pi^2}^{\infty} q^2 \mathrm{Im}\, F_2(q^2)dq^2 = 0$



$$F(q^2_{\mathrm{SL}}) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}F(q^2_{\mathrm{TL}})}{q^2_{\mathrm{TL}} - q^2_{\mathrm{SL}}} dq^2_{\mathrm{TL}}$$

VMD Models

- VMD + quark form factors
  - DRs $\longrightarrow$ analytic VM propagators
    - Time-like $|G_E| - |G_M|$ separation

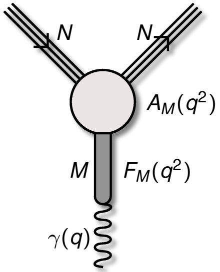
  $\Longrightarrow$ $G_E^{p,n}$ and $G_M^{p,n}$ in space and time-like region

Analyticitization of VMD-based models

PRC66,045501
PLB43,191

The **Lomon** and **Iachello** parameterizations for nucleon FF's are based on VMD, and include:

- coupling to the photons through vector meson exchange [VMD in terms of **propagators $F_M(q^2)$**, $M = \rho, \omega, \phi, \rho', \omega'$]
- **hadron/quark form factors $A_M(q^2)$** at vector meson-nucleon (quark) vertices to control transition to perturbative QCD at high momentum transfers

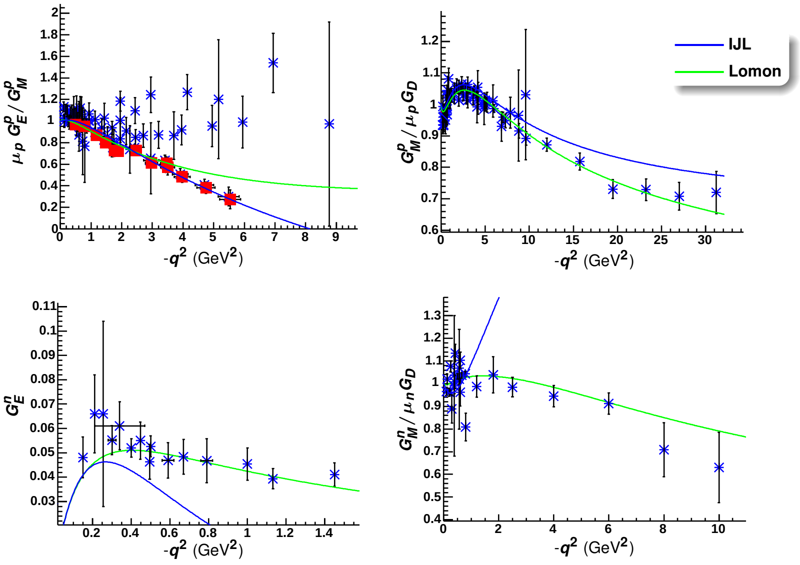


Analytic extension: space-like $\longrightarrow$ time-like

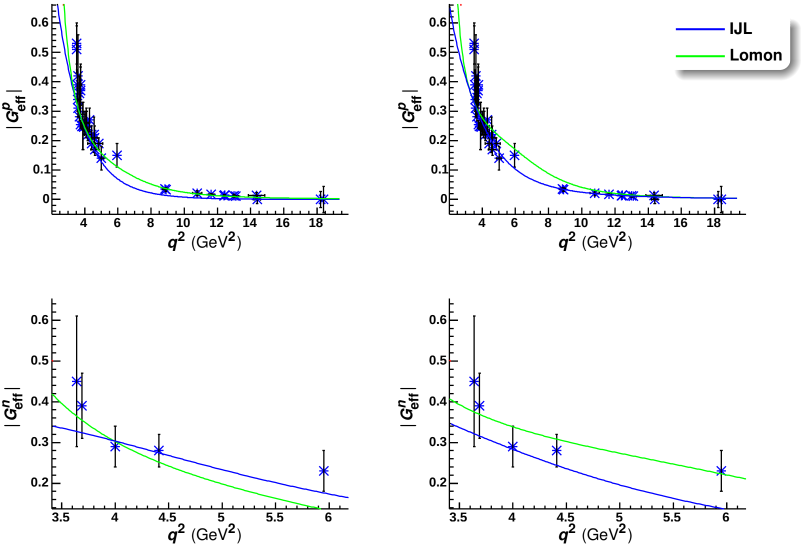
- ***F_M* for broad mesons**:
  simple poles $\longrightarrow$ poles with finite energy-dependent widths
- **Dispersion relations**:
  rigorous analytic continuation of F_M from time-like to space-like region

Space-like fits

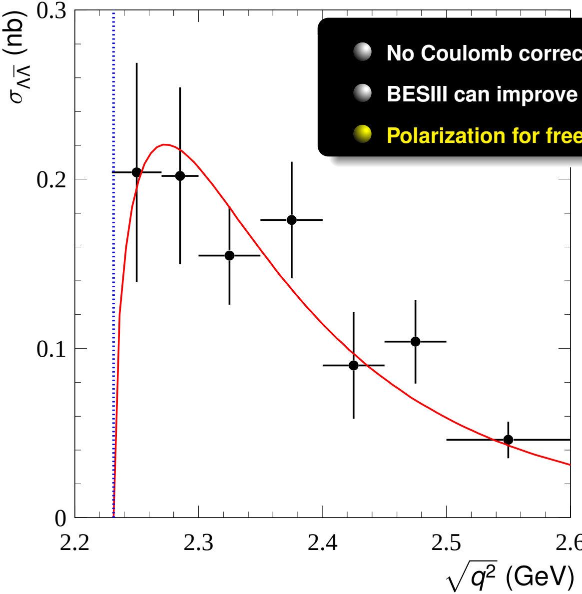
E. Tomasi-Gustafsson, *et al.*
EPJA24,419(05)



E. Tomasi-Gustafsson, *et al.*
EPJA24,419(05)



BABAR: $e^+e^- \longrightarrow \Lambda\bar{\Lambda}$



- No Coulomb correction (?)
- BESIII can improve this result
- Polarization for free: $\Lambda \longrightarrow p\pi^-$