

Experimental and theoretical aspects of nucleon form factors

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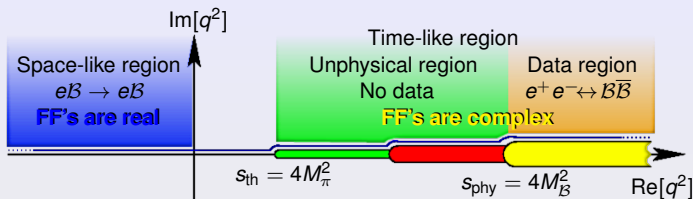
*Scattering and annihilation
electromagnetic processes*

18 - 22 February 2013
ECT* - Trento

- **Last News on Baryon FF near threshold**
- **The Neutral Baryon Puzzle**
- **Spacelike - Timelike Relationship**
- **Conclusions and Perspectives**

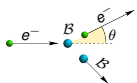


Cross sections and analyticity



Time-like: had. helicity = $\begin{cases} 1 \Rightarrow |G_E| \\ 0 \Rightarrow |G_M| \end{cases}$

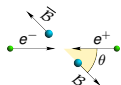
$$G_E(4M_B^2) = G_M(4M_B^2)$$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_e' \cos^2 \frac{\theta}{2}}{4E_e^3 \sin^4 \frac{\theta}{2}} \left[G_E^2 - \tau \left(1 + 2(1 - \tau) \tan^2 \frac{\theta}{2} \right) G_M^2 \right] \frac{1}{1 - \tau}$$

$$\tau = \frac{q^2}{4M_B^2}$$



Annihilation

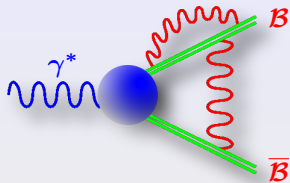
Coulomb correction

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta}{4q^2} \left[(1 + \cos^2 \theta) |G_M|^2 + \frac{1}{\tau} \sin^2 \theta |G_E|^2 \right]$$

$$\beta = \sqrt{1 - \frac{1}{\tau}}$$



The Coulomb Factor



$p\bar{p}$ Coulomb interaction as FSI

[Sommerfeld, Sakharov, Schwinger, Fadin, Khoze]

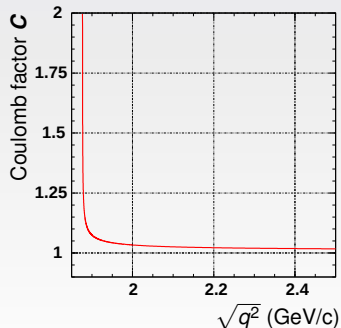
Distorted wave approximation

$$C = |\Psi_{\text{Coul}}(0)|^2$$

● S-wave: $C = \frac{\frac{\pi\alpha}{\beta}}{1 - \exp\left(-\frac{\pi\alpha}{\beta}\right)} \xrightarrow{\beta \rightarrow 0} \frac{\pi\alpha}{\beta}$

● D-wave: $C = 1$

No Coulomb factor for boson pairs (P-wave)



Sommerfeld Enhancement and Resummation Factors

Coulomb Factor $\mathcal{C}$ for S-wave only:

- Partial wave FF: $G_S = \frac{2G_M \sqrt{q^2/4M^2} + G_E}{3}$ $G_D = \frac{G_M \sqrt{q^2/4M^2} - G_E}{3}$

- Cross section: $\sigma(q^2) = 2\pi\alpha^2\beta \frac{4M^2}{(q^2)^2} [\mathcal{C} |G_S(q^2)|^2 + 2|G_D(q^2)|^2]$

$$\mathcal{C} = \mathcal{E} \times \mathcal{R}$$

- Enhancement factor: $\mathcal{E} = \pi\alpha/\beta$

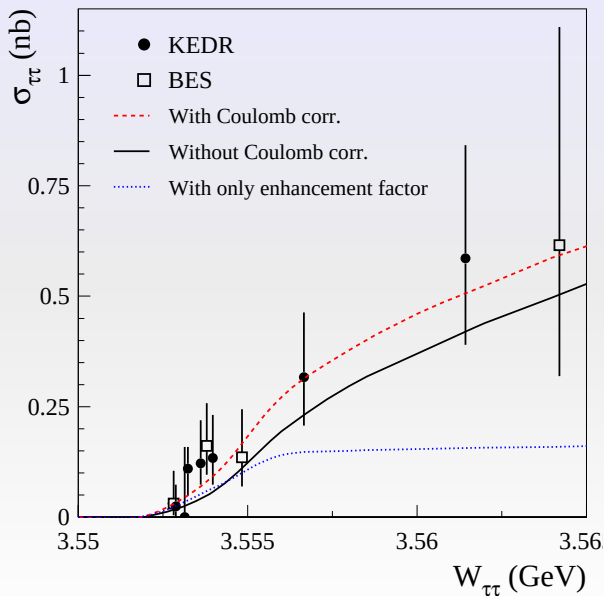
- Step at threshold: $\sigma(4M^2) = \frac{\pi^2\alpha^3}{2M^2} \frac{\cancel{\beta}}{\cancel{\beta}} |G_S(4M^2)|^2 = 0.85 |G_S(4M^2)|^2 \text{ nb}$

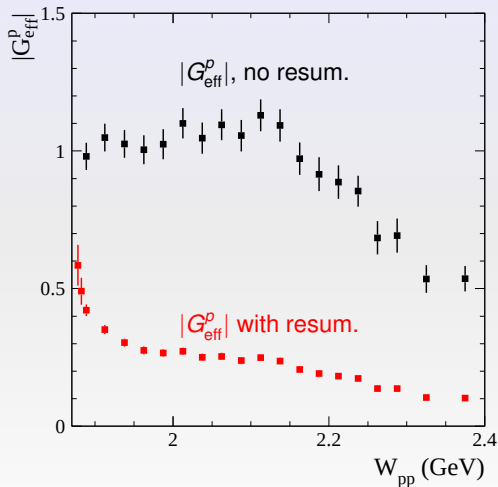
- Resummation factor: $\mathcal{R} = 1/[1 - \exp(-\pi\alpha/\beta)]$

- Few MeV above threshold: $\mathcal{C} \simeq 1 \Rightarrow \sigma(q^2) \propto \beta |G_S(q^2)|^2$



The $e^+e^- \rightarrow \tau^+\tau^-$ case







Pointlike Baryons?

R. Baldini Ferroli, S. Pacetti,
A. Zallo and A. Zichichi

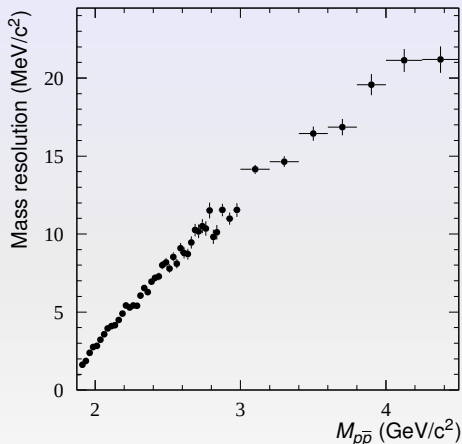
Advantages

- All q at the same time $\Rightarrow$ Better control on systematics
- c.m. boost $\Rightarrow$ at threshold **efficiency $\neq 0$ + $\sigma_W \sim 1 \text{ MeV}$**
- Detected ISR $\gamma \Rightarrow$ full $p\bar{p}$ angular coverage

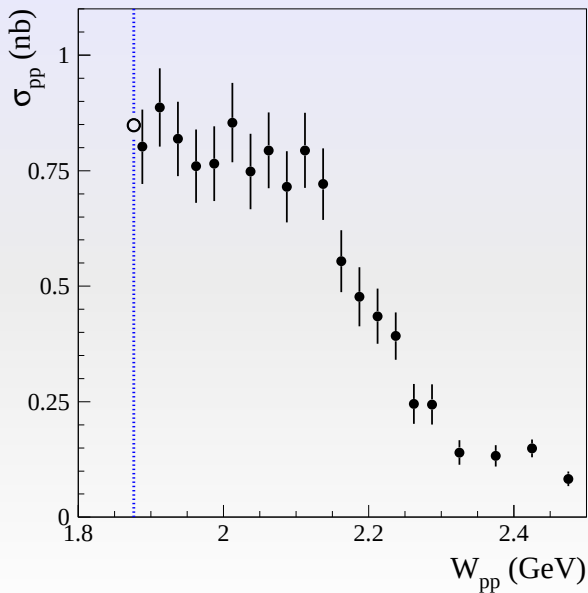
Drawbacks

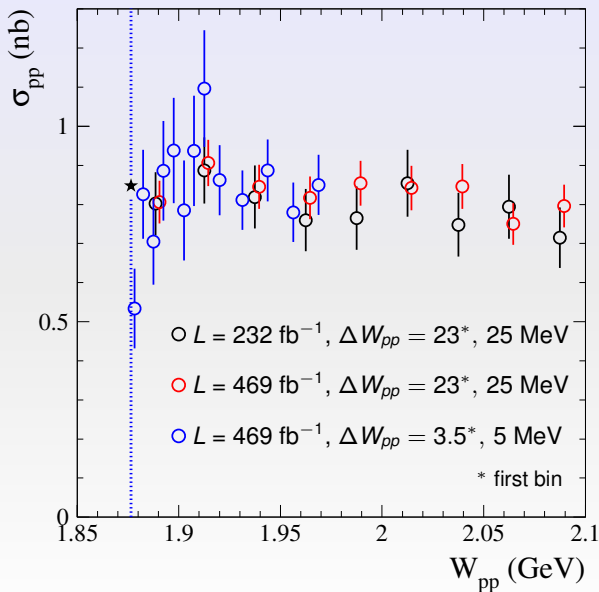
- $\mathcal{L} \propto$ invariant mass bin Δw
- More background

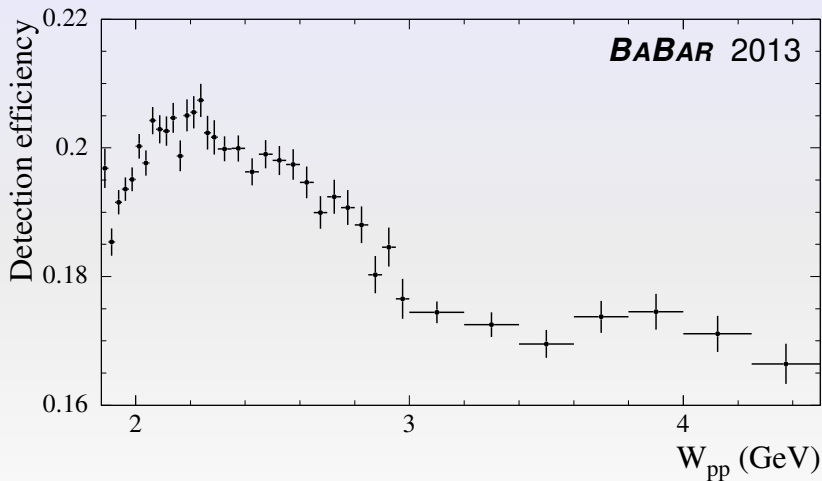
Mass resolution



Incredibly good at threshold ($\sim 1 \text{ MeV}/c^2$), as e^+e^- c.m.
 $\Delta p_T/p_T \sim 0.5\%$ at 1 GeV







Proton form factor at $q^2 = 4M_p^2$

$$\sigma(e^+e^- \rightarrow p\bar{p})(4M_p^2) = 0.83 \pm 0.05 \text{ nb}$$

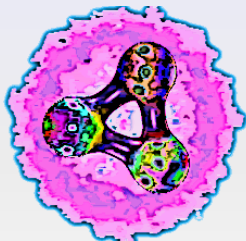
BA BAR

$$\sigma(e^+e^- \rightarrow p\bar{p})(4M_p^2) = \frac{\pi^2 \alpha^3}{2M_p^2} \cancel{\beta} |G^p(4M_p^2)|^2 = 0.85 |G^p(4M_p^2)|^2 \text{ nb}$$

$$|G^p(4M_p^2)| \equiv 1$$

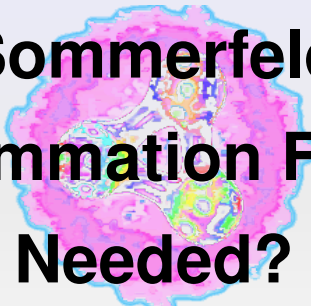
$$|G^p(4M_p^2)| = 0.99 \pm 0.04(\text{stat}) \pm 0.03(\text{syst})$$

$$|G^p(4M_p^2)| \equiv 1$$



**At $q^2 = 4M_p^2$ protons behave
as pointlike fermions!**

Sommerfeld Resummation Factor Needed?

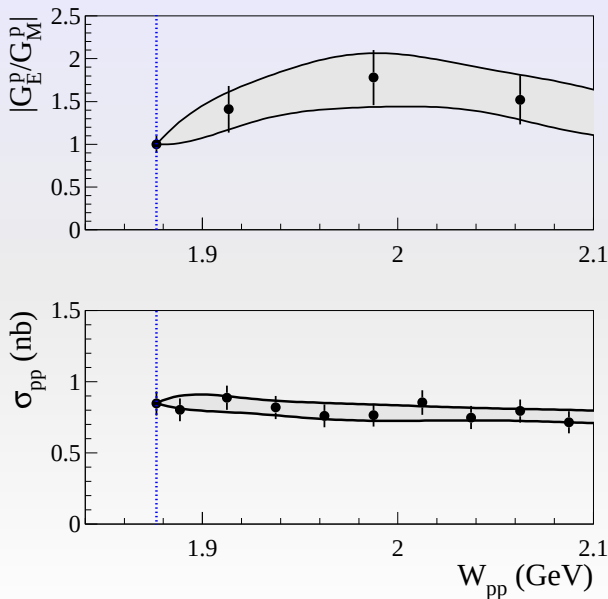


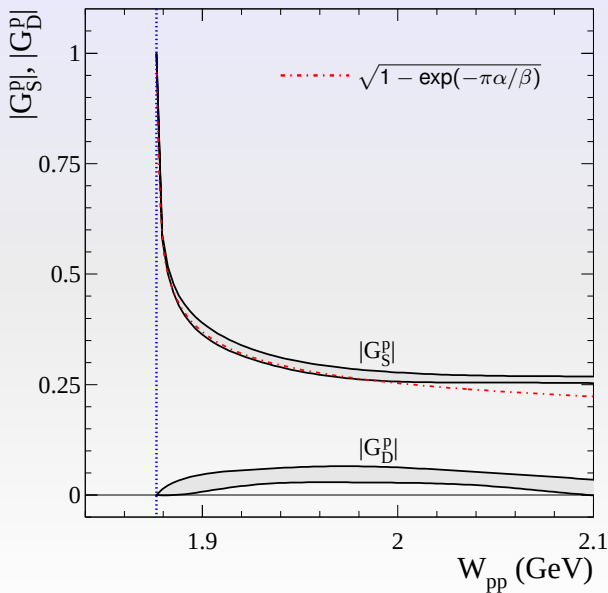
Resummation Factor Needed?

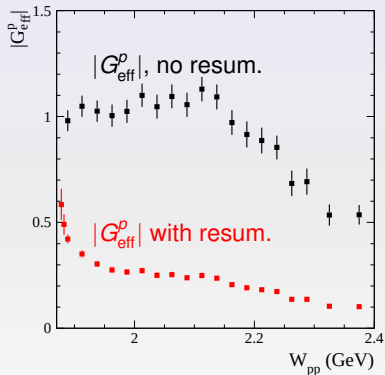
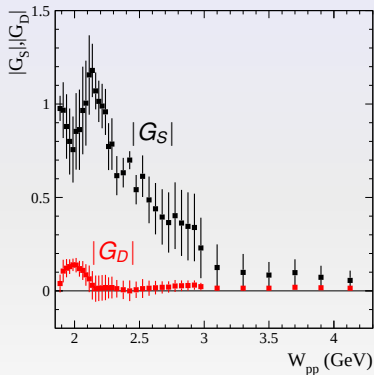
- At threshold: $G_E/G_M = 1 \Rightarrow \begin{cases} G_S \in \mathbb{R} \\ G_D = 0 \in \mathbb{R} \end{cases}$
- $\sigma(q^2), |G_E/G_M| \rightarrow G_S, G_D$
- $G_S = \sqrt{1 - \exp(-\pi\alpha/\beta)}$
- **No need of Resummation Factor**

For a wide energy range (~ 200 MeV):

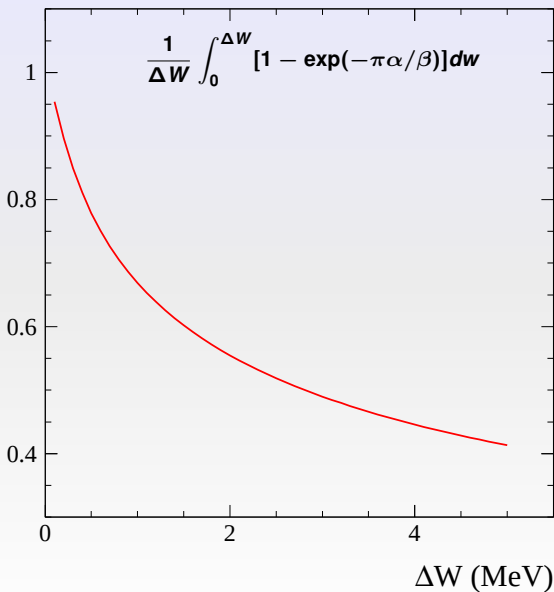
- Proton behaves as a pointlike particle
- e.m. dominance, no strong interaction?
- Mild sensitivity to $B\bar{B}$ invariant mass resolution

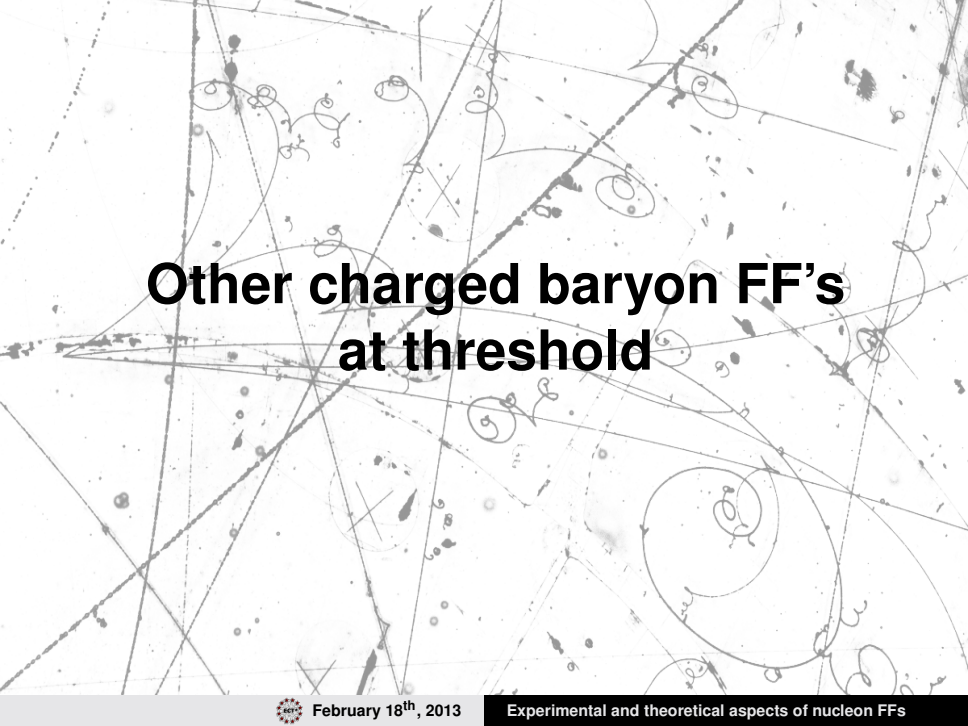






Integrated Sommerfeld factor

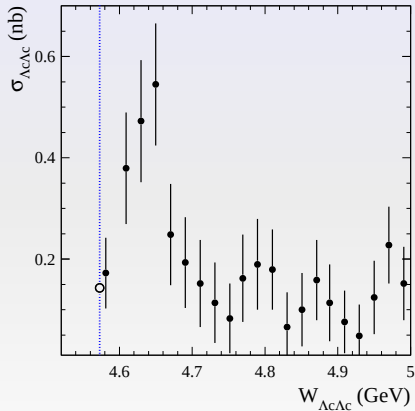




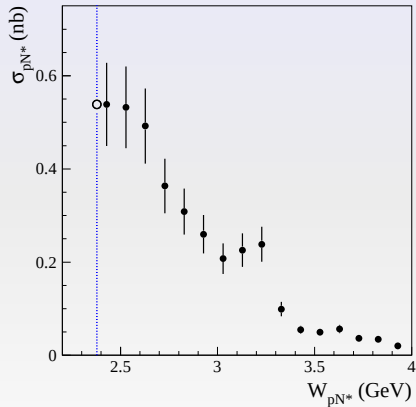
Other charged baryon FF's at threshold

$$e^+e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^- \text{ and } e^+e^- \rightarrow p \bar{N}(1440) + \text{c.c.}$$

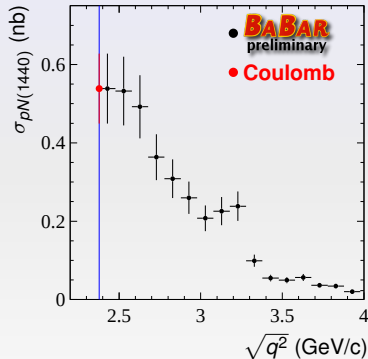
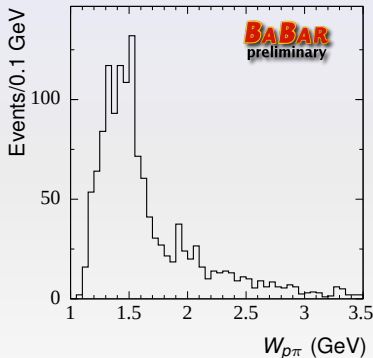
[Belle PRL101, 172001]



[BABAR PRD73, 012005]



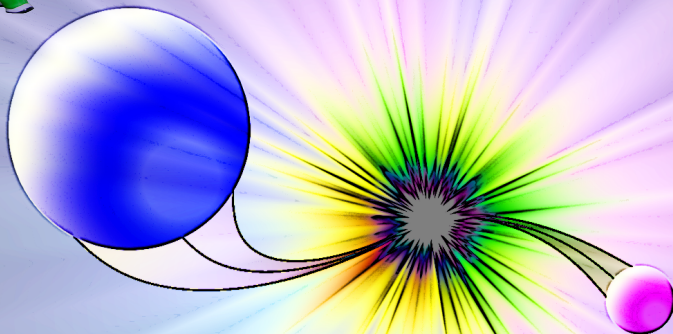
$$\sigma_{\text{Coulomb}} = \frac{16\pi^2 \alpha^3 M_p^{3/2} M_{N(1440)}^{3/2}}{(M_p + M_{N(1440)})^5} |G^{pN(1440)}|^2 = |G^{pN(1440)}|^2 \times 0.49 \text{ nb}$$



$$|G^{pN(1440)}| = 1.04 \pm 0.09$$

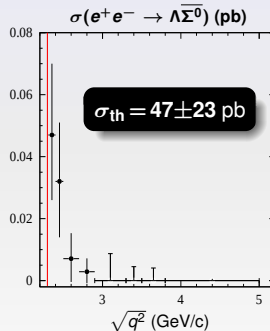
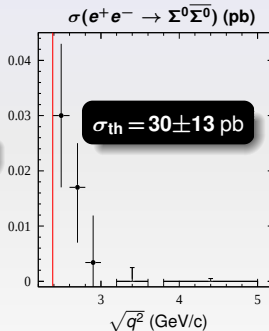
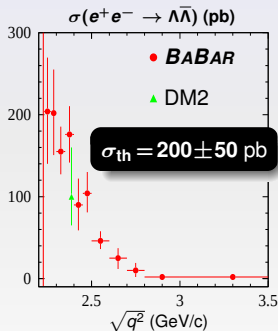


The neutral baryons puzzle



$$\sigma(e^+e^- \rightarrow B^0 \bar{B}^0) = \frac{4\pi\alpha^2\beta C_0}{3q^2} \left[|G_M^{B^0}|^2 + \frac{2M_{B^0}^2}{q^2} |G_E^{B^0}|^2 \right] \xrightarrow{\sqrt{q^2} \rightarrow 2M_{B^0}} \frac{\pi\alpha^2\beta}{2M_{B^0}^2} |G^{B^0}|^2 \rightarrow 0$$

No Coulomb correction at hadron level: $C_0 = 1$



Like a remnant of
Coulomb interactions
at quark level?

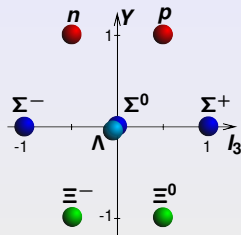


$C_0 \propto \beta^{-1}$
as $\sqrt{q^2} \rightarrow 2M_{B^0}$



For any neutral baryon

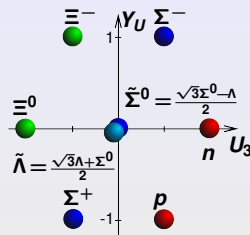
$$\sqrt{\sigma_{B^0 \bar{B}^0}} \propto \frac{|G^{B^0}|}{M_{B^0}}$$



$$(Y, I_3) \rightarrow (Y_U, U_3)$$

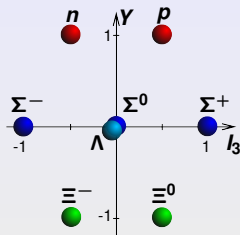
$$U_3 = -\frac{1}{2}I_3 + \frac{3}{4}Y$$

$$Y_U = -Q$$



U-spin relation: $G^{\Sigma^0} - G^{\Lambda} + \frac{2}{\sqrt{3}}G^{\Lambda\Sigma^0} = 0$

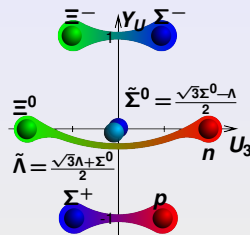
$$M_{\Sigma^0} \sqrt{\sigma_{\Sigma^0 \Sigma^0}} - M_{\Lambda} \sqrt{\sigma_{\Lambda \Lambda}} + \frac{2}{\sqrt{3}} M_{\Lambda \Sigma^0} \sqrt{\sigma_{\Lambda \Sigma^0}} = (-0.06 \pm 6.0) \times 10^{-4}$$



$$(Y, I_3) \rightarrow (Y_U, U_3)$$

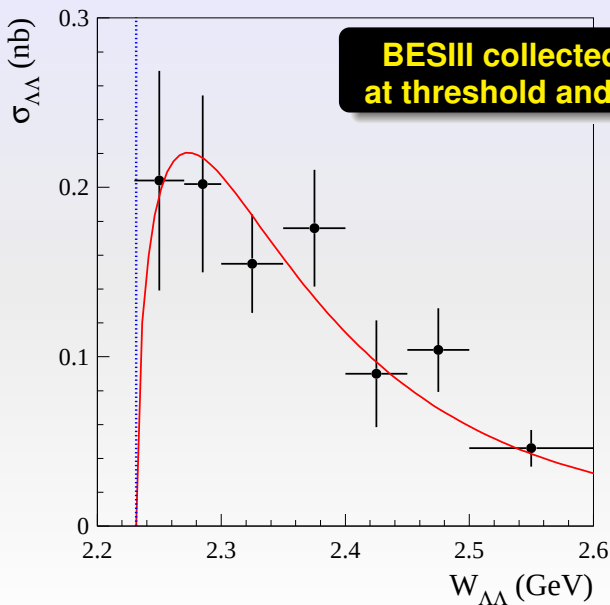
$$U_3 = -\frac{1}{2}I_3 + \frac{3}{4}Y$$

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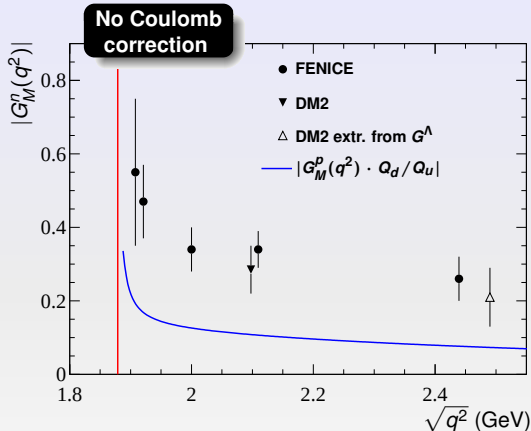


U-spin relation: $G^{\Sigma^0} - G^{\Lambda} + \frac{2}{\sqrt{3}} G^{\Lambda\Sigma^0} = 0$

$$M_{\Sigma^0} \sqrt{\sigma_{\Sigma^0 \Sigma^0}} - M_{\Lambda} \sqrt{\sigma_{\Lambda \Lambda}} + \frac{2}{\sqrt{3}} M_{\Lambda \Sigma^0} \sqrt{\sigma_{\Lambda \Sigma^0}} = (-0.06 \pm 6.0) \times 10^{-4}$$



Time-like $|G_M^n|$ measurements

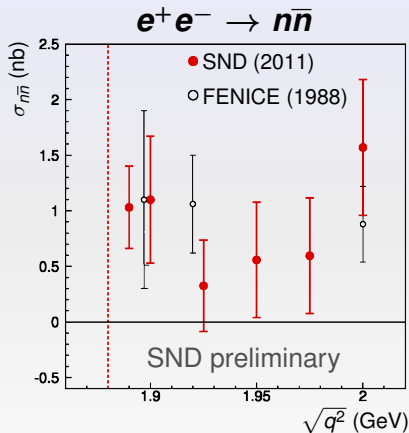


	$ G_M^n / G_M^p $
Data	~ 1.5
Naively	$\sim Q_d / Q_u $
pQCD	< 1
Soliton models	~ 1
VMD (Dubnicka)	$\gg 1$

Only SND, CMD2(?) and BESIII can measure this cross section

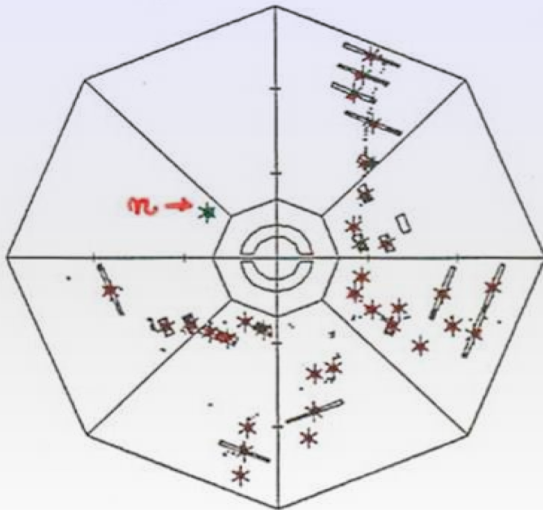
No other experiments at present and in near future will be able to perform such a measurement

$e^+e^- \rightarrow n\bar{n}$: preliminary result from SND

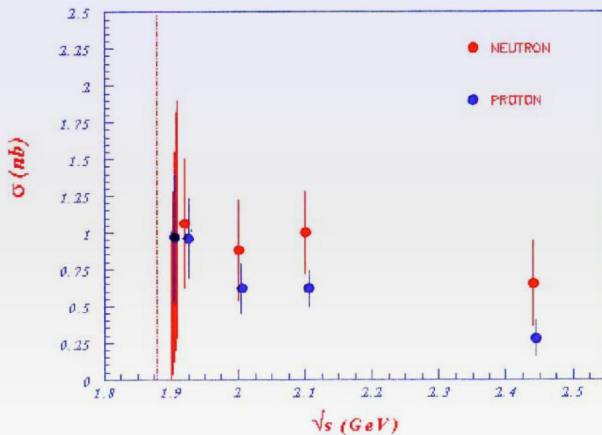


- Scan 2011
- Maximum energy: **2 GeV**
- Efficiency $\sim$ **30%**
- Above $n\bar{n}$ threshold:
 $\sigma_{n\bar{n}} = 0.8 \pm 0.2$ nb

$$e^+e^- \rightarrow n\bar{n}$$

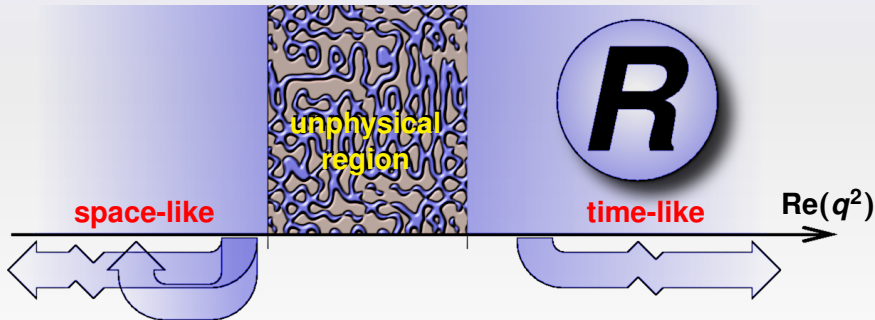


$$e^+e^- \rightarrow n\bar{n} \text{ (FENICE)}$$

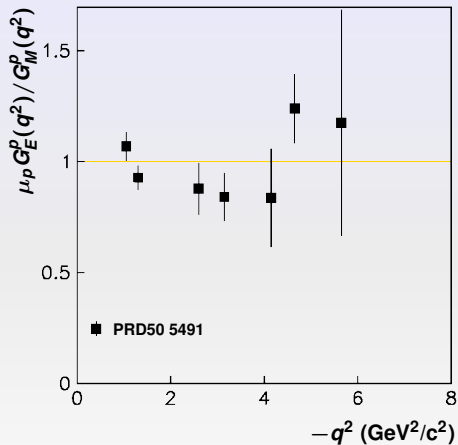


Dispersive analysis of the ratio $R = \mu_p \frac{G_E^p}{G_M^p}$

Eur. Phys. J. A32, 421
R. Baldini, S. Pacetti and A. Zallo



Space-like G_E^p/G_M^p measurements



$$G_E^p = F_1^p + \frac{q^2}{4M_p^2} F_2^p$$

$$G_M^p = F_1^p + F_2^p$$

Space-like

$$F_1 / \frac{q^2}{4M_p^2} F_2 \text{ cancellation}$$

$$\frac{G_E^p(q^2)}{G_M^p(q^2)} < 1$$

Time-like

$$F_1 / \frac{q^2}{4M_p^2} F_2 \text{ enhancement}$$

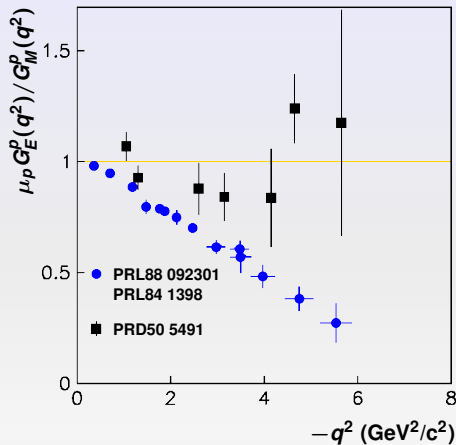
$$\left| \frac{G_E^p(q^2)}{G_M^p(q^2)} \right| > 1$$

Radiative corrections of
polarization technique



Radiative corrections in
Rosenbluth method

Space-like G_E^p/G_M^p measurements



$$G_E^p = F_1^p + \frac{q^2}{4M_p^2} F_2^p$$

$$G_M^p = F_1^p + F_2^p$$

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Radiative corrections of
polarization technique

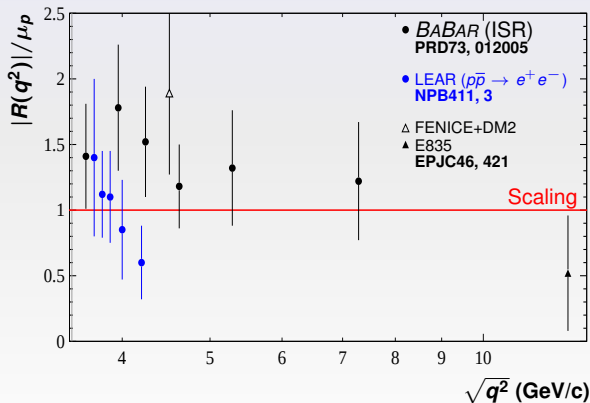


Radiative corrections in
Rosenbluth method

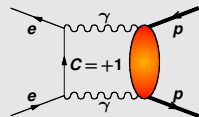
Time-like $|G_E^p/G_M^p|$ measurements

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta C}{2q^2} |G_M^p|^2 \left[(1 + \cos^2\theta) + \frac{4M_p^2}{q^2\mu_p^2} \sin^2\theta |R|^2 \right]$$

$$R(q^2) = \mu_p \frac{G_E^p(q^2)}{G_M^p(q^2)}$$



$\gamma\gamma$ exchange



$\gamma\gamma$ exchange interferes with the Born term

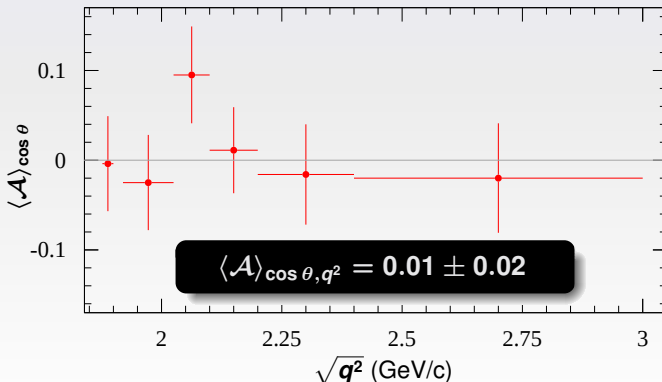


Asymmetry in angular distributions
[PLB659, 197]

$\gamma\gamma$ exchange from $e^+e^- \rightarrow p\bar{p}\gamma$ *BABAR* data

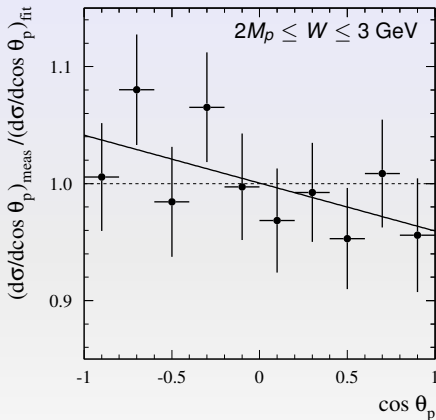
E. Tomasi-Gustafsson,
E. A. Kuraev, S. Bakmaev, SP
PLB659, 197

$$\mathcal{A}(\cos \theta, q^2) = \frac{\frac{d\sigma}{d\Omega}(\cos \theta, q^2) - \frac{d\sigma}{d\Omega}(-\cos \theta, q^2)}{\frac{d\sigma}{d\Omega}(\cos \theta, q^2) + \frac{d\sigma}{d\Omega}(-\cos \theta, q^2)}$$



$\gamma\gamma$ exchange from $e^+e^- \rightarrow p\bar{p}\gamma$ *BABAR* 2013 data

PLB659-197, arXiv:1302.0055



- Integrated over the energy from threshold up to 3 GeV
- The MC-fit assumes **one-photon exchange**
- **Slope = $-0.041 \pm 0.026 \pm 0.005$**

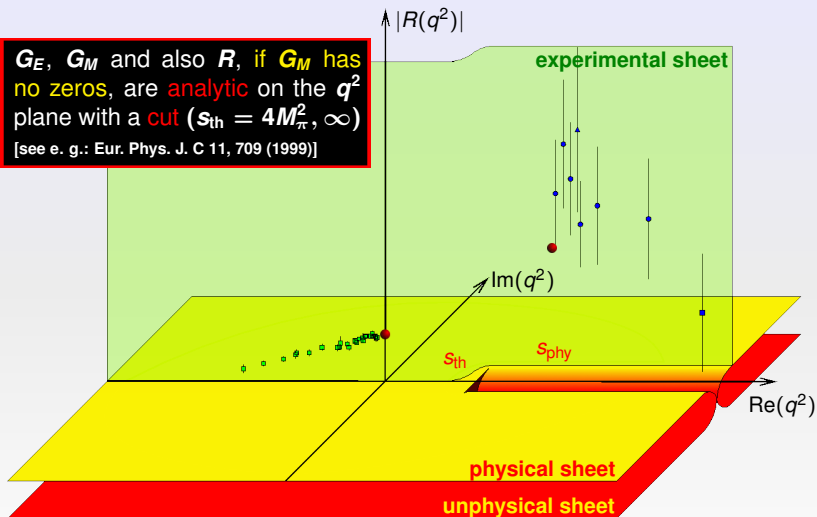
Integral asymmetry

$$\langle \mathcal{A} \rangle_{\cos \theta_p} = \frac{\sigma(\cos \theta_p > 0) - \sigma(\cos \theta_p < 0)}{\sigma(\cos \theta_p > 0) + \sigma(\cos \theta_p < 0)} = -0.025 \pm 0.014 \pm 0.003$$

$\sigma(\cos \theta_p \gtrless 0)$ is the cross section integrated with $W \leq 3$ GeV and $\cos \theta_p \gtrless 0$

$R(q^2)$ in the complex plane

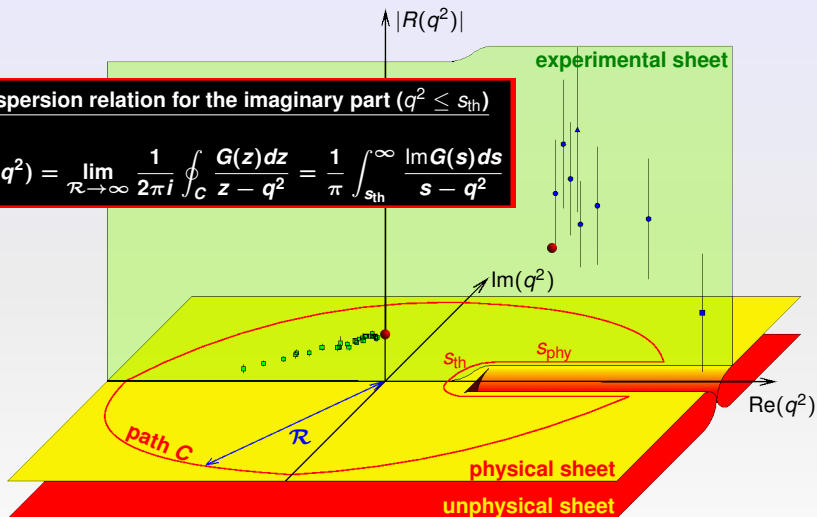
G_E , G_M and also R , if G_M has no zeros, are analytic on the q^2 plane with a cut ($s_{th} = 4M_\pi^2, \infty$) [see e. g.: Eur. Phys. J. C 11, 709 (1999)]



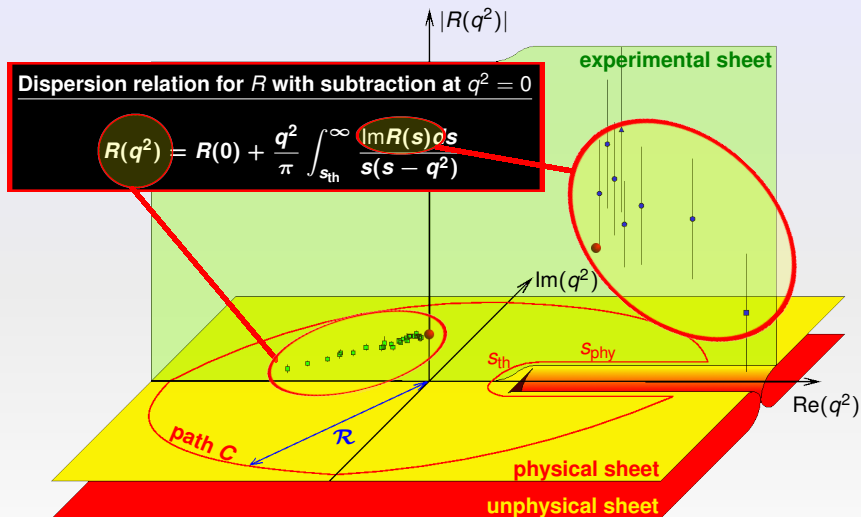
$R(q^2)$ in the complex plane

Dispersion relation for the imaginary part ($q^2 \leq s_{th}$)

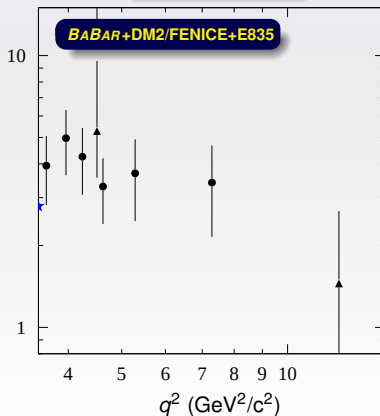
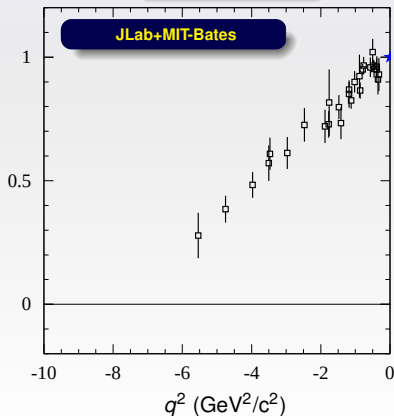
$$G(q^2) = \lim_{\mathcal{R} \rightarrow \infty} \frac{1}{2\pi i} \oint_C \frac{G(z) dz}{z - q^2} = \frac{1}{\pi} \int_{s_{th}}^{\infty} \frac{\text{Im} G(s) ds}{s - q^2}$$



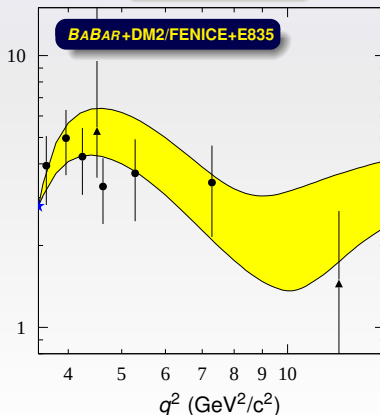
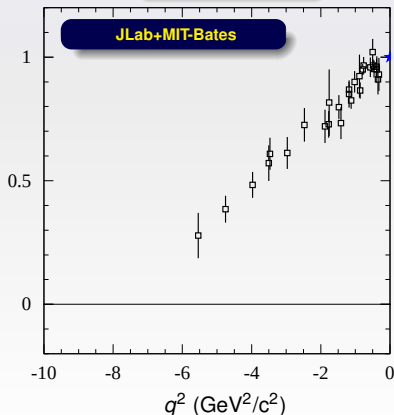
$R(q^2)$ in the complex plane



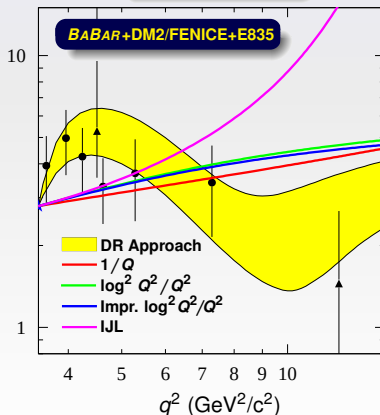
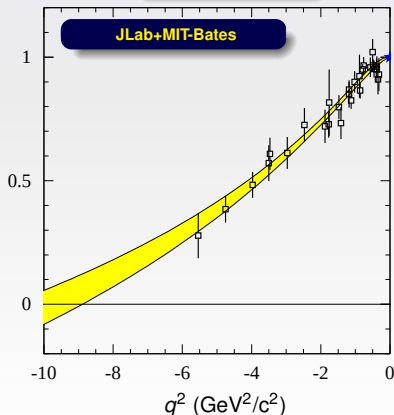
$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}R(s)}{s(s - q^2)} ds$$

 $\text{Re}q^2$ $R(q^2)$ space-like $|R(q^2)|$ time-like

$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}R(s)}{s(s - q^2)} ds$$

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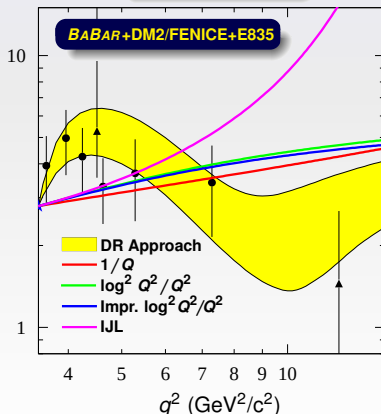
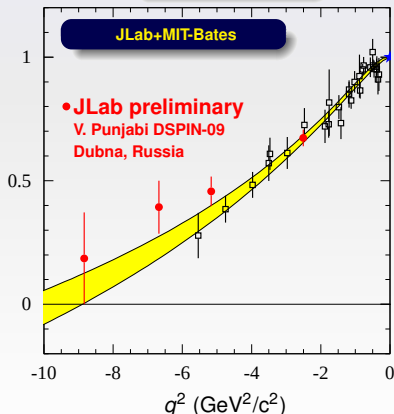
 $\text{Re}q^2$ $R(q^2)$ space-like $|R(q^2)|$ time-like

$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}R(s)}{s(s - q^2)} ds$$

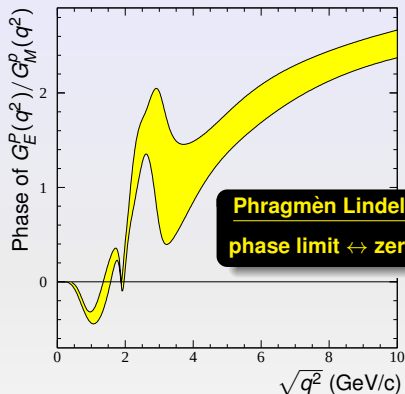
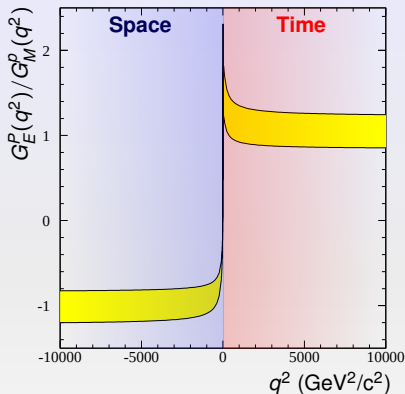
$R(q^2)$ space-like

$|R(q^2)|$ time-like

$\text{Re}q^2$



Asymptotic $G_E^P(q^2)/G_M^P(q^2)$ and phase



Phragmén Lindelöf
phase limit $\leftrightarrow$ zeros

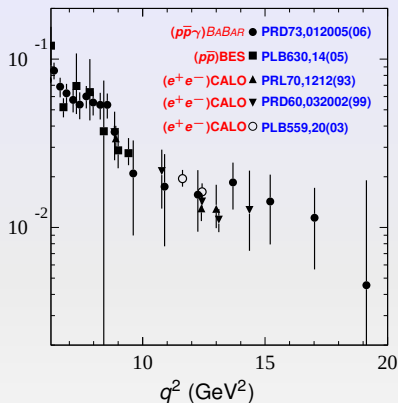
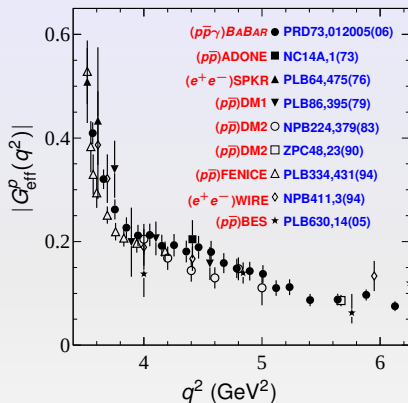
pQCD prediction

$$\frac{G_E^P(q^2)}{G_M^P(q^2)} \xrightarrow{|q^2| \rightarrow \infty} -1$$

Phase from DR

$$\phi(q^2) = -\frac{\sqrt{q^2 - s_0}}{\pi} \text{Pr} \int_{s_0}^{\infty} \frac{\ln |R(s)| ds}{\sqrt{s - s_0}(s - q^2)}$$

Time-like magnetic proton form factor



Data obtained assuming $|G_M^p| = |G_E^p| \equiv |G_{\text{eff}}^p|$ (true only at threshold)

$$|G_{\text{eff}}^p|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{16\pi\alpha^2 C_e}{3} \frac{\sqrt{1-1/\tau}}{4q^2} \left(1 + \frac{1}{2\tau}\right)}$$

Dispersion relation subtracted at $t = 0$

$$\ln G(t) = \frac{t\sqrt{s_{\text{th}} - t}}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\ln |G(s)| ds}{s\sqrt{s - s_{\text{th}}(s - t)}}$$

- Less dependent on the asymptotic behavior of the FF
- $\ln G(0) = 0 \Rightarrow$ no further terms have to be considered

Splitting the integral $\int_{s_{\text{th}}}^{\infty}$ into $\int_{s_{\text{th}}}^{s'_{\text{phy}}} + \int_{s'_{\text{phy}}}^{\infty}$ we obtain the integral equation

$$\overbrace{\ln G(t) - I_{\text{phy}}^{\infty}(t)}^{\text{Data and Theory}} = \frac{t\sqrt{s_{\text{th}} - t}}{\pi} \int_{s_{\text{th}}}^{s'_{\text{phy}}} \overbrace{\frac{\ln |G(s)| ds}{s\sqrt{s - s_{\text{th}}(s - t)}}}^{\text{Unknown}}$$

- To avoid instabilities around $s_{\text{phy}} = 4M_N^2$, the upper boundary has been shifted to $s'_{\text{phy}} = s_{\text{phy}} + \Delta$, with $\Delta \simeq 0.5 \text{ GeV}^2$
- We impose continuity of the FF at s'_{phy} and s_{th} , in addition, at the upper boundary s'_{phy} , continuity of the first derivative is also required
- A regularization, depending on a **free parameter** τ , is introduced by requiring the FF total curvature in the unphysical region to be limited

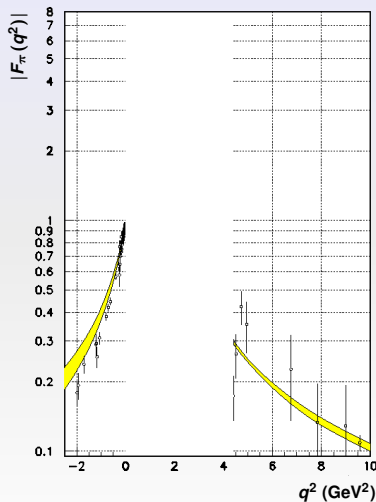
Solving procedure

Minimize: $\chi^2 = \chi_{\text{data}}^2 + \chi_{\text{theory}}^2 + \tau^6 \cdot \chi_{\text{regu}}^2$

$$\chi_{\text{regu}}^2 = \int_{s_{\text{th}}}^{s'_{\text{phy}}} \left[\frac{d^2 \ln |G(s)|}{ds^2} \right]^2 ds \propto \left[\text{total curvature in } [s_{\text{th}}, s'_{\text{phy}}] \right]$$

Pion FF to fix the **regularization parameter τ**

Space-like (DR) and time-like data (yellow bands) have been used as input in the integral equation to retrieve the time-like FF in the nucleon unphysical region.



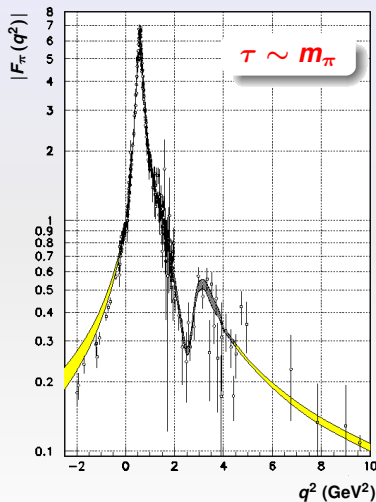
Solving procedure

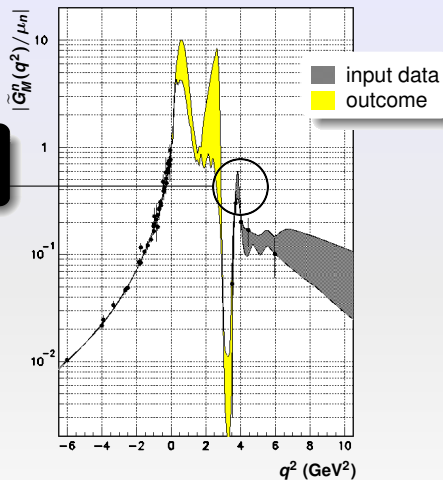
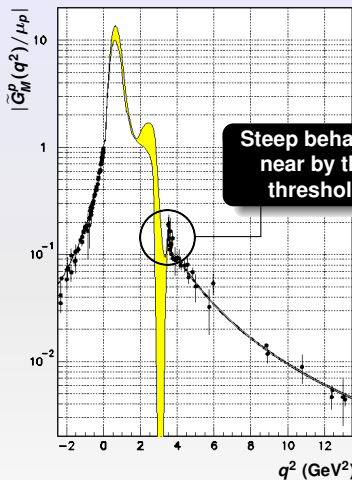
Minimize: $\chi^2 = \chi_{\text{data}}^2 + \chi_{\text{theory}}^2 + \tau^6 \cdot \chi_{\text{regu}}^2$

$$\chi_{\text{regu}}^2 = \int_{s_{\text{th}}}^{s'_{\text{phy}}} \left[\frac{d^2 \ln |G(s)|}{ds^2} \right]^2 ds \propto \left[\text{total curvature in } [s_{\text{th}}, s'_{\text{phy}}] \right]$$

Pion FF to fix the **regularization parameter τ**

Space-like (DR) and time-like data (yellow bands) have been used as input in the integral equation to retrieve the time-like FF in the nucleon unphysical region (gray band).





$$M_1 \sim 770 \text{ MeV} \quad \Gamma_1 \sim 350 \text{ MeV}$$

$$M_2 \sim 1600 \text{ MeV} \quad \Gamma_2 \sim 350 \text{ MeV}$$

Conclusions

- Pointlike Behavior at and well above threshold
- No Sommerfeld Resummation Factor
- Neutral baryon non zero cross section at threshold?
- G_E^p space-like $\rightarrow -1$ asymptotically?

Perspectives

- BESIII: ISR and scan
- Data from SND and CMD2 up to 2 GeV
- PANDA could explore FFs below threshold through $p\bar{p} \rightarrow \pi^0 l^+ l^-$
- SuperTauCharm ?