Time-like Baryon Form Factors

Rinaldo Baldini Ferroli, S. Pacetti and A. Zallo

Laboratori Nazionali di Frascati

CENTRO STUDI E RICERCHE Enrico Fermi

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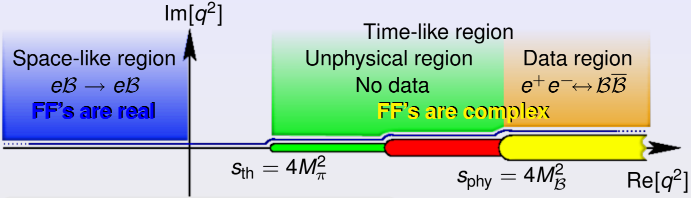
Outline

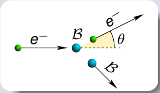
- **Last News on Baryon FF near threshold**
- **The Neutral Baryon Puzzle**
- **Spacelike - Timelike Relationship**
- **Interference Pattern in $J/\psi \to p\bar{p}$**
- **Conclusions and Perspectives**

Cross sections and analyticity



Time-like: had. helicity $= \begin{cases} 1 \Rightarrow |G_E| \\ 0 \Rightarrow |G_M| \end{cases}$

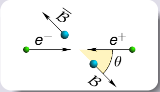
$$G_E(4M_\mathcal{B}^2) = G_M(4M_\mathcal{B}^2)$$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_e' \cos^2\frac{\theta}{2}}{4E_e^3 \sin^4\frac{\theta}{2}} \left[G_E^2 - \tau\left(1 + 2(1-\tau)\tan^2\frac{\theta}{2}\right)G_M^2\right]\frac{1}{1-\tau}$$

$$\tau = \frac{q^2}{4M_\mathcal{B}^2}$$



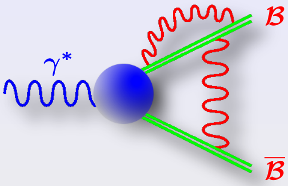
Annihilation

Coulomb correction

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta|G_E|^2\right]$$

$$\beta = \sqrt{1 - \frac{1}{\tau}}$$

The Coulomb Factor



$p\bar{p}$ Coulomb interaction as FSI

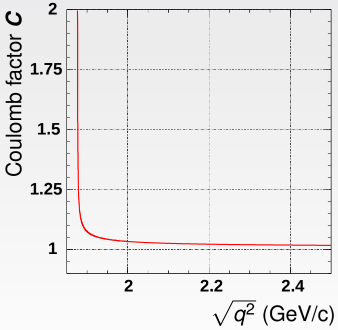
[Sommerfeld, Sakharov, Schwinger, Fadin, Khoze]

Distorted wave approximation

$$C = |\Psi_{\text{Coul}}(0)|^2$$

- **S-wave:** $C = \dfrac{\frac{\pi\alpha}{\beta}}{1 - \exp\left(-\frac{\pi\alpha}{\beta}\right)} \xrightarrow[\beta\to 0]{} \dfrac{\pi\alpha}{\beta}$
- **D-wave:** $C = 1$

No Coulomb factor for boson pairs (P-wave)



Sommerfeld Enhancement and Resummation Factors

Coulomb Factor $\mathcal{C}$ for S-wave only:

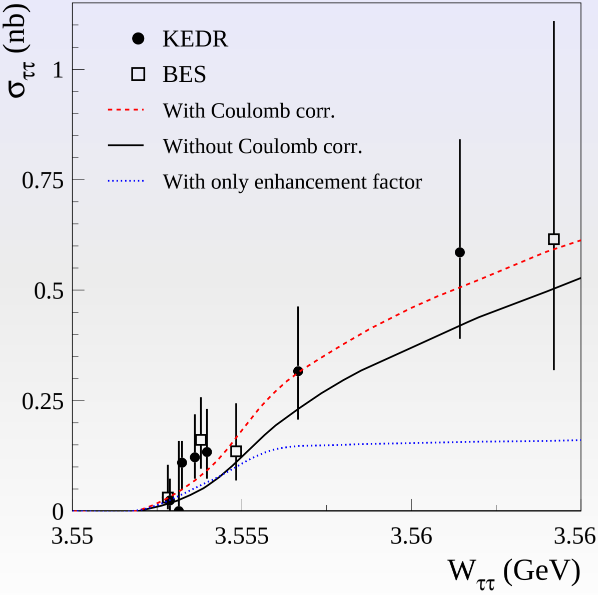
- **Partial wave FF:** $G_S = \dfrac{2G_M\sqrt{q^2/4M^2} + G_E}{3}$ $G_D = \dfrac{G_M\sqrt{q^2/4M^2} - G_E}{3}$
- **Cross section:** $\sigma(q^2) = 2\pi\alpha^2\beta\,\dfrac{4M^2}{(q^2)^2}\left[\mathcal{C}\,|G_S(q^2)|^2 + 2|G_D(q^2)|^2\right]$

$$\mathcal{C} = \mathcal{E} \times \mathcal{R}$$

- **Enhancement factor:** $\mathcal{E} = \pi\alpha/\beta$
- **Step at threshold:** $\sigma(4M^2) = \dfrac{\pi^2\alpha^3}{2M^2}\dfrac{\cancel{\beta}}{\cancel{\beta}}|G_S(4M^2)|^2 = 0.85\,|G_S(4M^2)|^2$ nb

- **Resummation factor:** $\mathcal{R} = 1/[1 - \exp(-\pi\alpha/\beta)]$
- **Few MeV above threshold:** $\mathcal{C} \simeq 1 \;\Rightarrow\; \sigma(q^2) \propto \beta\,|G_S(q^2)|^2$

The $e^+e^- \to \tau^+\tau^-$ case



Pointlike Baryons?

R. Baldini Ferroli, S. Pacetti,
A. Zallo and A. Zichichi

I.S.R. versus c.m.

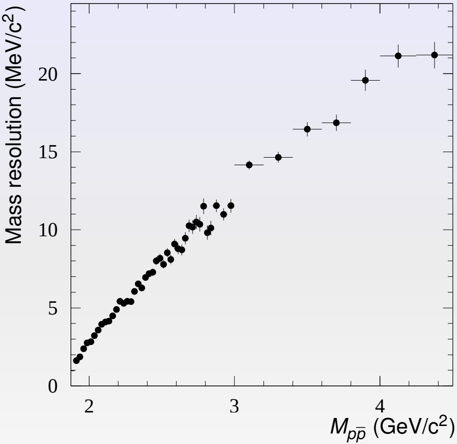
Advantages

- All q at the same time $\Longrightarrow$ Better control on systematics
- c.m. boost $\Longrightarrow$ at threshold efficiency $\neq 0 + \sigma_W \sim 1$ *MeV*
- Detected ISR $\gamma \Longrightarrow$ full $p\bar{p}$ angular coverage

Drawbacks

- $\mathcal{L} \propto$ invariant mass bin Δw
- More background

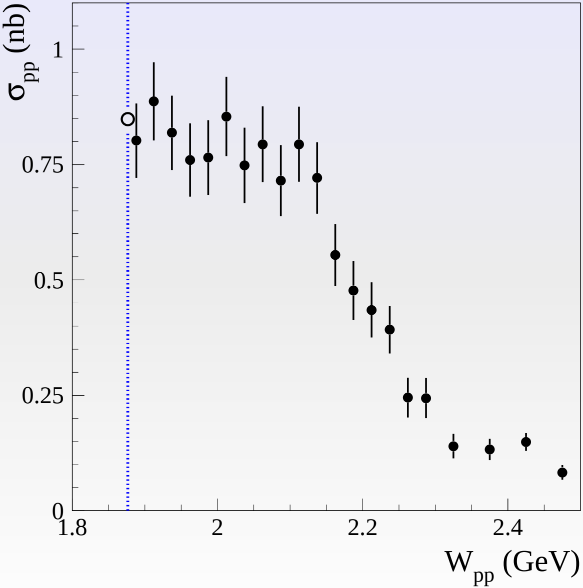
Mass resolution



Incredibly good at threshold ($\sim$ 1 MeV/c^2), as e^+e^- c.m.

$\Delta p_T / p_T \sim 0.5\%$ at 1 GeV

BABAR: $e^+e^- \to p\bar{p}$



Proton form factor at $q^2 = 4M_p^2$

$$\sigma(e^+e^- \to p\bar{p})(4M_p^2) = 0.83 \pm 0.05 \text{ nb}$$

BABAR

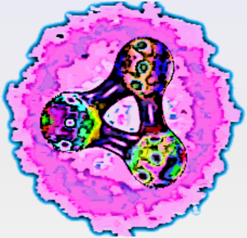
$$\sigma(e^+e^- \to p\bar{p})(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\frac{\beta}{\beta}|G^p(4M_p^2)|^2 = 0.85\,|G^p(4M_p^2)|^2 \text{ nb}$$

$$|G^p(4M_p^2)| \equiv 1$$

$$|G^p(4M_p^2)| = 0.99 \pm 0.04(\text{stat}) \pm 0.03(\text{syst})$$

Proton form factor at $q^2 = 4M_p^2$

$$|G^p(4M_p^2)| \equiv 1$$



At $q^2 = 4M_P^2$ protons behave as pointlike fermions!

Sommerfeld Resummation Factor Needed?

Resummation Factor Needed?

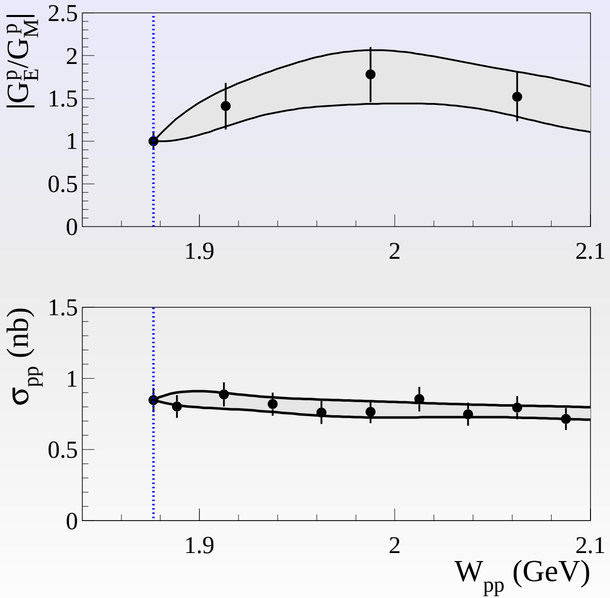
- **At threshold:** $G_E/G_M = 1 \Rightarrow \begin{cases} G_S \in \mathbb{R} \\ G_D = 0 \in \mathbb{R} \end{cases}$
- $\sigma(q^2)$, $|G_E/G_M| \longrightarrow G_S, G_D$
- $G_S = 1/\sqrt{1 - \exp(-\pi\alpha/\beta)}$
- **No need of Resummation Factor**

For a wide energy range ($\sim$ 200 MeV):

- **Proton behaves as a pointlike particle**
- **e.m. dominance, no strong interaction?**
- **Mild sensitivity to $\mathcal{B}\overline{\mathcal{B}}$ invariant mass resolution**

BABAR: $|G_E^p/G_M^p|$ and $\sigma(e^+e^- \longrightarrow p\bar{p})$

[PRD73, 012005]



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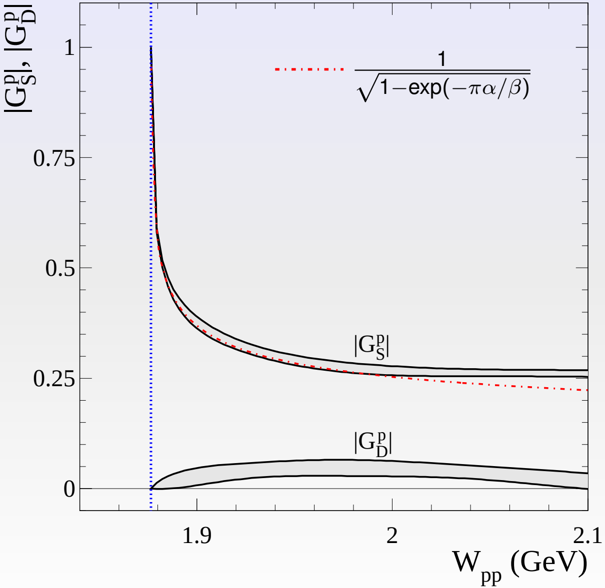
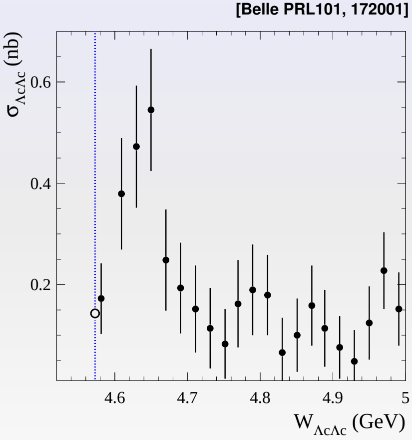
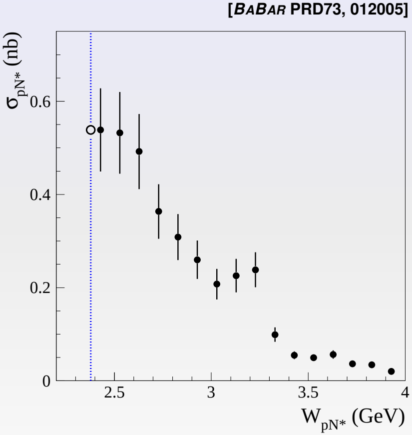
[PRD73, 012005]



Other charged baryon FF's at threshold

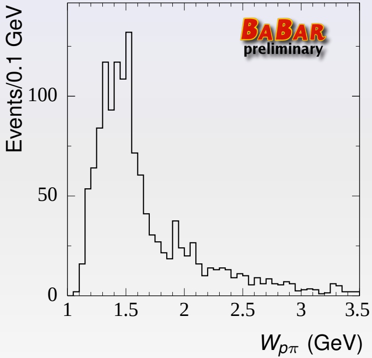
$e^+e^- \longrightarrow \Lambda_c^+\overline{\Lambda}_c^-$ and $e^+e^- \longrightarrow p\overline{N}(1440)$+c.c.

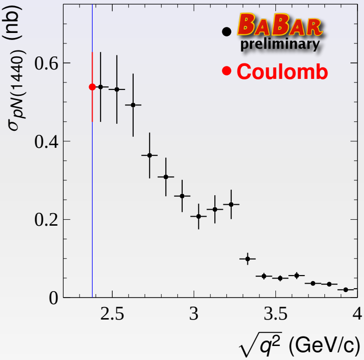




$e^+e^- \to p\overline{N}(1440)$+c.c.

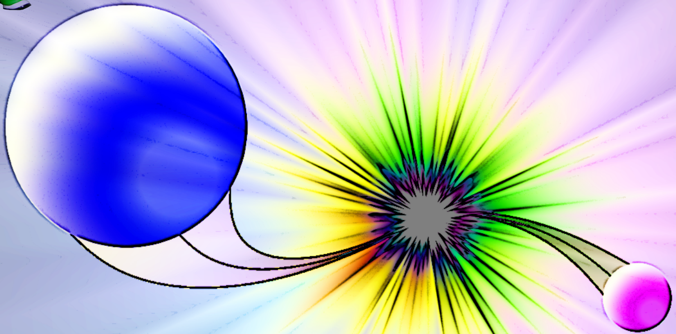
$$\sigma^{\text{Coulomb}} = \frac{16\pi^2\alpha^3 M_p^{3/2} M_{N(1440)}^{3/2}}{(M_p + M_{N(1440)})^5}\left|G^{pN(1440)}\right|^2 = \left|G^{pN(1440)}\right|^2 \times 0.49\ \text{nb}$$





$$\left|G^{pN(1440)}\right| = 1.04 \pm 0.09$$

The neutral baryons puzzle

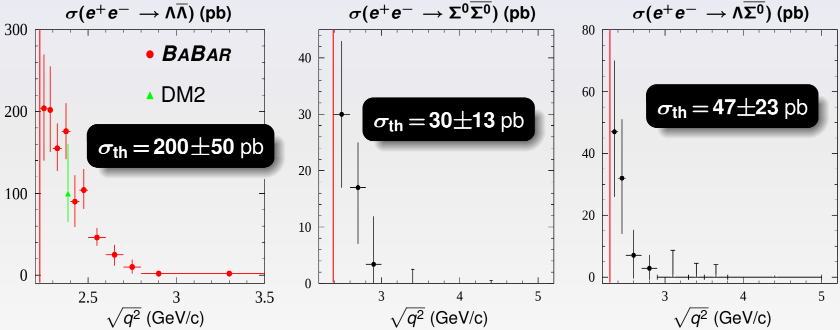


Neutral Baryons puzzle (*BABAR*)

[PRD76, 092006]

$$\sigma(e^+e^- \to \mathcal{B}^0\overline{\mathcal{B}}^0) = \frac{4\pi\alpha^2\beta C_0}{3q^2}\left[|G_M^{\mathcal{B}^0}|^2 + \frac{2M_{\mathcal{B}^0}^2}{q^2}|G_E^{\mathcal{B}^0}|^2\right] \xrightarrow[\sqrt{q^2}\to 2M_{\mathcal{B}^0}]{} \frac{\pi\alpha^2\beta}{2M_{\mathcal{B}^0}^2}|G^{\mathcal{B}^0}|^2 \to 0$$

No Coulomb correction at hadron level: $C_0 = 1$



Like a remnant of Coulomb interactions at quark level?

$\Longrightarrow$

$C_0 \propto \beta^{-1}$ as $\sqrt{q^2} \to 2M_{\mathcal{B}^0}$

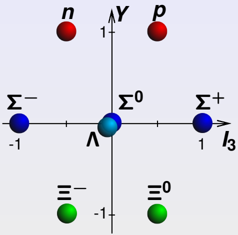
$\Longrightarrow$

For any neutral baryon

$$\sqrt{\sigma_{\mathcal{B}^0\overline{\mathcal{B}}^0}} \propto \frac{|G^{\mathcal{B}^0}|}{M_{\mathcal{B}^0}}$$

Baryon octet and U-spin

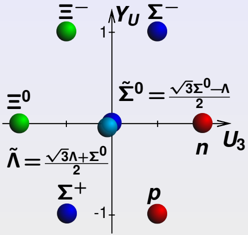
arXiv:0812.3283



$$(Y, I_3) \longrightarrow (Y_U, U_3)$$

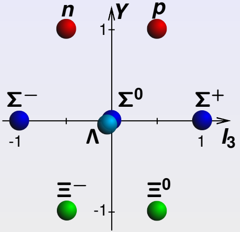
$$U_3 = -\tfrac{1}{2} I_3 + \tfrac{3}{4} Y$$

$$Y_U = -Q$$



U-spin relation: $G^{\Sigma^0} - G^{\Lambda} + \frac{2}{\sqrt{3}} G^{\Lambda\Sigma^0} = 0$

$$M_{\Sigma^0}\sqrt{\sigma_{\Sigma^0\overline{\Sigma^0}}} - M_{\Lambda}\sqrt{\sigma_{\Lambda\overline{\Lambda}}} + \frac{2}{\sqrt{3}}\overline{M_{\Lambda\Sigma^0}}\sqrt{\sigma_{\Lambda\overline{\Sigma^0}}} = (-0.06 \pm 6.0) \times 10^{-4}$$



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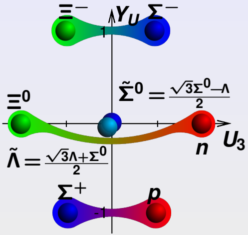

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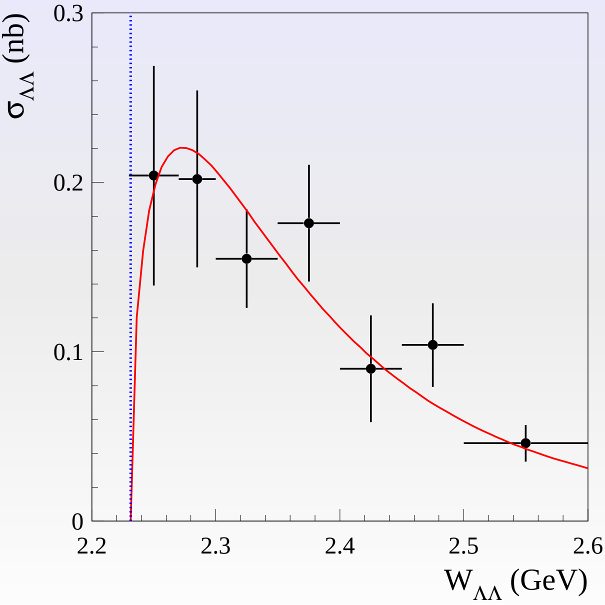
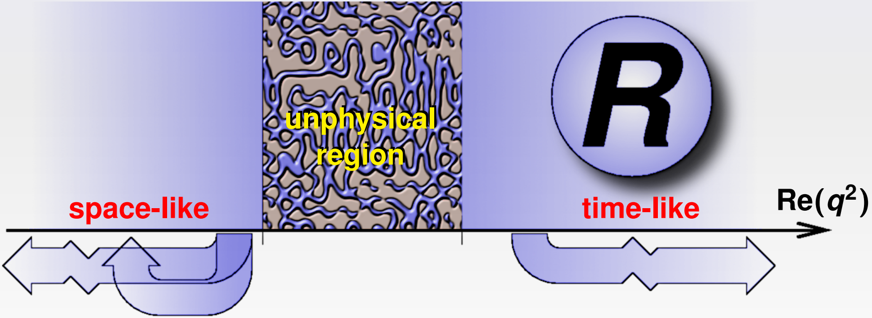
BABAR: $e^+e^- \longrightarrow \Lambda\bar{\Lambda}$

[PRD76, 092006]

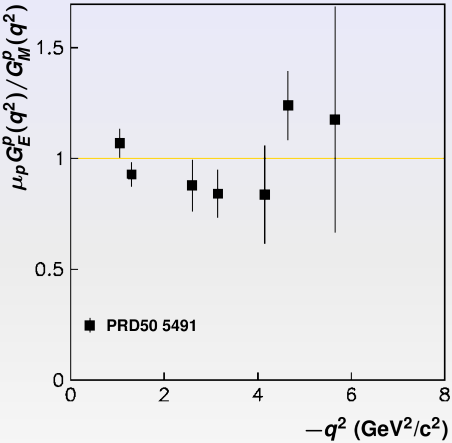


Dispersive analysis of the ratio $R = \mu_p \frac{G_E^p}{G_M^p}$

Eur. Phys. J. A32, 421
R. Baldini, S. Pacetti and A. Zallo



Space-like G_E^p / G_M^p measurements



$$G_E^p = F_1^p + \frac{q^2}{4M_p^2} F_2^p$$

$$G_M^p = F_1^p + F_2^p$$

Space-like

F_1 / $\frac{q^2}{4M_p^2} F_2$ cancellation

$$\frac{G_E^p(q^2)}{G_M^p(q^2)} < 1$$

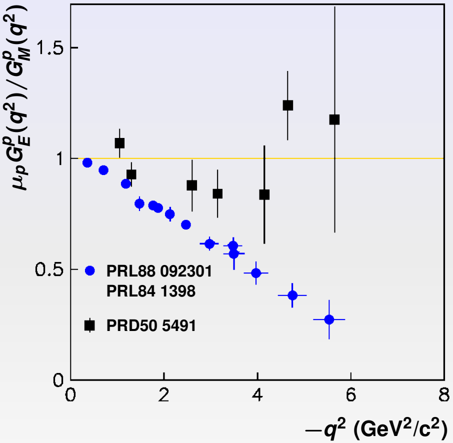
Time-like

F_1 / $\frac{q^2}{4M_p^2} F_2$ enhancement

$$\left|\frac{G_E^p(q^2)}{G_M^p(q^2)}\right| > 1$$

Radiative corrections of polarization technique $\ll$ Radiative corrections in Rosenbluth method

Space-like G_E^p/G_M^p measurements



$$G_E^p = F_1^p + \frac{q^2}{4M_p^2} F_2^p$$

$$G_M^p = F_1^p + F_2^p$$

Space-like

F_1 / $\frac{q^2}{4M_p^2} F_2$ **cancellation**

$$\frac{G_E^p(q^2)}{G_M^p(q^2)} < 1$$

Time-like

F_1 / $\frac{q^2}{4M_p^2} F_2$ **enhancement**

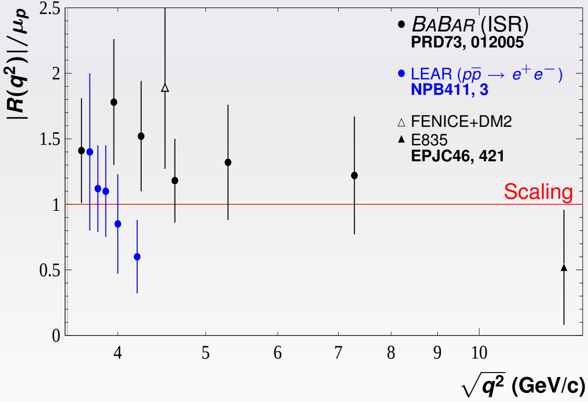
$$\left|\frac{G_E^p(q^2)}{G_M^p(q^2)}\right| > 1$$

Radiative corrections of polarization technique $\ll$ **Radiative corrections in Rosenbluth method**

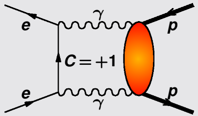
Time-like $|G_E^p/G_M^p|$ measurements

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta C}{2q^2}|G_M^p|^2\left[(1+\cos^2\theta)+\frac{4M_p^2}{q^2\mu_p^2}\sin^2\theta|R|^2\right]$$

$$R(q^2) = \mu_p \frac{G_E^p(q^2)}{G_M^p(q^2)}$$



$\gamma\gamma$ exchange



$\gamma\gamma$ exchange interferes with the Born term

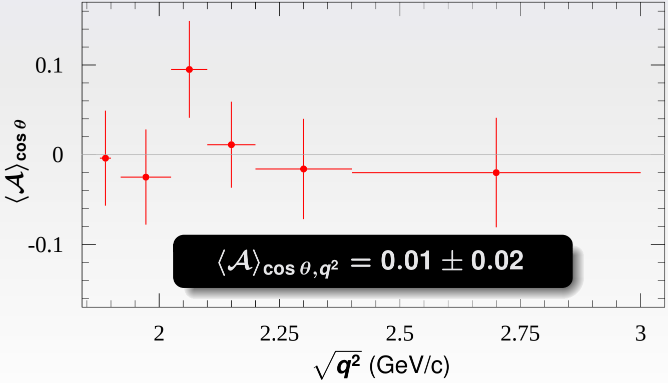
Asymmetry in angular distributions

[PLB659, 197]

$\gamma\gamma$ exchange from $e^+e^- \to p\bar{p}\gamma$ BaBar data

E. Tomasi-Gustafsson,
E. A. Kuraev, S. Bakmaev, SP
PLB659, 197

$$\mathcal{A}(\cos\theta, q^2) = \frac{\frac{d\sigma}{d\Omega}(\cos\theta, q^2) - \frac{d\sigma}{d\Omega}(-\cos\theta, q^2)}{\frac{d\sigma}{d\Omega}(\cos\theta, q^2) + \frac{d\sigma}{d\Omega}(-\cos\theta, q^2)}$$

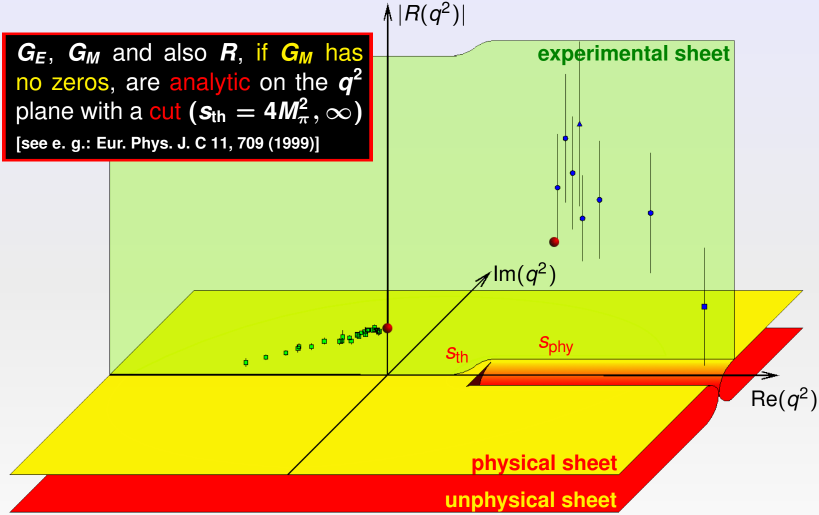


$$\langle\mathcal{A}\rangle_{\cos\theta, q^2} = 0.01 \pm 0.02$$

$R(q^2)$ in the complex plane

G_E, G_M and also R, if G_M has no zeros, are analytic on the q^2 plane with a cut $(s_{th} = 4M_\pi^2, \infty)$

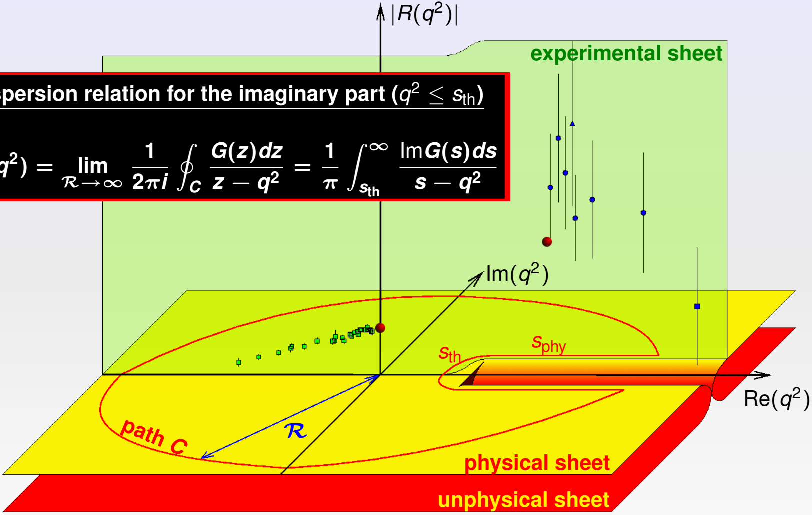
[see e. g.: Eur. Phys. J. C 11, 709 (1999)]



$R(q^2)$ in the complex plane

Dispersion relation for the imaginary part ($q^2 \leq s_{th}$)

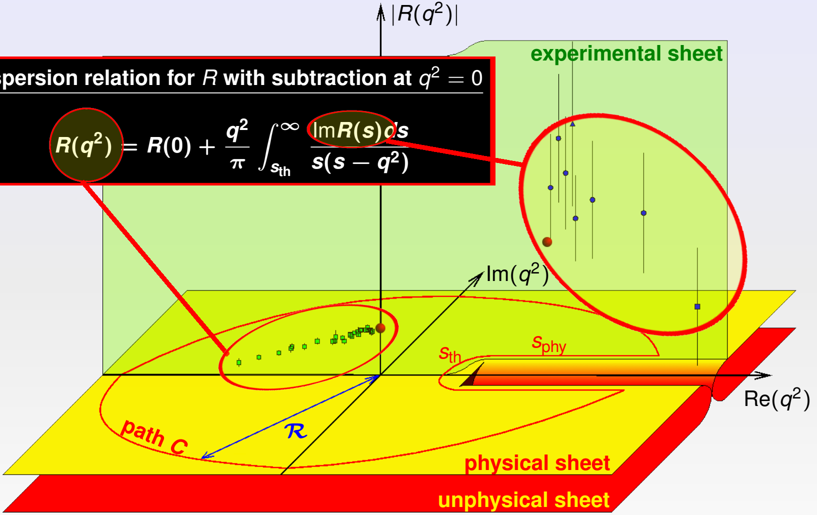
$$G(q^2) = \lim_{\mathcal{R}\to\infty} \frac{1}{2\pi i}\oint_C \frac{G(z)dz}{z-q^2} = \frac{1}{\pi}\int_{s_{th}}^{\infty} \frac{\mathrm{Im}G(s)ds}{s-q^2}$$



$R(q^2)$ in the complex plane

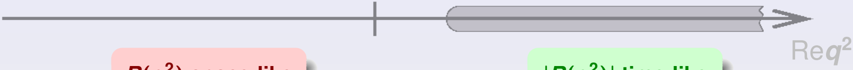
Dispersion relation for R with subtraction at $q^2 = 0$

$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{s_{\rm th}}^{\infty}\frac{{\rm Im}R(s)\,ds}{s(s-q^2)}$$



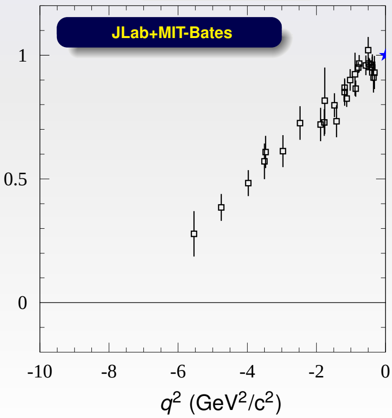
$R(q^2)$

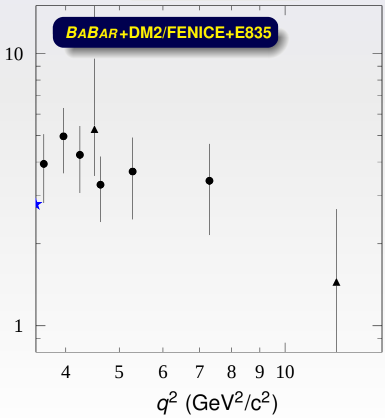
$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{Im}R(s)}{s(s-q^2)}ds$$



$R(q^2)$ space-like

$|R(q^2)|$ time-like





$R(q^2)$

EPJA32 421

$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}\, R(s)}{s(s-q^2)} ds$$

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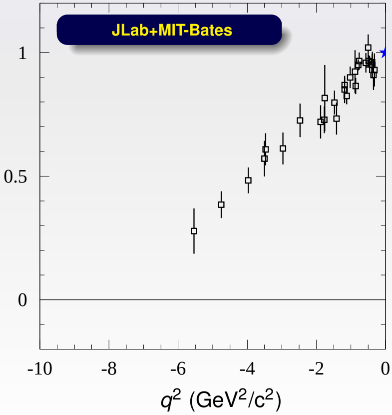
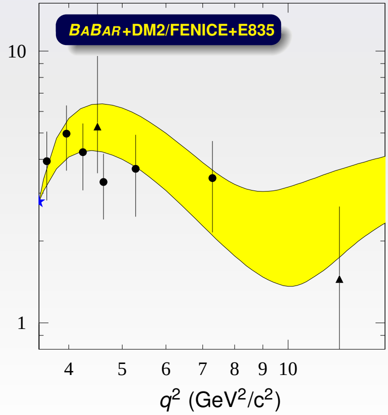
$|R(q^2)|$ time-like





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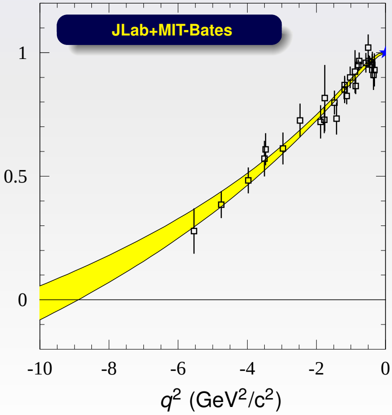
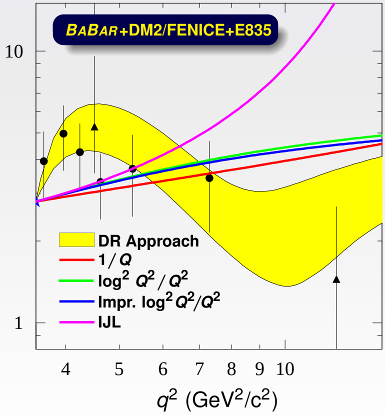
$R(q^2)$ space-like



$|R(q^2)|$ time-like



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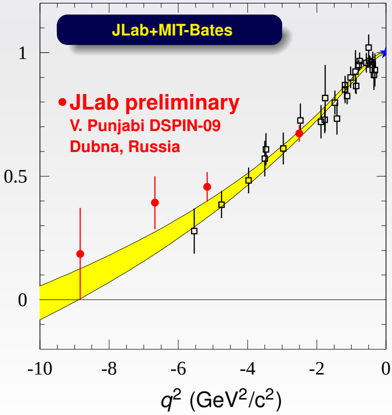
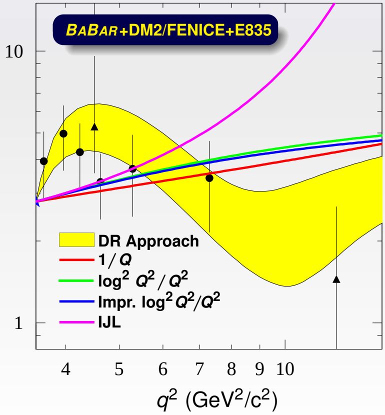
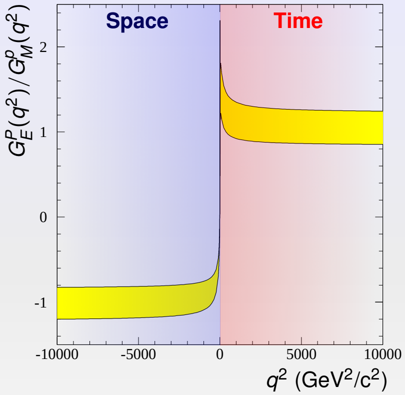
$R(q^2)$ space-like

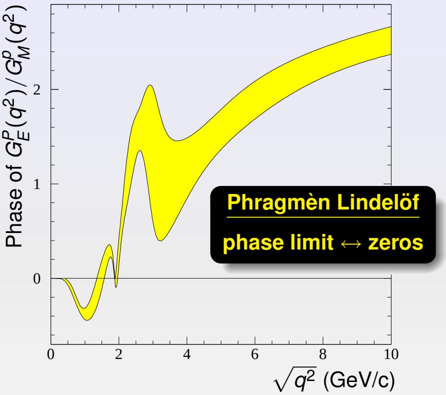
$|R(q^2)|$ time-like





Asymptotic $G_E^P(q^2)/G_M^p(q^2)$ and phase





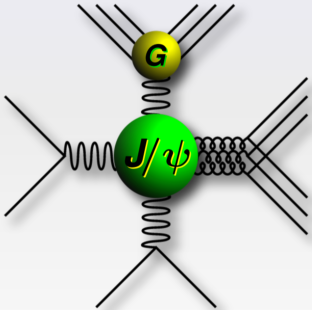
pQCD prediction

$$\frac{G_E^p(q^2)}{G_M^p(q^2)} \underset{|q^2|\to\infty}{\longrightarrow} -1$$

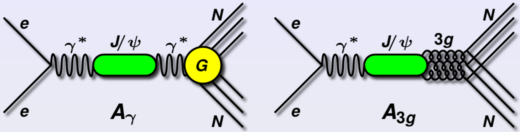
Phase from DR

$$\phi(q^2) = -\frac{\sqrt{q^2 - s_0}}{\pi} \, \mathrm{Pr} \int_{s_0}^{\infty} \frac{\ln |R(s)| ds}{\sqrt{s - s_0}(s - q^2)}$$

J/ψ strong and electromagnetic phase



J/ψ decays: strong and electromagnetic



cross section $\sim |A_\gamma + A_{3g}|^2 = |A_\gamma|^2 + |A_{3g}|^2 + \underbrace{2\,\mathrm{Re}[A_\gamma^* A_{3g}]}_{\text{interference term}}$

According to pQCD: A_γ and A_{3g} are real $\Rightarrow$ interference

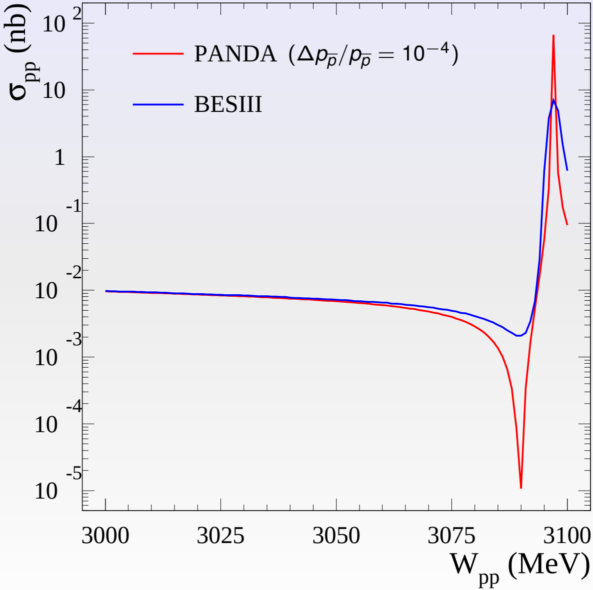
On the contrary data suggest:

$J/\psi \rightarrow J_1^P J_2^P$	$\frac{A_\gamma}{A_{3g}}$ phase
$1^- 0^-$	$106^o \pm 10^o$
$1^- 1^-$	$138^o \pm 37^o$
$0^- 0^-$	$90^o \pm 10^o$
$n\bar{n}$	$89^o \pm 15^o$

But these conclusions have been obtained modeling SU(3) breaking, or using poorly measured $n\bar{n}$ cross section outside J/ψ

Interference with the continuum measures the relative phase in an independent way

Full interference as seen by PANDA or BESIII

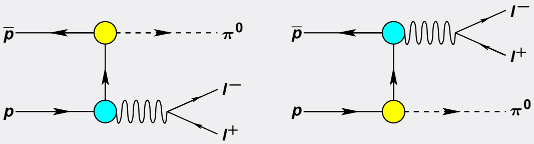


Conclusions

- Pointlike Behavior at and well above threshold
- No Sommerfeld Resummation Factor

Perspectives

- More data from *BABAR* ($\times 2$) and Belle (?)
- BESIII: ISR now, scan 2012-2013
- PANDA unique opportunity: FF below threshold exploting $p\bar{p} \to \pi^0 l^+ l^-$



Expected narrow forward/backward peaks