Baryon Form Factors at threshold

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CENTRO STUDI E RICERCHE Enrico Fermi

The Low-Energy Frontier of the Standard Model

From Quarks and Gluons to Hadrons and Nuclei

Kick-Off Meeting

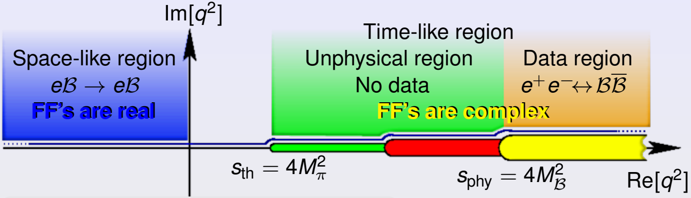
Mainz, September 3rd-5th, 2012

JG|U

Outline

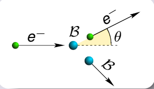
- **Last News on Baryon FF near threshold**
- **The Neutral Baryon Puzzle**
- **Spacelike - Timelike Relationship**
- **Interference Pattern in $J/\psi \to p\bar{p}$**
- **Conclusions and Perspectives**

Cross sections and analyticity



Time-like: had. helicity $= \begin{cases} 1 \Rightarrow |G_E| \\ 0 \Rightarrow |G_M| \end{cases}$

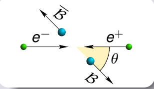
$$G_E(4M_\mathcal{B}^2) = G_M(4M_\mathcal{B}^2)$$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_e' \cos^2\frac{\theta}{2}}{4E_e^3 \sin^4\frac{\theta}{2}} \left[G_E^2 - \tau\left(1 + 2(1-\tau)\tan^2\frac{\theta}{2}\right)G_M^2\right]\frac{1}{1-\tau}$$

$$\tau = \frac{q^2}{4M_\mathcal{B}^2}$$



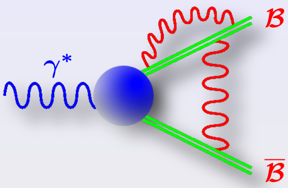
Annihilation

Coulomb correction

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta|G_E|^2\right]$$

$$\beta = \sqrt{1 - \frac{1}{\tau}}$$

The Coulomb Factor



$p\bar{p}$ Coulomb interaction as FSI

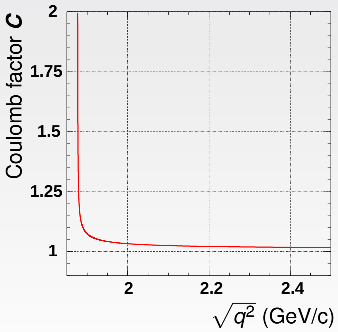
[Sommerfeld, Sakharov, Schwinger, Fadin, Khoze]

Distorted wave approximation

$$C = |\Psi_{\text{Coul}}(0)|^2$$

- **S-wave:** $C = \dfrac{\frac{\pi\alpha}{\beta}}{1 - \exp\left(-\frac{\pi\alpha}{\beta}\right)} \xrightarrow[\beta\to 0]{} \dfrac{\pi\alpha}{\beta}$
- **D-wave:** $C = 1$

No Coulomb factor for boson pairs (P-wave)



Sommerfeld Enhancement and Resummation Factors

Coulomb Factor $\mathcal{C}$ for S-wave only:

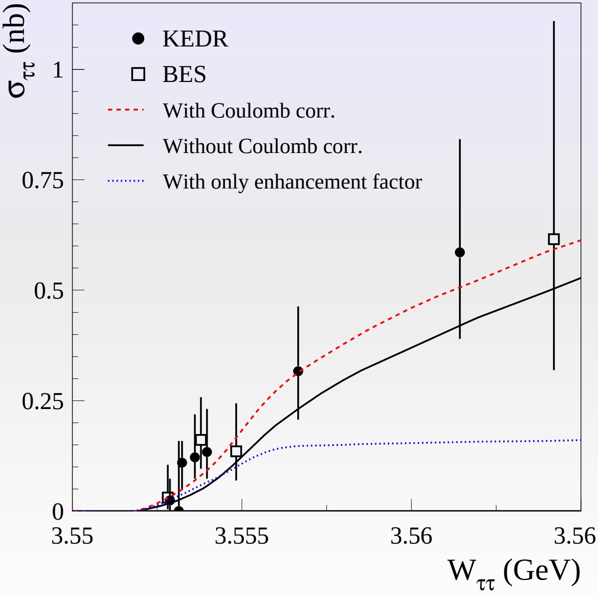
- **Partial wave FF:** $G_S = \dfrac{2G_M\sqrt{q^2/4M^2} + G_E}{3}$ $\quad G_D = \dfrac{G_M\sqrt{q^2/4M^2} - G_E}{3}$
- **Cross section:** $\sigma(q^2) = 2\pi\alpha^2\beta\,\dfrac{4M^2}{(q^2)^2}\left[\mathcal{C}\,|G_S(q^2)|^2 + 2|G_D(q^2)|^2\right]$

$$\mathcal{C} = \mathcal{E} \times \mathcal{R}$$

- **Enhancement factor:** $\mathcal{E} = \pi\alpha/\beta$
- **Step at threshold:** $\sigma(4M^2) = \dfrac{\pi^2\alpha^3}{2M^2}\dfrac{\not\beta}{\not\beta}|G_S(4M^2)|^2 = 0.85\,|G_S(4M^2)|^2$ nb

- **Resummation factor:** $\mathcal{R} = 1/[1 - \exp(-\pi\alpha/\beta)]$
- **Few MeV above threshold:** $\mathcal{C} \simeq 1 \;\Rightarrow\; \sigma(q^2) \propto \beta\,|G_S(q^2)|^2$

The $e^+e^- \to \tau^+\tau^-$ case



Pointlike Baryons?

R. Baldini Ferroli, S. Pacetti,
A. Zallo and A. Zichichi

I.S.R. versus c.m.

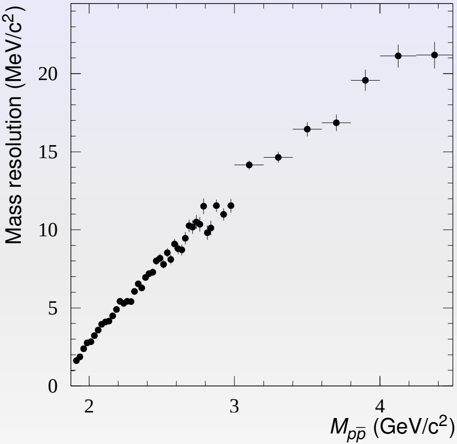
Advantages

- All q at the same time $\Longrightarrow$ Better control on systematics
- c.m. boost $\Longrightarrow$ at threshold efficiency $\neq 0 + \sigma_W \sim 1\ MeV$
- Detected ISR $\gamma \Longrightarrow$ full $p\bar{p}$ angular coverage

Drawbacks

- $\mathcal{L} \propto$ invariant mass bin Δw
- More background

Mass resolution

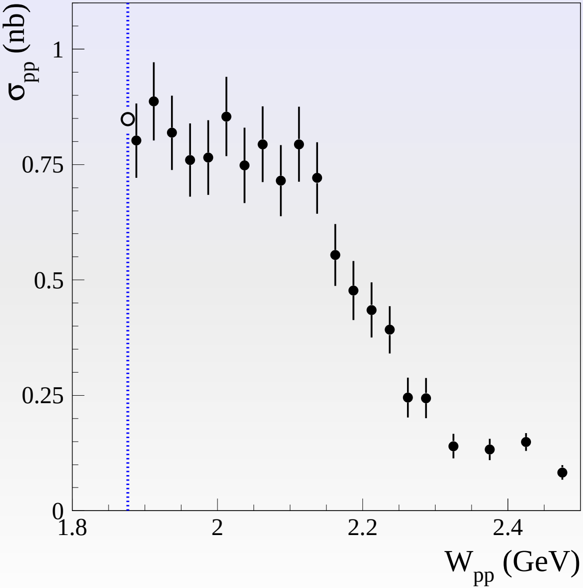


Incredibly good at threshold ($\sim$ 1 MeV/c^2), as e^+e^- c.m.

$\Delta p_T/p_T \sim 0.5\%$ at 1 GeV

BABAR: $e^+e^- \to p\bar{p}$

[PRD73, 012005]



Proton form factor at $q^2 = 4M_p^2$

$$\sigma(e^+e^- \to p\bar{p})(4M_p^2) = 0.83 \pm 0.05 \text{ nb}$$

BABAR

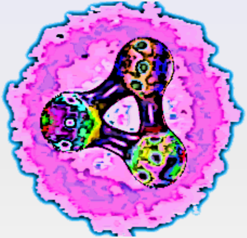
$$\sigma(e^+e^- \to p\bar{p})(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\frac{\not{\beta}}{\not{\beta}}|G^p(4M_p^2)|^2 = 0.85\,|G^p(4M_p^2)|^2 \text{ nb}$$

$$|G^p(4M_p^2)| \equiv 1$$

$$|G^p(4M_p^2)| = 0.99 \pm 0.04(\text{stat}) \pm 0.03(\text{syst})$$

Proton form factor at $q^2 = 4M_p^2$

$$|G^p(4M_p^2)| \equiv 1$$



At $q^2 = 4M_P^2$ protons behave as pointlike fermions!

Sommerfeld Resummation Factor Needed?

Resummation Factor Needed?

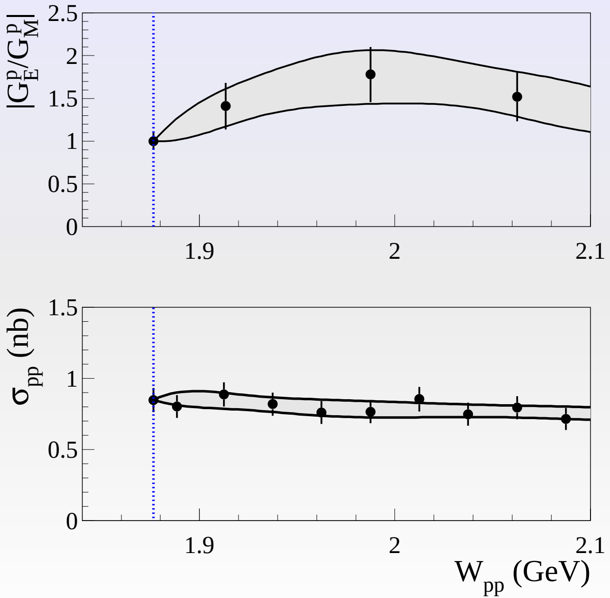
- At threshold: $G_E/G_M = 1 \Rightarrow \begin{cases} G_S \in \mathbb{R} \\ G_D = 0 \in \mathbb{R} \end{cases}$
- $\sigma(q^2)$, $|G_E/G_M| \to G_S, G_D$
- $G_S = \sqrt{1 - \exp(-\pi\alpha/\beta)}$
- No need of Resummation Factor

For a wide energy range ($\sim$ 200 MeV):

- Proton behaves as a pointlike particle
- e.m. dominance, no strong interaction?
- Mild sensitivity to $\mathcal{B}\overline{\mathcal{B}}$ invariant mass resolution

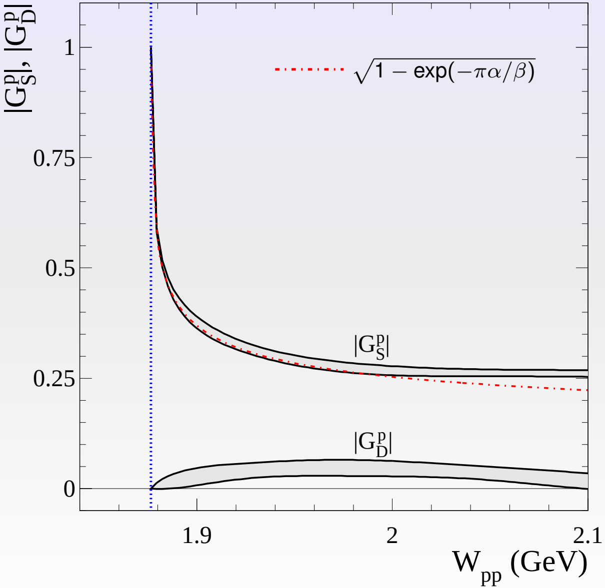
$B\!A\!B\!A\!R$: $|G_E^p/G_M^p|$ and $\sigma(e^+e^- \to p\bar{p})$

[PRD73, 012005]



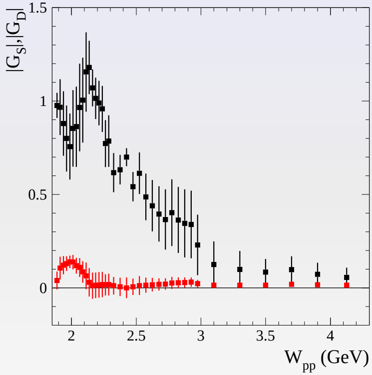
BABAR: $|G_E^p/G_M^p|$ and $\sigma(e^+e^- \to p\bar{p})$

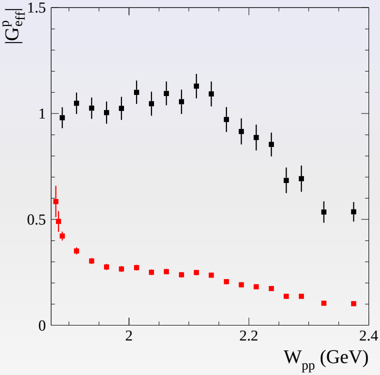
[PRD73, 012005]



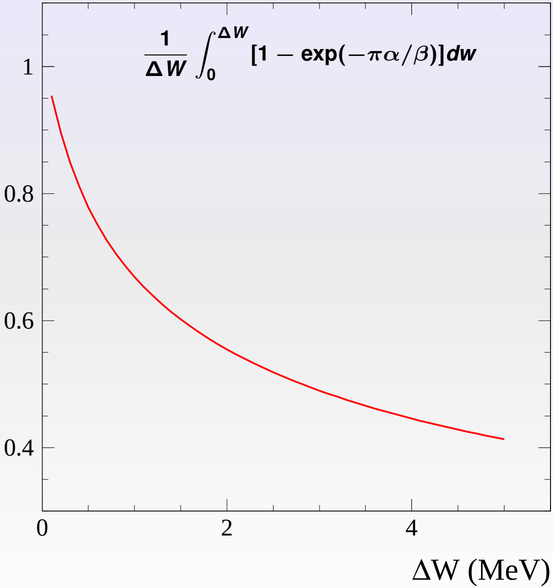
BABAR: G_S, G_D and G_{eff}

[PRD73, 012005]





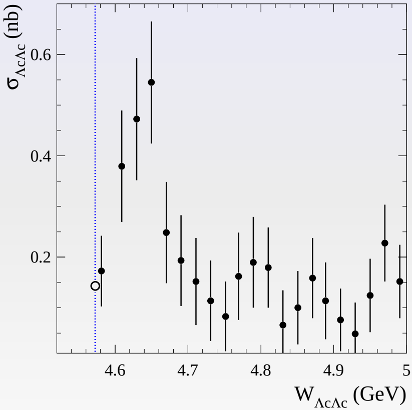
Integrated Sommerfeld factor



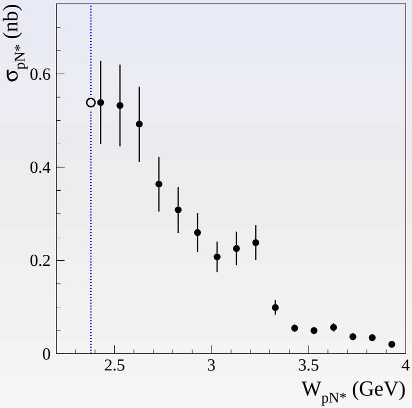
Other charged baryon FF's at threshold

$e^+e^- \to \Lambda_c^+\overline{\Lambda}_c^-$ and $e^+e^- \to p\overline{N}(1440)$+c.c.

[Belle PRL101, 172001]

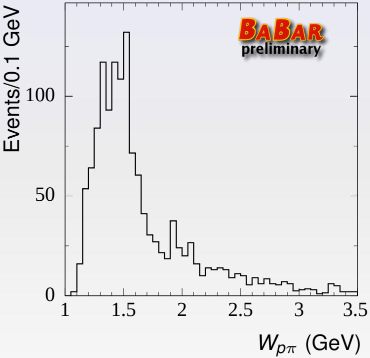


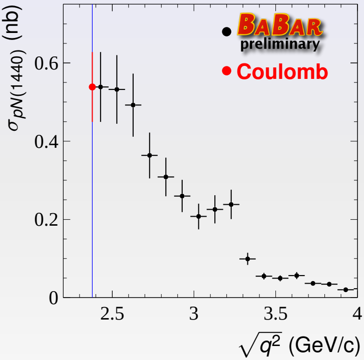
[BABAR PRD73, 012005]



$e^+e^- \to p\overline{N}(1440)$+c.c.

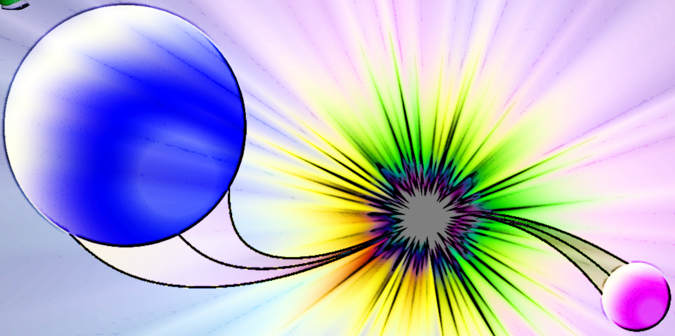
$$\sigma^{\text{Coulomb}} = \frac{16\pi^2\alpha^3 M_p^{3/2} M_{N(1440)}^{3/2}}{(M_p + M_{N(1440)})^5} \left|G^{pN(1440)}\right|^2 = \left|G^{pN(1440)}\right|^2 \times 0.49\text{ nb}$$





$$\left|G^{pN(1440)}\right| = 1.04 \pm 0.09$$

The neutral baryons puzzle

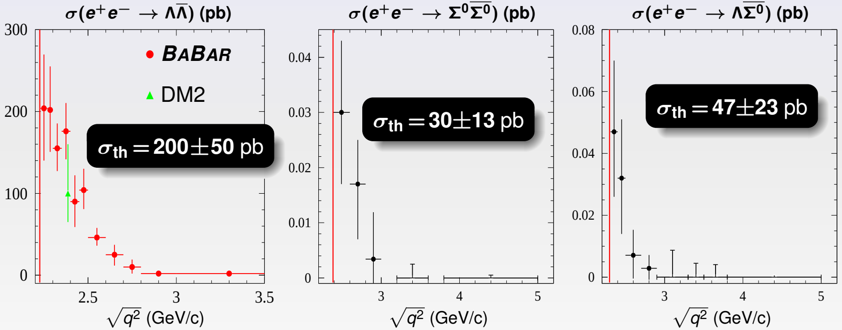


Neutral Baryons puzzle (BABAR)

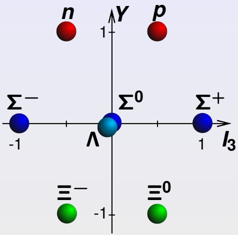
[PRD76, 092006]

$$\sigma(e^+e^- \to \mathcal{B}^0\overline{\mathcal{B}}^0) = \frac{4\pi\alpha^2\beta C_0}{3q^2}\left[|G_M^{\mathcal{B}^0}|^2 + \frac{2M_{\mathcal{B}^0}^2}{q^2}|G_E^{\mathcal{B}^0}|^2\right] \xrightarrow[\sqrt{q^2}\to 2M_{\mathcal{B}^0}]{} \frac{\pi\alpha^2\beta}{2M_{\mathcal{B}^0}^2}|G^{\mathcal{B}^0}|^2 \to 0$$

No Coulomb correction at hadron level: $C_0 = 1$



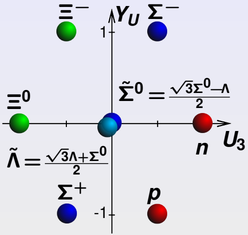
Like a remnant of Coulomb interactions at quark level? $\Longrightarrow$ $C_0 \propto \beta^{-1}$ as $\sqrt{q^2} \to 2M_{\mathcal{B}^0}$ $\Longrightarrow$ For any neutral baryon $\sqrt{\sigma_{\mathcal{B}^0\overline{\mathcal{B}}^0}} \propto \frac{|G^{\mathcal{B}^0}|}{M_{\mathcal{B}^0}}$



$$\underline{(Y, I_3) \rightarrow (Y_U, U_3)}$$

$$U_3 = -\tfrac{1}{2} I_3 + \tfrac{3}{4} Y$$

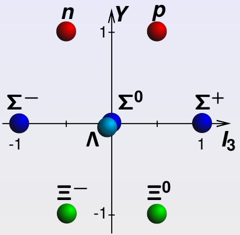
$$Y_U = -Q$$



U-spin relation: $G^{\Sigma^0} - G^{\Lambda} + \frac{2}{\sqrt{3}} G^{\Lambda\Sigma^0} = 0$

$$M_{\Sigma^0}\sqrt{\sigma_{\Sigma^0\overline{\Sigma^0}}} - M_{\Lambda}\sqrt{\sigma_{\Lambda\overline{\Lambda}}} + \frac{2}{\sqrt{3}}\overline{M_{\Lambda\Sigma^0}}\sqrt{\sigma_{\Lambda\overline{\Sigma^0}}} = (-0.06 \pm 6.0) \times 10^{-4}$$

Baryon octet and U-spin



$(Y, I_3) \to (Y_U, U_3)$

$$U_3 = -\frac{1}{2} I_3 + \frac{3}{4} Y$$

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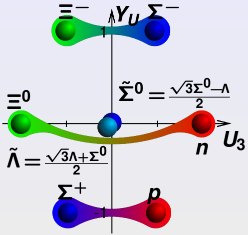
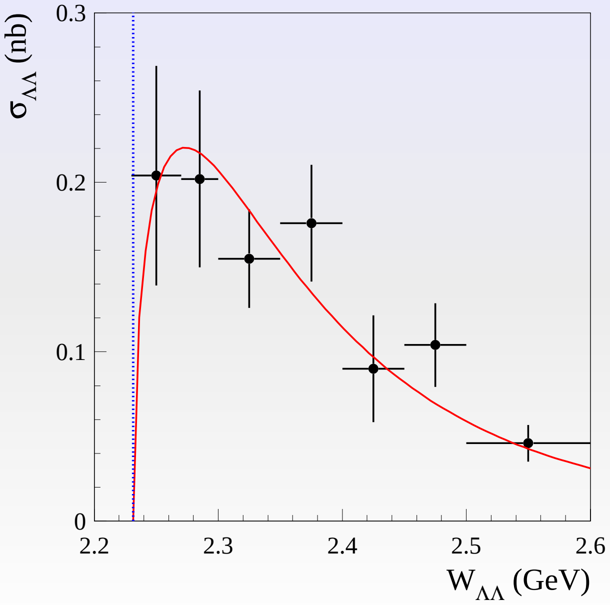


U-spin relation: $G^{\Sigma^0} - G^{\Lambda} + \frac{2}{\sqrt{3}} G^{\Lambda\Sigma^0} = 0$

$$M_{\Sigma^0}\sqrt{\sigma_{\Sigma^0\overline{\Sigma^0}}} - M_{\Lambda}\sqrt{\sigma_{\Lambda\overline{\Lambda}}} + \frac{2}{\sqrt{3}}\overline{M_{\Lambda\Sigma^0}}\sqrt{\sigma_{\Lambda\overline{\Sigma^0}}} = (-0.06 \pm 6.0) \times 10^{-4}$$

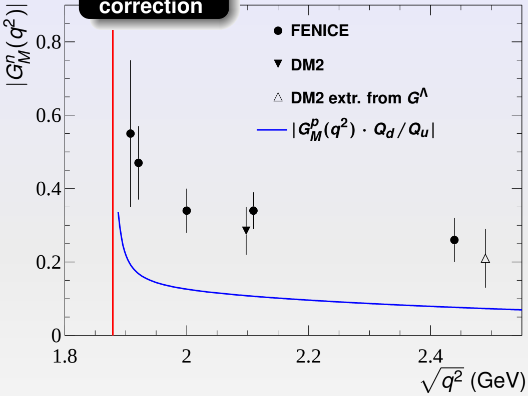
BaBar: $e^+e^- \to \Lambda\bar{\Lambda}$

[PRD76, 092006]



Time-like $|G_M^n|$ measurements

No Coulomb correction

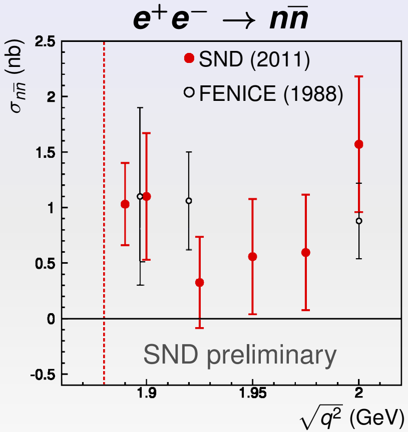


	$\|G_M^n/G_M^p\|$
Data	~ 1.5
Naively	$\sim \|Q_d/Q_u\|$
pQCD	< 1
Soliton models	~ 1
VMD (Dubnicka)	$\gg 1$

Only SND, CMD2(?) and BESIII can measure this cross section

No other experiments at present and in near future will be able to perform such a measurement

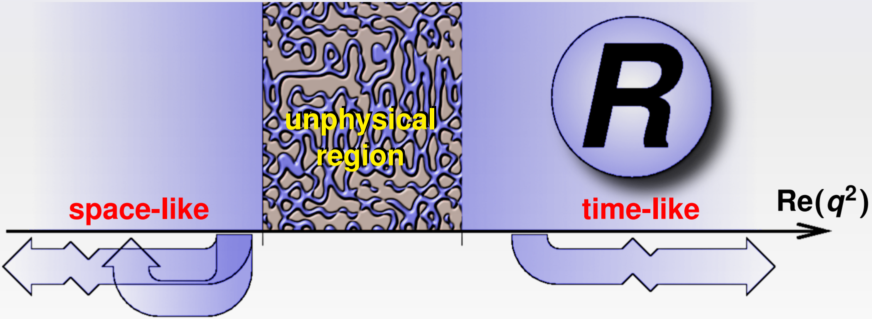
$e^+e^- \to n\bar{n}$: preliminary result from SND



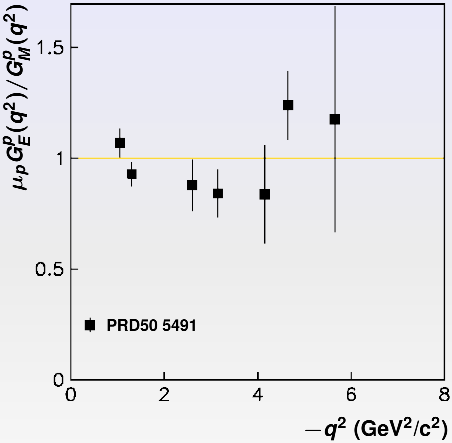
- **Scan 2011**
- **Maximum energy: 2 GeV**
- **Efficiency $\sim$ 30%**
- **Above $n\bar{n}$ threshold:**
  $\sigma_{n\bar{n}} = 0.8 \pm 0.2$ **nb**

Dispersive analysis of the ratio $R = \mu_p \frac{G_E^p}{G_M^p}$

Eur. Phys. J. A32, 421
R. Baldini, S. Pacetti and A. Zallo



Space-like G_E^p / G_M^p measurements



$$G_E^p = F_1^p + \frac{q^2}{4M_p^2} F_2^p$$

$$G_M^p = F_1^p + F_2^p$$

Space-like

F_1 / $\frac{q^2}{4M_p^2} F_2$ **cancellation**

$$\frac{G_E^p(q^2)}{G_M^p(q^2)} < 1$$

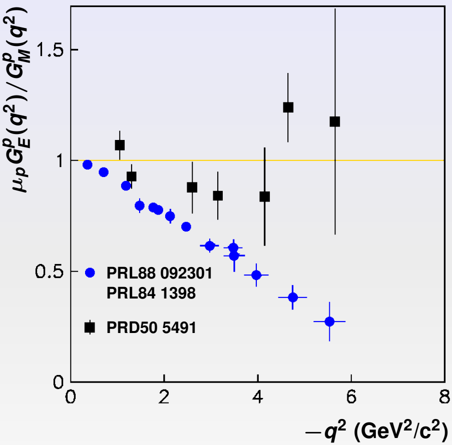
Time-like

F_1 / $\frac{q^2}{4M_p^2} F_2$ **enhancement**

$$\left|\frac{G_E^p(q^2)}{G_M^p(q^2)}\right| > 1$$

Radiative corrections of polarization technique $\ll$ **Radiative corrections in Rosenbluth method**

Space-like G_E^p / G_M^p measurements



$$G_E^p = F_1^p + \frac{q^2}{4M_p^2} F_2^p$$

$$G_M^p = F_1^p + F_2^p$$

Space-like

F_1 / $\frac{q^2}{4M_p^2} F_2$ cancellation

$$\frac{G_E^p(q^2)}{G_M^p(q^2)} < 1$$

Time-like

F_1 / $\frac{q^2}{4M_p^2} F_2$ enhancement

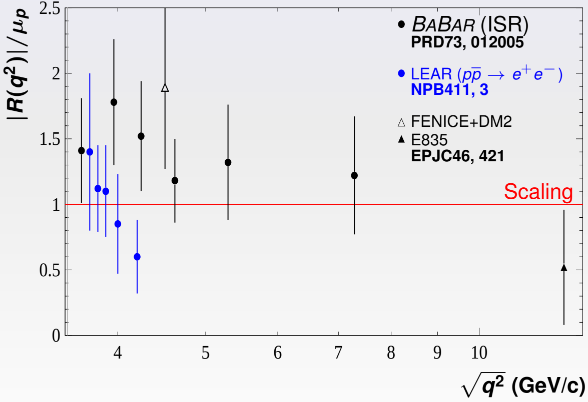
$$\left|\frac{G_E^p(q^2)}{G_M^p(q^2)}\right| > 1$$

Radiative corrections of polarization technique $\ll$ **Radiative corrections in Rosenbluth method**

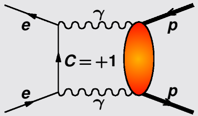
Time-like $|G_E^p/G_M^p|$ measurements

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta C}{2q^2}|G_M^p|^2\left[(1+\cos^2\theta)+\frac{4M_p^2}{q^2\mu_p^2}\sin^2\theta|R|^2\right]$$

$$R(q^2) = \mu_p\frac{G_E^p(q^2)}{G_M^p(q^2)}$$



$\gamma\gamma$ exchange



$\gamma\gamma$ exchange interferes with the Born term

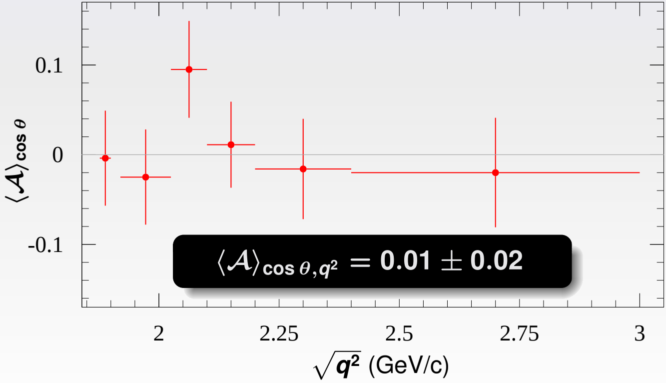
Asymmetry in angular distributions

[PLB659, 197]

$\gamma\gamma$ exchange from $e^+e^- \to p\bar{p}\gamma$ BABAR data

E. Tomasi-Gustafsson,
E. A. Kuraev, S. Bakmaev, SP
PLB659, 197

$$\mathcal{A}(\cos\theta, q^2) = \frac{\frac{d\sigma}{d\Omega}(\cos\theta, q^2) - \frac{d\sigma}{d\Omega}(-\cos\theta, q^2)}{\frac{d\sigma}{d\Omega}(\cos\theta, q^2) + \frac{d\sigma}{d\Omega}(-\cos\theta, q^2)}$$

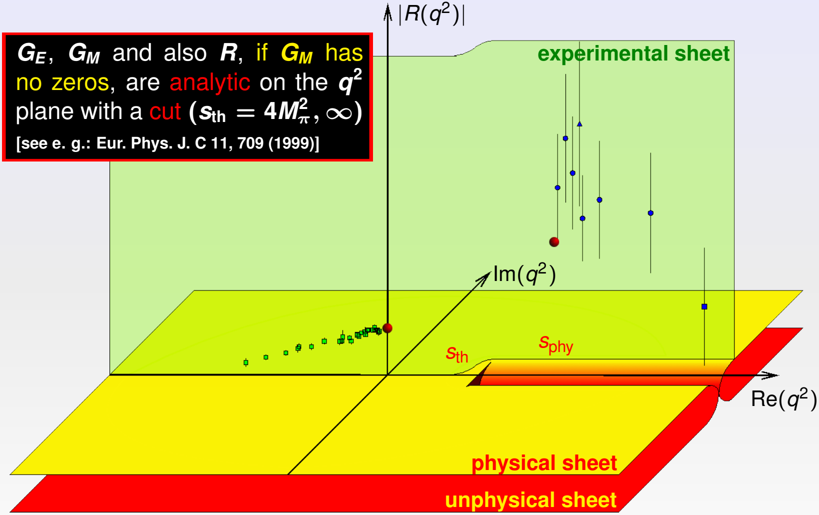


$$\langle\mathcal{A}\rangle_{\cos\theta, q^2} = 0.01 \pm 0.02$$

$R(q^2)$ in the complex plane

G_E, G_M and also R, if G_M has no zeros, are analytic on the q^2 plane with a cut $(s_{th} = 4M_\pi^2, \infty)$

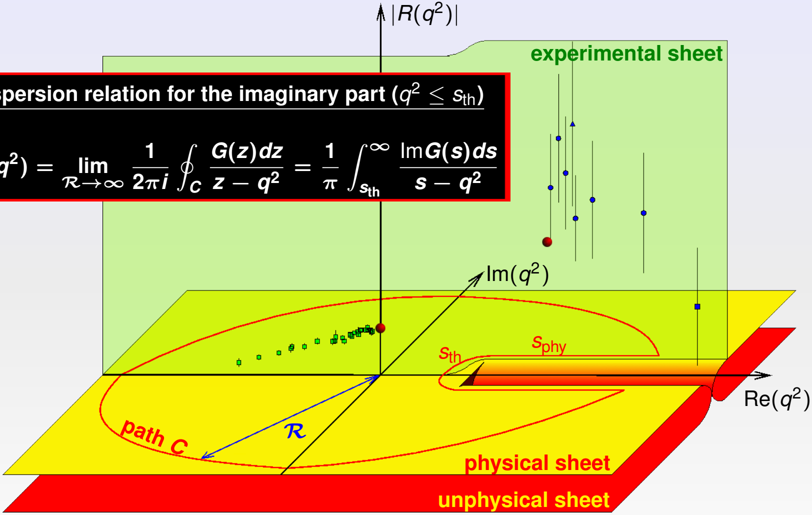
[see e. g.: Eur. Phys. J. C 11, 709 (1999)]



$R(q^2)$ in the complex plane

Dispersion relation for the imaginary part ($q^2 \leq s_{th}$)

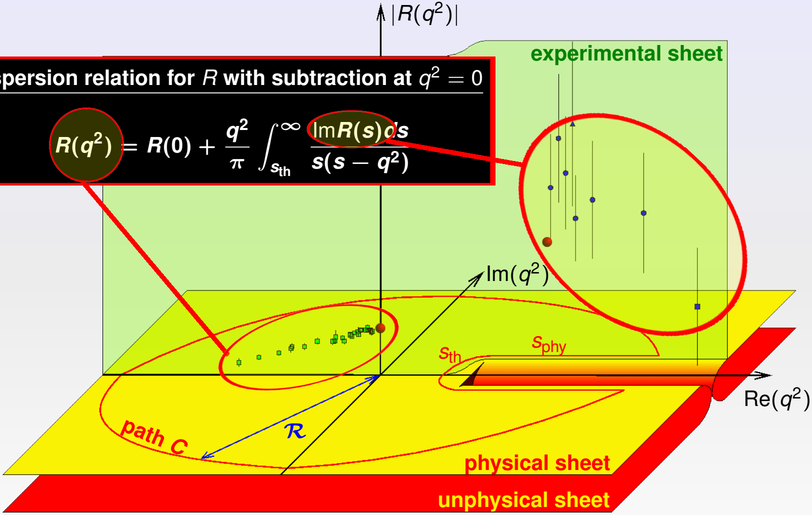
$$G(q^2) = \lim_{\mathcal{R}\to\infty} \frac{1}{2\pi i} \oint_C \frac{G(z)dz}{z - q^2} = \frac{1}{\pi} \int_{s_{th}}^{\infty} \frac{\mathrm{Im}G(s)ds}{s - q^2}$$



$R(q^2)$ in the complex plane

Dispersion relation for R with subtraction at $q^2 = 0$

$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{s_{\rm th}}^{\infty} \frac{{\rm Im}R(s)\,ds}{s(s-q^2)}$$

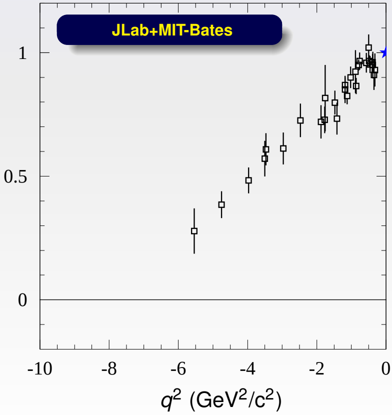


$R(q^2)$

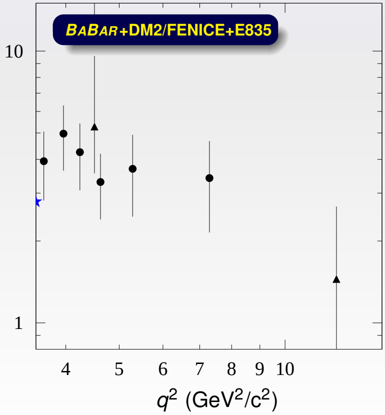
$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}R(s)}{s(s-q^2)}ds$$

Req^2

$R(q^2)$ space-like



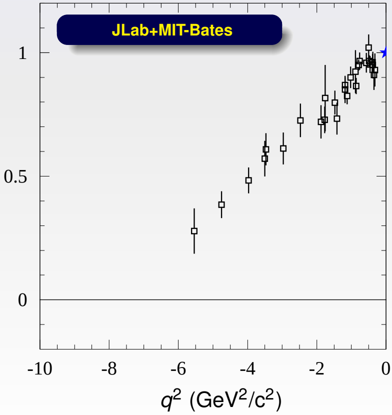
$|R(q^2)|$ time-like



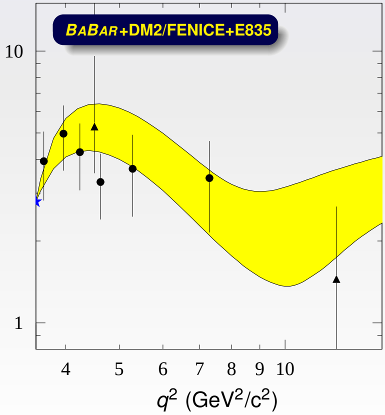
$R(q^2)$

$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{Im}R(s)}{s(s-q^2)}ds$$

$R(q^2)$ space-like



$|R(q^2)|$ time-like



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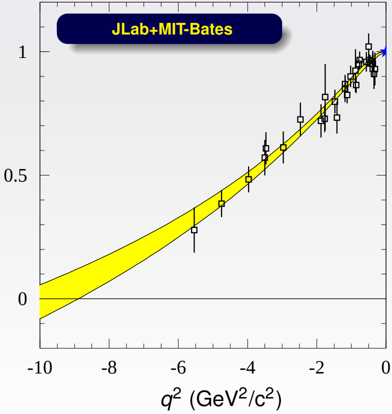
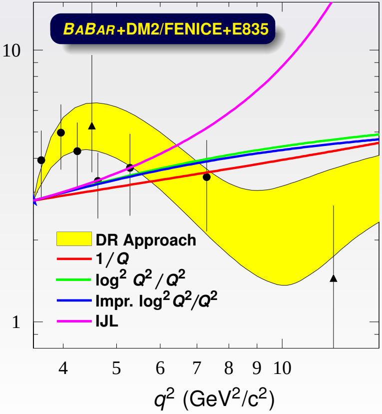


$|R(q^2)|$ time-like



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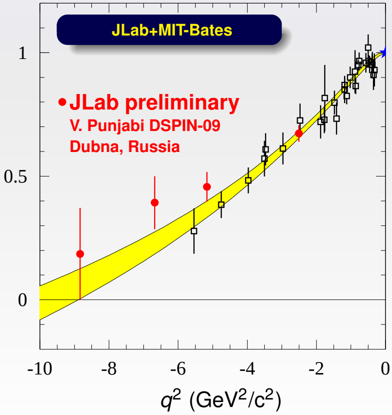
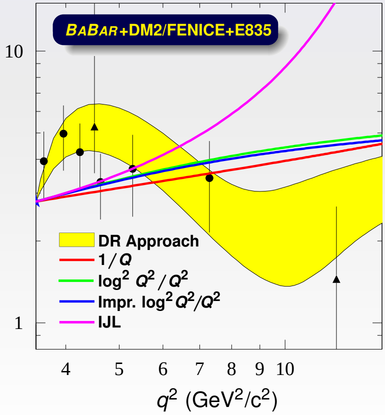
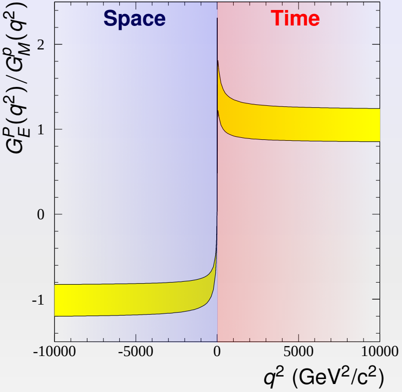
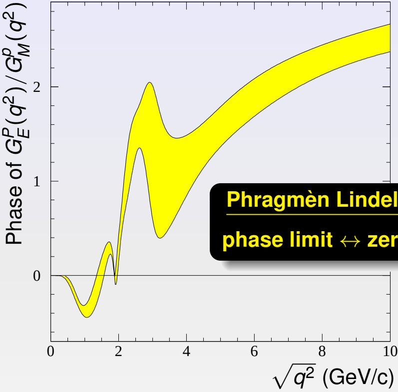


$|R(q^2)|$ time-like



Asymptotic $G_E^P(q^2)/G_M^p(q^2)$ and phase





Phragmèn Lindelöf

phase limit $\leftrightarrow$ zeros

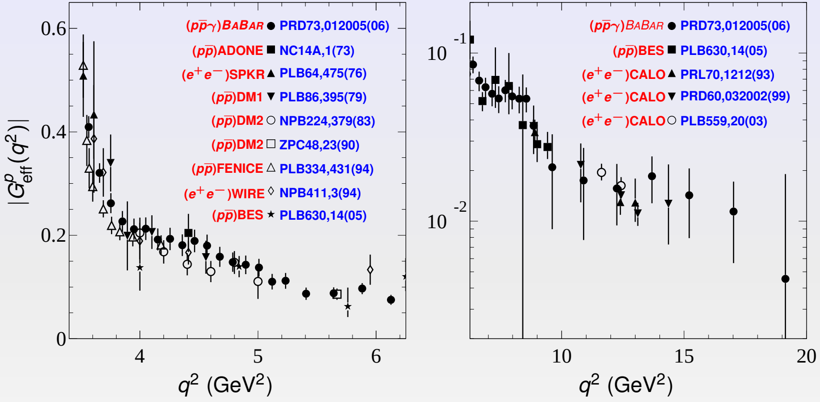
pQCD prediction

$$\frac{G_E^p(q^2)}{G_M^p(q^2)} \underset{|q^2|\to\infty}{\longrightarrow} -1$$

Phase from DR

$$\phi(q^2) = -\frac{\sqrt{q^2 - s_0}}{\pi} \, \mathrm{Pr} \int_{s_0}^{\infty} \frac{\ln|R(s)|ds}{\sqrt{s - s_0}(s - q^2)}$$

Time-like magnetic proton form factor



Data obtained assuming $|G_M^p| = |G_E^p| \equiv |G_{\rm eff}^p|$ (true only at threshold)

$$|G_{\rm eff}^p|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{16\pi\alpha^2 C_e}{3}\,\frac{\sqrt{1-1/\tau}}{4q^2}\left(1+\frac{1}{2\tau}\right)}$$

The integral equation for G_M

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Dispersion relation subtracted at $t = 0$

$$\ln G(t) = \frac{t\sqrt{s_{\rm th} - t}}{\pi} \int_{s_{\rm th}}^{\infty} \frac{\ln|G(s)|\,ds}{s\sqrt{s - s_{\rm th}}(s - t)}$$

- Less dependent on the asymptotic behavior of the FF
- $\ln G(0) = 0 \Rightarrow$ no further terms have to be considered

Splitting the integral $\int_{s_{\rm th}}^{\infty}$ into $\int_{s_{\rm th}}^{s'_{\rm phy}} + \int_{s'_{\rm phy}}^{\infty}$ we obtain the integral equation

$$\overbrace{\ln G(t) - I_{\rm phy}^{\infty}(t)}^{\text{Data and Theory}} = \frac{t\sqrt{s_{\rm th} - t}}{\pi} \int_{s_{\rm th}}^{s'_{\rm phy}} \frac{\overbrace{\ln|G(s)|}^{\text{Unknown}}\,ds}{s\sqrt{s - s_{\rm th}}(s - t)}$$

- To avoid instabilities around $s_{\rm phy} = 4M_N^2$, the upper boundary has been shifted to $s'_{\rm phy} = s_{\rm phy} + \Delta$, with $\Delta \simeq 0.5$ GeV^2
- We impose continuity of the FF at $s'_{\rm phy}$ and $s_{\rm th}$, in addition, at the upper boundary $s'_{\rm phy}$, continuity of the first derivative is also required
- A regularization, depending on a free parameter τ, is introduced by requiring the FF total curvature in the unphysical region to be limited

Solving procedure and test

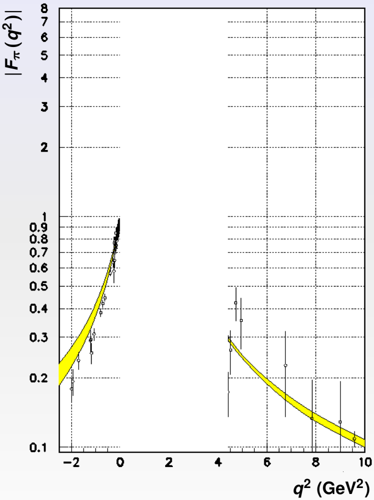
Solving procedure

Minimize: $\chi^2 = \chi^2_{\text{data}} + \chi^2_{\text{theory}} + \tau^6 \cdot \chi^2_{\text{regu}}$

$$\chi^2_{\text{regu}} = \int_{s_{\text{th}}}^{s'_{\text{phy}}} \left[\frac{d^2 \ln |G(s)|}{ds^2}\right]^2 ds \propto \left[\begin{array}{c}\text{total curvature}\\ \text{in } [s_{\text{th}}, s'_{\text{phy}}]\end{array}\right]$$

Pion FF to fix the regularization parameter τ

Space-like (DR) and time-like data (yellow bands) have been used as input in the integral equation to retrieve the time-like FF in the nucleon unphysical region.



Solving procedure and test

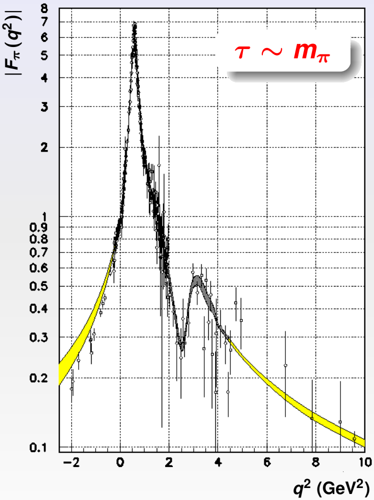
Solving procedure

Minimize: $\chi^2 = \chi^2_{\text{data}} + \chi^2_{\text{theory}} + \tau^6 \cdot \chi^2_{\text{regu}}$

$$\chi^2_{\text{regu}} = \int_{s_{\text{th}}}^{s'_{\text{phy}}} \left[\frac{d^2 \ln |G(s)|}{ds^2} \right]^2 ds \propto \left[\begin{array}{c} \text{total curvature} \\ \text{in } [s_{\text{th}}, s'_{\text{phy}}] \end{array} \right]$$

Pion FF to fix the regularization parameter τ

Space-like (DR) and time-like data (yellow bands) have been used as input in the integral equation to retrieve the time-like FF in the nucleon unphysical region (gray band).



Nucleon magnetic form factors

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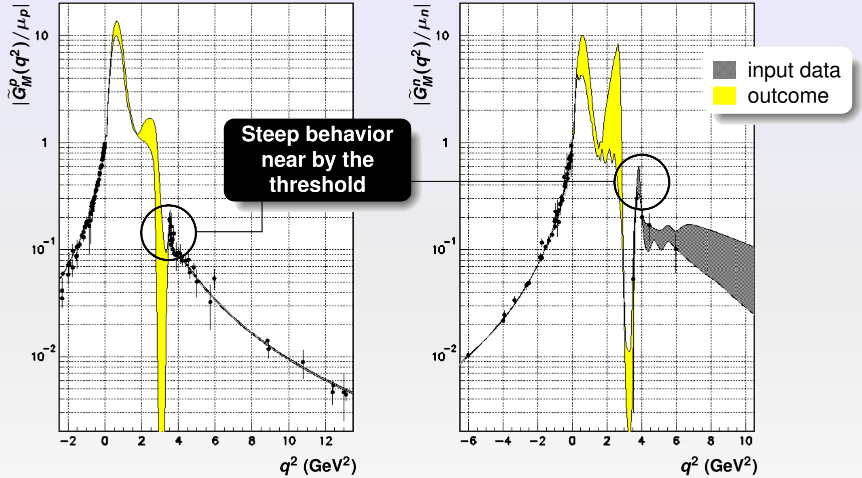
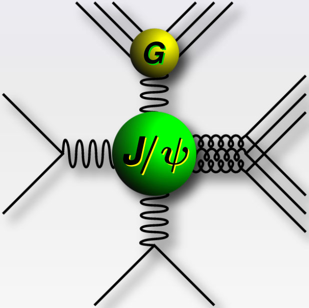


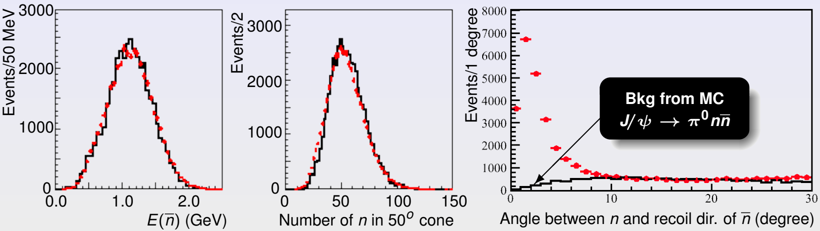
$M_1 \sim 770$ MeV	$\Gamma_1 \sim 350$ MeV
$M_2 \sim 1600$ MeV	$\Gamma_2 \sim 350$ MeV

J/ψ strong and electromagnetic phase



BESIII preliminary results: $J/\psi \to p\bar{p}$, $n\bar{n}$

$n\bar{n}$ identification



BESIII

$$B(J/\psi \to n\bar{n}) = (2.07 \pm 0.01 \pm 0.14) \cdot 10^{-3}$$

$$B(J/\psi \to p\bar{p}) = (2.112 \pm 0.004 \pm 0.027) \cdot 10^{-3}$$

PDG

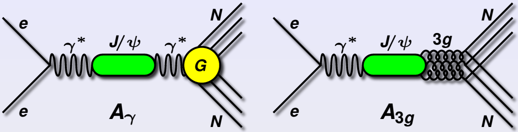
$$B(J/\psi \to n\bar{n}) = (2.2 \pm 0.4) \cdot 10^{-3}$$

$$B(J/\psi \to p\bar{p}) = (2.17 \pm 0.07) \cdot 10^{-3}$$

$$B(J/\psi \to p\bar{p}) \simeq B(J/\psi \to n\bar{n})$$

suggests a phase $\sim 90^\circ$ between strong and e.m. amplitudes!

J/ψ decays: strong and electromagnetic



cross section $\sim |A_\gamma + A_{3g}|^2 = |A_\gamma|^2 + |A_{3g}|^2 + 2\,\mathrm{Re}[A_\gamma^* A_{3g}]$

interference term

According to pQCD: A_γ and A_{3g} are real $\Rightarrow$ interference

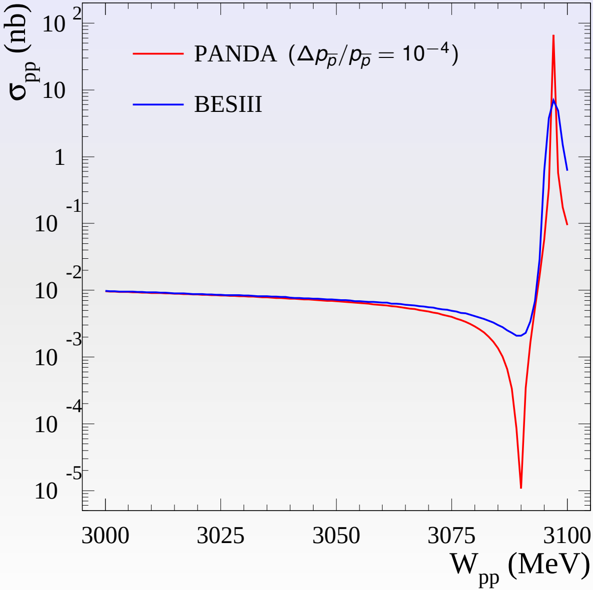
On the contrary data suggest:

$J/\psi \to J_1^P J_2^P$	$\frac{A_\gamma}{A_{3g}}$ phase
1^-0^-	$106^o \pm 10^o$
1^-1^-	$138^o \pm 37^o$
0^-0^-	$90^o \pm 10^o$
$n\bar{n}$	$89^o \pm 15^o$

But these conclusions have been obtained modeling SU(3) breaking, or using poorly measured $n\bar{n}$ cross section outside J/ψ

Interference with the continuum measures the relative phase in an independent way

Full interference as seen by PANDA or BESIII



Conclusions

- Pointlike Behavior at and well above threshold
- No Sommerfeld Resummation Factor
- Neutral baryon non zero cross section at threshold?
- G_E^p space-like $\rightarrow -1$ asymptotically?
- Imaginary J/ψ strong decay amplitude?

Perspectives

- Data from SND and CMD2
- More data from *BaBar* ($\times$2) and Belle (?)
- BESIII: ISR now, scan 2012-2013
- PANDA could explore FFs below threshold through $p\bar{p} \rightarrow \pi^0 l^+ l^-$