Time-like Baryon Form Factors at threshold

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CENTRO STUDI E RICERCHE Enrico Fermi

Ferrara International School Niccolò Cabeo
on hadron structure and interactions

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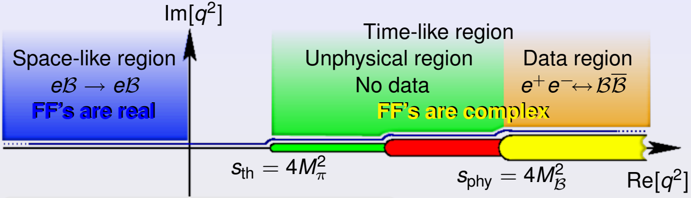
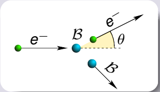
- **Last News on Baryon FF near threshold**
- **The Neutral Baryon Puzzle**
- **Interference Pattern in $J/\psi \rightarrow p\bar{p}$**
- **Conclusions and Perspectives**

Cross sections and analyticity



Time-like: had. helicity $= \begin{cases} 1 \Rightarrow |G_E| \\ 0 \Rightarrow |G_M| \end{cases}$

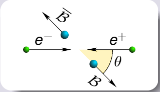
$$G_E(4M_\mathcal{B}^2) = G_M(4M_\mathcal{B}^2)$$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_e' \cos^2\frac{\theta}{2}}{4E_e^3 \sin^4\frac{\theta}{2}} \left[G_E^2 - \tau\left(1 + 2(1-\tau)\tan^2\frac{\theta}{2}\right) G_M^2 \right] \frac{1}{1-\tau}$$

$$\tau = \frac{q^2}{4M_\mathcal{B}^2}$$

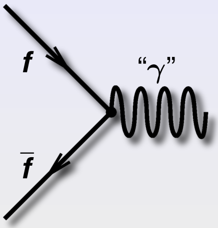


Annihilation Coulomb correction

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta |G_E|^2\right]$$

$$\beta = \sqrt{1 - \frac{1}{\tau}}$$

$f\bar{f}$ electromagnetic current structure$_1$

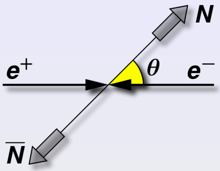


$J_0 \equiv 0$ $\qquad$ $J_z \propto M$ $\qquad$ $J_x \pm iJ_y \propto E$

"$|1, 0\rangle$" $\qquad$ "$|1, \pm 1\rangle$"

e^+e^- : $m_e << E_e \Rightarrow$ "γ" polarized like a real γ

$f\bar{f}$ electromagnetic current structure$_2$



$$|1,1\rangle_z = \frac{1+\cos\theta}{2}|+\rangle - \frac{\sin\theta}{\sqrt{2}}|0\rangle + \frac{1-\cos\theta}{2}|-\rangle$$

Space-like Breit frame:

N_{in}	Helicity Alignement
$\frac{2M}{W} G_E$	Opposite
G_M	Same

Time-like c.m. frame:

$\overline{N}_{\text{out}}(-p) \equiv N_{\text{out}}(p)$

N	Helicity Alignement	
G_E	Same	$\|0\rangle$
G_M	Opposite	$\|\pm\rangle$

$$\left(\frac{1+\cos\theta}{2}\right)^2 |G_M|^2 + \frac{\cos^2\theta}{2}\frac{4M^2}{W^2}|G_E|^2 + \left(\frac{1-\cos\theta}{2}\right)^2 |G_M|^2$$

S and D wave

$$\begin{cases} P_\gamma = -1 & P_{N\overline{N}} = (-1)^L \times (-1) \;\; \Rightarrow \;\; L = 0, 2 \\ J_\gamma = 1 & S = 0: \;\; L = 1 \text{ forbidden} \longrightarrow S = 1 \end{cases}$$

$$G_E = G_S - 2G_D \qquad G_M = \frac{G_S + G_D}{W/2M}$$

At threshold S wave only: $G_E = G_M$

$$\begin{cases} G_E = F_1 + \dfrac{W^2}{4M^2} F_2 \\ G_M = F_1 + F_2 \end{cases} \Longrightarrow \quad G_E = G_M$$

Detailed balance

$$\sigma(e^+e^- \longrightarrow p\bar{p}) \cdot \frac{\beta_p}{\beta_e} = \sigma(p\bar{p} \longrightarrow e^+e^-) \cdot \frac{\beta_e}{\beta_p}$$

$p\bar{p} \longrightarrow e^+e^-$ at threshold

$\sigma(p\bar{p} \longrightarrow e^+e^-)$ divergent $\longrightarrow$ normalization? $p\bar{p}$ atom?

$e^+e^- \longrightarrow p\bar{p}$ at threshold

experimental energy cut, but **ISR** OK

Polarized beams

e^+e^-:

Longitudinal

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \equiv \left|\tfrac{1}{2},\text{-}\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},\text{-}\tfrac{1}{2}\right\rangle$$

Unless P violation (weak interaction)

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},\text{-}\tfrac{1}{2}\right\rangle \equiv \left|\tfrac{1}{2},\text{-}\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \equiv \mathbf{0} \Rightarrow \text{al } \times 2\ ?$$

Transverse

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle + e^{\pm i\phi} \left|\tfrac{1}{2},\text{-}\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},\text{-}\tfrac{1}{2}\right\rangle \Rightarrow |G_M|^2 \times \sin^2\theta \cos^2\phi$$

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},\text{-}\tfrac{1}{2}\right\rangle + e^{i\phi} \left|\tfrac{1}{2},\text{-}\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \equiv \mathbf{0} \quad \text{no info on } \mathrm{Re}[G_E^* G_M]$$

$p\overline{p}$: transverse G_M same as e^+e^-

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},\text{-}\tfrac{1}{2}\right\rangle + e^{i\phi} \left|\tfrac{1}{2},\text{-}\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \equiv \mathbf{0} \longrightarrow \frac{M}{E} G_E\,|1,0\rangle$$

but

$$G_E\,|1,0\rangle = \underbrace{\frac{\sin\theta}{\sqrt{2}} G_E\,|+\rangle} + \cos\theta\, G_E\,|0\rangle + \underbrace{\frac{\sin\theta}{2} G_E\,|-\rangle}$$

Interference with G_M contribution: $\mathrm{Re}[G_E^* G_M]$
$G_E \neq G_M \Rightarrow \cos(\phi_E - \phi_M)$

Final State Coulomb Interaction

$$T_{\text{fi}} = \langle \chi_{\text{f}} | \, V \, | \psi_i^+ \rangle$$

$$V = W + U$$

$$\psi^\pm = \chi + \frac{1}{E - H_0 + \epsilon} V \Phi^\pm$$

$$\phi^\pm = \chi + \frac{1}{E - H_0 + \epsilon} U \phi^\pm$$

$$\Phi^\pm = \chi + \frac{1}{E - H_0 + \epsilon} W \Phi^\pm$$

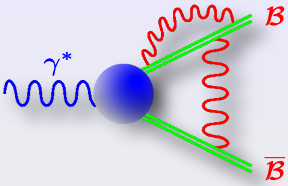
$$T_{\text{fi}} = \left\langle \phi_{\text{f}}^- \right| V \left| \Phi_i^+ \right\rangle + \cancel{\left\langle \psi_{\text{f}}^- - \phi_{\text{f}}^- \right| U \left| \Phi_i^+ \right\rangle}$$

$$T_{\text{fi}} \sim \int d^3 q' \left\langle \phi_{\text{f}}^- \right| \chi_f \rangle \langle \chi_{\text{f}} | \, V \left| \Phi_i^+ \right\rangle \sim \phi(0) \cdot T_{\text{fi}}^{\text{No Coulomb}}$$

$\phi(0)$: Coulomb scattering wave funcition at the origin

$$\sigma(e^+e^- \to p\bar{p}) = \frac{\pi\alpha/\beta}{1 - e^{-\pi\alpha/\beta}} \sigma_{\text{Born}}(e^+e^- \to p\bar{p})$$

The Coulomb Factor



$p\bar{p}$ Coulomb interaction as FSI

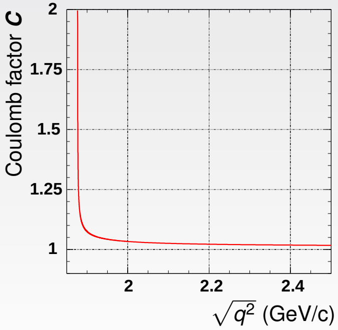
[Sommerfeld, Sakharov, Schwinger, Fadin, Khoze]

Distorted wave approximation

$$C = |\Psi_{\mathrm{Coul}}(0)|^2$$

- **S-wave:** $C = \dfrac{\frac{\pi\alpha}{\beta}}{1 - \exp\left(-\frac{\pi\alpha}{\beta}\right)} \xrightarrow[\beta\to 0]{} \dfrac{\pi\alpha}{\beta}$
- **D-wave:** $C = 1$

No Coulomb factor for boson pairs (P-wave)



Sommerfeld Enhancement and Resummation Factors

Coulomb Factor $\mathcal{C}$ for S-wave only:

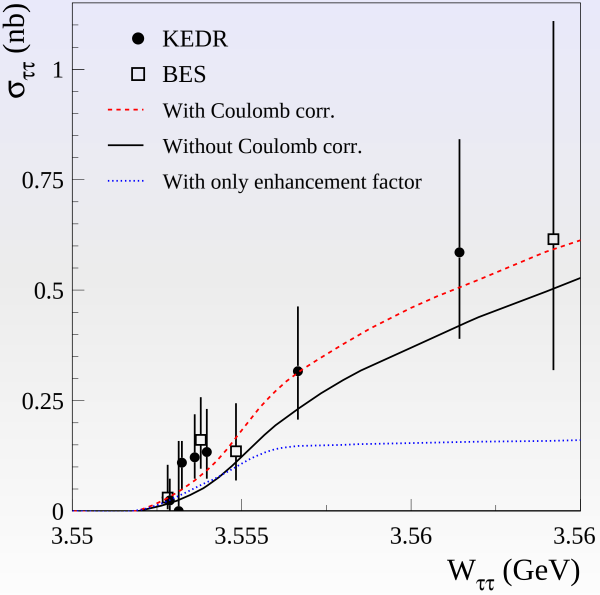
- **Partial wave FF:** $G_S = \dfrac{2G_M\sqrt{q^2/4M^2} + G_E}{3}$ $\qquad G_D = \dfrac{G_M\sqrt{q^2/4M^2} - G_E}{3}$
- **Cross section:** $\sigma(q^2) = 2\pi\alpha^2\beta\,\dfrac{4M^2}{(q^2)^2}\left[\mathcal{C}\,|G_S(q^2)|^2 + 2|G_D(q^2)|^2\right]$

$$\mathcal{C} = \mathcal{E} \times \mathcal{R}$$

- **Enhancement factor:** $\mathcal{E} = \pi\alpha/\beta$
- **Step at threshold:** $\sigma(4M^2) = \dfrac{\pi^2\alpha^3}{2M^2}\dfrac{\cancel{\beta}}{\cancel{\beta}}|G_S(4M^2)|^2 = 0.85\,|G_S(4M^2)|^2$ nb

- **Resummation factor:** $\mathcal{R} = 1/[1 - \exp(-\pi\alpha/\beta)]$
- **Few MeV above threshold:** $\mathcal{C} \simeq 1 \quad\Rightarrow\quad \sigma(q^2) \propto \beta\,|G_S(q^2)|^2$

The $e^+e^- \to \tau^+\tau^-$ case



Pointlike Baryons?

R. Baldini Ferroli, S. Pacetti,
A. Zallo and A. Zichichi

I.S.R. versus c.m.

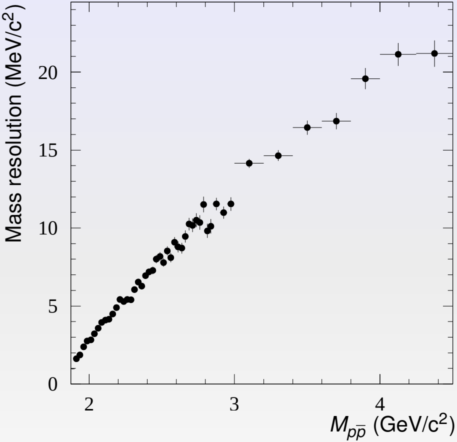
Advantages

- All q at the same time $\Longrightarrow$ Better control on systematics
- c.m. boost $\Longrightarrow$ at threshold efficiency $\neq 0$ + $\sigma_W \sim 1$ *MeV*
- Detected ISR $\gamma \Longrightarrow$ full $p\bar{p}$ angular coverage

Drawbacks

- $\mathcal{L} \propto$ invariant mass bin Δw
- More background

Mass resolution

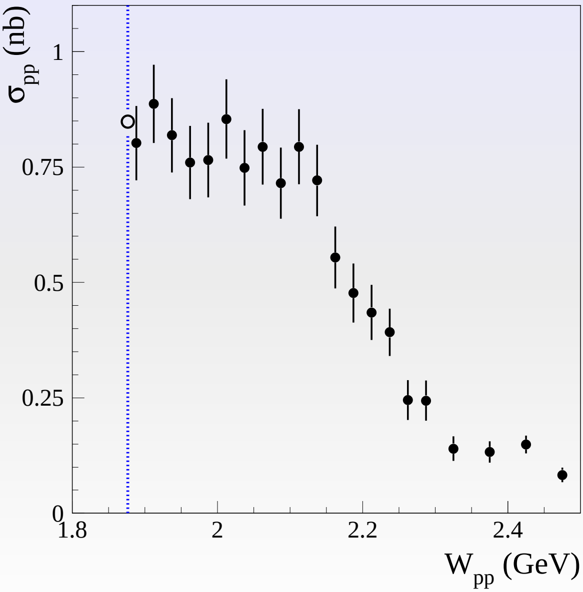


Incredibly good at threshold ($\sim$ 1 MeV/c^2), as e^+e^- c.m.

$\Delta p_T/p_T \sim 0.5\%$ at 1 GeV

BABAR: $e^+e^- \longrightarrow p\bar{p}$

[PRD73, 012005]



Proton form factor at $q^2 = 4M_p^2$

$$\sigma(e^+e^- \to p\bar{p})(4M_p^2) = 0.83 \pm 0.05 \text{ nb}$$

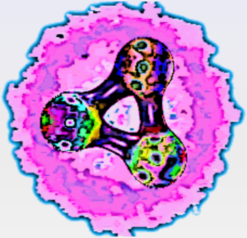
BABAR

$$\sigma(e^+e^- \to p\bar{p})(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\frac{\cancel{\beta}}{\cancel{\beta}}|G^p(4M_p^2)|^2 = 0.85\,|G^p(4M_p^2)|^2 \text{ nb}$$

$$|G^p(4M_p^2)| \equiv 1$$

$$|G^p(4M_p^2)| = 0.99 \pm 0.04(\text{stat}) \pm 0.03(\text{syst})$$

$$|G^p(4M_p^2)| \equiv 1$$



At $q^2 = 4M_P^2$ protons behave as pointlike fermions!

Sommerfeld Resummation Factor Needed?

Resummation Factor Needed?

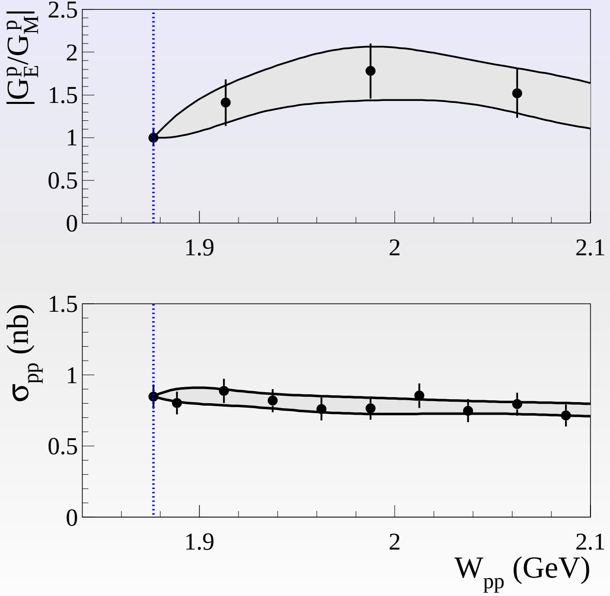
- **At threshold:** $G_E/G_M = 1 \Rightarrow \begin{cases} G_S \in \mathbb{R} \\ G_D = 0 \in \mathbb{R} \end{cases}$
- $\sigma(q^2)$, $|G_E/G_M| \longrightarrow G_S$, G_D
- $G_S = 1/\sqrt{1 - \exp(-\pi\alpha/\beta)}$
- **No need of Resummation Factor**

For a wide energy range ($\sim$ 200 MeV):

- **Proton behaves as a pointlike particle**
- **e.m. dominance, no strong interaction?**
- **Mild sensitivity to $\mathcal{B}\overline{\mathcal{B}}$ invariant mass resolution**

BABAR: $|G_E^p/G_M^p|$ and $\sigma(e^+e^- \longrightarrow p\bar{p})$

[PRD73, 012005]



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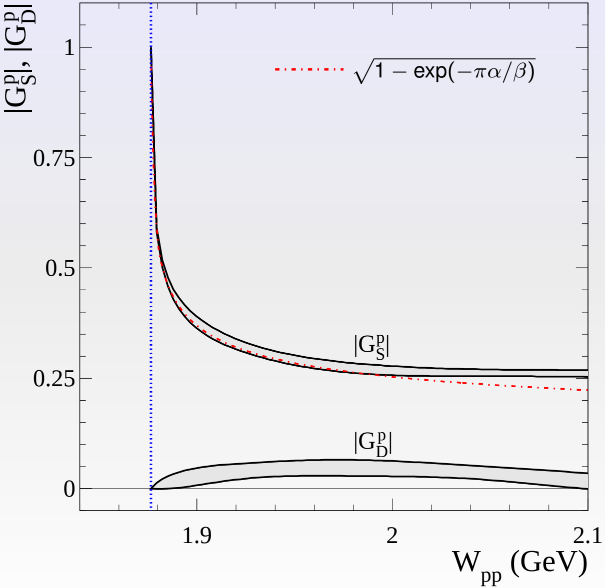
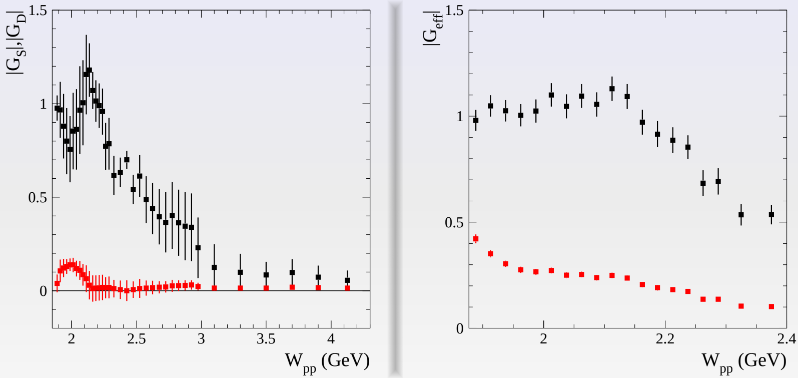
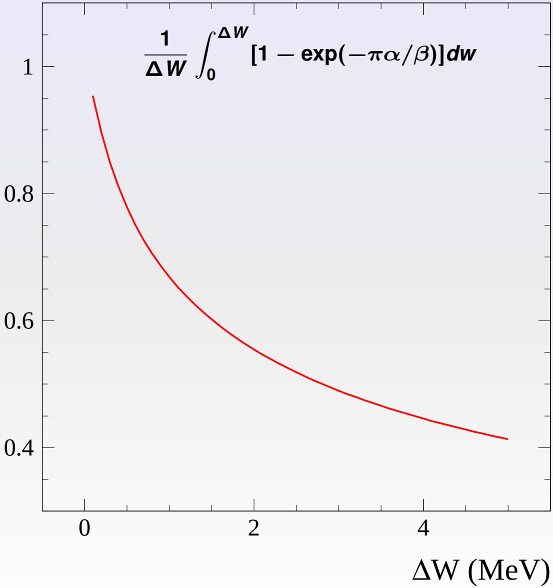


BABAR: G_S, G_D and G_{eff}



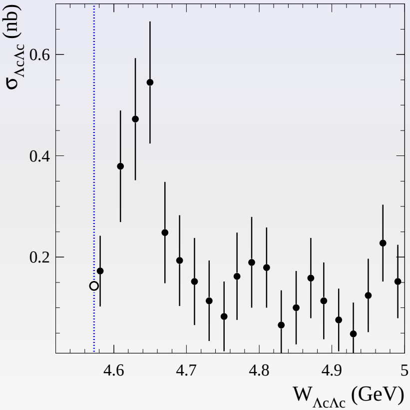
Integrated Sommerfeld factor



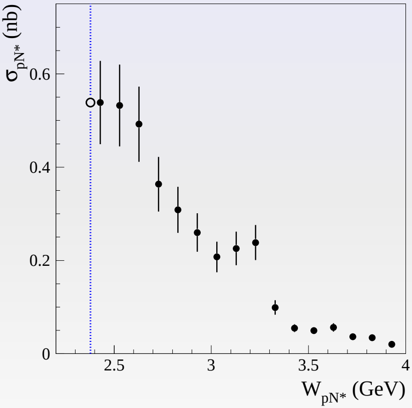
Other charged baryon FF's at threshold

$e^+e^- \to \Lambda_c^+\overline{\Lambda}_c^-$ and $e^+e^- \to p\overline{N}(1440)$+c.c.

[Belle PRL101, 172001]

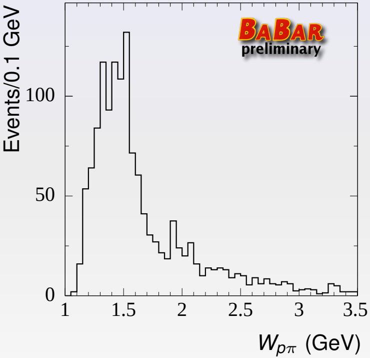


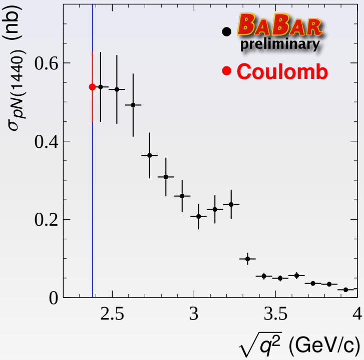
[BABAR PRD73, 012005]



$e^+e^- \to p\overline{N}(1440)$+c.c.

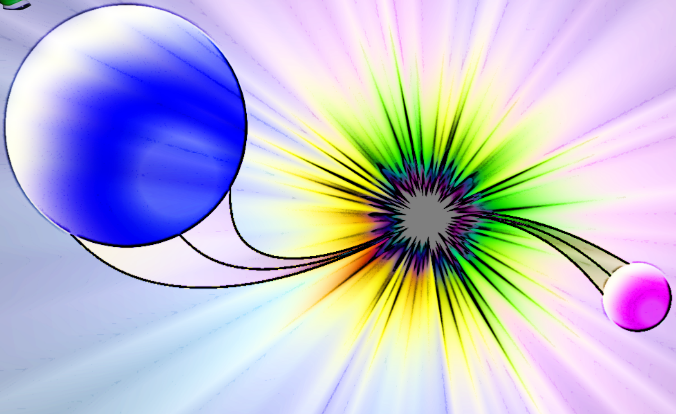
$$\sigma^{\text{Coulomb}} = \frac{16\pi^2\alpha^3 M_p^{3/2} M_{N(1440)}^{3/2}}{(M_p + M_{N(1440)})^5} \left|G^{pN(1440)}\right|^2 = \left|G^{pN(1440)}\right|^2 \times 0.49\ \text{nb}$$





$$\left|G^{pN(1440)}\right| = 1.04 \pm 0.09$$

The neutral baryons puzzle



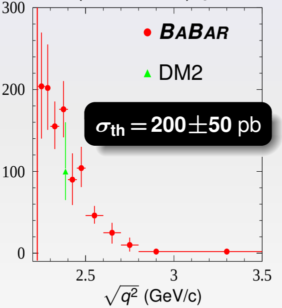
Neutral Baryons puzzle (*BaBar*)

[PRD76, 092006]

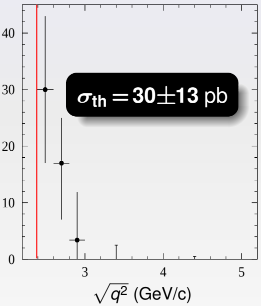
$$\sigma(e^+e^-\to \mathcal{B}^0\overline{\mathcal{B}}^0)=\frac{4\pi\alpha^2\beta C_0}{3q^2}\left[|G_M^{\mathcal{B}^0}|^2+\frac{2M^2_{\mathcal{B}^0}}{q^2}|G_E^{\mathcal{B}^0}|^2\right]\xrightarrow[\sqrt{q^2}\to 2M_{\mathcal{B}^0}]{}\frac{\pi\alpha^2\beta}{2M^2_{\mathcal{B}^0}}|G^{\mathcal{B}^0}|^2\to 0$$

No Coulomb correction at hadron level: $C_0 = 1$

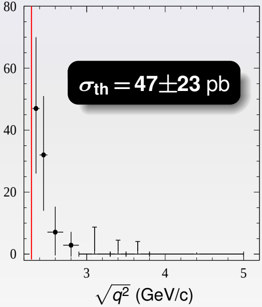
$\sigma(e^+e^- \to \Lambda\overline{\Lambda})$ (pb)



$\sigma(e^+e^- \to \Sigma^0\overline{\Sigma^0})$ (pb)



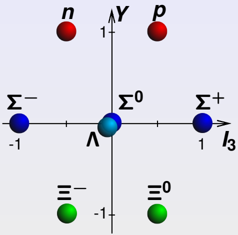
$\sigma(e^+e^- \to \Lambda\overline{\Sigma^0})$ (pb)



Like a remnant of Coulomb interactions at quark level? $\Longrightarrow$ $C_0 \propto \beta^{-1}$ as $\sqrt{q^2} \to 2M_{\mathcal{B}^0}$ $\Longrightarrow$ For any neutral baryon

$$\sqrt{\sigma_{\mathcal{B}^0\overline{\mathcal{B}}^0}} \propto \frac{|G^{\mathcal{B}^0}|}{M_{\mathcal{B}^0}}$$

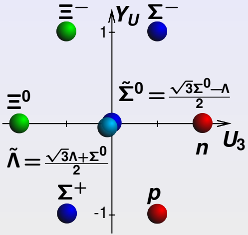
Baryon octet and U-spin



$$(Y, I_3) \longrightarrow (Y_U, U_3)$$

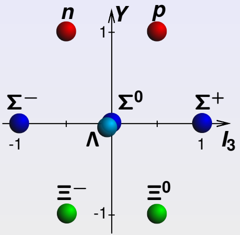
$$U_3 = -\tfrac{1}{2} I_3 + \tfrac{3}{4} Y$$

$$Y_U = -Q$$



U-spin relation: $G^{\Sigma^0} - G^{\Lambda} + \frac{2}{\sqrt{3}} G^{\Lambda\Sigma^0} = 0$

$$M_{\Sigma^0}\sqrt{\sigma_{\Sigma^0\overline{\Sigma^0}}} - M_{\Lambda}\sqrt{\sigma_{\Lambda\overline{\Lambda}}} + \frac{2}{\sqrt{3}}\overline{M_{\Lambda\Sigma^0}}\sqrt{\sigma_{\Lambda\overline{\Sigma^0}}} = (-0.06 \pm 6.0) \times 10^{-4}$$



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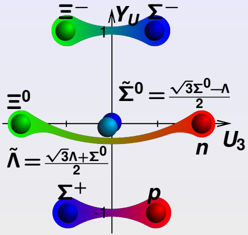

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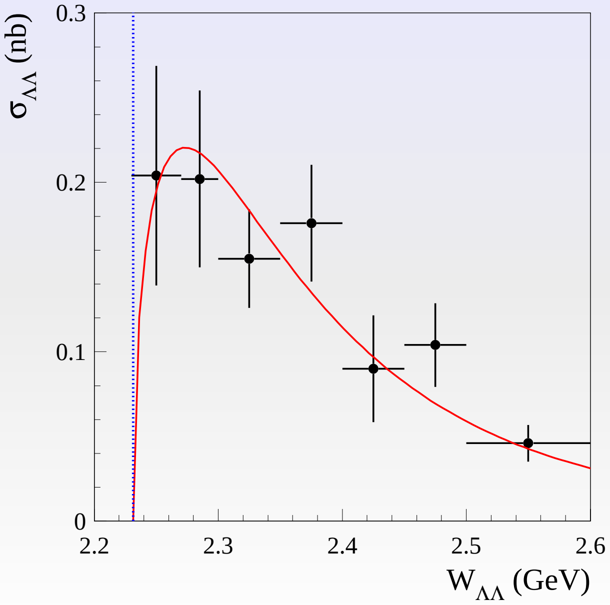
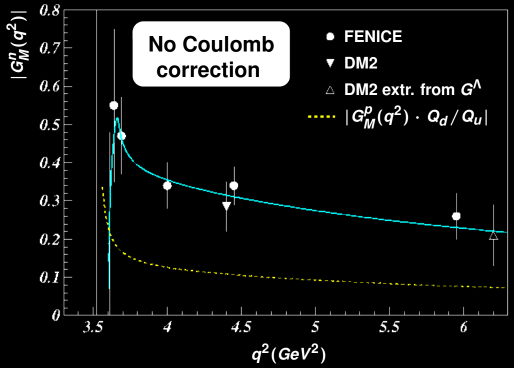
BABAR: $e^+e^- \to \Lambda\bar{\Lambda}$



Time-like $|G_M^n|$ measurements

Only two measurements by FENICE and DM2



	$\|G_M^n/G_M^p\|$
Data	~ 1.5
Naively	$\sim \|Q_d/Q_u\|$
pQCD	< 1
Soliton models	~ 1
VMD	$\gg 1$

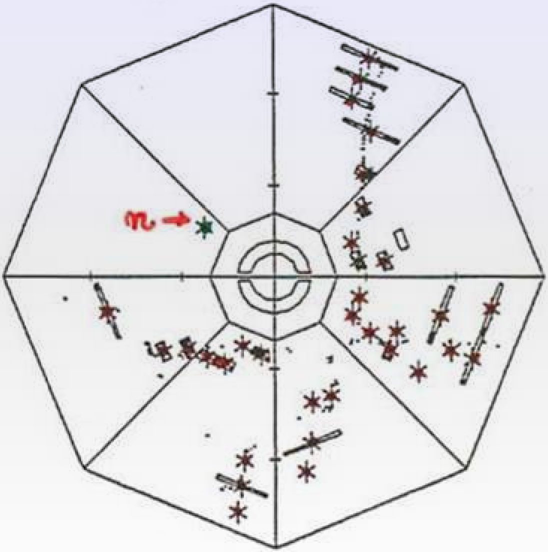
Threshold behaviour from angular distribution

$$G_M^n(4M_n^2) = G_E^n(4M_n^2) = 0?$$

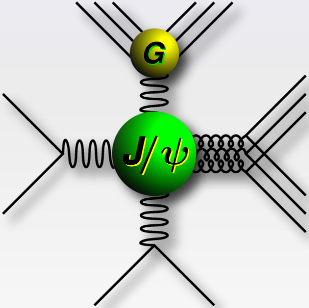
BaBar does agree with FENICE

$$\text{Large } G_M^\Lambda \overset{\text{U-spin}}{\Longrightarrow} \text{large } G_M^n$$

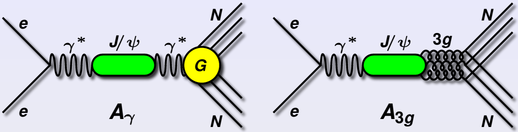
$e^+e^- \longrightarrow n\bar{n}$ (FENICE)



J/ψ strong and electromagnetic phase



J/ψ decays: strong and electromagnetic



cross section $\sim |A_\gamma + A_{3g}|^2 = |A_\gamma|^2 + |A_{3g}|^2 + \underbrace{2\,\mathrm{Re}[A_\gamma^* A_{3g}]}_{\text{interference term}}$

According to pQCD: A_γ and A_{3g} are real $\Rightarrow$ interference

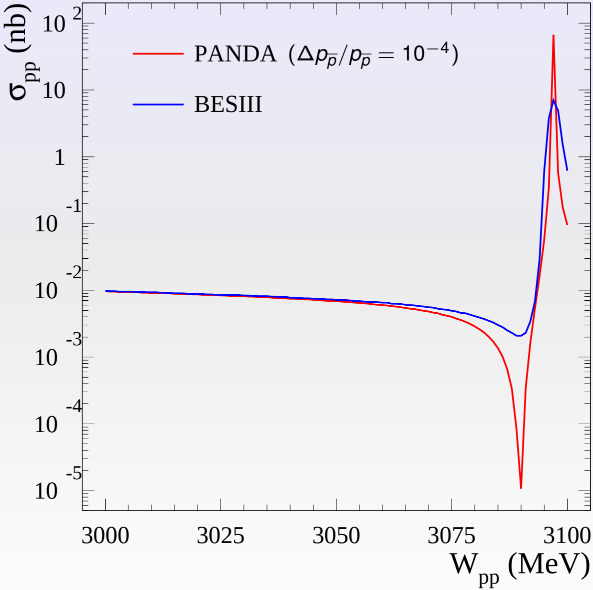
On the contrary data suggest:

$J/\psi \to J_1^P J_2^P$	$\frac{A_\gamma}{A_{3g}}$ phase
$1^- 0^-$	$106^o \pm 10^o$
$1^- 1^-$	$138^o \pm 37^o$
$0^- 0^-$	$90^o \pm 10^o$
$n\bar{n}$	$89^o \pm 15^o$

But these conclusions have been obtained modeling SU(3) breaking, or using poorly measured $n\bar{n}$ cross section outside J/ψ

Interference with the continuum measures the relative phase in an independent way

Full interference as seen by PANDA or BESIII

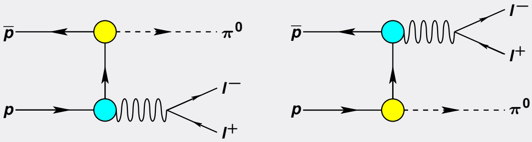


Conclusions

- Pointlike Behavior at and well above threshold
- No Sommerfeld Resummation Factor
- Neutral Baryons Puzzle

Perspectives

- More data from *BABAR* ($\times 2$) and Belle (?)
- BESIII: ISR now, scan 2012-2013
- PANDA unique opportunity: FF below threshold exploting $p\bar{p} \to \pi^0 l^+ l^-$



Expected narrow forward/backward peaks