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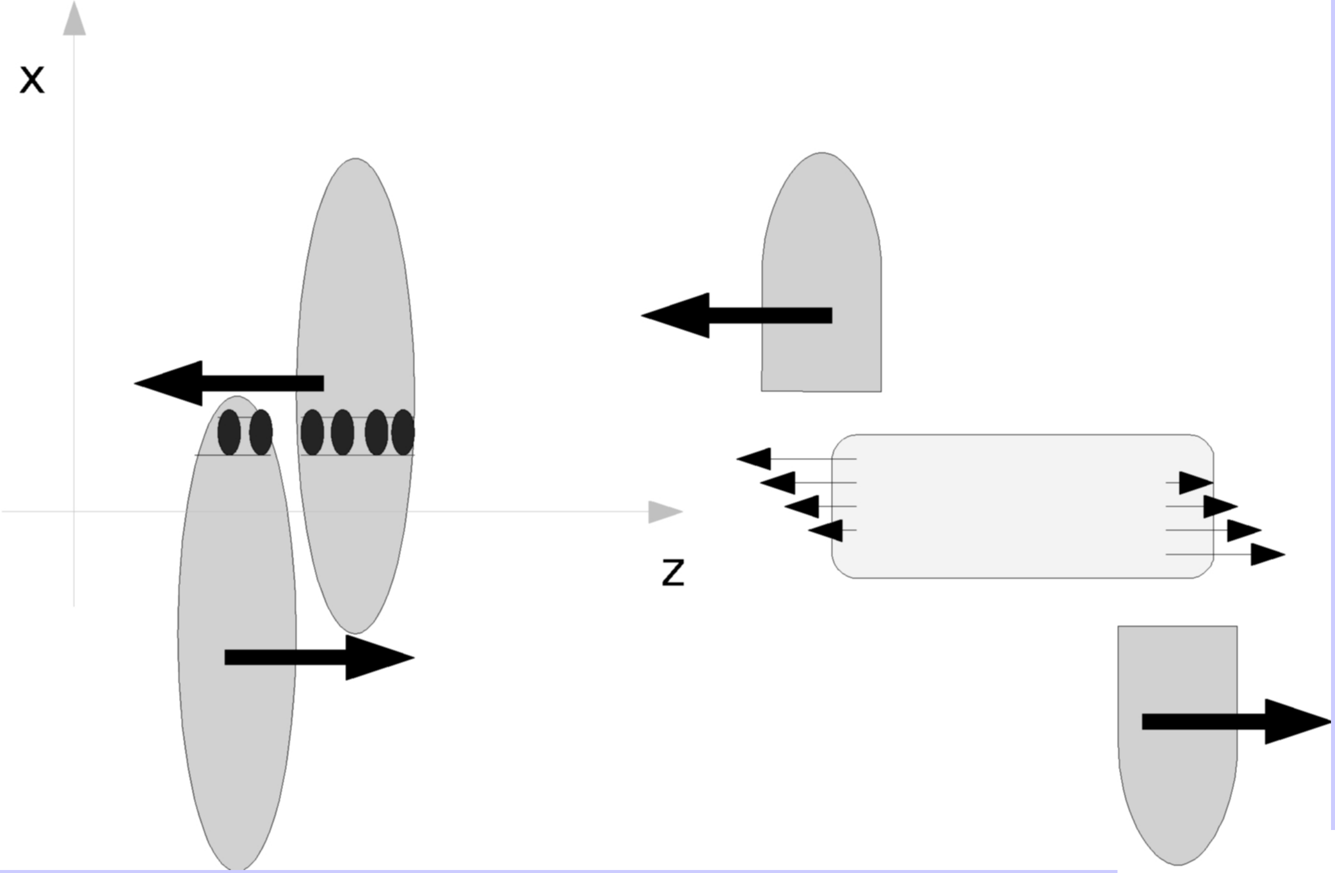
The effects of angular momentum conservation in relativistic heavy ion collisions at the LHC

F. B., F. Piccinini arXiv:0710.5694
F. B., F. Piccinini, J. Rizzo arXiv:0711.1253

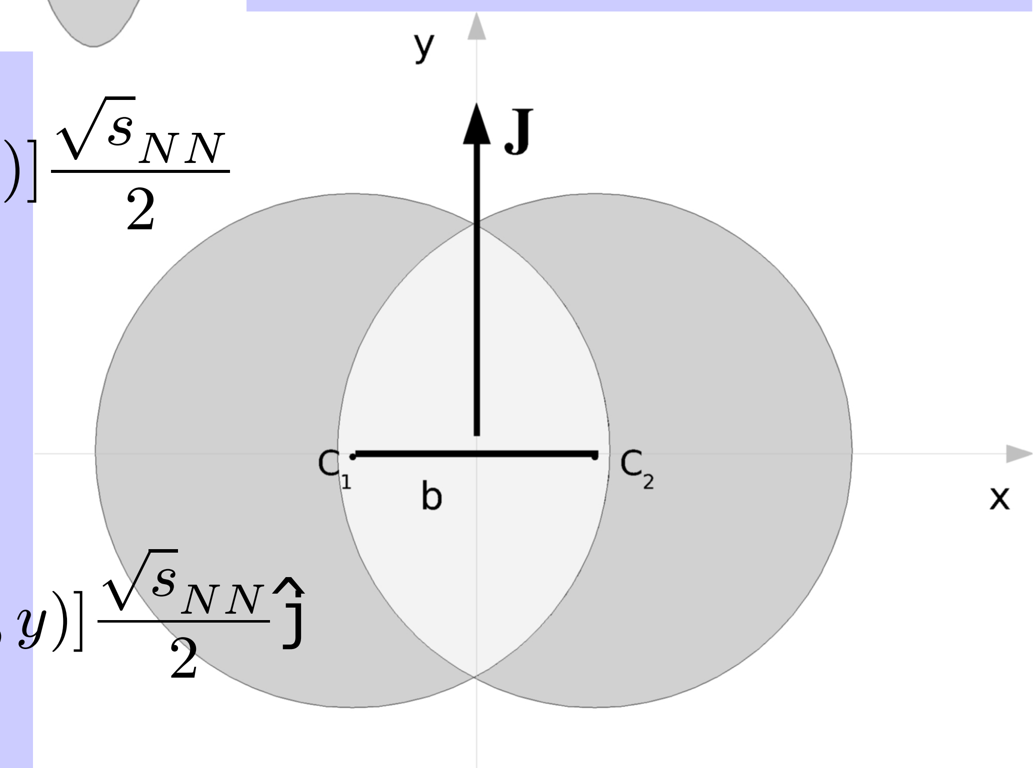
OUTLINE

- Angular momentum conservation in HIC
- Hydrodynamical scheme: enhancement of elliptic flow
- Thermodynamics of rotating systems
- Polarization induced by accelerated motion



Because of the inhomogenety of the thickness function, the momentum of different strips is non-vanishing at the collision time

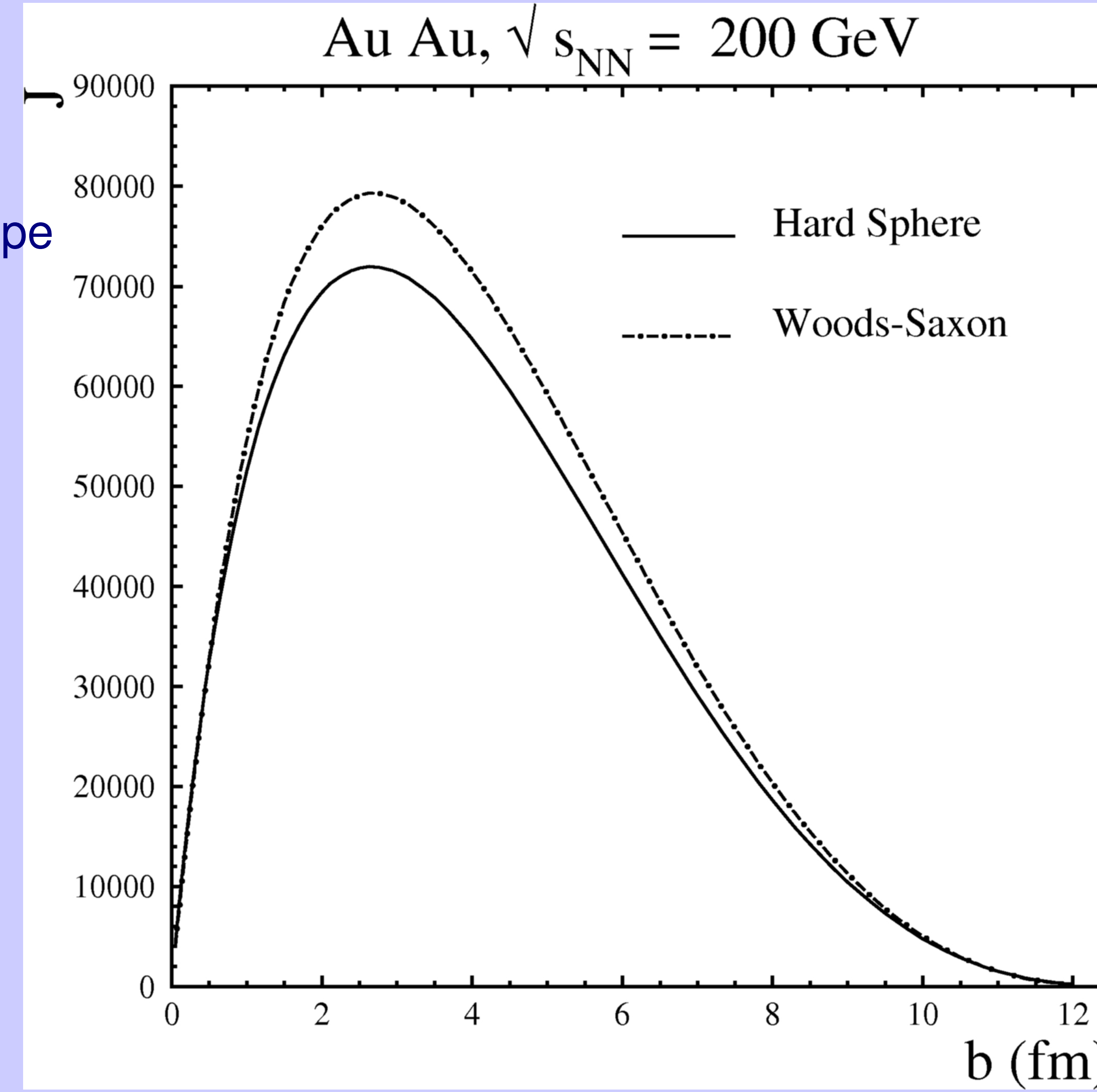
$$\frac{dP}{dxdy} = [T(x - b/2, y) - T(x + b/2, y)]\frac{\sqrt{s}_{NN}}{2}$$



$$\mathbf{J} = \int dS\; x[T(x - b/2, y) - T(x + b/2, y)]\frac{\sqrt{s}_{NN}}{2}\hat{\jmath}$$

Angular momentum of the interaction region

At the LHC, the same shape scaled by a factor 27.5



Hydrodynamics

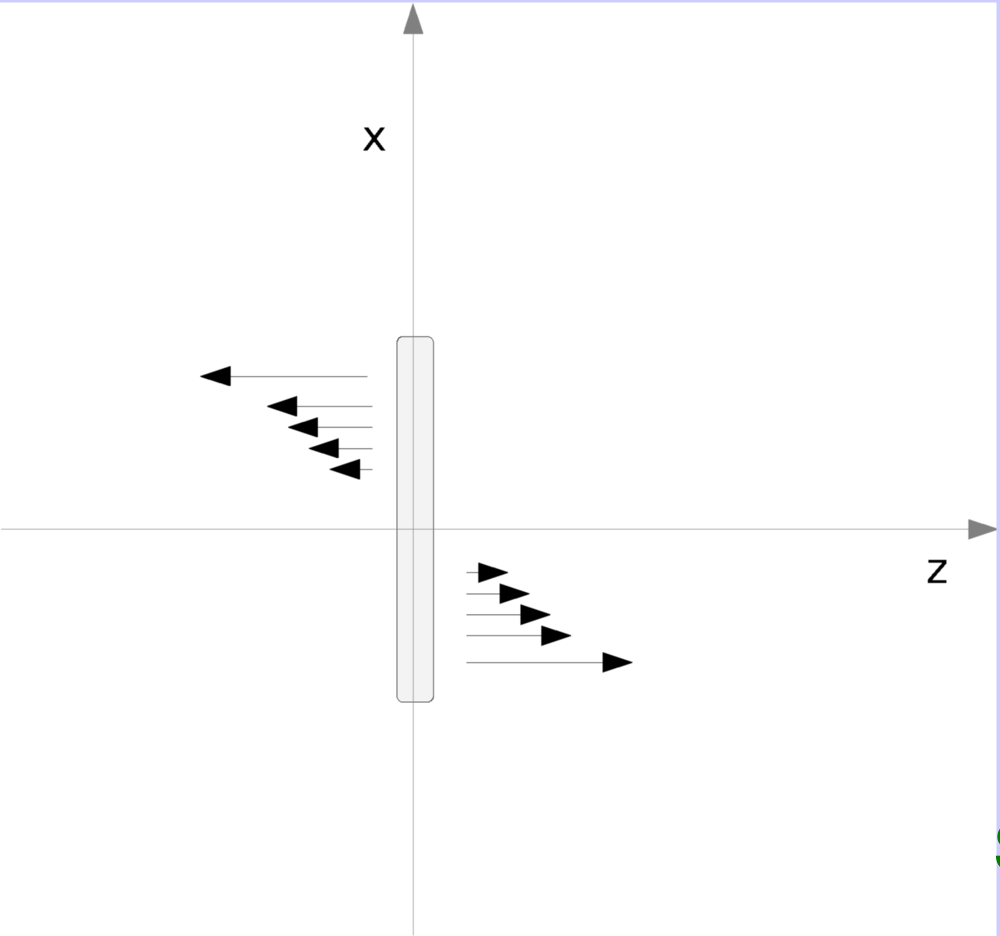
Breaking Bjorken scaling is required unless the proper energy density has an asymmetric dependence on x, which is quite unnatural

$$-\int dV\ x\, T^{0z} = -\int dV\ x(\rho + p)\gamma^2 v_z(x) = J$$

<u>Vorticity is non-vanishing</u>

$$\boldsymbol{\omega} = \frac{1}{2}\nabla \times \mathbf{v} \Rightarrow \omega_y = -\frac{1}{2}\frac{\partial v_z}{\partial x} \neq 0$$

A simple hydrodynamical scheme



Perfect fluid with p=ρ/3

$$(\rho+p)\gamma^2 v_z(t=0) = \frac{1}{\Delta z}\frac{dP}{dxdy}$$

$$(\rho+p)\gamma^2 - p(t=0) = \frac{1}{\Delta z}\frac{dE}{dxdy} =$$

$$\frac{1}{\Delta z}\left[T(x-b/2,y)+T(x+b/2,y)\right]\frac{\sqrt{s}_{NN}}{2}$$

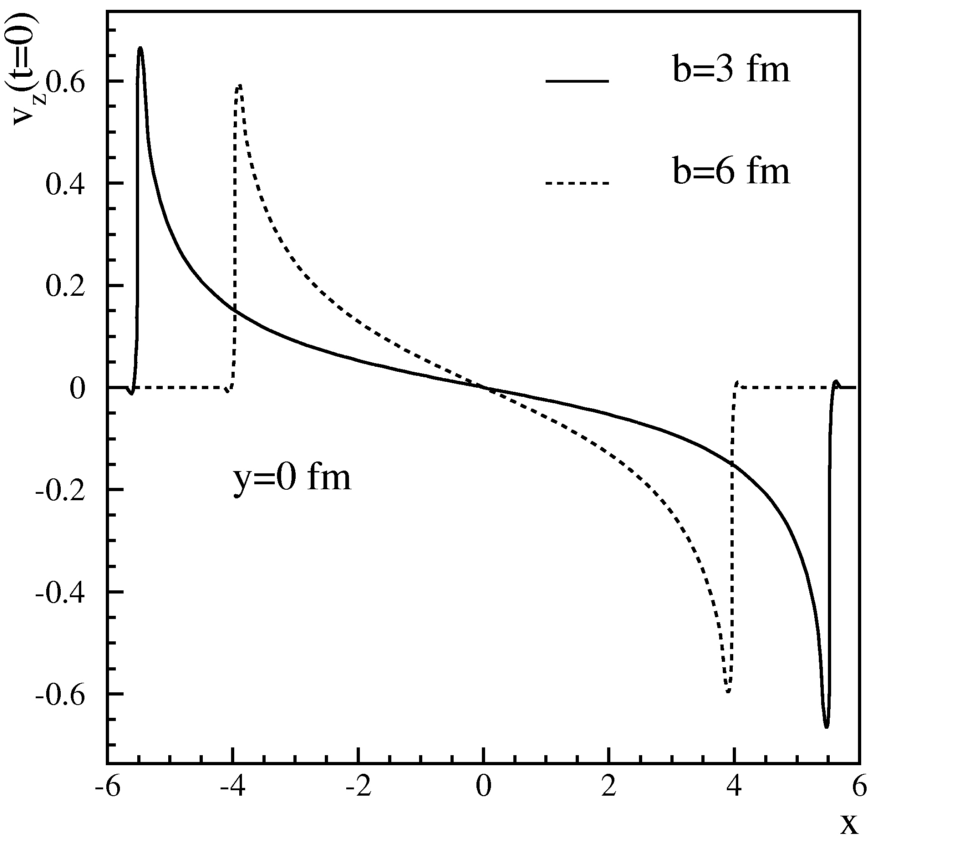
Study initial tranverse expansion rate

vorticity term: speeds up expansion, more in x than y

$$\rho\gamma\frac{\partial u_i}{\partial t}\Big|_{t=0} = -\frac{1}{4}\frac{\partial \rho}{\partial x_i}\Big|_{t=0} = -\frac{1}{4\gamma^2}\frac{\partial \rho\gamma^2}{\partial x_i}\Big|_{t=0} + \frac{1}{2}\rho\gamma^2 v_{z0}\frac{\partial v_{z0}}{\partial x_i}\Big|_{t=0}$$

$$\rho_0 = \frac{1}{\Delta z}\sqrt{4\left(\frac{dE}{dxdy}\right)^2 - 3\left(\frac{dP}{dxdy}\right)^2} - \frac{1}{\Delta z}\frac{dE}{dxdy}$$

$$v_{z0} = \frac{3\frac{dP}{dxdy}}{\sqrt{4\left(\frac{dE}{dxdy}\right)^2 - 3\left(\frac{dP}{dxdy}\right)^2} + 2\frac{dE}{dxdy}}$$



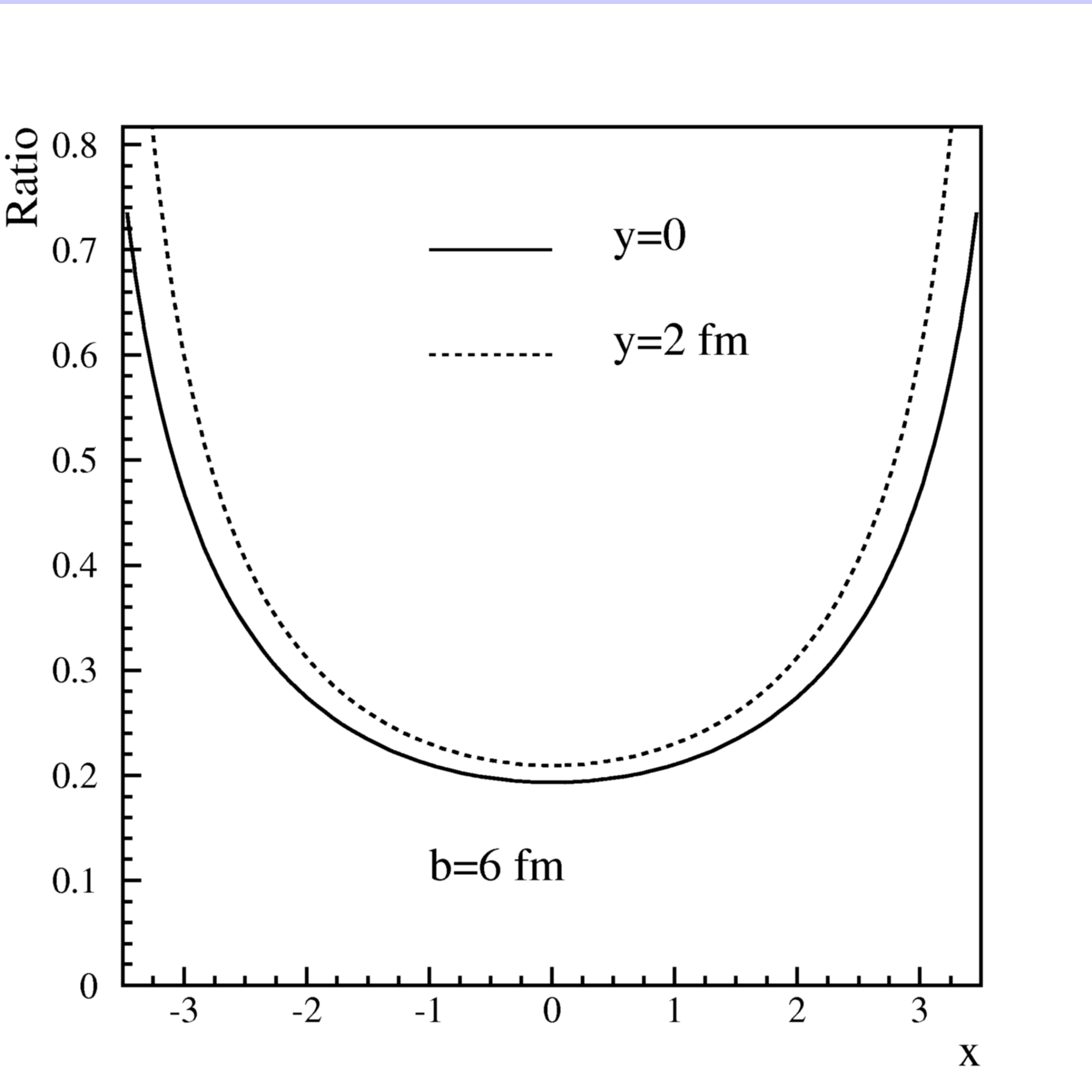
$$\left|\frac{\partial v_{z0}}{\partial x}\right| > \left|\frac{\partial v_{z0}}{\partial y}\right|$$

ENHANCEMENT OF THE ELLIPTIC FLOW!

Hard sphere nuclei, R=7fm

The additional term speeds up expansion and it is not equivalent to making the proper energy density asymmetric in x

Hirano, Tsuda, PRC 66, 054905 (2002),



1. Enforce dJ/dV to be the same

$$\frac{4}{3}\tilde{\rho}\tilde{\gamma}^2\tilde{v}_{z0} = \frac{4}{3}\rho\gamma^2 v_{z0}$$

2. Obtain the initial exp. rate

$$\left.\frac{\partial u_x}{\partial t}\right|_{t=0} = -\frac{1}{4\gamma\rho}\frac{\partial\rho}{\partial x} =$$

$$-\frac{1}{4\tilde{\rho}\tilde{\gamma}}\frac{\partial\tilde{\rho}}{\partial x}\frac{\tilde{\gamma}}{\gamma} + \frac{1}{4\gamma^3 v_{z0}}\frac{\partial\gamma^2 v_{z0}}{\partial x}$$

it is not the same!

(partial) CONCLUSIONS

- Initial angular momentum (reasonably) implies Bjorken scaling breaking and appearance of vorticity

- The additional vorticity term speeds up expansion rate, more in the reaction plane than ortogonally to it, thus enhancing elliptic flow (=centrifugal effect of the angular momentum)

- This may cure the elliptic flow deficit observed recently* when turning on a minimal viscosity; angular momentum conservation is not affected by viscosity

* P. Romatschke and U.Romatschke, ``How perfect is the RHIC fluid?,'' arXiv:0706.1522.
H.Song and U. Heinz, ``Suppression of elliptic flow in a minimally viscous quark-gluon plasma,'' arXiv:0709.0742.

Study of an equilibrated spinning system

Motivations

- Viscosity produces entropy
- Entropy production leads the system towards the full equilibrium configuration
- For a system with fixed, finite, large angular momentum, this is a rigidly rotating fluid

The quick expansion will prevent this to happen, but the system will try to evolve towards that configuration

P. Castorina, D. Kharzeev, H. Satz, Eur. Phys. J. C 52 187 (2007) : ARGUED POSSIBLE FORMATION OF A SPINNING SYSTEM

Rotating system in statistical equilibrium

Grand-canonical partition function with fixed angular momentum – Classical limit (J large)

$$Z = \frac{1}{(2\pi)^3} \int \mathrm{d}^3\phi \, \exp\left[i\mathsf{J}\cdot\phi + \sum_j \frac{\lambda_j}{(2\pi)^3} \int \mathrm{d}^3\mathrm{x} \int \mathrm{d}^3\mathrm{p} \, \mathrm{tr} D^{S_j}(\mathsf{R}_{\hat\phi}(\phi)) \mathrm{e}^{-\varepsilon_j/T} \mathrm{e}^{-i\phi\cdot(\mathsf{x}\times\mathsf{p})} \right]$$

F. B., L. Ferroni, *The microcanonical ensemble of the relativistic quantum gas with angular momentum conservation*, arXiv: 0707.0793.

Saddle-point expansion for *J* and *V* large: introduction of a rotational potential (=angular velocity)

$$\nabla_\phi \left[i\mathsf{J}\cdot\phi + \sum_j \frac{\lambda_j}{(2\pi)^3} \int \mathrm{d}^3\mathrm{x} \int \mathrm{d}^3\mathrm{p} \, \mathrm{tr} D^{S_j}(\mathsf{R}_{\hat\phi}(\phi)) \mathrm{e}^{-\varepsilon_j/T} \mathrm{e}^{-i\phi\cdot(\mathsf{x}\times\mathsf{p})} \right] = 0$$

$$\begin{aligned} \mathsf{J} &= \sum_j \frac{\lambda_j}{(2\pi)^3} \int \mathrm{d}^3\mathrm{x} \int \mathrm{d}^3\mathrm{p} \, \mathrm{tr} D^{S_j}(\mathsf{R}_{\hat\phi}(\phi)) (\mathsf{x}\times\mathsf{p}) \mathrm{e}^{-\varepsilon_j/T} \mathrm{e}^{-i\phi\cdot(\mathsf{x}\times\mathsf{p})} \\ &+ \sum_j \frac{\lambda_j}{(2\pi)^3} \int \mathrm{d}^3\mathrm{x} \int \mathrm{d}^3\mathrm{p} \left[\nabla_\phi \mathrm{tr} D^{S_j}(\mathsf{R}_{\hat\phi}(\phi)) \right] \mathrm{e}^{-\varepsilon_j/T} \mathrm{e}^{-i\phi\cdot(\mathsf{x}\times\mathsf{p})} \end{aligned}$$

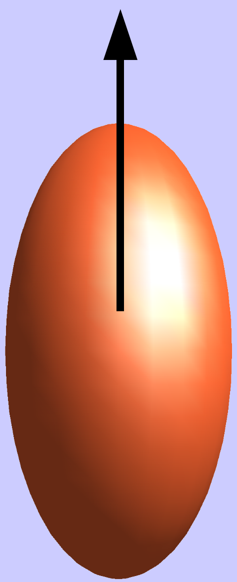
L

S

Rotating system in statistical equilibrium (2)

If the region is symmetric with respect to the J axis:

$$\hat{\phi} = \hat{\mathsf{J}}$$



Definition of the ``rotational potential" ω

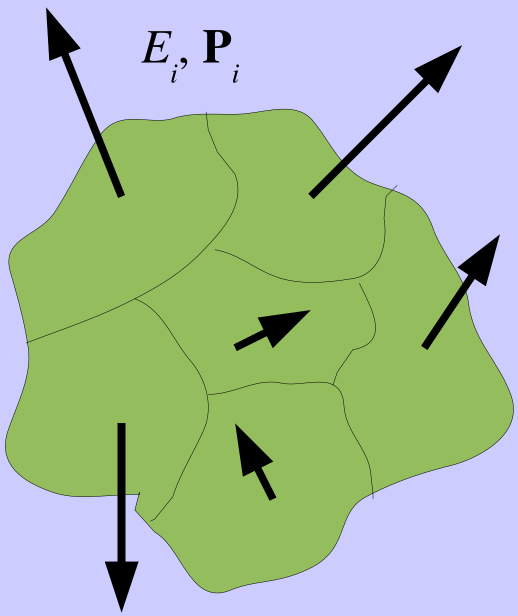
$$\phi \equiv i\omega/T$$

Grand-canonical-rotational partition function:

$$Z_\omega = \exp\left[\sum_j \frac{\lambda_j}{(2\pi)^3}\int \mathrm{d}^3\mathrm{x}\int \mathrm{d}^3\mathrm{p}\,\mathrm{tr}D^{S_j}(\mathsf{R}_{\hat{\mathsf{J}}}(i\omega/T))\mathrm{e}^{-\varepsilon_j/T}\mathrm{e}^{\omega\cdot(\mathsf{x}\times\mathsf{p})}\right]$$

$$S = \frac{U}{T} - \frac{\omega\cdot\mathsf{J}}{T} + \log Z_\omega - \frac{\sum_i \mu_i Q_i}{T}$$

Landau's argument on the equilibrium of macroscopic bodies



$$S = \sum_i S_i(\sqrt{E_i^2 - \mathsf{P}_i^2})$$

$$\frac{\partial S_i}{\partial E_i} = \frac{E_i}{M_i}\frac{\partial S_i}{\partial M_i} = \frac{\gamma_i}{T_i}$$

Maximize entropy with constraints

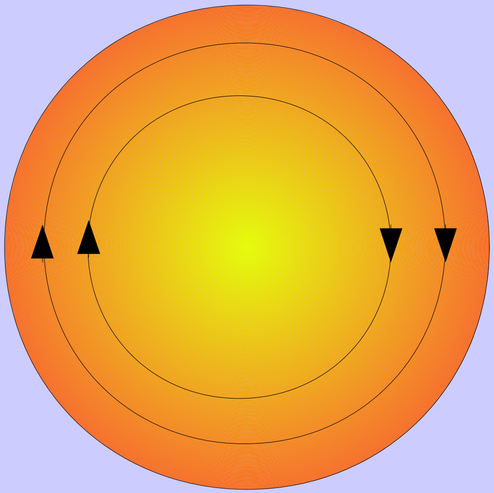
$$\sum_i S_i - \frac{\beta}{T}\cdot\sum_i \mathsf{P}_i - \frac{1}{T}(\sum_i E_i - E_0) - \frac{\omega}{T}\cdot(\sum_i \mathsf{x}_i \times \mathsf{P}_i - \mathsf{J})$$

$$\frac{\gamma_i}{T_i} = \frac{1}{T} \qquad \forall i$$

$$\beta_i = \omega \times \mathsf{x}_i \qquad \forall i$$

$$T_i = \frac{T}{\sqrt{1-(\omega \times \mathsf{x}_i)^2}}$$

Local temperature



Rotating <u>relativistic</u> system in thermodynamical equilibrium:

outer layers are HOTTER than inner layers

see e.g. W. Israel, Ann. Phys. 100 (1976) 310

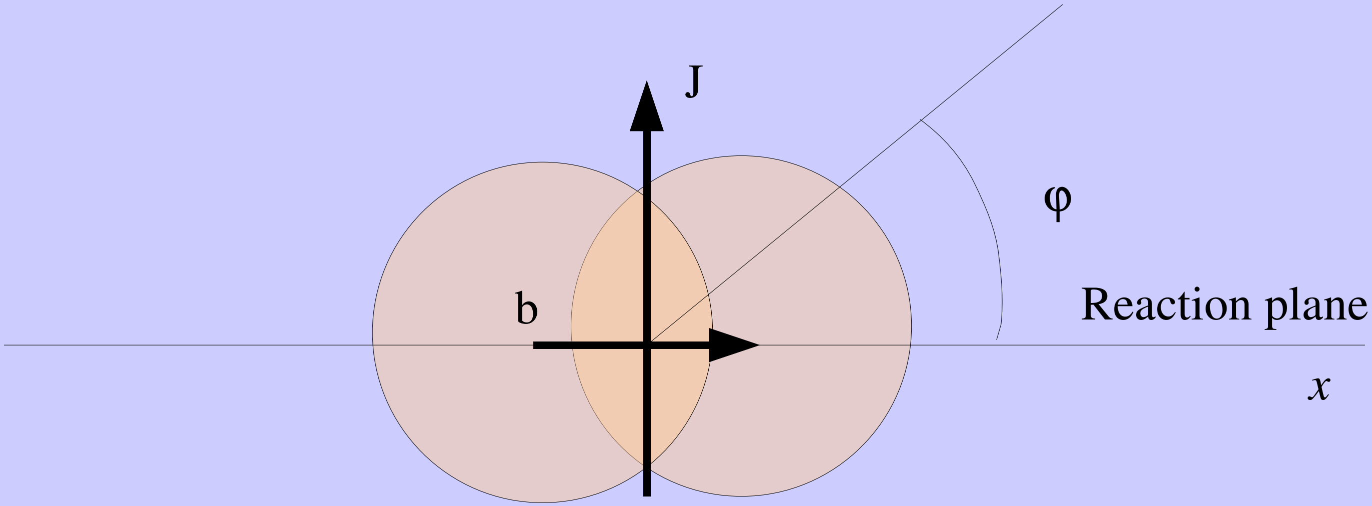
$$T_{loc}(\mathbf{x}) = \frac{T}{\sqrt{1-(\omega \times \mathbf{x})^2}} \qquad (\omega \times \mathbf{x})^2 < 1$$

BEWARE the distinction between LOCAL temperature (in the local rest frame) and GLOBAL temperature

IF THE SYSTEM DECOUPLES AT THE CRITICAL (LOCAL) TEMPERATURE :

$$T = T_c \sqrt{1-\omega^2 R_{max}^2} \qquad \omega R_{max} < 1$$

Spectra (primary hadrons)



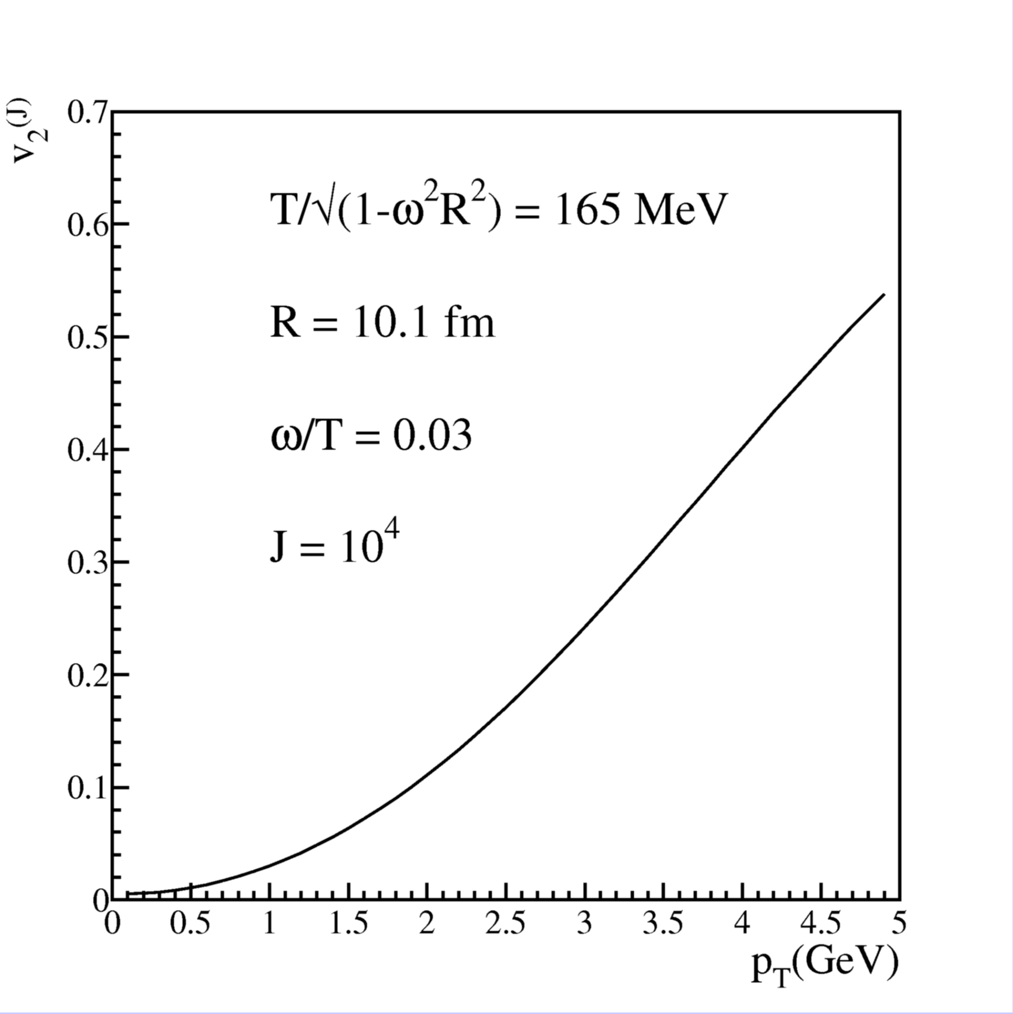
Azimuthal anisotropy

$$\frac{\mathrm{d}n_j}{\mathrm{d}p_T\mathrm{d}\varphi} = \frac{\mathrm{tr}D^{S_j}(\mathsf{R}_{\hat{\jmath}}(i\omega/T))\lambda_j}{4\pi^3}\int \mathrm{d}^3x \frac{p_T m_T \mathrm{K}_1(m_T\sqrt{1-|\omega\times\mathsf{x}|_\parallel^2}/T)}{\sqrt{1-|\omega\times\mathsf{x}|_\parallel^2}} \exp\left[\frac{p_T|\omega\times\mathsf{x}|_\perp\cos\varphi}{T}\right]$$

$$\frac{\mathrm{d}n_j}{\mathrm{d}p_T} = \frac{\mathrm{tr}D^{S_j}(\mathsf{R}_{\hat{\jmath}}(i\omega/T))\lambda_j}{2\pi^2}\int \mathrm{d}^3x \frac{p_T m_T \mathrm{K}_1(m_T\sqrt{1-|\omega\times\mathsf{x}|_\parallel^2}/T)}{\sqrt{1-|\omega\times\mathsf{x}|_\parallel^2}} \mathrm{I}_0\left(\frac{p_T|\omega\times\mathsf{x}|_\perp}{T}\right)$$

Elliptic flow

$$v_2^{(J)} = \frac{\int \mathrm{d}^3x \frac{\mathrm{K}_1(m_T\sqrt{1-|\omega\times\mathsf{x}|_\parallel^2}/T)}{\sqrt{1-|\omega\times\mathsf{x}|_\parallel^2}} \mathrm{I}_2\left(\frac{p_T z\omega}{T}\right)}{\int \mathrm{d}^3x \frac{\mathrm{K}_1(m_T\sqrt{1-|\omega\times\mathsf{x}|_\parallel^2}/T)}{\sqrt{1-|\omega\times\mathsf{x}|_\parallel^2}} \mathrm{I}_0\left(\frac{p_T z\omega}{T}\right)}$$



Polarization

Relation between angular momentum and polarization pointed out by

Z. T. Liang and X. N. Wang, Phys. Rev. Lett. 94, 102301 (2005). PQCD CALCULATION

Qualitative relation between polarization and vorticity

B. Betz, M. Gyulassy and G. Torrieri, Phys. Rev. C 76, 044901 (2007).

Very recent calculation of polarization of photons emitted by a QGP with polarized quarks

A. Ipp et al., ``Photon polarization as a probe for quark-gluon plasma dynamics,'' arXiv:0710.5700.

Polarization: statistical approach

Taking advantage of the formalism developed in:

F. B., L. Ferroni, *The microcanonical ensemble of the relativistic quantum gas with angular momentum conservation*, arXiv 0707.0793.

$$\rho = \frac{1}{2}\left[D^S([p]^{-1}\mathsf{R}_{\hat{\mathsf{j}}}(i\omega/T))[p]) + D^S([p]^\dagger \mathsf{R}_{\hat{\mathsf{j}}}(i\omega/T)[p]^{\dagger -1})\right]$$

$$\Pi = \frac{\mathrm{tr}(\hat{W}\hat{\rho}/m)}{\mathrm{tr}\hat{\rho}}$$

in the lab frame

Spin 1/2

$$\Pi = (0, \mathbf{\Pi}_0)$$

in the particle rest frame

$$2\mathbf{\Pi}_0 \cdot \hat{\mathsf{p}} = \tanh\frac{\omega}{2T}\,\frac{p_y}{p}$$

Longitudinal polarization (helicity) in the particle rest frame

$$2\mathbf{\Pi}_0 \cdot \hat{\mathsf{j}} = \tanh\frac{\omega}{2T}\left(\frac{\varepsilon}{m} - \frac{p_y^2}{m^2 + \varepsilon m}\right)$$

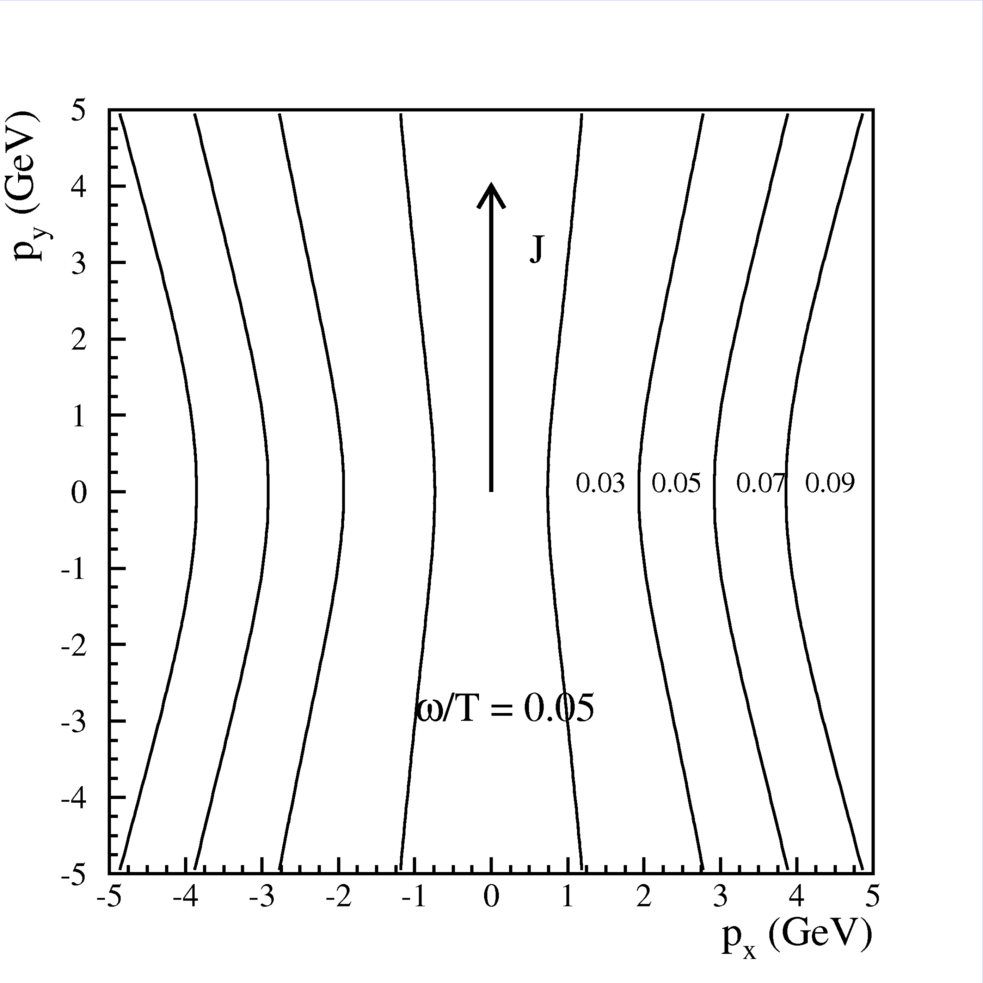
Polarization along the J direction in the particle rest frame

Polarization (2)

Rapidity average

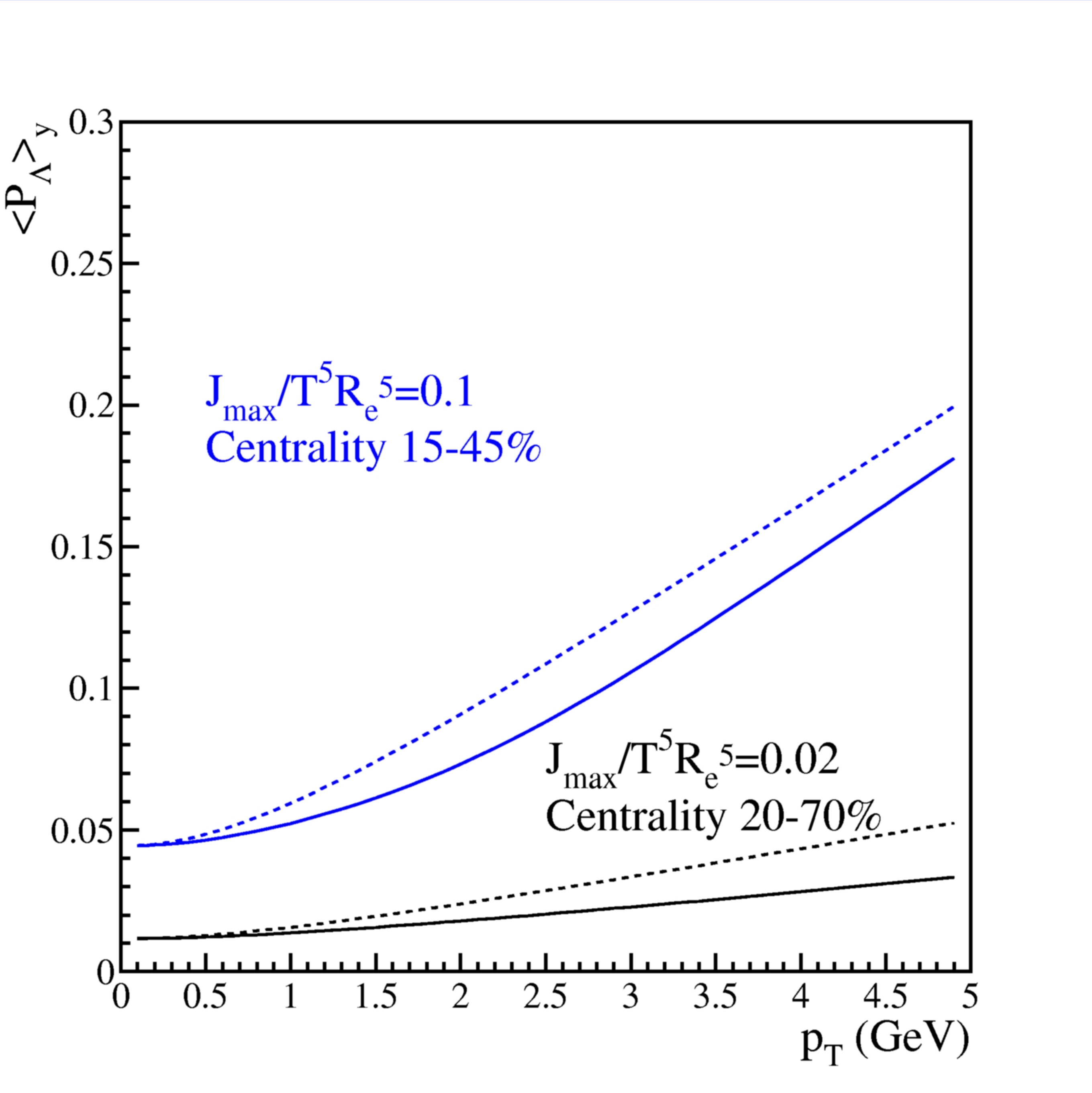
$$\langle 2\mathbf{\Pi}_0 \cdot \hat{\mathsf{p}} \rangle_y \simeq \tanh \frac{\omega}{2T} \; \sin\varphi$$

$$\langle 2\mathbf{\Pi}_0 \cdot \hat{\mathsf{j}} \rangle_y \simeq \tanh \frac{\omega}{2T} \; \left(\frac{m_T}{m} - \frac{p_T^2}{m^2 + m_T m} \sin^2\varphi \right)$$



Polarization along the **J** direction in the particle rest frame

(primary) Λ polarization along *J*



Polarization vector for a generic spin S

$$\mathbf{\Pi}_0 = \frac{\sum_{n=-S}^{S} n e^{n\omega/T}}{\sum_{n=-S}^{S} e^{n\omega/T}} \left(\frac{\varepsilon}{m} \hat{\mathsf{j}} - \frac{\mathsf{p}\cdot\hat{\mathsf{j}}}{m^2 + m\varepsilon} \mathsf{p} \right)$$

ρ_{00} for spin 1

$$\rho_{00}(p) \simeq \frac{1}{3} + \frac{1 - 3(\hat{\mathsf{p}}\cdot\hat{\boldsymbol{\omega}})^2}{9} \frac{\omega^2}{T^2}$$

CAVEATS: decays, jets production

What about general accelerated fluid ?

We conjecture that $\boldsymbol{\omega}$ angular velocity is to be replaced by its local generalization

$$\frac{1}{v^2}\mathsf{a} \times \mathsf{v}$$

Measuring particle polarization and alignment (vectors) can give important information about vorticity in the plasma and, more in general, on the fluid motion.