The Dark Side of the Universe

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The bright side

$e \quad \mu \quad \tau \qquad W^{\pm} \; Z^{0}$

$\nu_e \quad \nu_\mu \quad \nu_\tau \qquad \gamma$

$u \quad u \quad u$

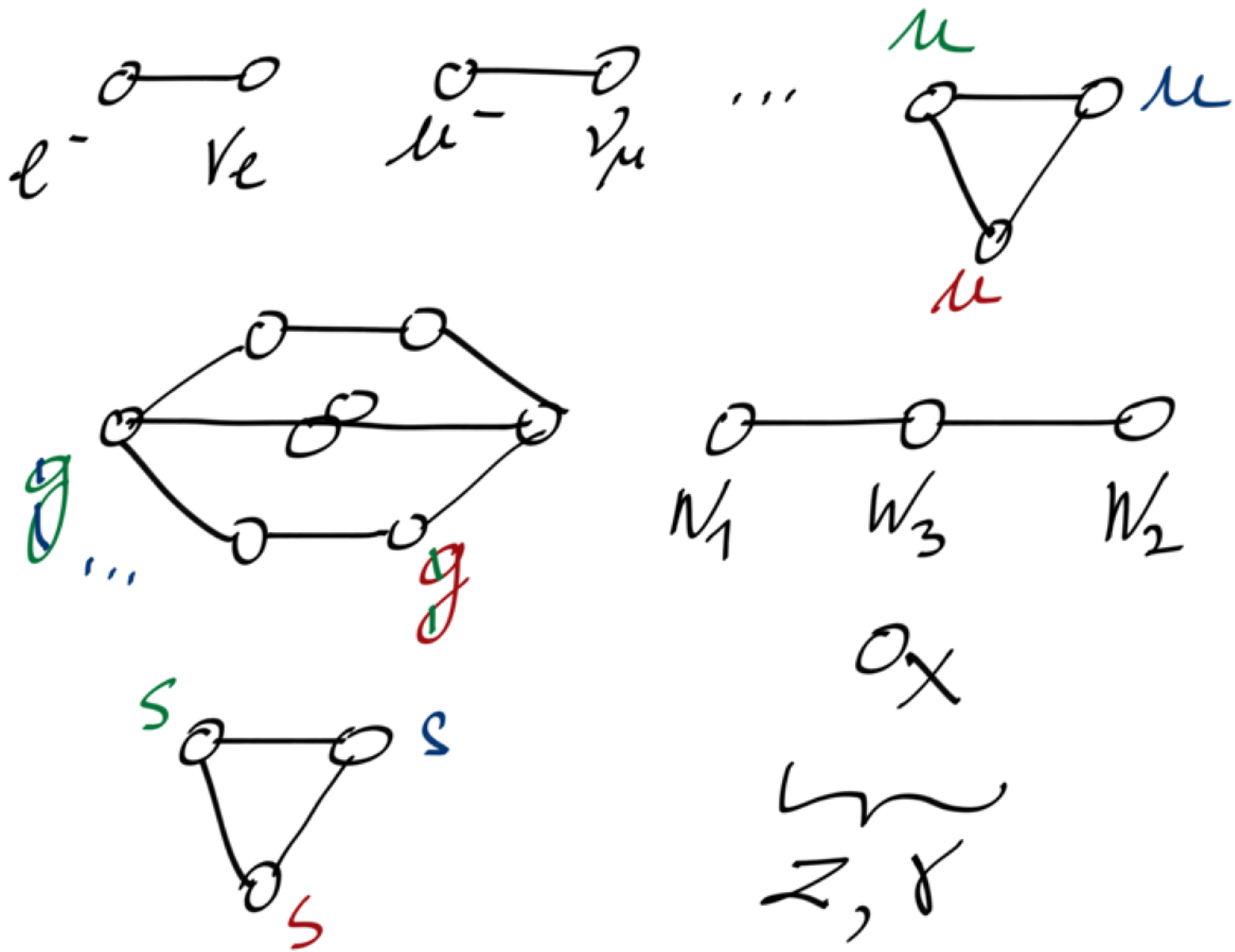
$d \quad d \quad d \qquad g$

$s \quad s \quad s$

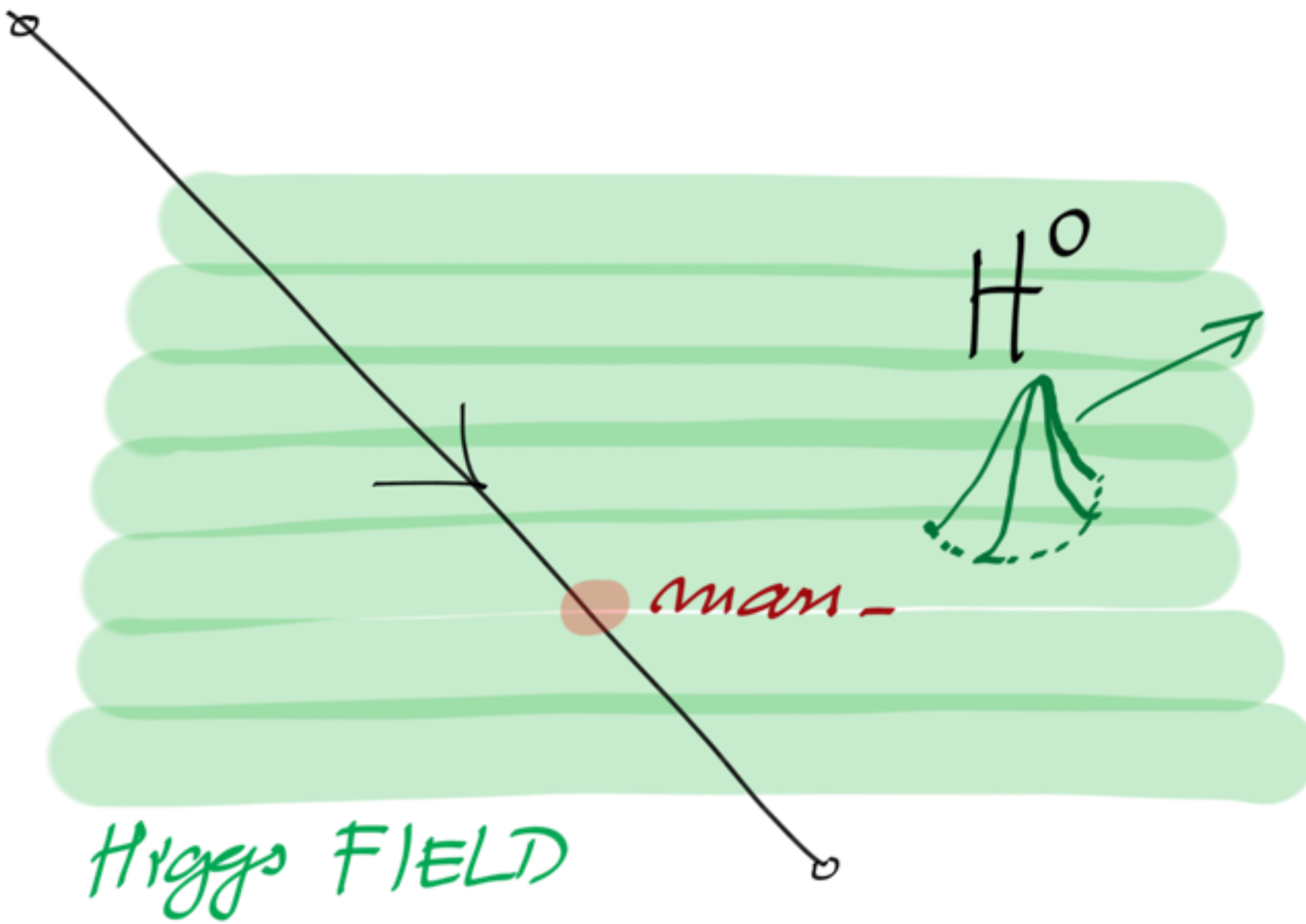
$c \ldots \quad b \ldots \quad t \ldots$

+ antiparticles –

Relations



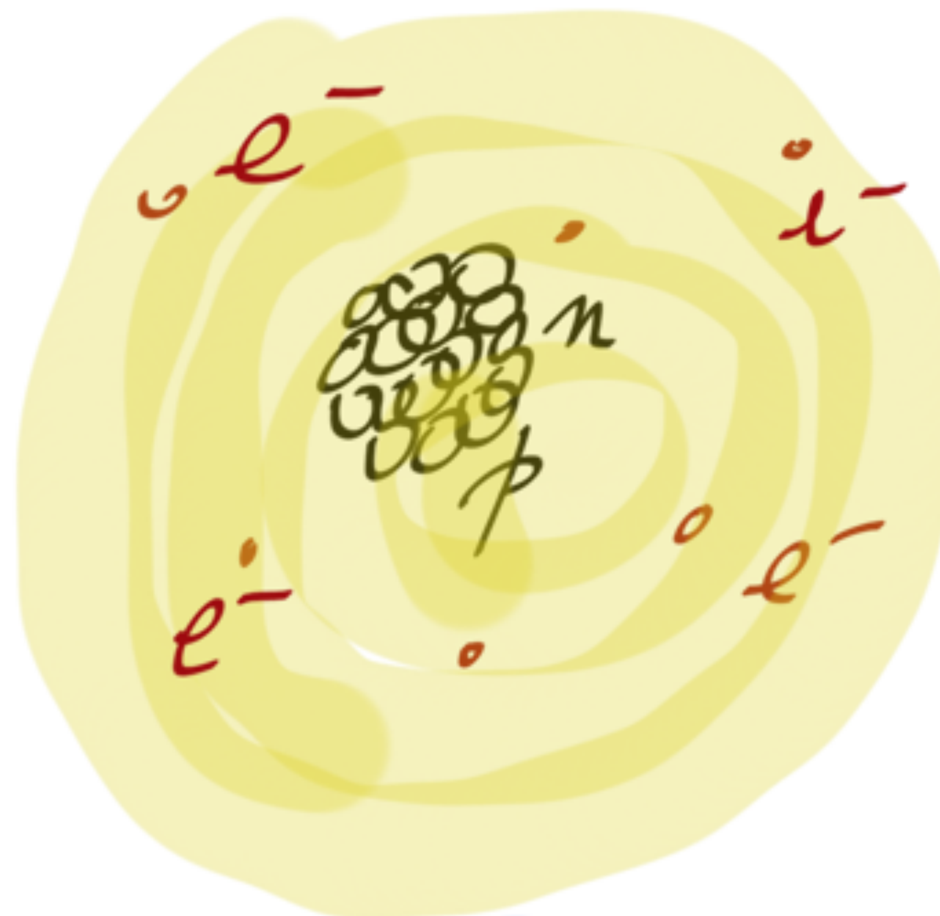
Brout-Englert-Higgs



— All particles couple to it

Composite Particles

Hydrogen



p ? & n $\equiv$

Sizes: Atoms

$$\hbar c = 197\ \text{MeV}\cdot\text{fm}$$

Atomic Physics $\qquad \dfrac{\hbar}{\alpha m_e c}$

$$\alpha = 1/137$$

$$a = \frac{\hbar c}{\alpha m_e c^2} = 137 \times 197 \times \frac{\text{MeV}\cdot\text{fm}}{0{,}5\,\text{MeV}}$$

$$a \sim 0{,}5 \times 10^{-10}\,\text{m}$$

Sizes: Nuclei

$$r \sim A^{1/3} r_0$$

$$r_0 \sim 1\,fm$$

Exercise: Compute the length

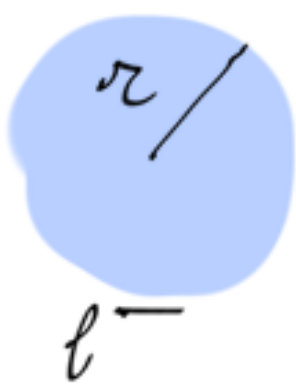
$$\ell \sim \frac{\hbar c}{\mathcal{E}}$$

when $\mathcal{E}$ is as large as 14 TeV

(LHC center of mass energy)

Electron (classical) radius

$$PE: -m\,G\,\frac{M}{(R+x)} \simeq m\,g\,(R+x)$$



$$e\,\frac{1}{4\pi\epsilon_0}\,\frac{e}{r} = \underbrace{\frac{e^2}{4\pi\epsilon_0\hbar c}}_{\alpha}\,\frac{\hbar c}{r}$$

$$\alpha\,\frac{\hbar c}{R} = m_e c^2 = 0{,}5\,MeV$$

$$R \simeq 3\,fm$$

like a nucleus with $A = 27$??

Gauss

$$\Phi_\Sigma = \frac{q_T}{\epsilon_0}$$

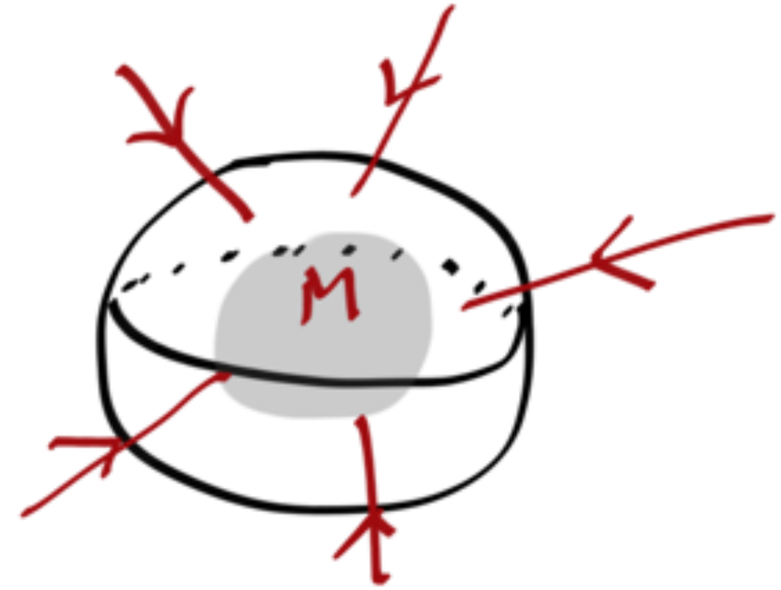
$$= \frac{4\pi}{4\pi\epsilon_0} q_T =$$

$$= 4\pi \left(k q_T \right)$$

Similarly $\Phi_\Sigma = 4\pi \left(G M \right)$

$$\Phi_\Sigma = -\left(4\pi R^2 \right) g$$

$$g = -\frac{GM}{R^2} \left(= -G \frac{4}{3}\pi R \rho \right)$$



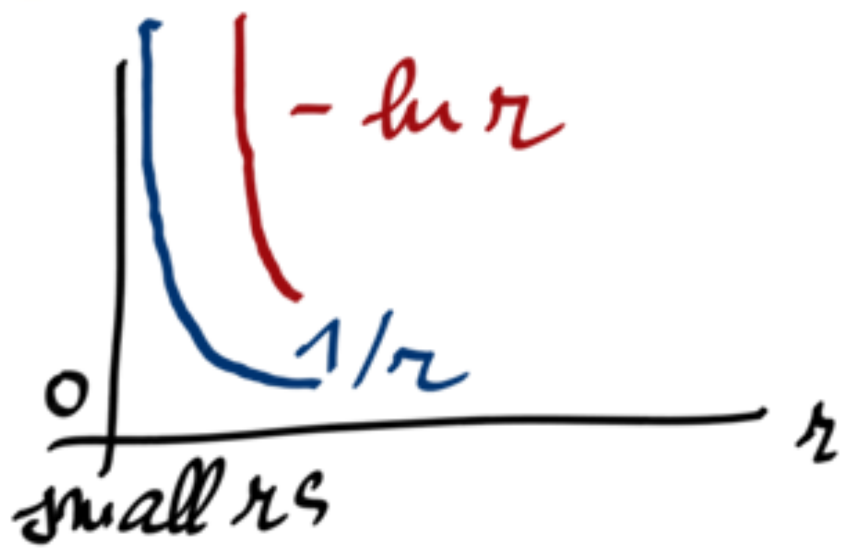
$F = m g$

$(F = qE)$

Antiparticles

In the theory with e^- e^+ γ (QED)

$$U \sim -\ln(r) \quad \text{as } r \to 0$$



So that the electron can be as small as a "point"

e^+ μ^+ τ^- $\bar{u}$ $\bar{d}$ $\bar{s}$ $\bar{c}$ $\bar{b}$ $\bar{t}$ ....

Exercises

I. Compute the numerical value of the electron's classical radius

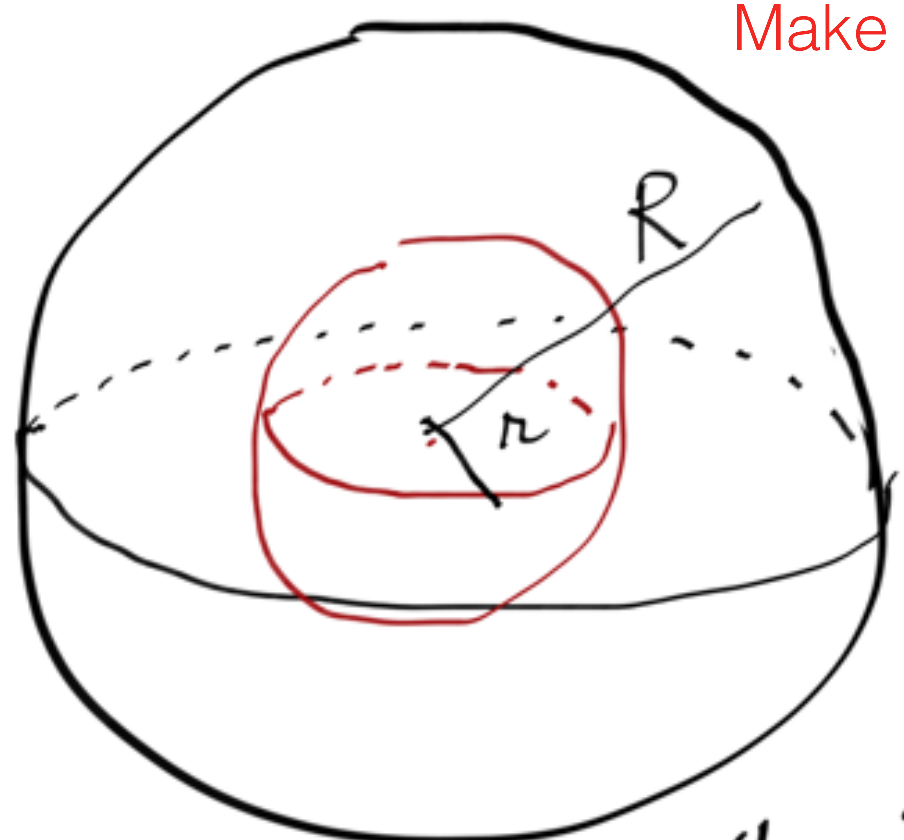
II. If r has dimensions of a distance can we write $\ln(r)$?

What about $\ln\left(\frac{mc^2}{\hbar c} r\right)$?

or $\ln\left(\frac{\hbar}{mc} r\right)$?

Rotation curves

Make use of the Gauss theorem

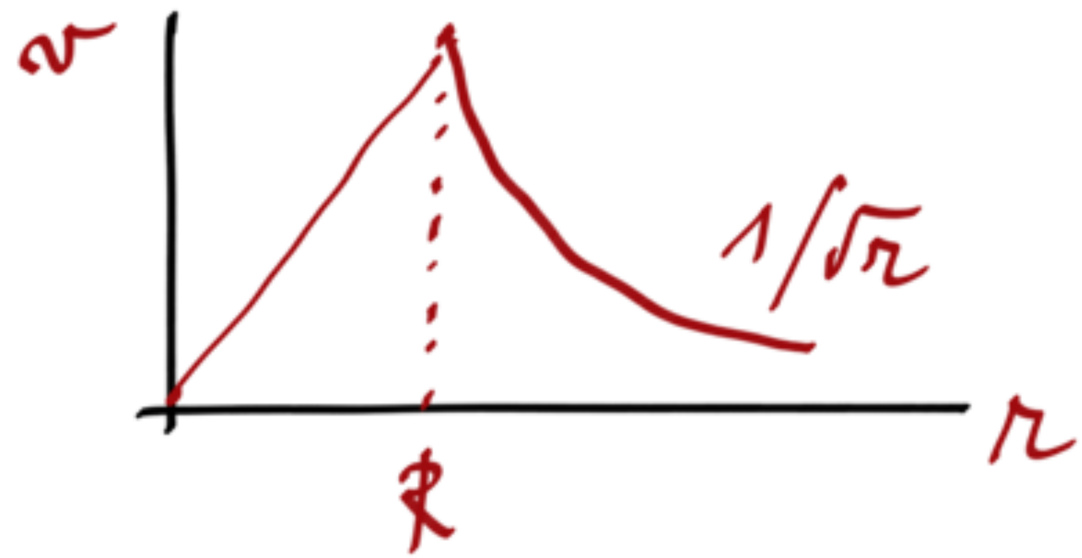


$$
r < R \begin{cases} g(r) = \dfrac{GM}{r^2} = G\,\dfrac{\frac{4}{3}\pi r^3 \rho}{r^2} \sim r \\[2ex] g(r) = \dfrac{v^2}{r} \quad \text{thus} \quad \boxed{v \sim r} \end{cases}
$$

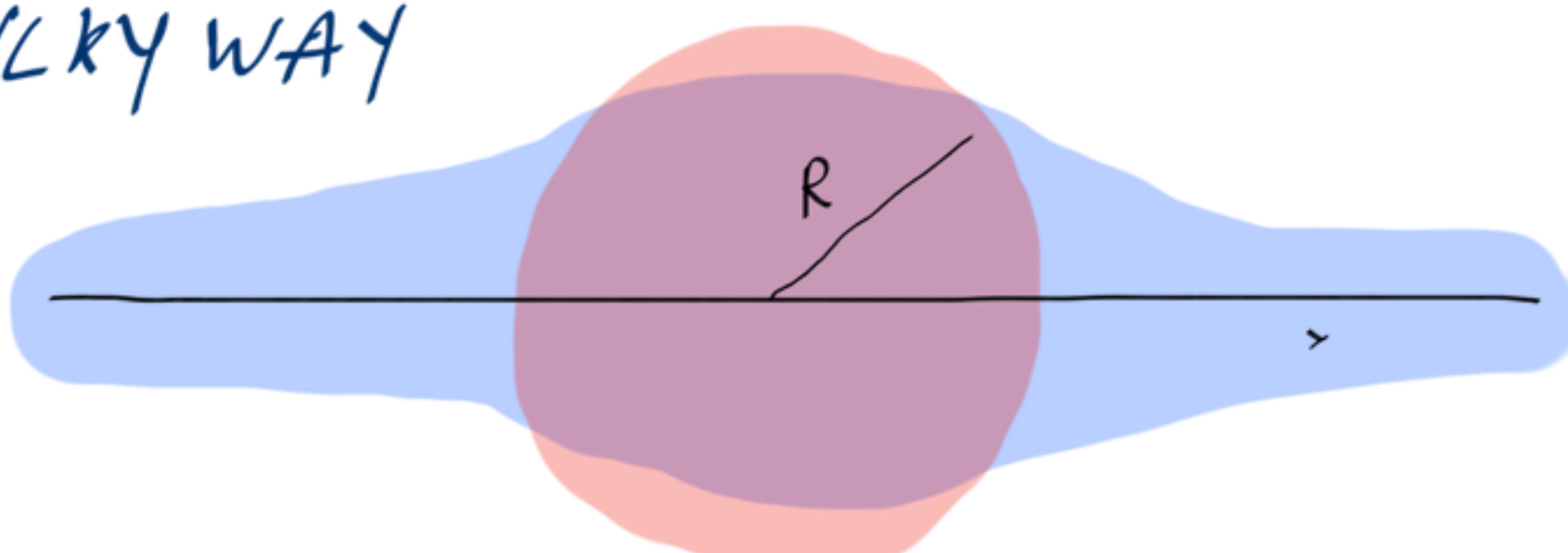
Rotation curves II.

$$r > R \quad \begin{cases} g(r) = \dfrac{GM}{r^2} \\[2ex] g(r) = \dfrac{v^2}{r} \end{cases}$$

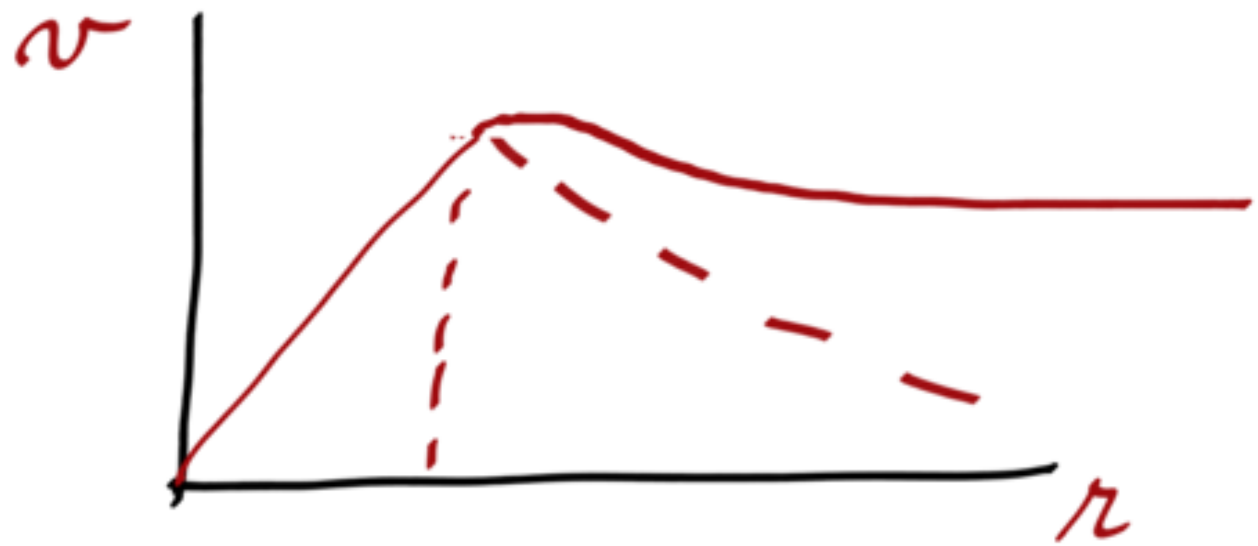
$$v \sim \frac{1}{\sqrt{r}}$$



MILKY WAY



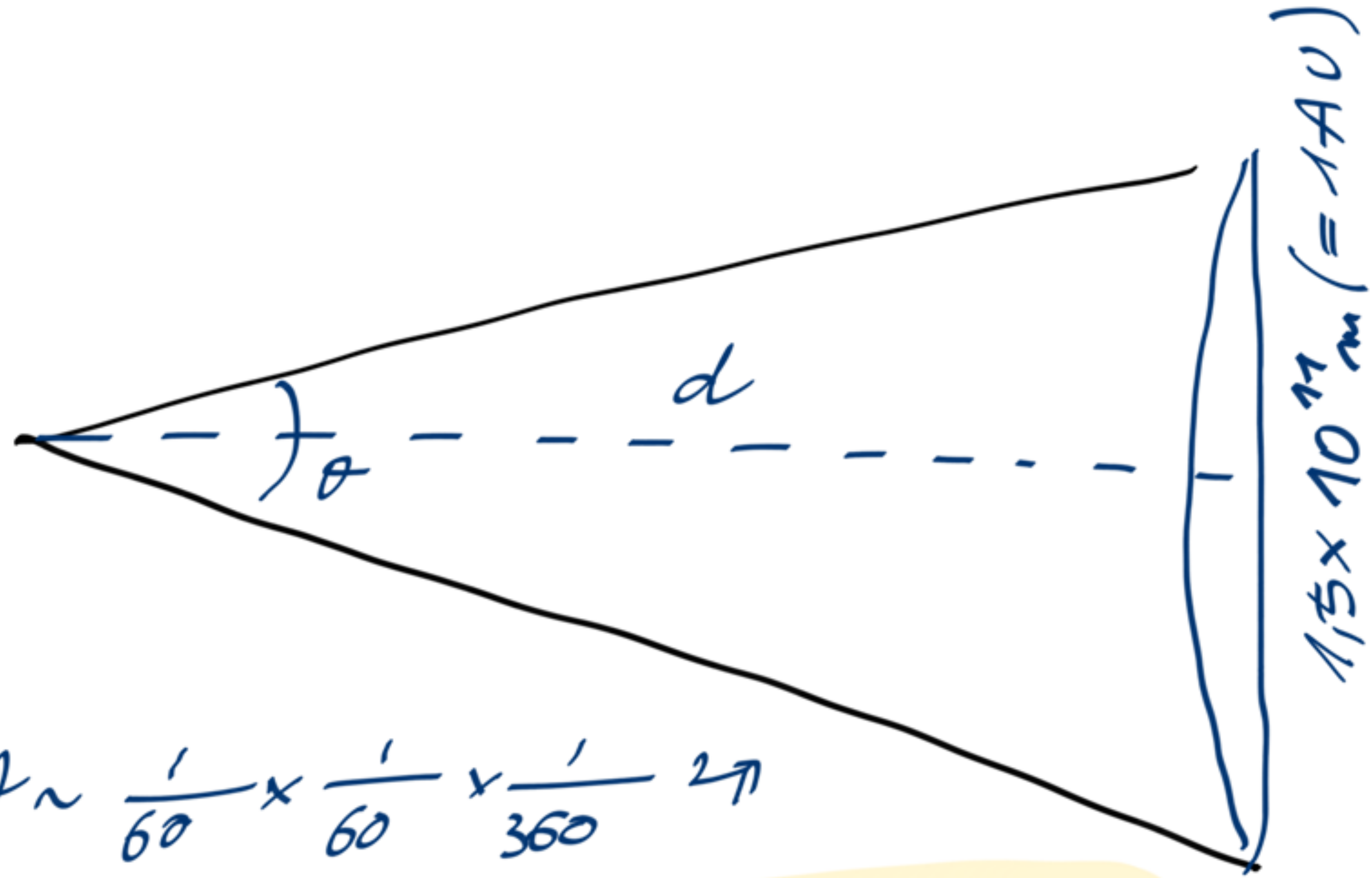
Rotation curves III.



The Sun moves faster (220 km/sec) than what expected (160 km/sec)

THE GRAVITY PULL IS WAY LARGER THAN WHAT CAN BE ESTIMATED AS DUE TO STARS.

PARSEC

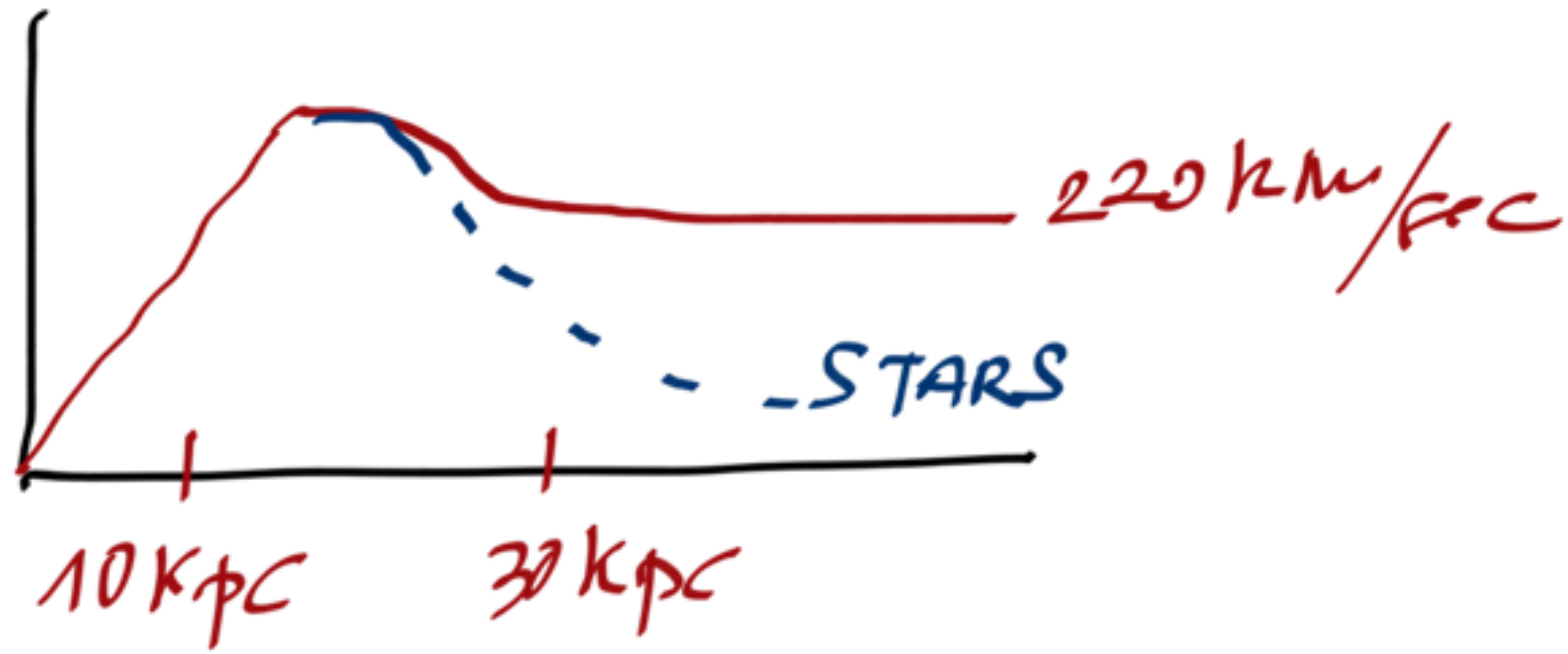


$$\theta \sim \frac{1}{60} \times \frac{1}{60} \times \frac{1}{360} 2\pi$$

$= 1$ arcsec.

$d \sim 3.26$ ly

Rotation curves IV.

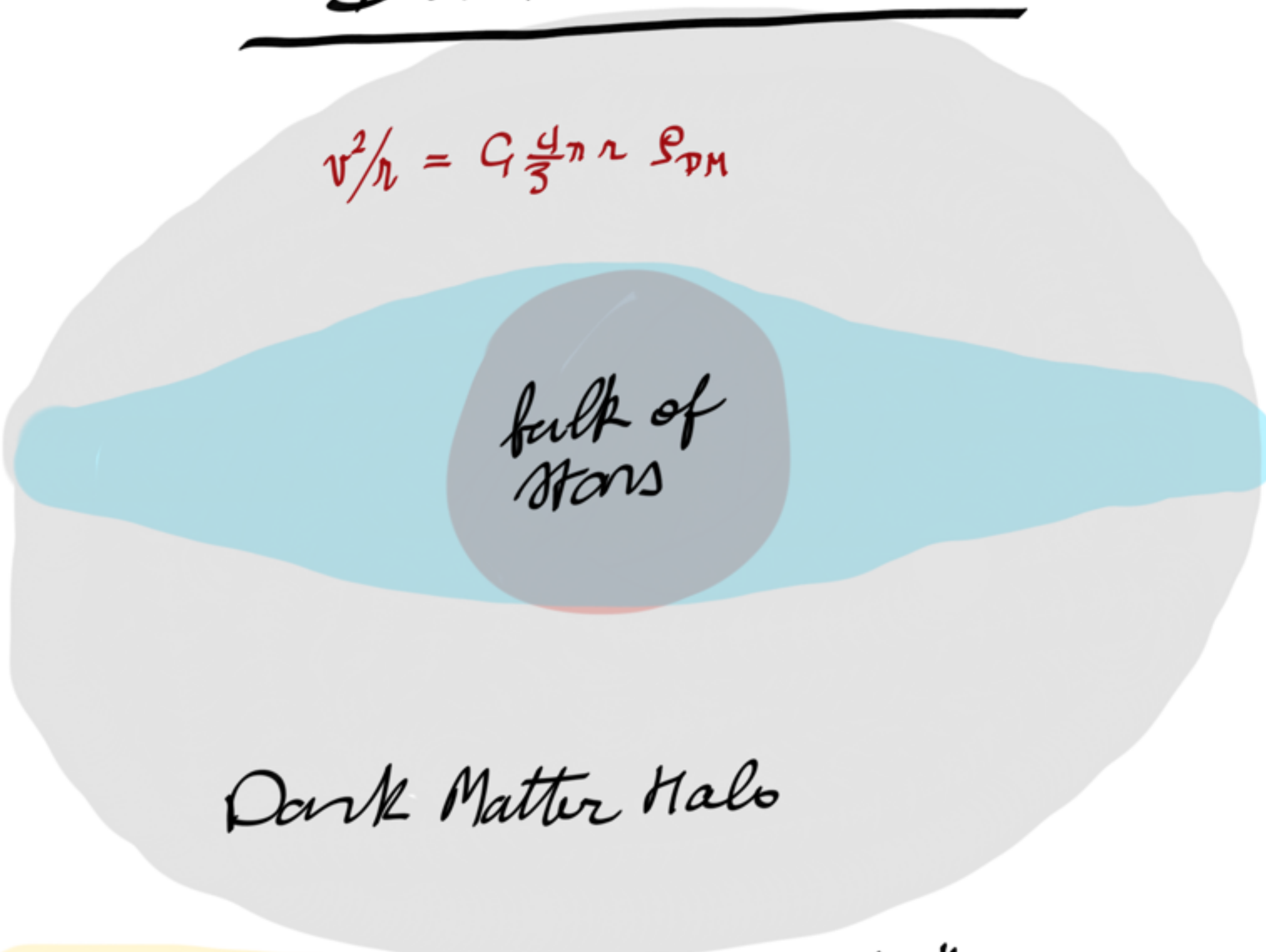


HYP. DIFFUSE HALO OF NON-LUMINOUS MATTER (DARK)

Actually also planets are DARK....

Dwarfs are DARK...

Dark Matter



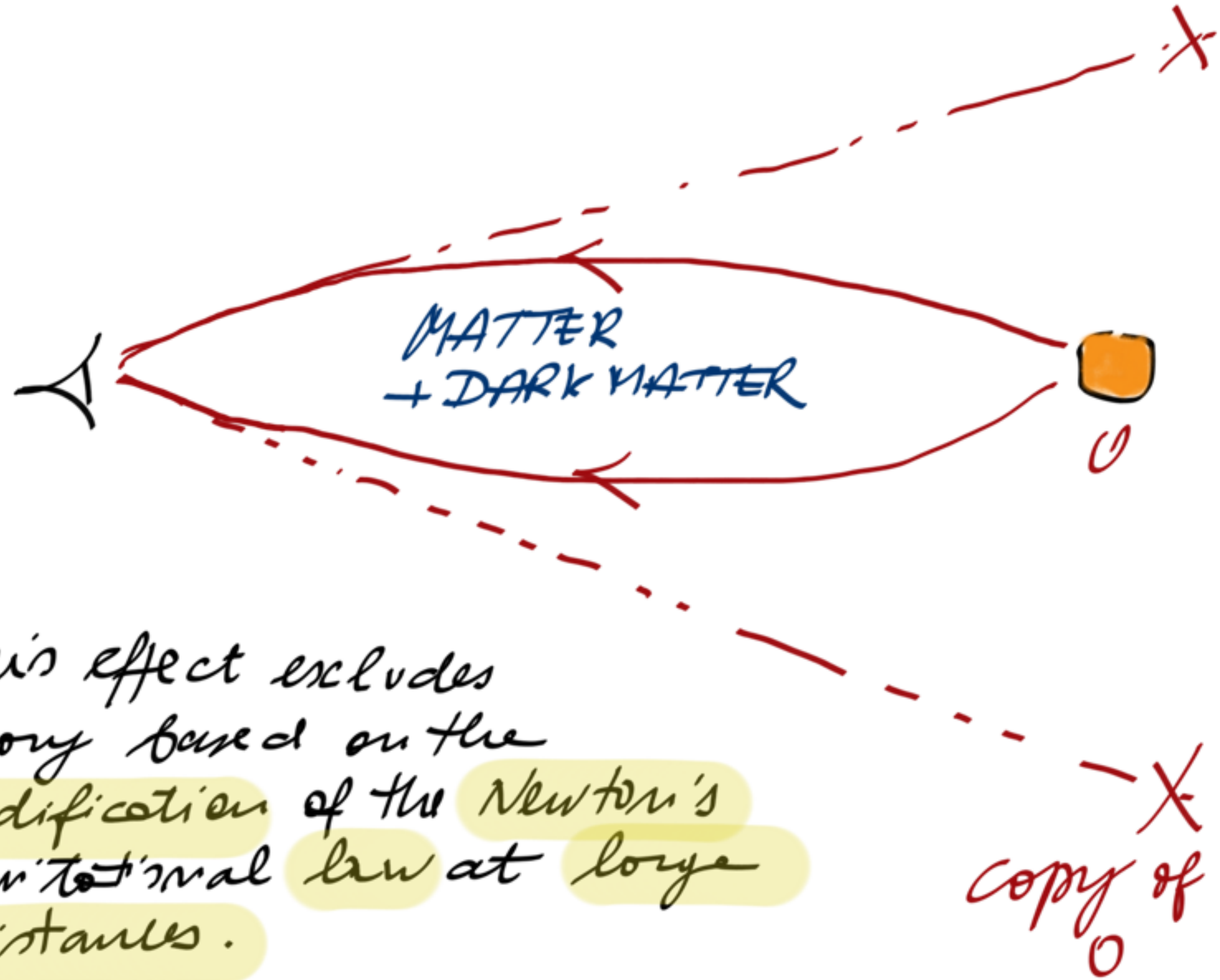
$\rho_{DM}^{loc} \sim 5 \times 10^{-25}\, gr/cm^3$ vs $\rho_{\oplus}^{earth} \sim 5\, gr/cm^3$

IDENTIKIT

1. Interacts gravitationally
2. Does not emit UV, IR, X, RADIO, ...
3. Probably pervades universe ~ uniformly
4. Should be cold
5. No planets // Rocks // Dust

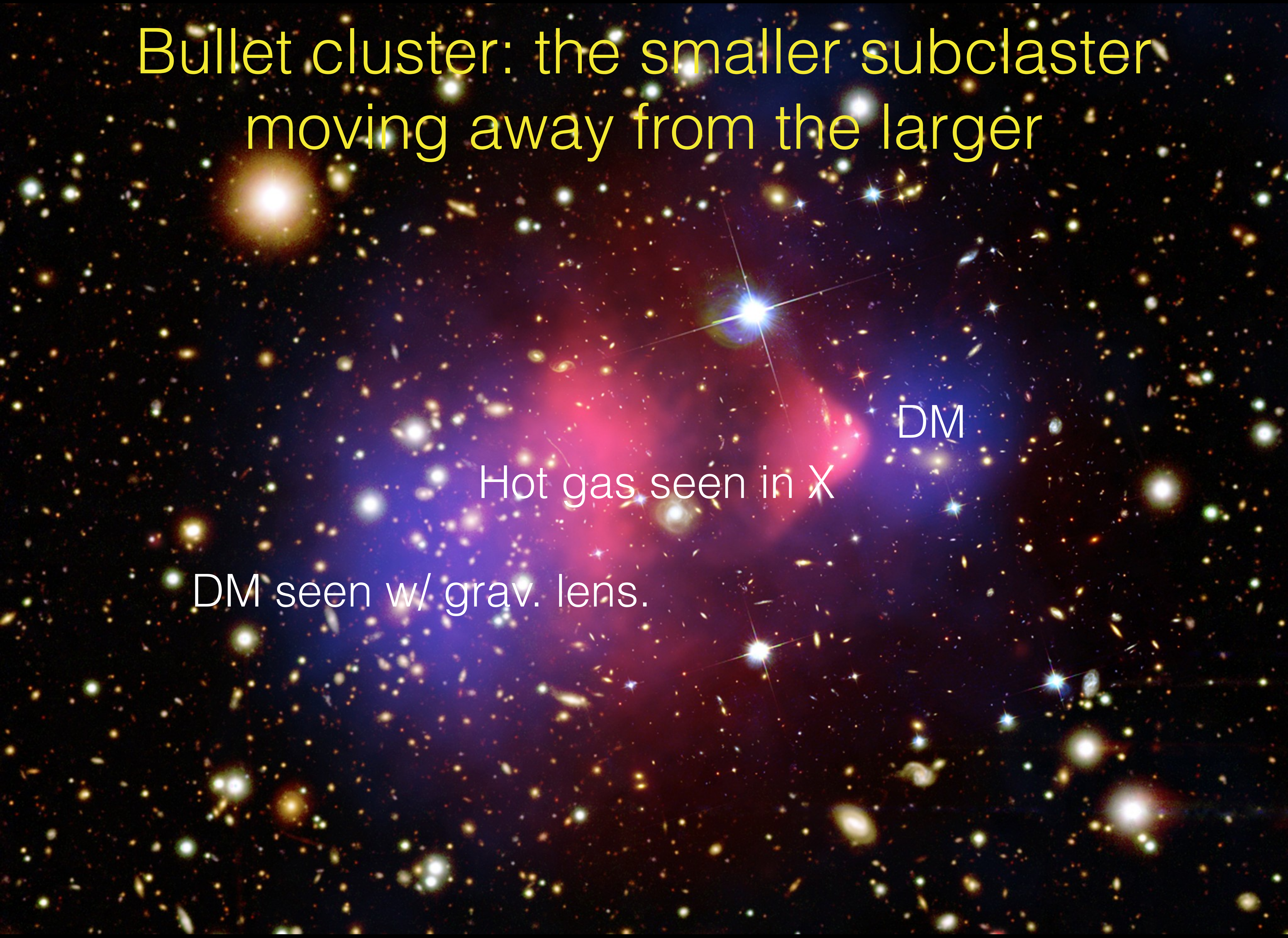
   (distant objects would look more opaque)

Gravitational Lensing



This effect excludes theory based on the modification of the Newton's gravitational law at large distances.

Bullet cluster: the smaller subclaster moving away from the larger



Density Ratios

$$\frac{\rho_f + \rho_\gamma + \rho_\nu + \rho_{DM}}{\rho_{crit}} + \Omega_\Lambda = 1$$

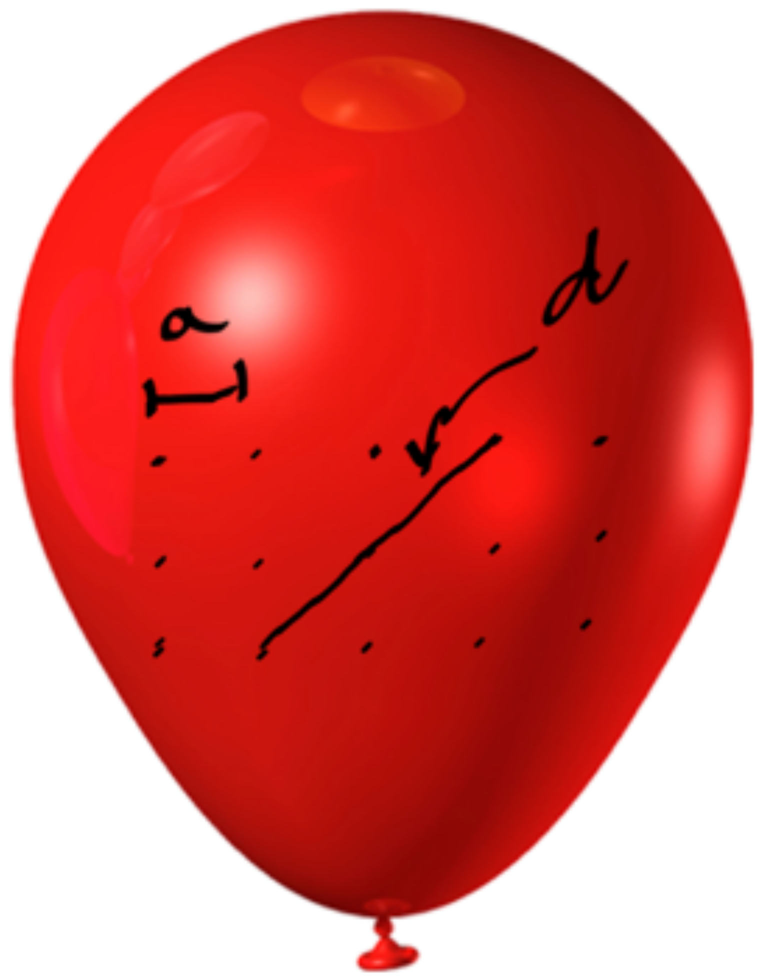
FLAT UNIVERSE

On large scales DM dominates the Universe

84.5% of matter is DARK

N.B. ρ_f includes barions, mesons, leptons

Scuit & the Expanding Universe



$a_0 = 1$

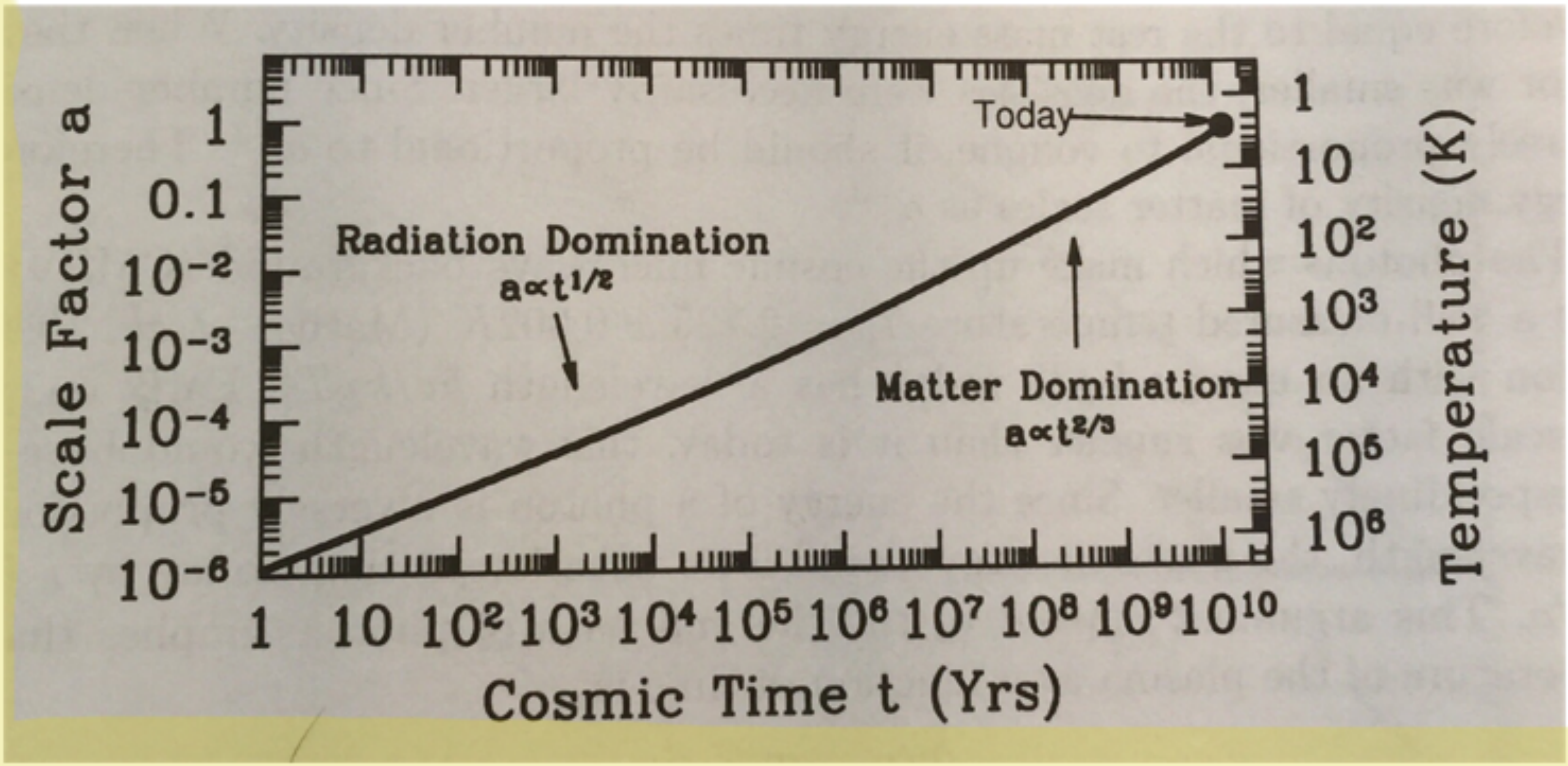
$$d = \sqrt{(2a)^2 + (2a)^2} = 2a\sqrt{2} \text{ (cm)}$$

but a could be

$$a = a(t)$$

Inflate balloon!

Evolution of a(t)



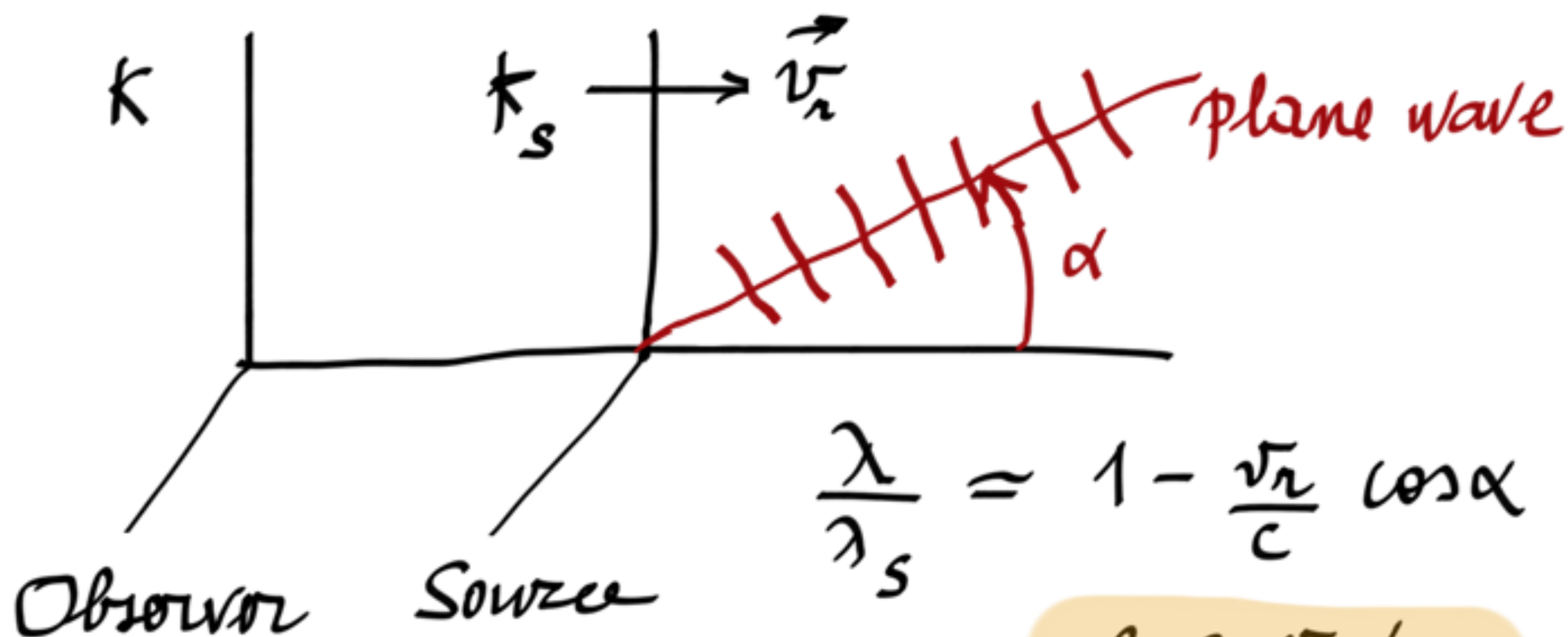
$$\rho_b \sim m_b \, n_b \sim 1/a^3$$

$$\rho_\gamma \sim \mathcal{E}_\gamma \, n_\gamma \sim 1/\lambda \cdot 1/a^3 \sim 1/a^4$$

Redshift

$$\frac{\lambda}{\lambda_s} = 1 + z \quad , \quad z > 1$$

from z one can reconstruct the recession velocity v_r



$$\frac{\lambda}{\lambda_s} \simeq 1 - \frac{v_r}{c} \cos\alpha$$

$$z \simeq v_r / c$$

Redshift II.

On the other hand

$$\frac{\lambda}{\lambda_s} \sim \frac{1}{a(t)}$$

thus we expect

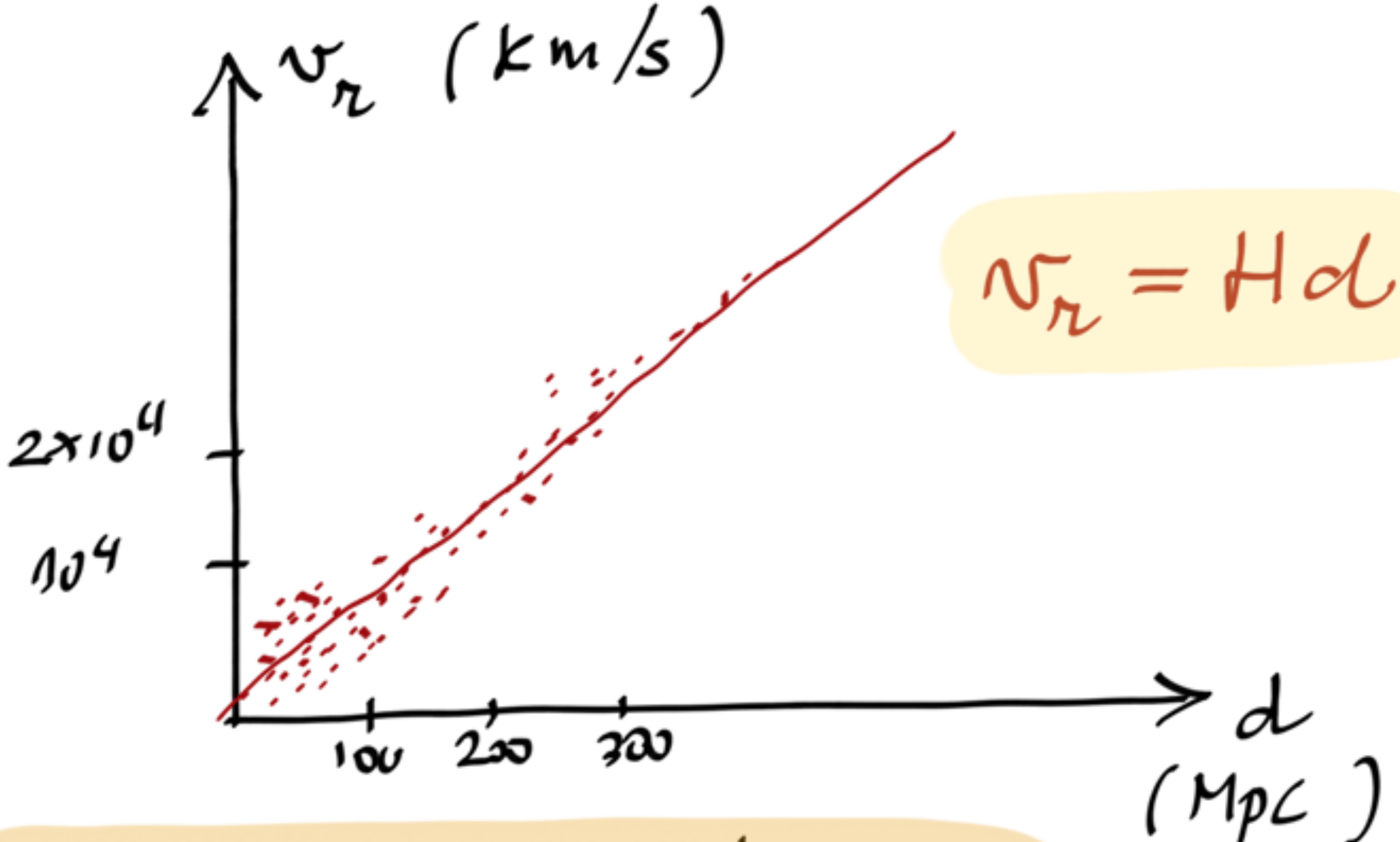
$$\nu \sim \frac{\#}{a}$$

Exercise. Again, what are the dimensions of a?

Hubble law

d = distance between two galaxies

$= x a$



$$v_r = H d$$

$$H_0 = 100\, h \text{ km/s Mpc}^{-1},\ h = 0.7$$

Hubble law II. & ρ_{crit}

$$v_r = \dot{d} = \dot{a}x + a\dot{x} = \dot{a}x$$
$$= Hax = Hd$$

$$\boxed{H = \frac{\dot{a}}{a}}$$

$$\boxed{\rho_{crit} = \frac{3H_0^2}{8\pi G} \simeq 5\,\mathrm{KeV/cm^3}}$$

$\rho = \rho_{crit} \rightarrow$ FLAT (EUCLIDEAN) UNIVERSE

$\rho \neq \rho_{crit} \rightarrow$ CURVED UNIVERSE

Density ratios II

$$\Omega_i = \rho_i / \rho_{crit}$$

$$\Omega_b = 0.049 \pm 0.02$$

$$\Omega_\gamma = \frac{\pi^2/15 \, T^4}{\rho_{crit}} = (5 \pm 0.2) \times 10^{-5}$$

$$\Omega_\nu = 0.001 \div 0.02$$

$$\Omega_{DM} = 0.267 \pm 0.01$$

... Overall, the dominant Ω is Ω_Λ

— DARK ENERGY —

other than matter & radiation.

Dark Matter particles?

1. Interacts gravitationally
2. Stable
3. Massive
4. Neutral
5. "Cold"

Neutrinos?

Early Universe: hot & dense

$$e^- p \rightleftarrows n \nu$$

Expansion & Cooling: few leftover ν's

relic ν's $\simeq 100/cm^3$

Left-Over Particles

$n_\chi \sim n_{\bar{\chi}}$ @ $t=0$

(1) $\chi\bar{\chi} \rightleftarrows$ STANDARD PARTICLES

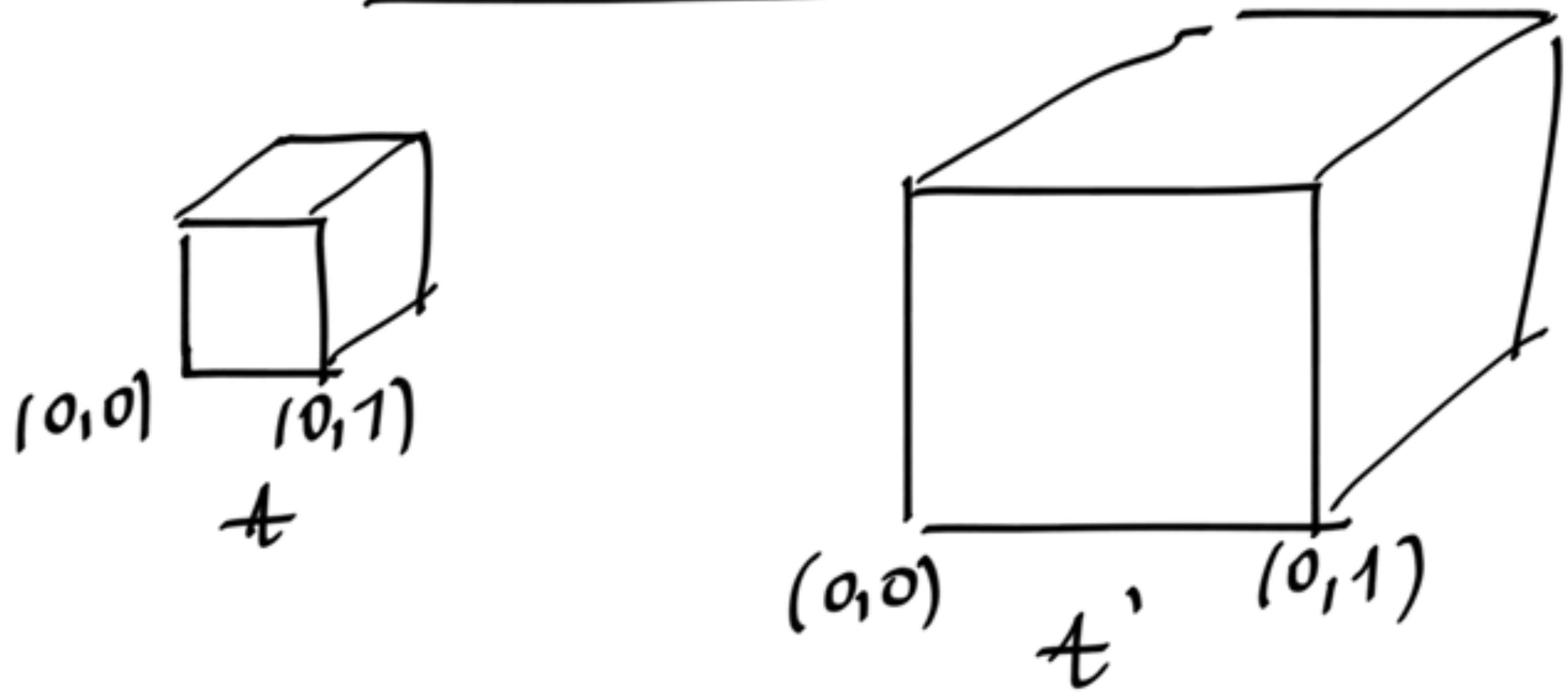
$n_{\chi,\bar{\chi}}$ DROPS DURING EXPANSION

(1) becomes ineffective at further reducing n

The number density $n_{\chi,\bar{\chi}}$ GETS FROZEN AT SOME VALUE

(This does not work for baryons n_B)

Left-Over particles II-



$\underline{t' > t}$

# of particles in a 'COMOVING' VOLUME $a^3 =$

$$= n\, a^3$$

(i.e. # of part. in the box in figure.)

L-O III.

This # may change with time due to

$\chi\bar{\chi} \to$ standard

thus is better to write

$$n(t)\, a^3(t)$$

Turns out

$$n(t)a^3(t) = \frac{n(t_0)\, a^3(t_0)}{1 + n(t_0)a^3(t_0) \underbrace{\int_{t_0}^{t} \frac{\langle v\sigma\rangle}{a^3(t')} dt'}_{I}}$$

$t \to \infty$

If $I = 0$, na^3 does not change

If $I = \infty$, $na^3 \to 0 \Rightarrow$ NO LEFT OVER

L-O IV

$\chi\bar{\chi} \to$ standard $\sim e^+e^- \to \gamma\gamma$

Exotermic Reaction:

$k \to 0$, $k' \to$ const.

— $v\sigma \sim k \cdot \dfrac{k^{2\ell+1} \cdot \text{const}}{k^2} \sim \text{const}$

— $\langle v\sigma \rangle_T \underset{\text{low}\,T}{\sim} \int_0^\infty dE\, e^{-E/kT}\, v\sigma(E)$

$\sim$ const.

(LATER TIME)

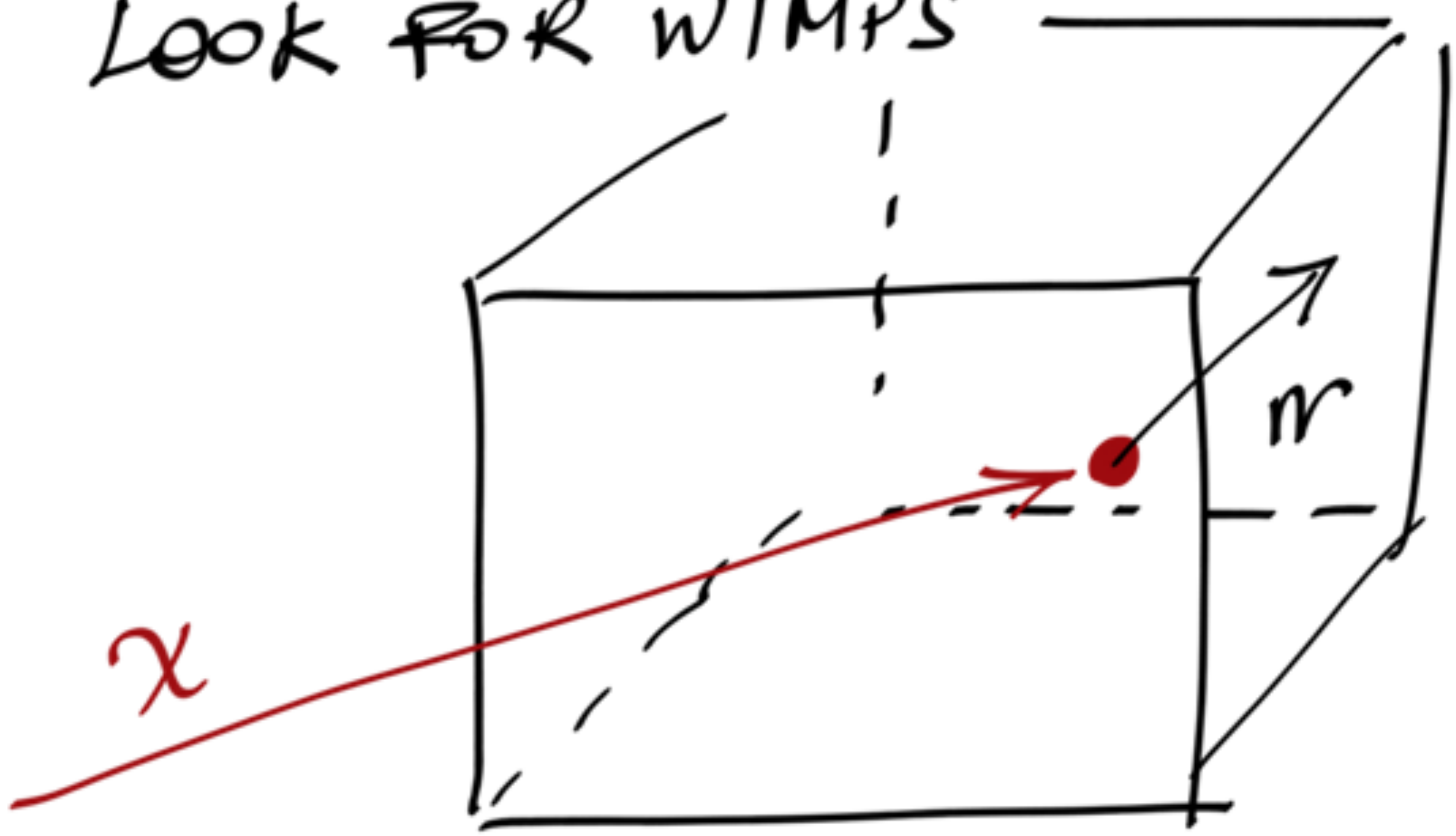
$$\int_{t_0}^{t} \frac{\langle v\sigma \rangle}{a^3(t')} dt' \sim \text{const} \int_{t_0}^{t} \frac{dt'}{(t/t)^2} \xrightarrow{t \to \infty} \text{finite}$$

WIMPS

χ of $M_\chi \approx 100\,\mathrm{GeV}$ weakly inter.
with $\langle v\sigma \rangle \approx 10^{-26}\,\mathrm{cm}^3/\mathrm{sec}$ give
a LEFT OVER Ω WHICH IS

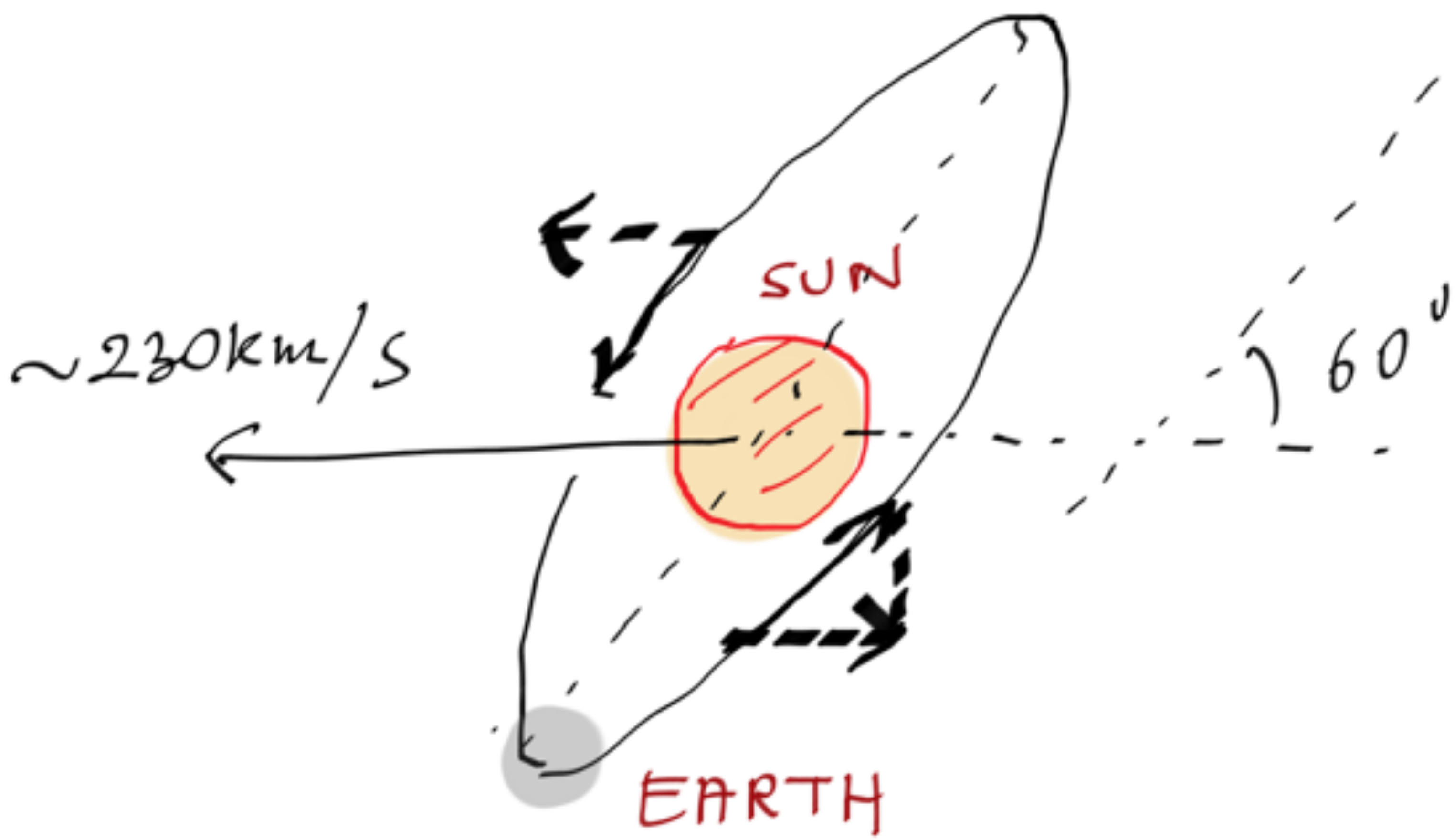
$$\Omega \approx \Omega_{DM}$$

LOOK FOR WIMPS



Crystal.

WIMP WIND



$$\vec{w}(t) = [232 + 15\cos\psi(t)]\,\hat{k}$$

$$\psi(t) = 2\pi\,\frac{t - 152.5}{365.25}$$

WIMP DISTRIBUTION

IN GALAXY $\quad f(\vec{v}\,') \sim \frac{1}{v_0^3}\, e^{-\vec{v}\,'^2/\vec{v}_0^{\,2}}$

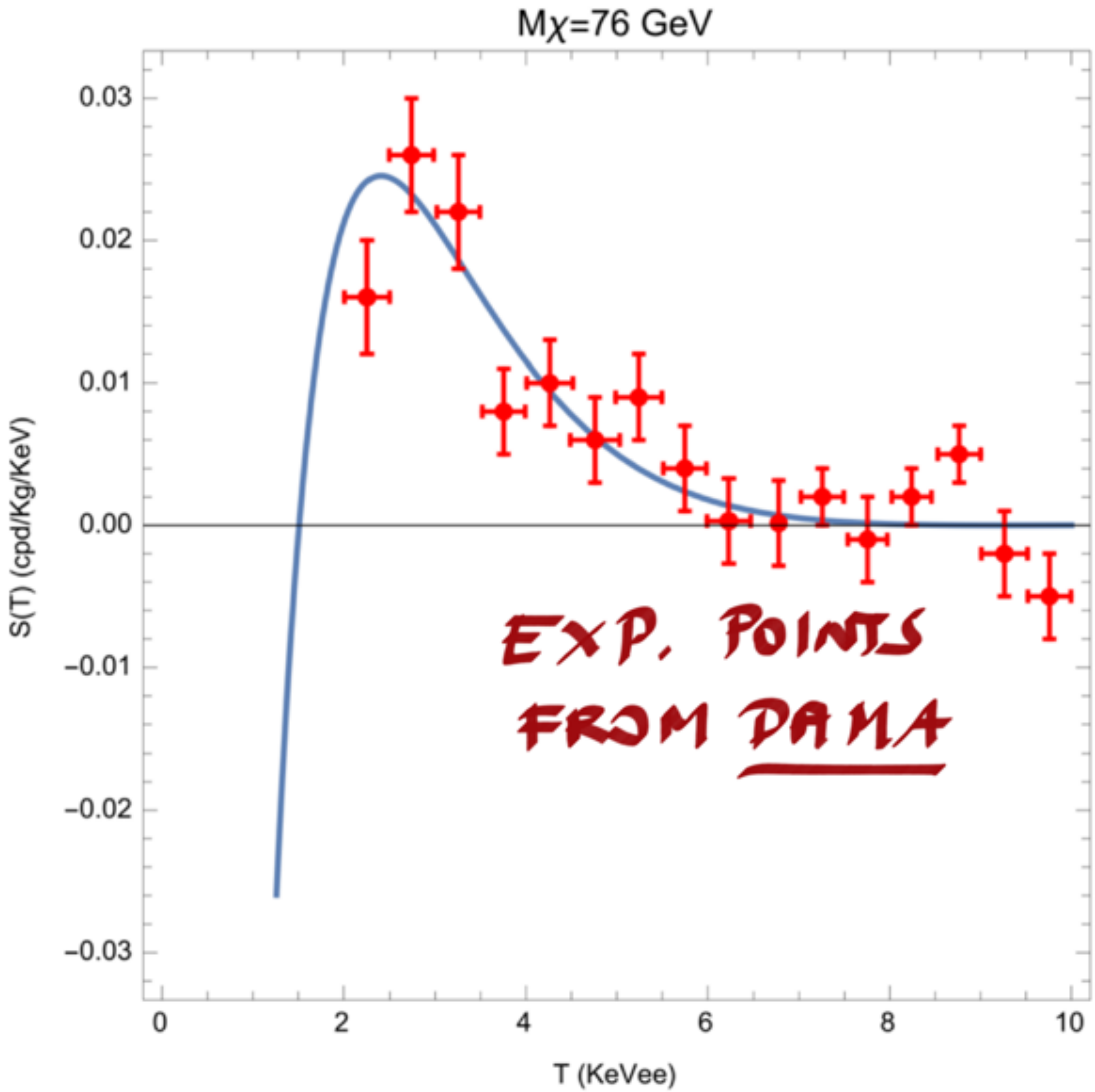
ON EARTH $\quad \vec{v} + \vec{w} = \vec{v}\,'$

? $\boxed{\dfrac{d\Gamma}{dT} = A(T) + S(T)\cos(wt+\varphi)}$?

$T =$ kinetic energy of the recoiled nucleus.

$w = 2\pi / 365\,\text{days}$

THE S(T) AMPLITUDE



Dark...