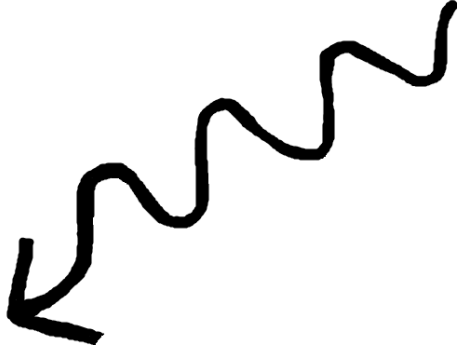
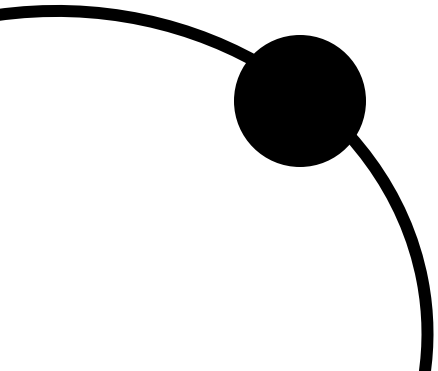


From Planck's Constant to Quantum Mechanics

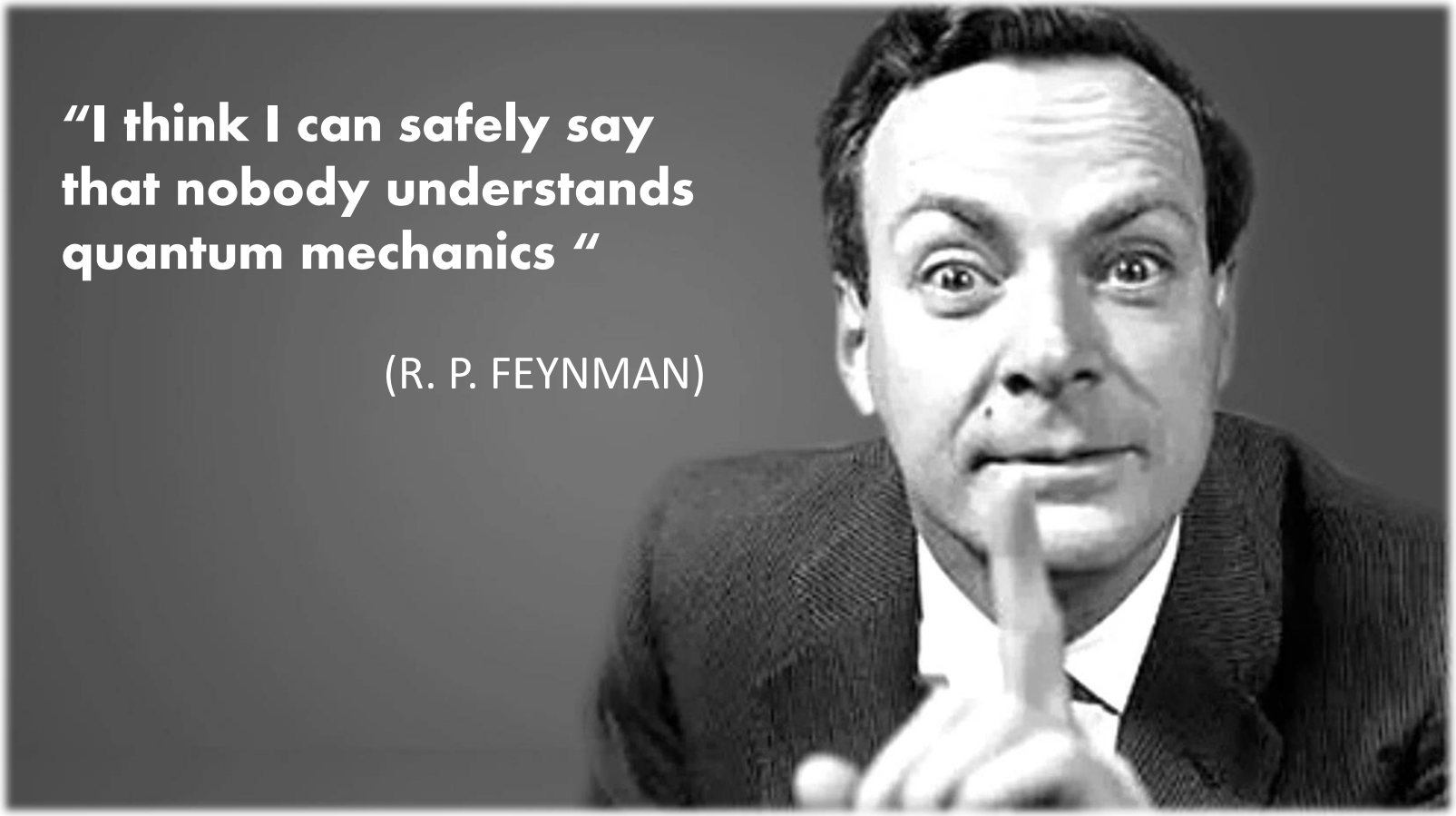
Matteo Mascolo



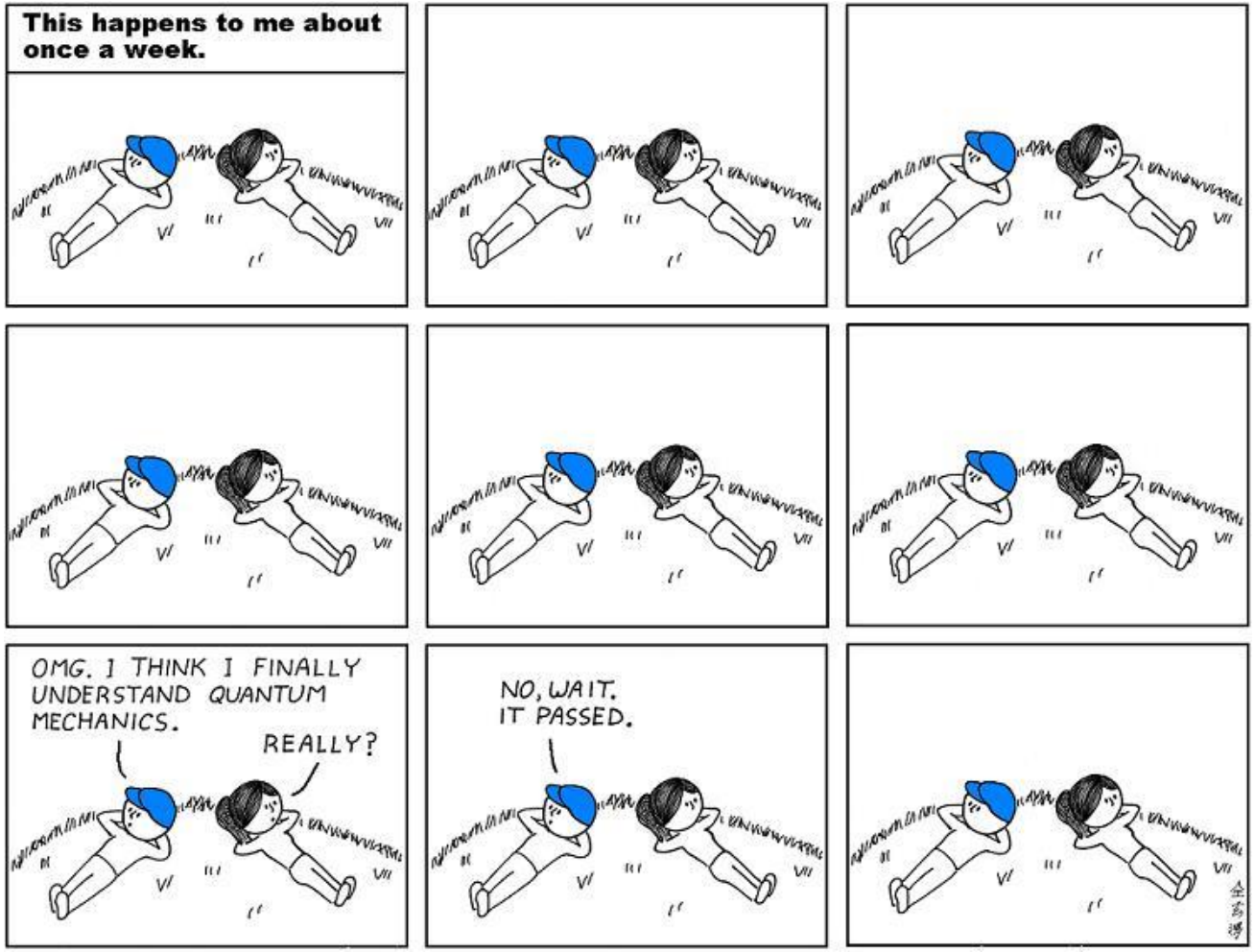
INFN
Istituto Nazionale
di Fisica Nucleare

Don't be afraid ...

... you are in good company !



If you don't understand QM ...



Let's start from the very beginning

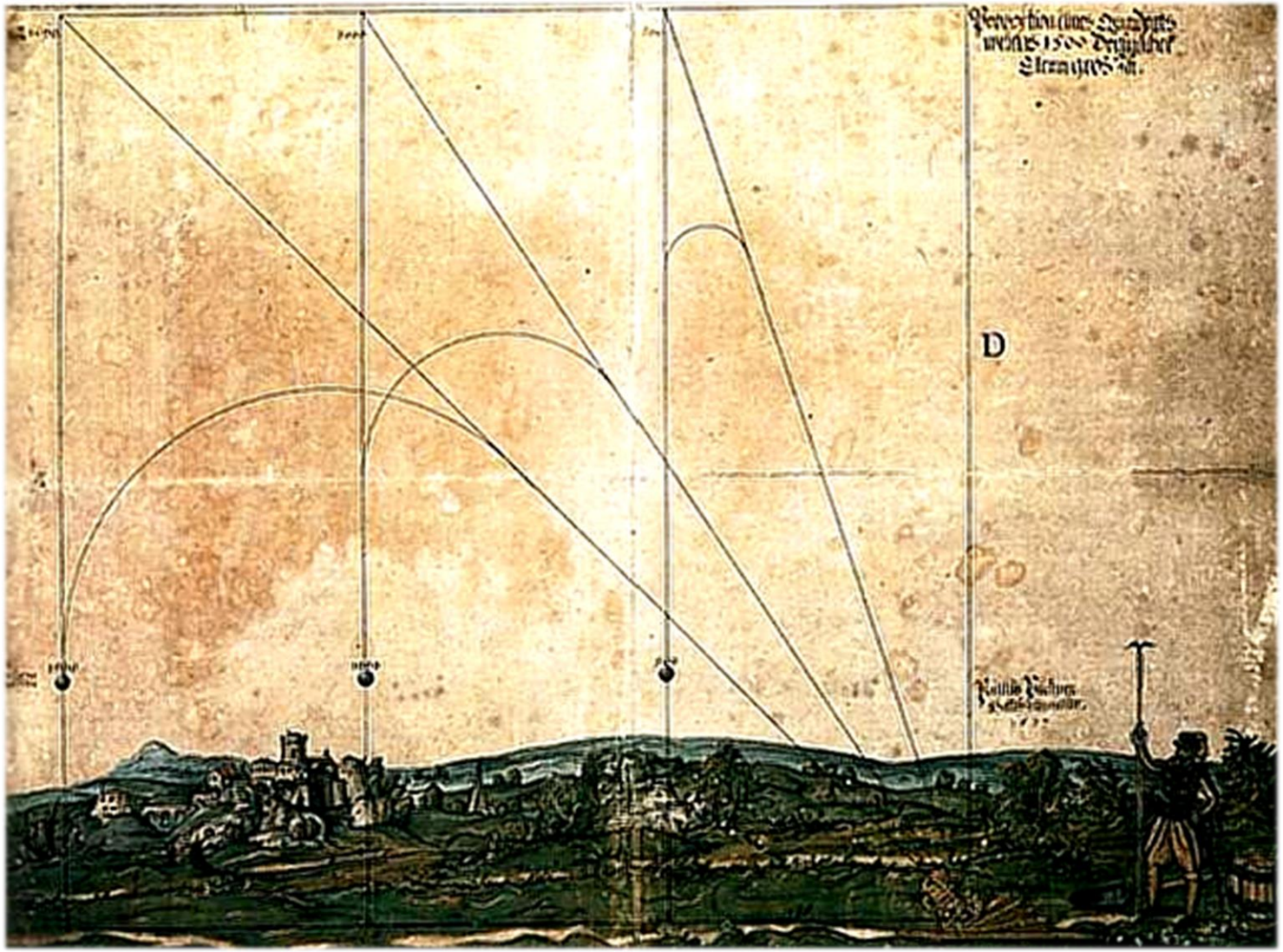
Physics before Galileo Galiei (1622). According to Aristotele....

A constant force produces a uniform motion

$$F = m\,v$$

The speed of a body in vacuum is infinity

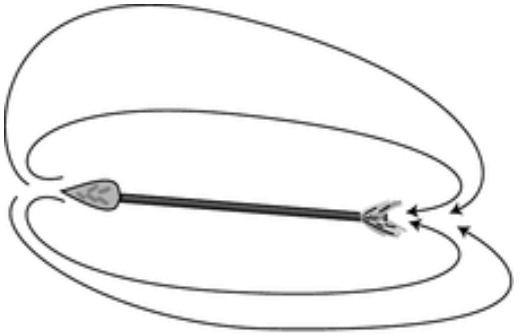
Something is evidently wrong



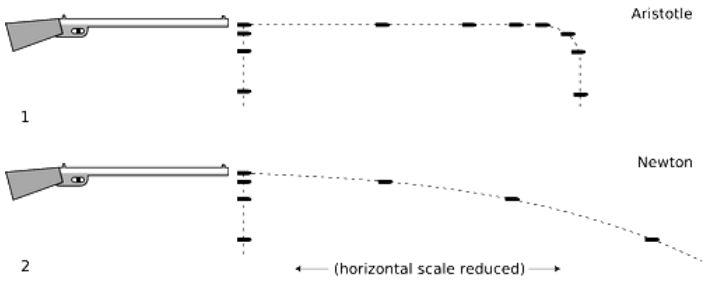
No way to understand it !!

Two easy observations can show the lacks of Aristotelian mechanics

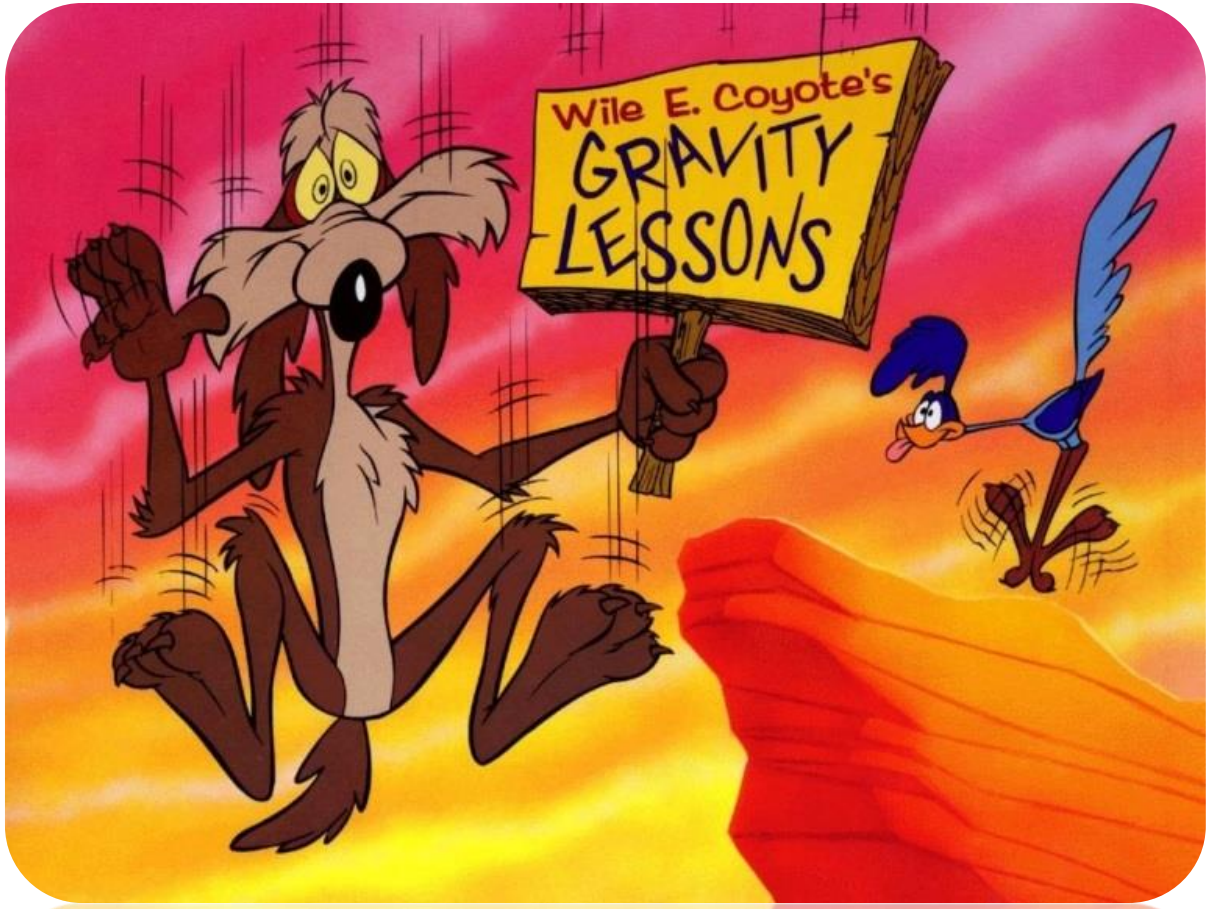
1) It hardly explains how an arrow keeps on flying after being shot (Aristotele tried with the theories of **“natural places”** and the **“horror vacui”**)



2) Trajectories can be easily disproved through a simple experiment (throwing an arrow and looking its motion!!)



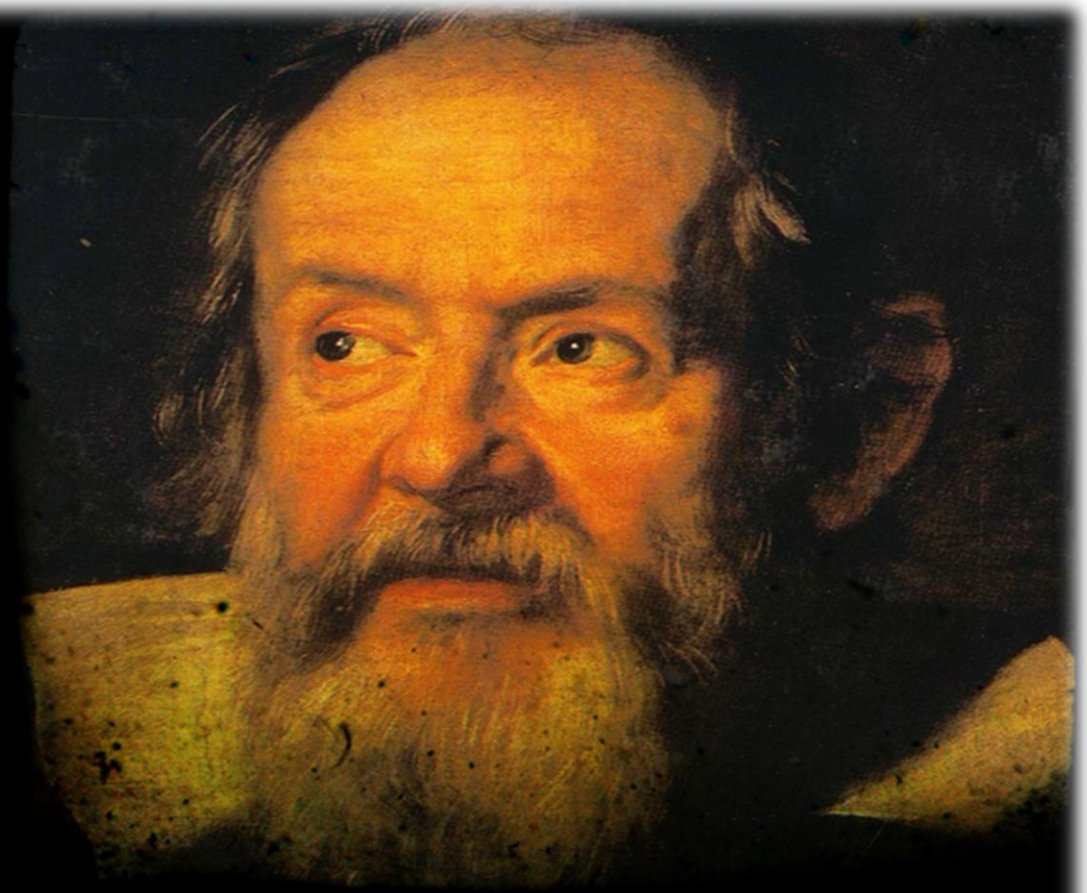
Aristotelian physics is WRONG!



One more easy thing …

SCIENTIFIC METHOD (1622)

- Observation of phenomenon
- Hypothesis and prediction
- Experiment
- Write a law using the "language of mathematics"
- **Falsifiability of the law**
- Prediction capabilities

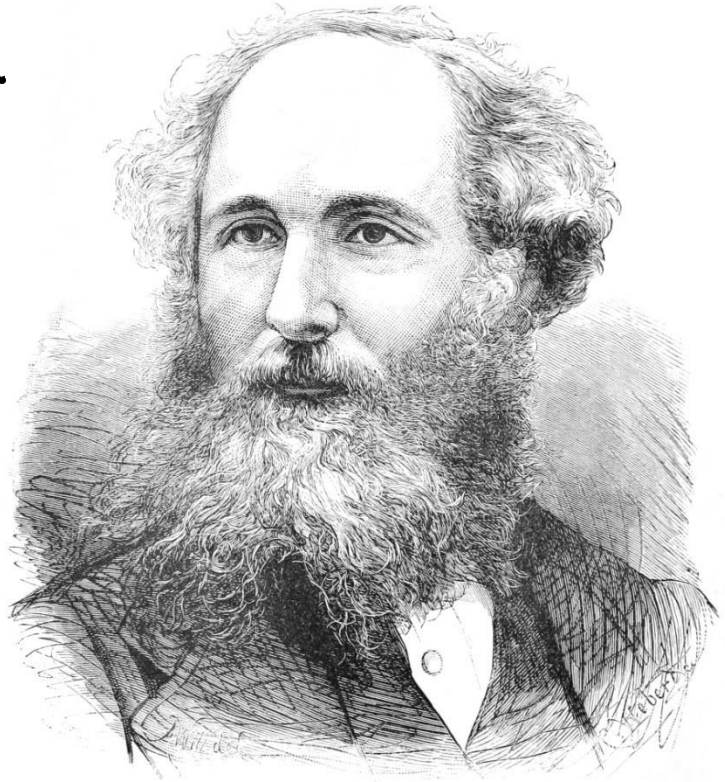


The best "EPIC FAIL" ever ...

" The task of physics is nearly completed. There are a few minor things left to do: measure some quantities with higer precision and find a theoretical justification for the ***emission*** *and* ***absoprtion spectra****, the* ***photoelectric effect*** *and the* ***black body radiation*** *..."*

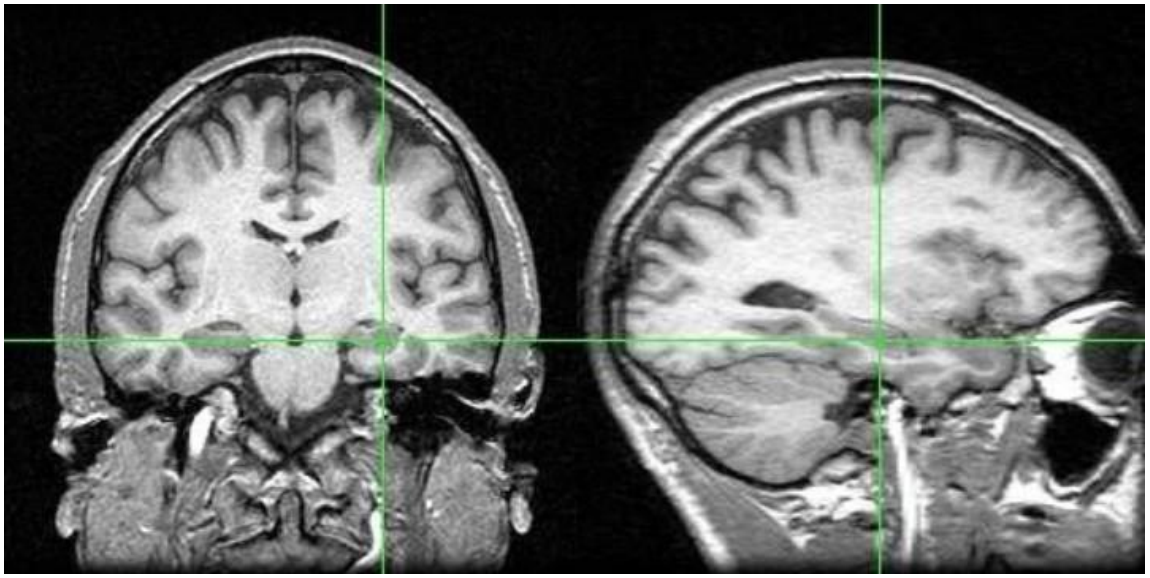
1874

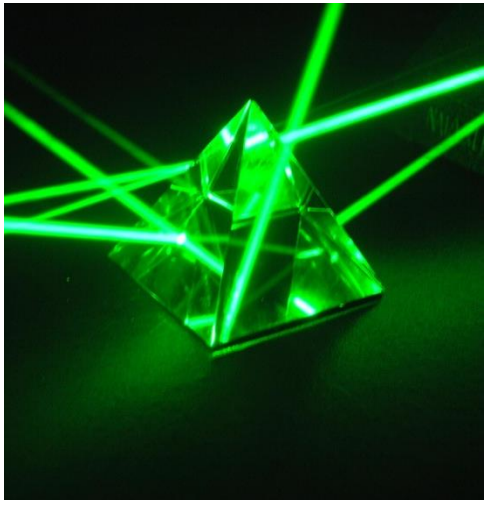
1643-1727

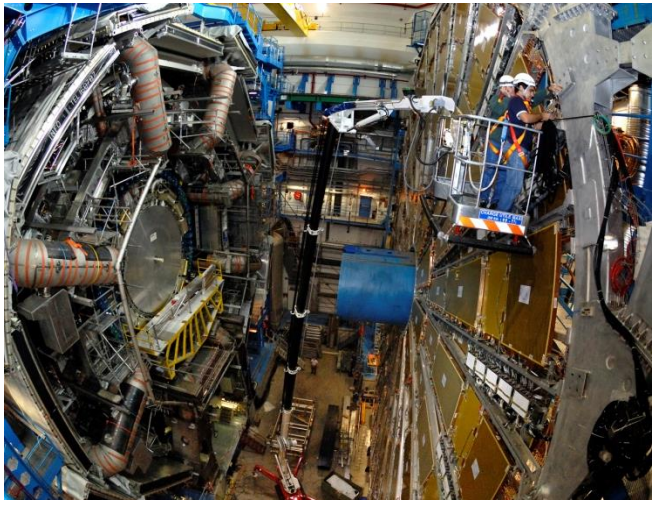


1831-1879

But ... we knowk what came after !



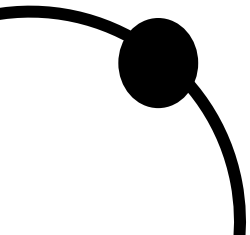




Than the questions are:

- Is classical mechanics as wrong as Aristotelian mechanics?
- How can classical mechanics, relativity, and quantum mechanics to reconcile?

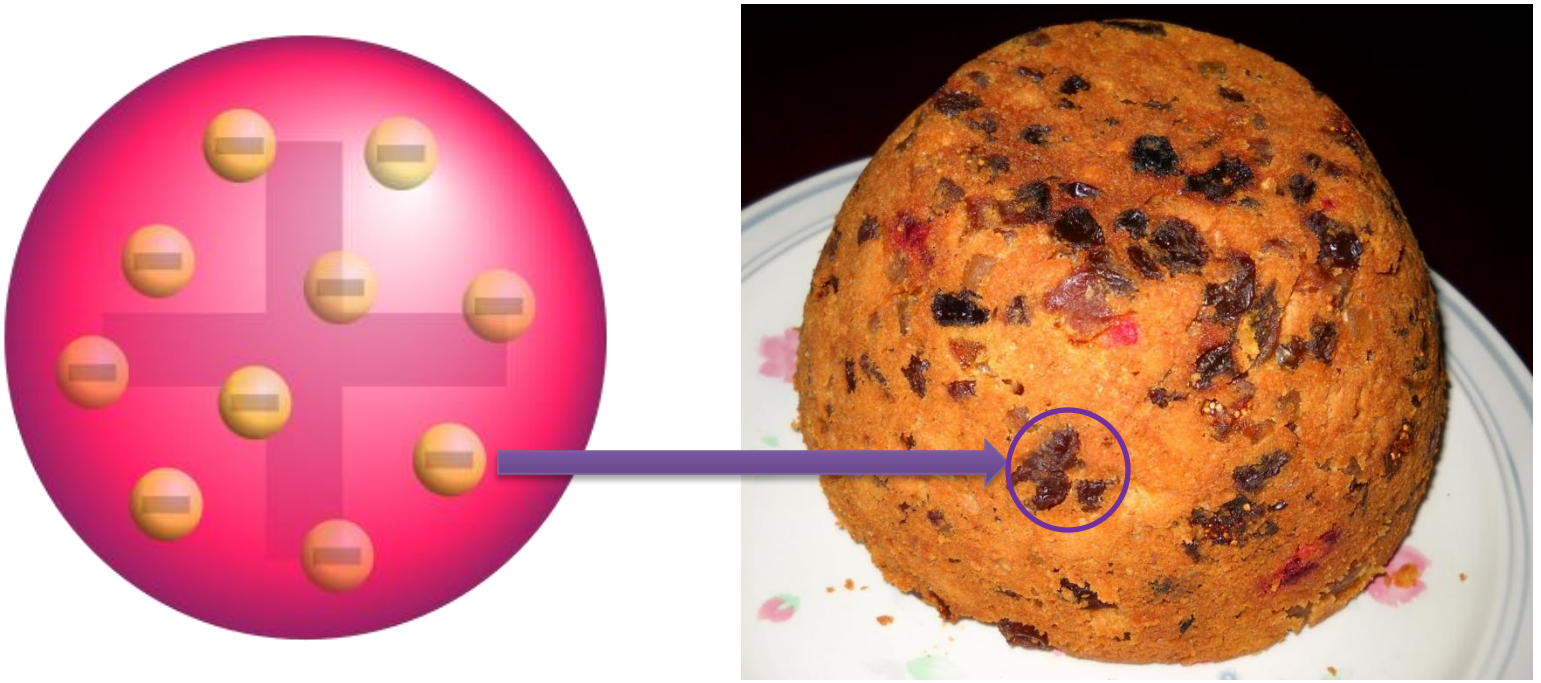
Every theory has its own scope of validity, one has to know its limits!



For example, how is the atom made?

The Thomson model

Also kwown as '**Plum Pudding**' model



"The atoms […] consist of a number of negatively electrified corpuscles enclosed in a sphere of uniform positive electrification…"

[J.J. Thomson]

The limits of Thomson model

- The electron is the only charged particle having mass, thus every atom should contain a very huge number of particles

- It does not explain the different attitude of chemical elements to ionize and combine together

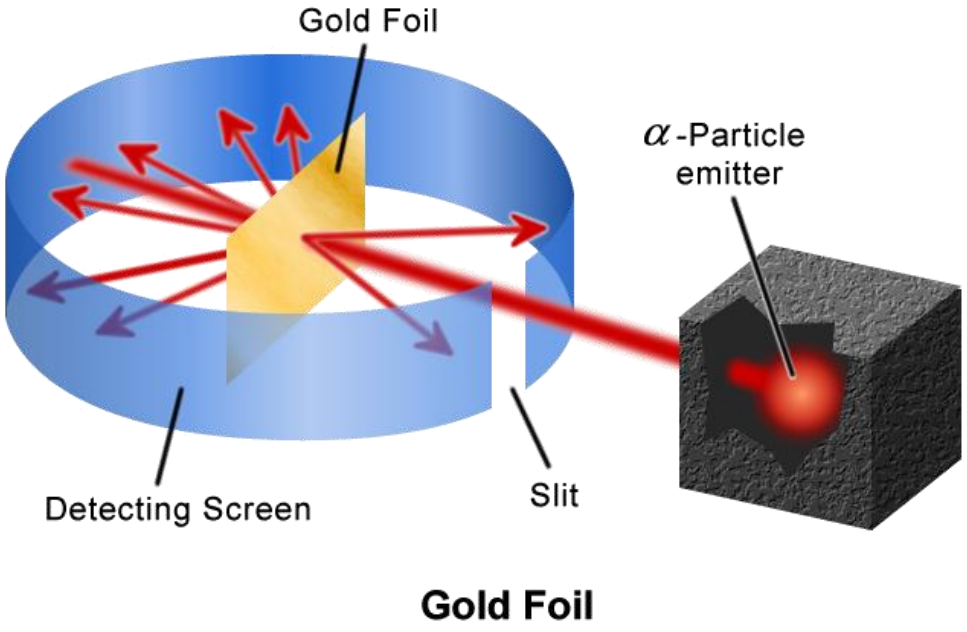
- It does not explain absorption and emission lines of elements

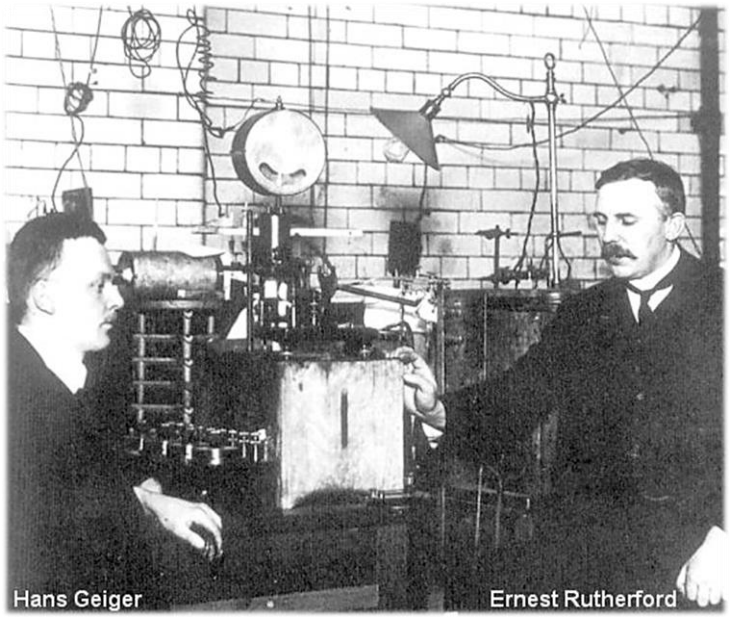
- It does not explain the results coming from **scattering experiments**

3 out of 4 points solved by the Thomson's pupil : prof. 'Crocodile'

The Geiger-Marsden experiment

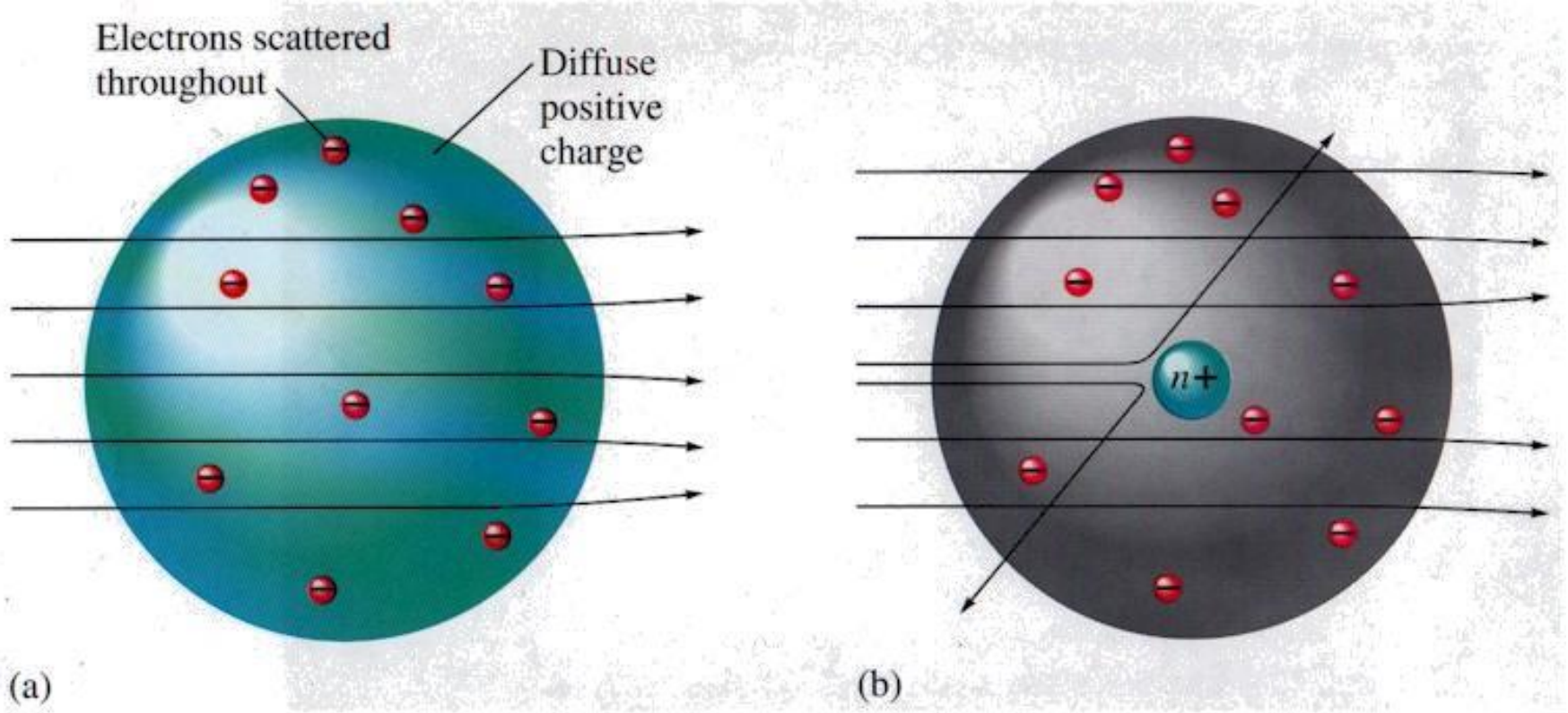
(aka the Rutherford experiment)





α-particles (nuclei of Helium) on a thin foil of gold : scattering effects were evaluated ''by eye'', working in absolute dark and **assuming drugs** that dilated the pupila

Thomson vs Rutherford

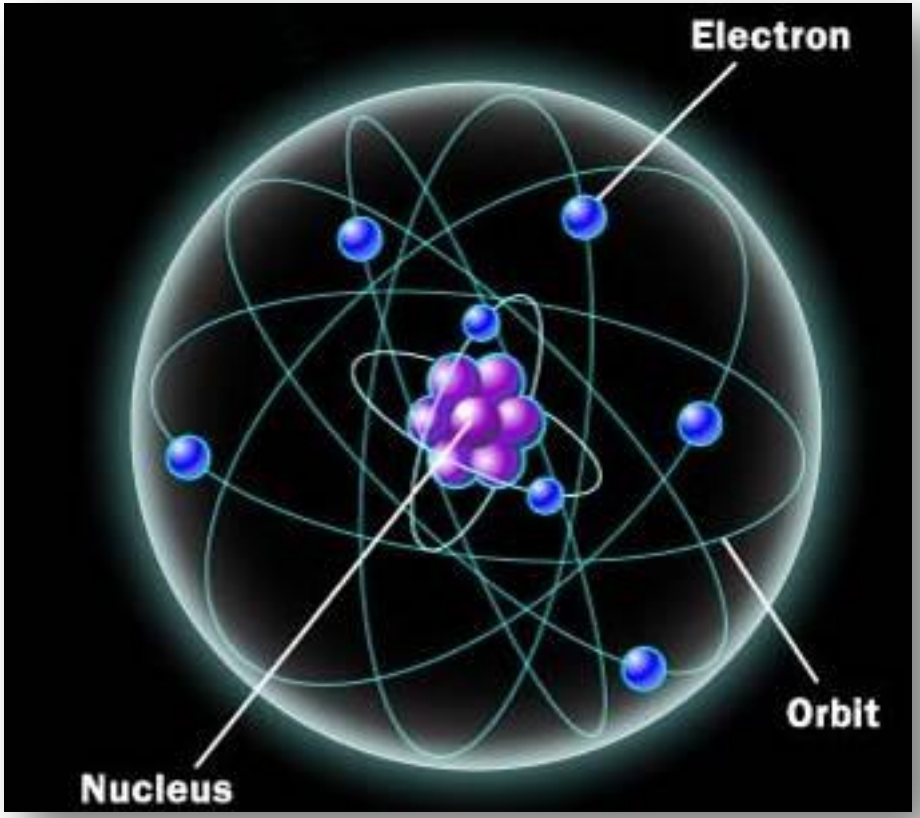


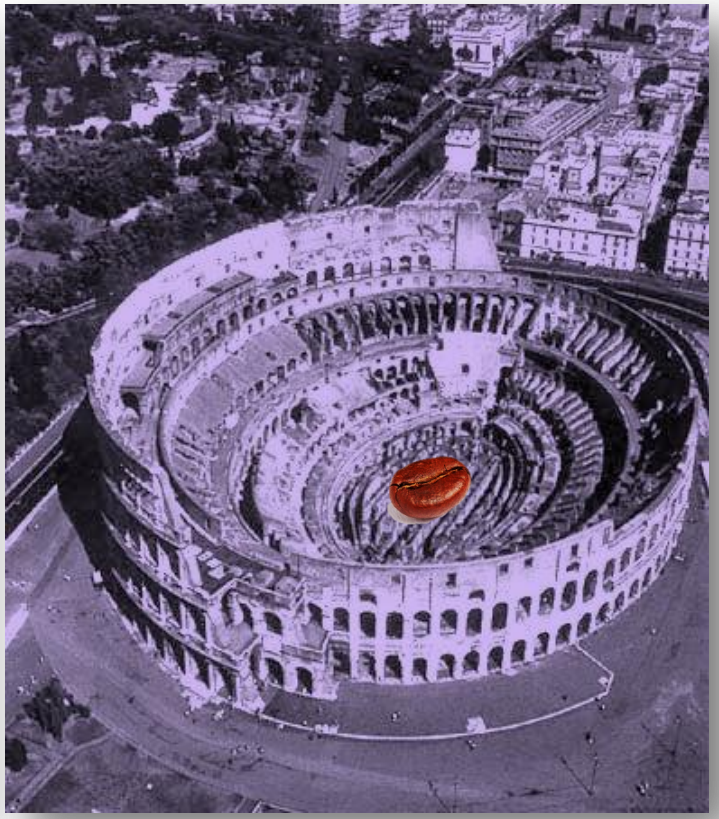
THOMSON ATOM

RUTHERFORD ATOM

Rutherford modeling of the atomù

(Nobel Prize 1908)



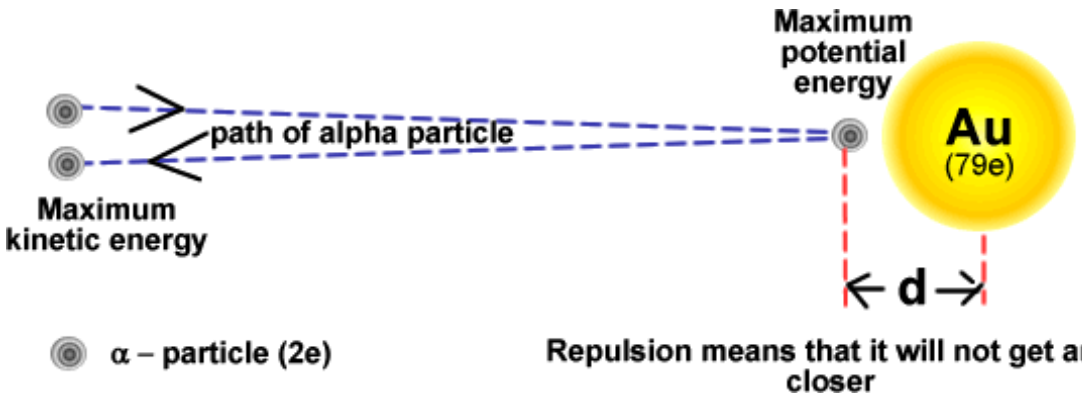


Negatively-charged electrons orbit around a positively charged nucleus (no neutrons yet!)

How big is an atom ??

$$E = T + V = \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

The total energy is the kinetic energy of the α-particle ***T*** plus the Coulomb-energy ***V*** due to the repulsion between charges of the same sign. When the particle turns back ***T=V***.



$$T = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$r = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{T}$$

How big is an atom ??

$$r = \frac{1}{4\pi\varepsilon_0}\frac{Z_\alpha Z_{Au} e^2}{T}$$

Given the elementary charge ***e***, the dielectric constant of vacuum $\boldsymbol{\varepsilon_0}$ and the kinetic energy ***T***

$$e = 1.6 \cdot 10^{-19} C$$

$$\varepsilon_0 = 8.85 \cdot 10^{-12}\ \mathrm{m^{-3}kg^{-1}s^4A^2}$$

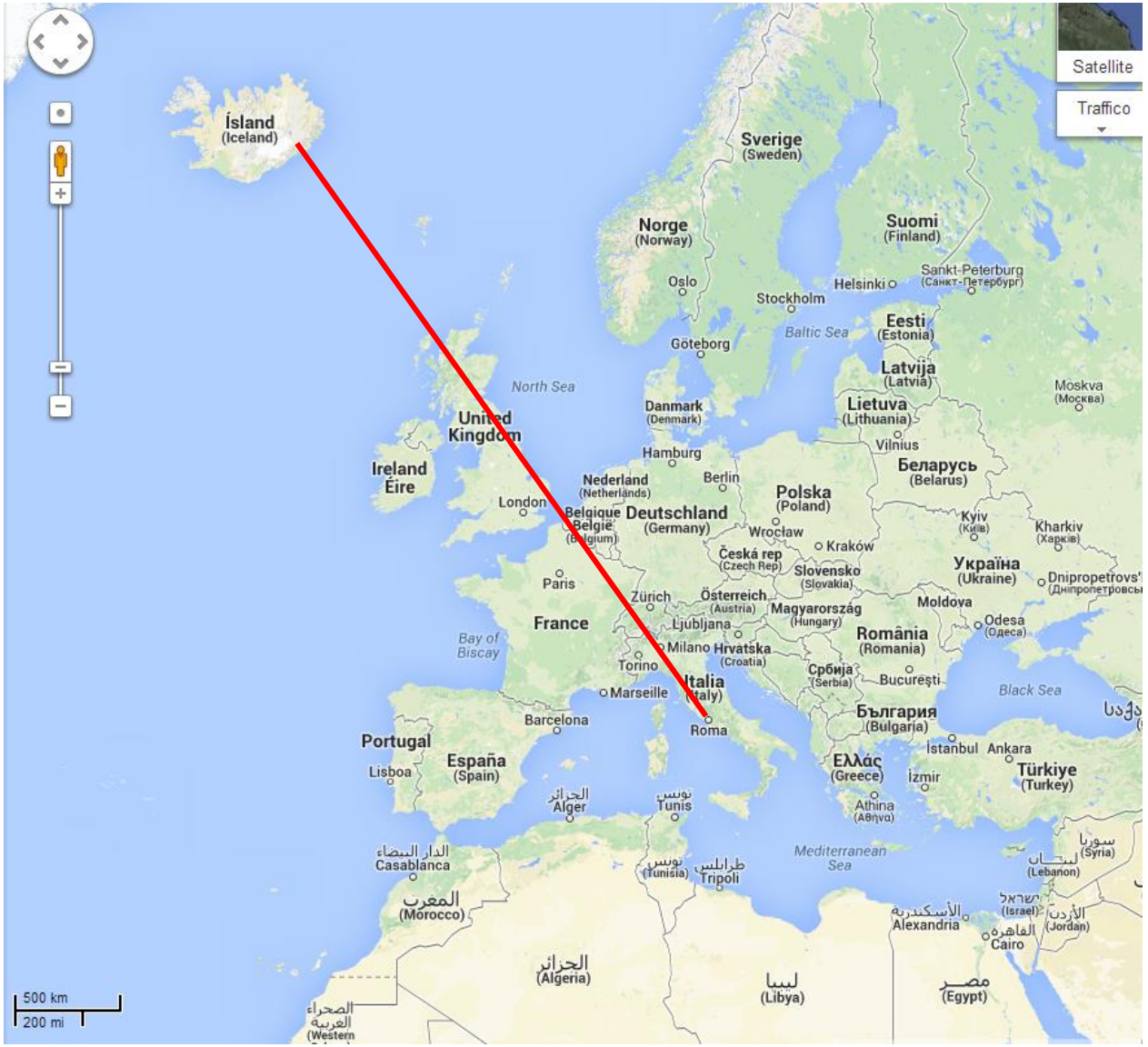
$$T = 5 MeV = 5 \cdot 10^6 \cdot 1.6 \cdot 10^{-19} J$$

$$r = \frac{1}{4\pi\varepsilon_0}\frac{z_\alpha \cdot z_{Au} \cdot e^2}{T} =$$

$$\approx 45.5 \cdot 10^{-15}\,\mathrm{m} = 45.5\,\mathrm{fm}$$

At that time, the radius of the atom of gold was well known: 150'000 fm. **It is 3000 times larger!**

3000 times larger it's a lot !!!



A new model of the atom

"It was quite the most incredible event that has ever happened to me in my life. It was almost as if you fired a 15-inch shell into a piece of tissue paper and it came back and hit you […]

[…] this scattering backward must be the result of a single collision, and when I made calculations I saw that it was impossible to get anything of that order of magnitude unless you took a system in which the greater part of the mass of the atom was concentrated in a minute nucleus. It was then that I had the idea of an atom with a minute massive center, carrying a charge. "

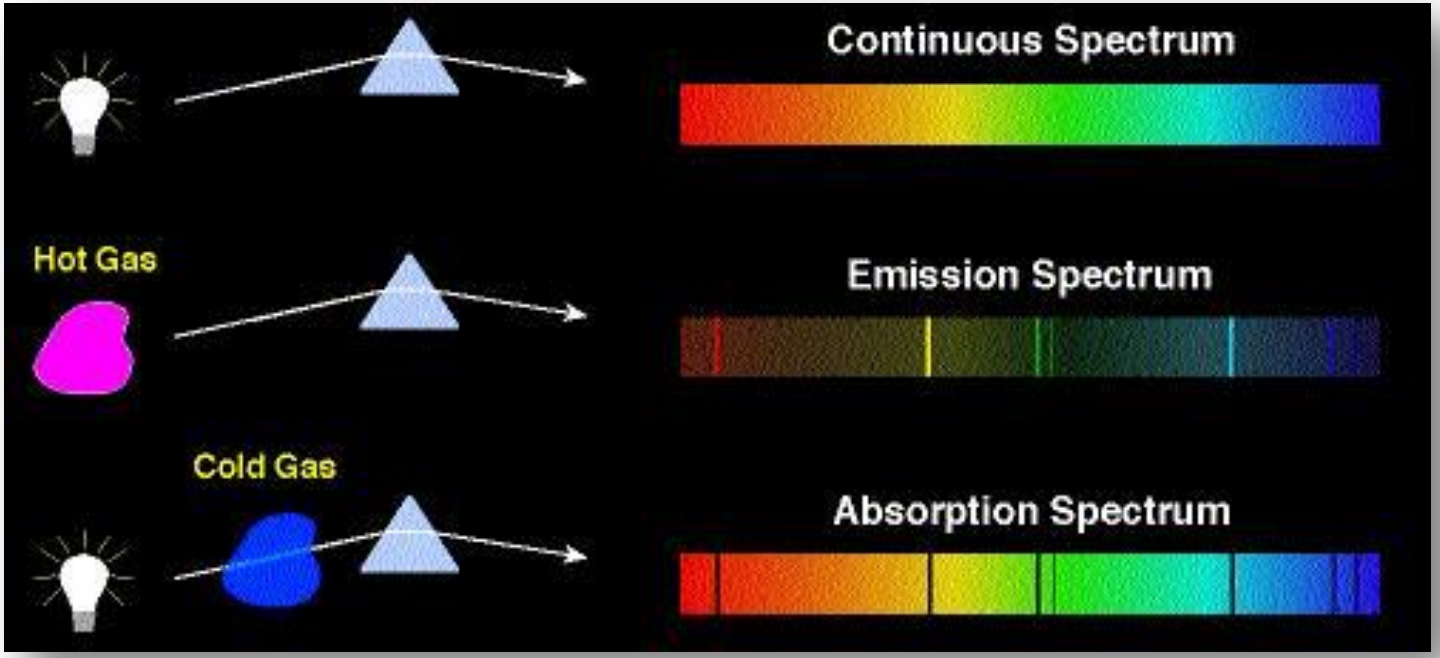
[E. Rutherford]

The limits of the Rutherford model

1. Electrons are charges in motion and they should lose energy by emitting radiation (Maxwell - Hertz 1888)
2. They should fall on the nucleus in 10^{-10} seconds
3. The puzzle of spectral lines is still unsolved !

"The great tragedy of Science: the slaying of a beautiful hypothesis by an ugly fact"

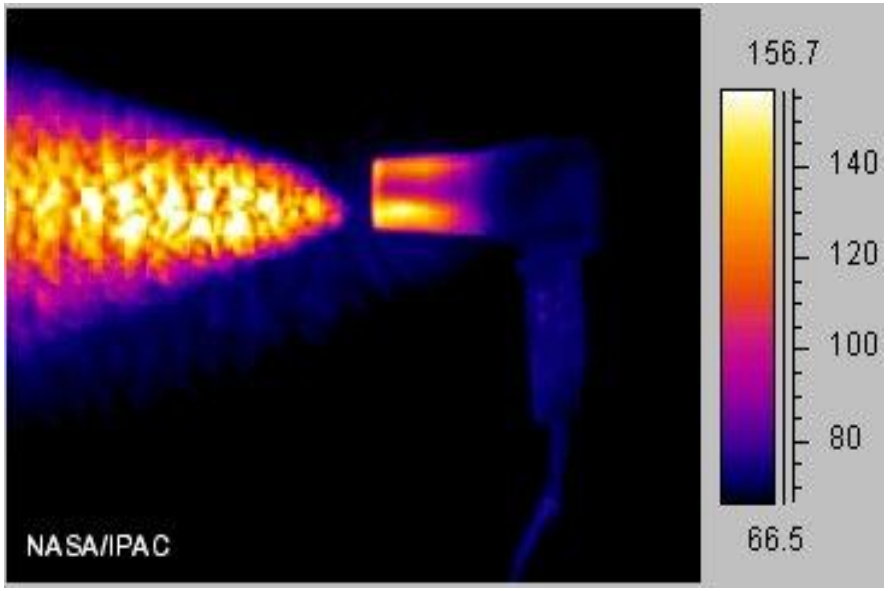
[T. Huxley]

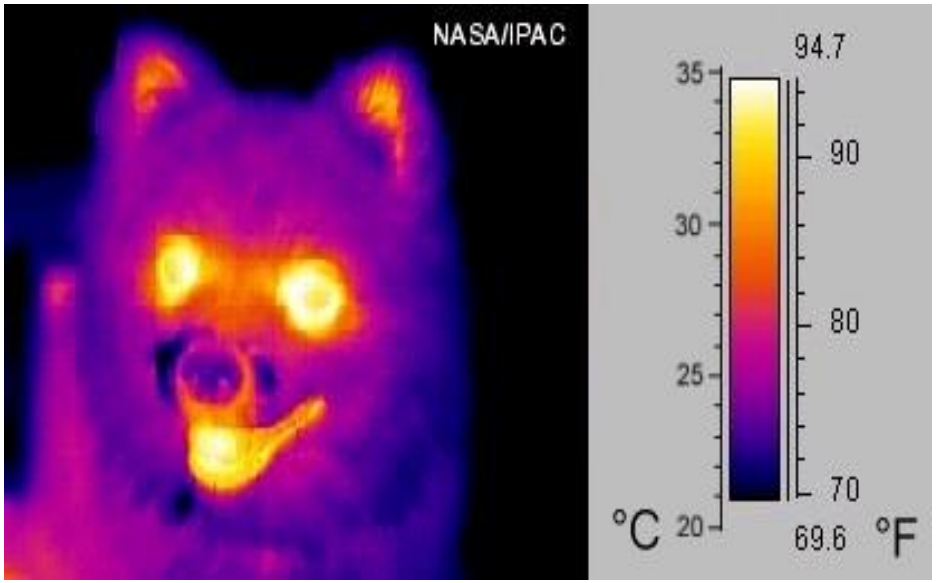


The emission of radiation

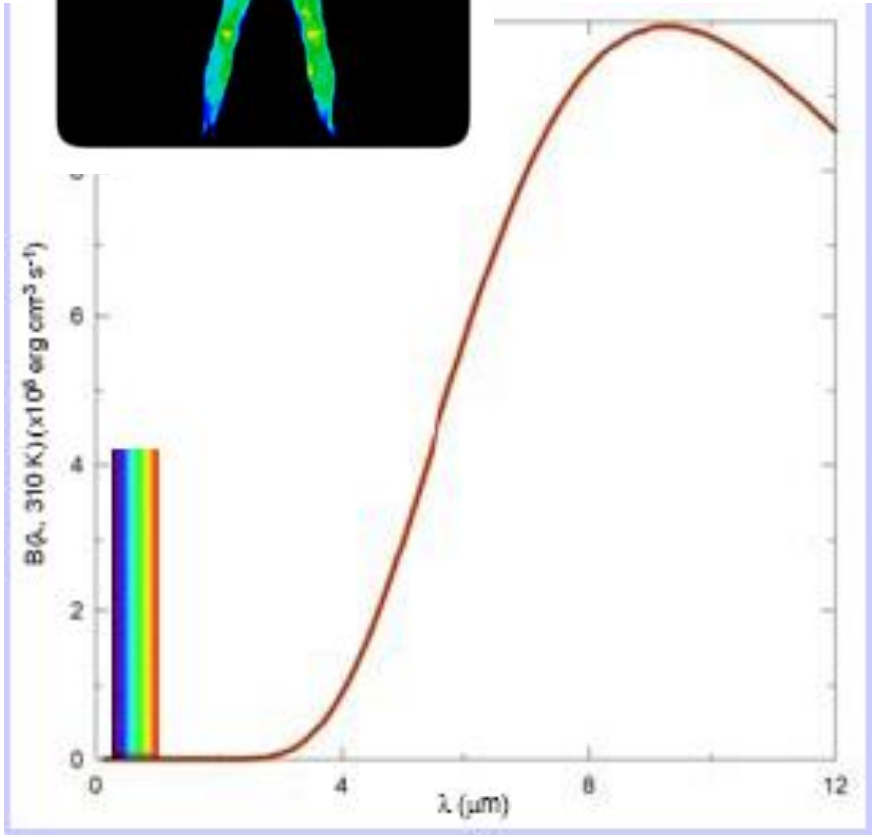
• Every object at a given temperature emits energy in the form of electromagnetic radiation (the common “heat”)

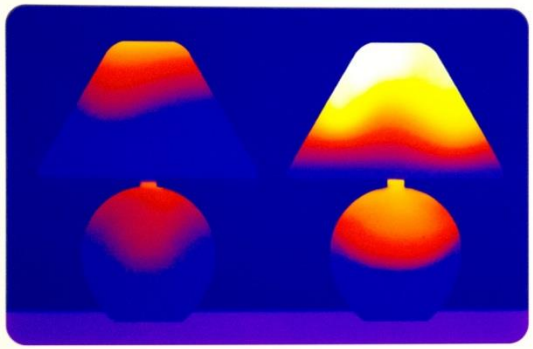
• An object emits radiation at **all** the wavelengths (λ), but the distribution of the emitted energy as a function of λ changes with the temperature (**T**).

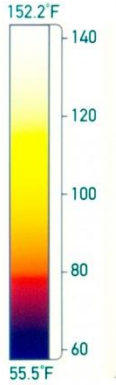


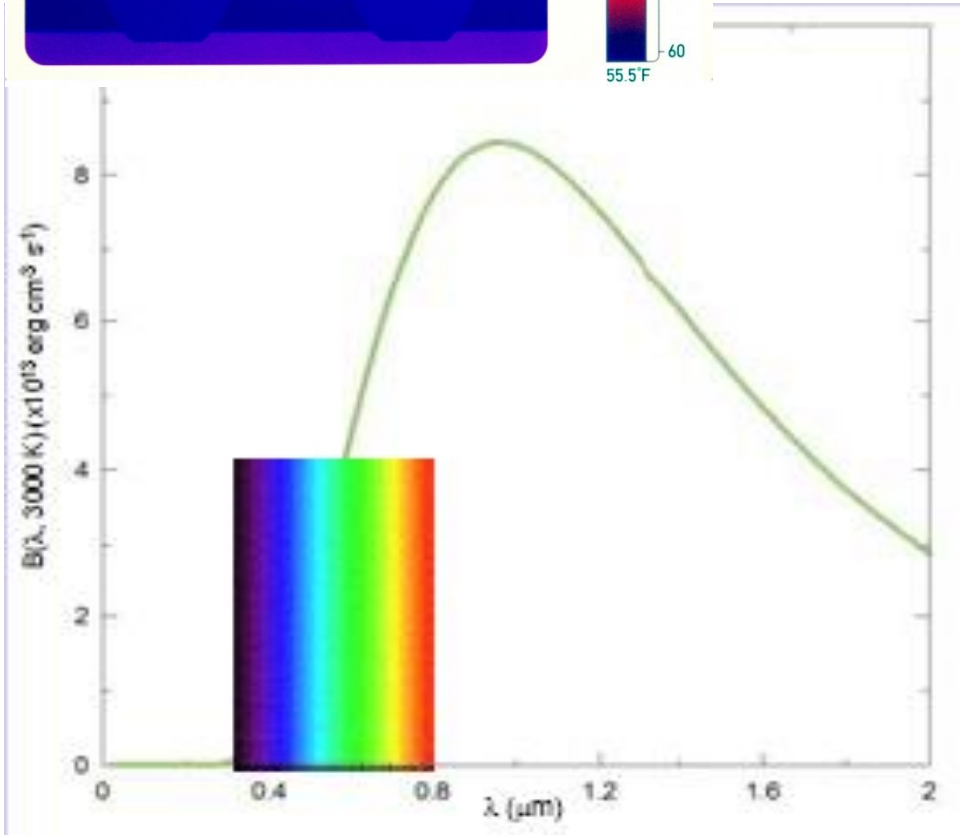


Some emission spectra









The "Black Body"

- A body is composed of many oscillating charges.
  The oscillations increase if temperature **T** increases

- Oscillating charge emit radiation and slow down.
  That is how bodies cool down

- All bodies at equilibrium have emissivity equal to absorption $e = a$ for every value of **T** and λ

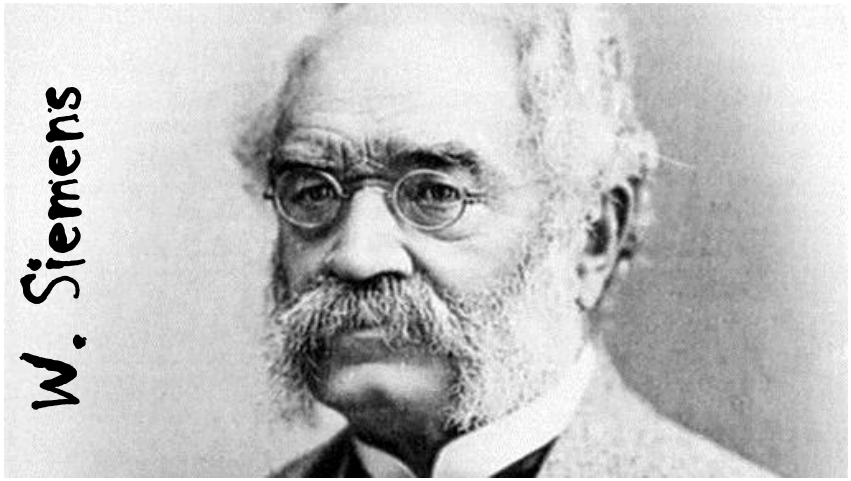
- A body at high temperature **T** that absorb **all** the energy that is able to emit (at equilibrium)

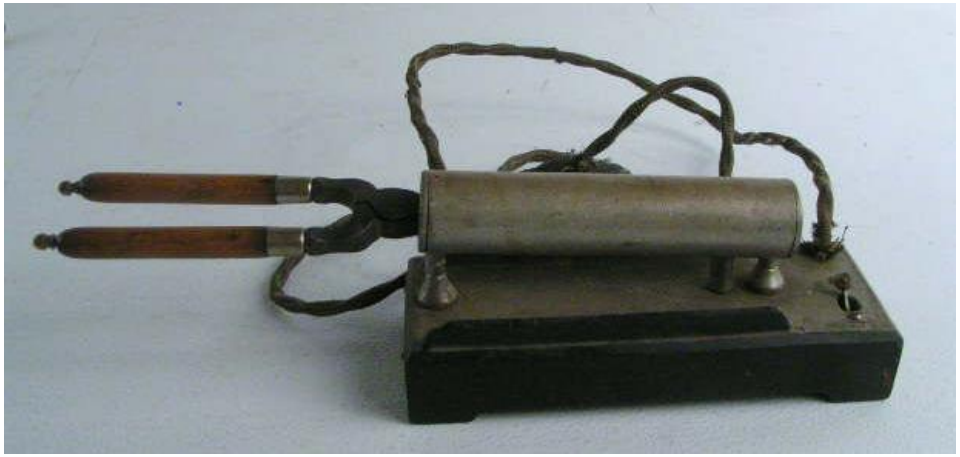
$$e = a = 1$$

is called a "**black body"** (it radiates not reflects radiation, that's why it was called a black body!)

Why the "Black Body" question??

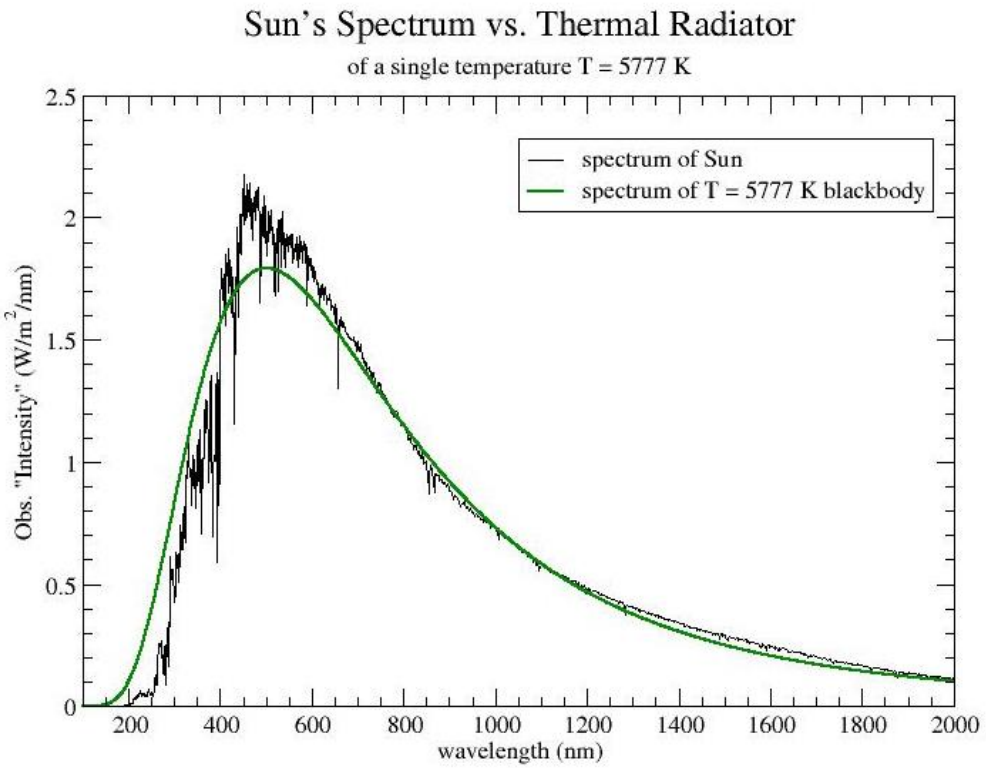
The answer is absolutely disappointing: **TRIVIAL INDUSTRIAL NEEDS**
(i.e. measure very the very high temperature of devices from their luminosity!)

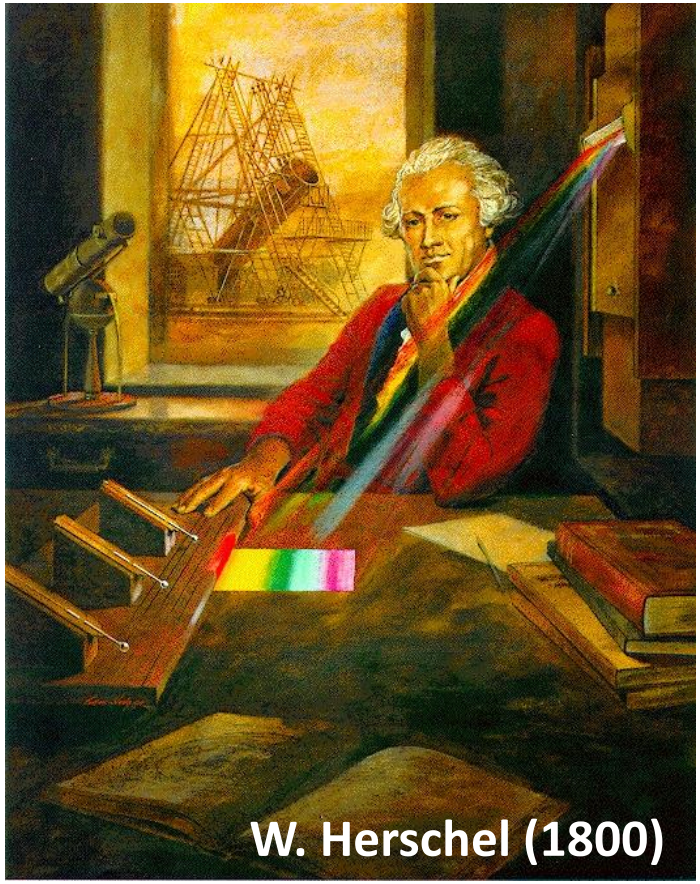




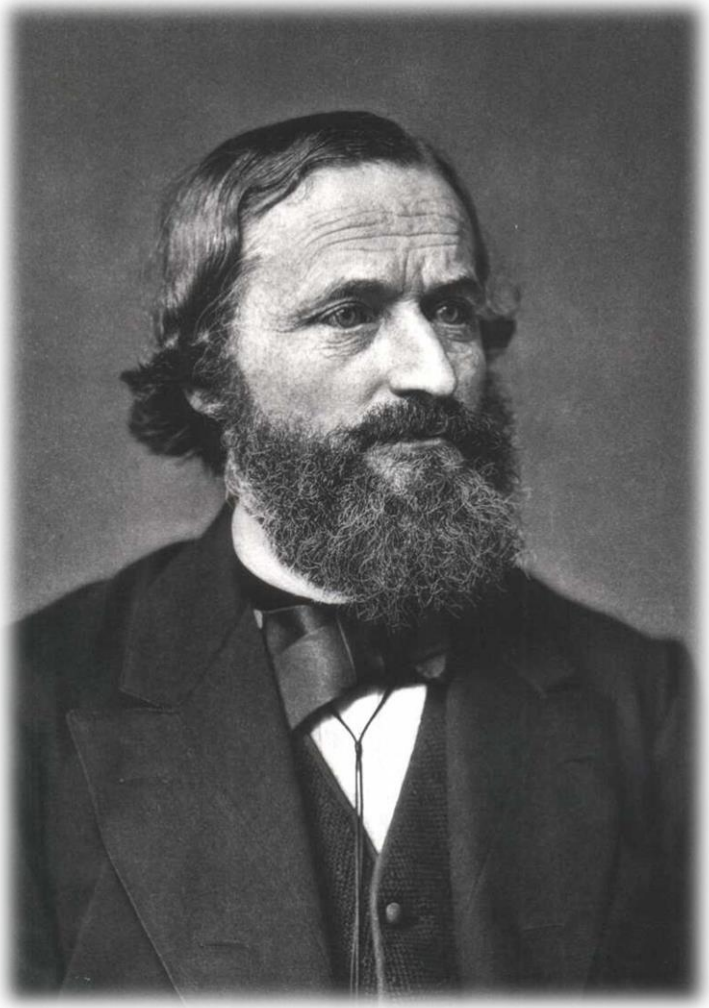
Why physicists ??

The spectrum of a hot body was already measurable with very high precision, but there was no way to connect it with theoretical predictions !!!

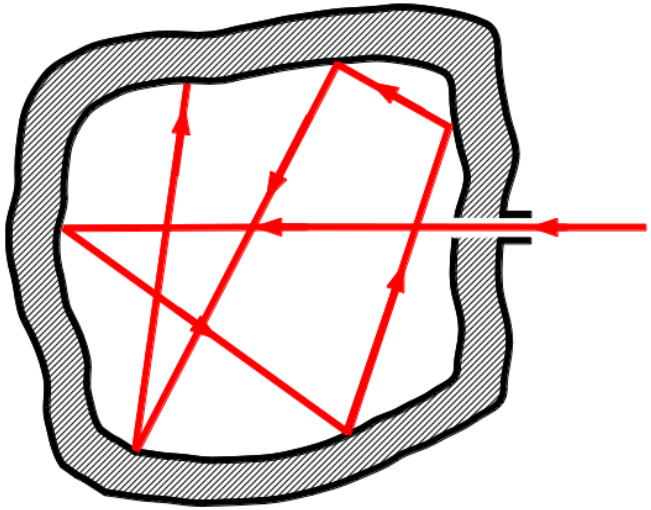




Kirchhoff and the black body



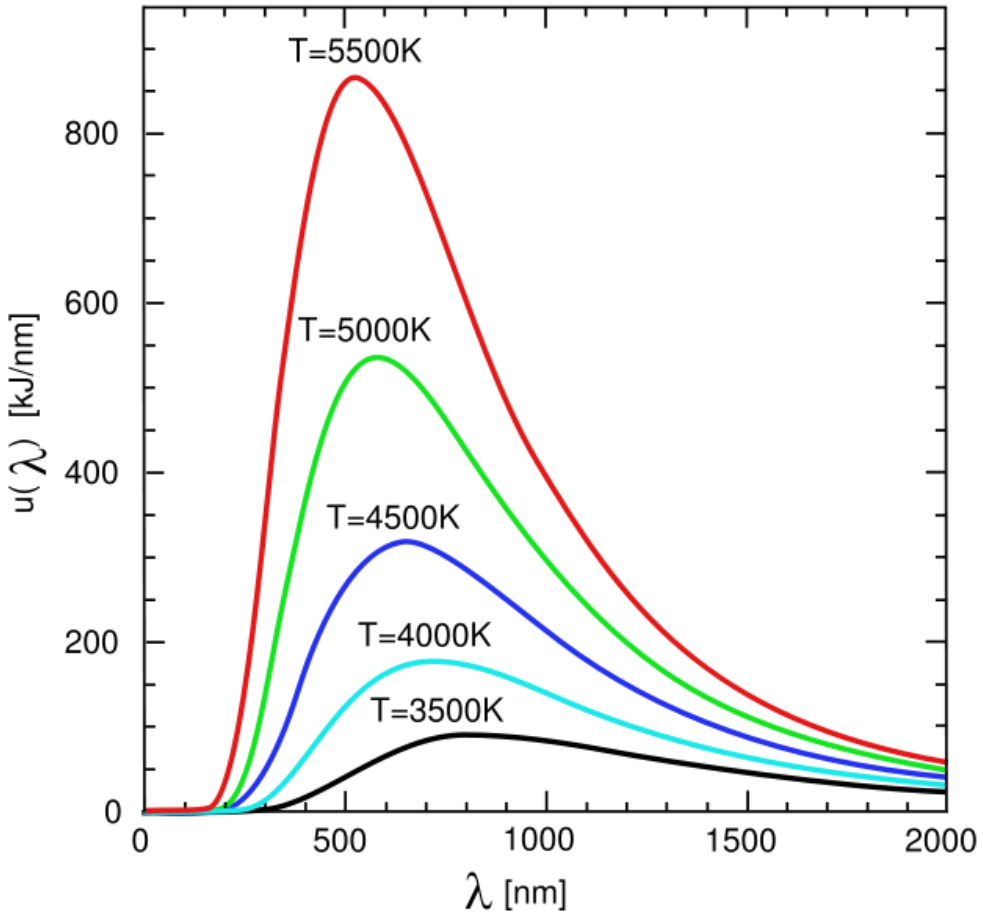
A **black body** is a cavity (e.g. an oven) with a tiny hole, kept at a constant temperature



The radiation entering the black body is reflected by the inner walls a huge number of times before getting out

The black body spectrum

All black bodies at the same temperature emit thermal radiation with the **same spectrum** (regardless of shape, dimensions and material)



From spectra observation:

$\lambda_{max} T = \text{constant}$

Wien's law

$M_{tot} \alpha T^4$

Stefan (1879)

From theoretical calculation:

$M_{tot} = \sigma T^4$

Boltzmann (1879)

The black body "puzzle"

The flux of the radiation within the cavity in every direction is zero, but there is energy transfer everywhere

This transfer is given by the density of radiated energy in the wavelength range $(\lambda, \lambda+d\lambda)$: $\Psi_\lambda d\lambda$ (Ψ_λ is also called *emission power*)

The calculation of Ψ_λ giving the shape of expected experimental spectrum, was very difficult from th theoretical point of view

First Attempt ... by Wien

$$\psi_\lambda = \frac{C}{\lambda^5 e^{C'/\lambda T}}$$

First "empirical" forumla (similar to Maxwell velocity distribution)

The black body "puzzle"

Wien's Ψ_λ does not work! It fits data at low λ not at high λ. Constants C e C' are completely arbitrary!

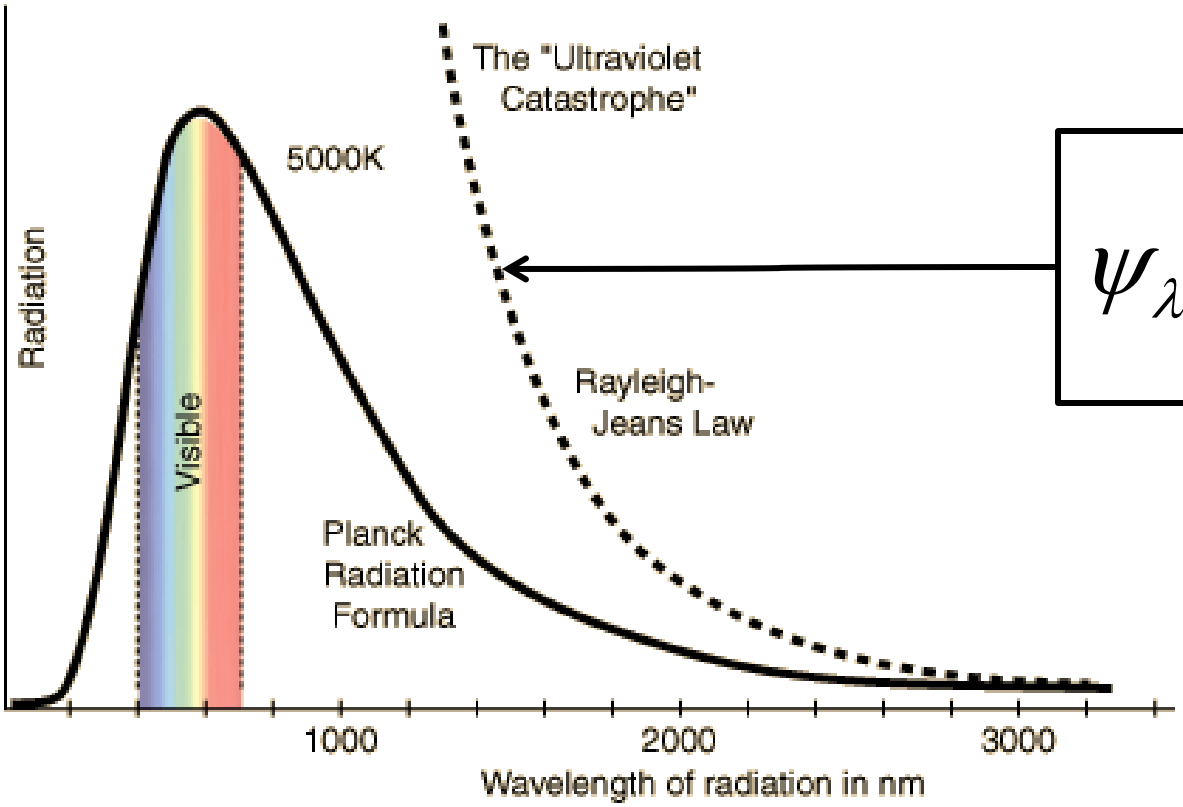
Second Attempt ...
by Rayleigh-Jeans

$$\psi_\lambda = \frac{8\pi}{\lambda^4} kT$$

Model with stationary waves inside the cavity. The energy density is evaluated as the density of modes with mean kinetic energy kT

It does not work! It fits data only at high λ. Smaller is λ greater is the number of possible stationary waves! But everything was calculated correctly…

The "ultraviolet catastrophe"



$$\psi_\lambda = \frac{8\pi}{\lambda^4} kT$$

- **Wien:** empirical formula, correct only for small values of λ
- **Rayleigh-Jeans:** consistent formula, correct for big λ

AND NOW?!?!

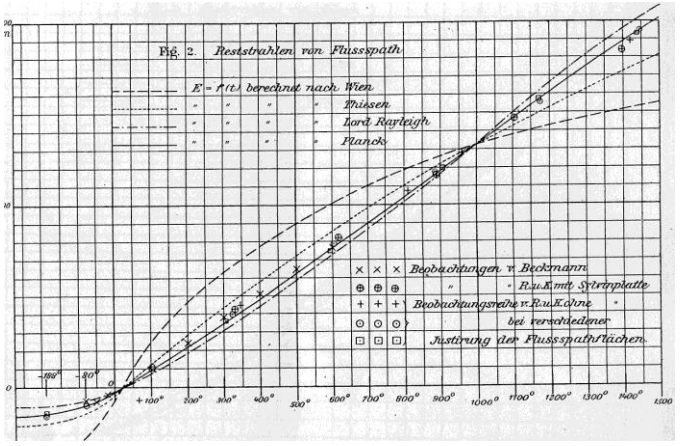
Spectroscopy at that time …


H. Rubens


H. Kayser (5000 spectra!)

"A triumph of observation, in a theoretical desert!"

[Unknown]



Planck's lucky solution

OCTOBER 1900

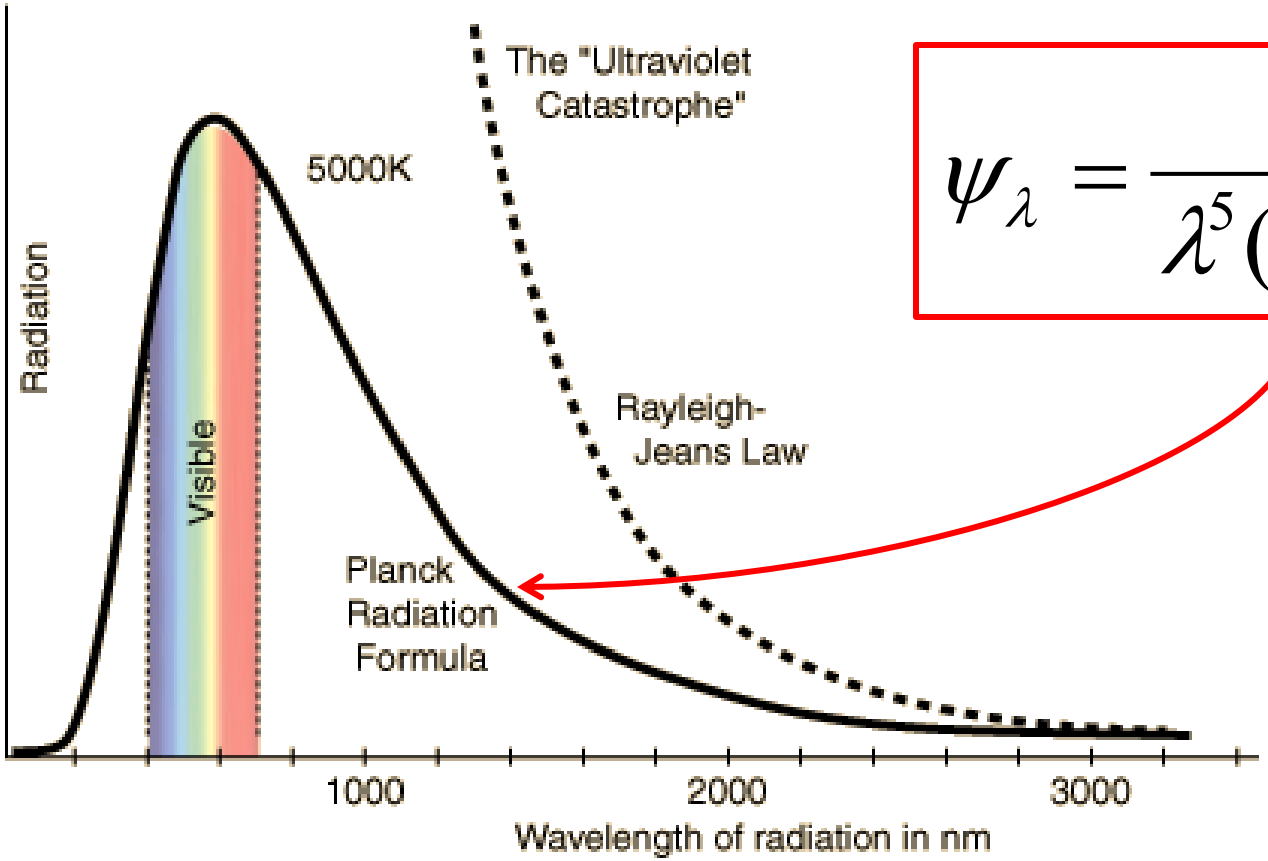
Max Planck solves the puzzle of the spectrum of black body radiation. With a new empirical formula:

$$\psi_\lambda = \frac{C}{\lambda^5 (e^{C'/\lambda T} - 1)}$$

- What are the constants C and C' !?
- It works for every value of λ!

Yeah but ... WHY?!?!

Planck's lucky solution



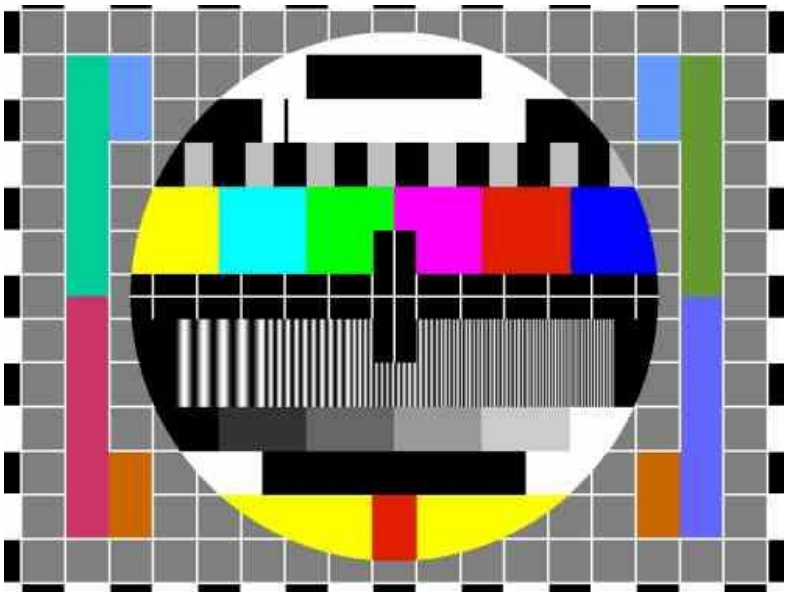
$$\psi_\lambda = \frac{C}{\lambda^5 (e^{C'/\lambda T} - 1)}$$

The agreement with Rubens' experimental data is striking.
Nevertheless Planck can't find a physical explanation...

A personal remark

When Planck started to study the black body radiation, he worked under the influence of the 'energetics' (see Rankine and Otswald).

He was quite convinced that atoms did not exist, and pretty sure that statistical interpretation of entropy, given by Boltzmann, was wrong.



The "desperate act"

"[...] something which started innocuously as the color of light from burning coal had developed into a phenomenon with much deeper meaning."

"A theoretical explanation must be found, no matter how far from the present knoledge"

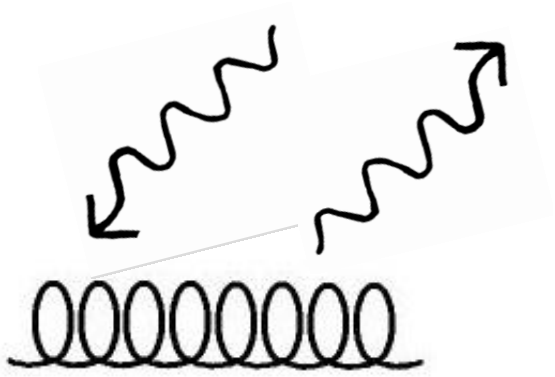
[Max Planck]

After countless attempts based on classic approach, Planck makes a "desperate act", **denying the continuity of Nature laws**.

It's **December 1900** (3 months after Ruben's measurements!)

Planck's idea ...

Oscillators of frequency ***f*** in the black body cavity can exchange only certain amounts of energy, multiple of a fixed value, called **quantum of energy** depending upon the frequency ***f*** :



$$E_n = nhf \qquad n \in N$$

(***h*** is the Planck's constant, extracted from data : [h] = [J][s] also called *"action"*)

This idea allows to limit the high-frequency part of the spectrum and solves the *ultraviolet catastrophe*: the larger is the value of $\mathbf{E_n}$, the more difficult is the exchange of energy, because it would change the equilibrium status of the cavity!

Getting ready for calculation …

Planck disagreed with atomic theories, and did not like statistical thermodynamics… Nevertheless, Planck took his start from Boltzmann's statistical approach, stating that the number of oscillators with energy between **E** and **(E + dE)** is given by

$$e^{-E/kT}$$

The mean value of the energy of the oscillators (**at fixed** λ) is:

$$< E > = \frac{Total\ \ Energy}{\#\ of\ oscillators}$$

The fraction of oscillator with energy in the range **(E, E+dE)** is:

$$N(E,\ E + dE) = N_T e^{-E/kT} dE$$

Planck's calculation

In the continuum hypothesis (pre-Planck) thus:

$$< E > = \frac{\int_0^\infty N_T e^{-E/kT} dE}{N_T} = \ldots = kT$$

Multiplying this mean energy (of the oscillators at a fixed frequency) by the number of ''modes'' (i.e. the number of possible frequencies), one obtains Rayleigh-Jeans formula!

In Planck's discrete hypothesis:

- N_1 particles with energy **h*f*** : $N_1 = N_0 e^{-hf/kT}$
- N_2 particles with energy **2h*f***: $N_2 = N_0 e^{-2hf/kT}$
- N_3 particles with energy **3h*f***: $N_3 = N_0 e^{-3hf/kT}$
- Etc … .....

Planck's calculation

$$N_1 = N_0 e^{-hf/kT} \qquad N_2 = N_0 e^{-2hf/kT} \qquad N_3 = N_0 e^{-3hf/kT} \qquad \ldots$$

$$< E > = \frac{N_0 E_0 + N_1 E_1 + N_2 E_2 + \ldots}{N_0 + N_1 + N_2 + \ldots} = \frac{N_0 hf\,(0 + e^{-hf/kT} + 2e^{-2hf/kT} + \ldots)}{N_0 (1 + e^{-hf/kT} + e^{-2hf/kT} + \ldots)}$$

Let's call: $x = e^{-hf/kT}$ and let's stop at 3rd order …

$$< E > = \frac{hf\,(0 + x + 2x^2 + 3x^3)}{(1 + x + x^2 + x^3)}$$

but: $(1-x)^{-1} = 1 + x + x^2 + x^3 \qquad (1-x)^{-2} = 1 + 2x + 3x^2$

$$< E > = hf \frac{(1-x)^{-2} x}{(1-x)^{-1}} = hf \frac{x}{(1-x)} = hf \frac{1}{(1/x - 1)} = \frac{hf}{e^{hf/kT} - 1}$$

LOOKS FAMILIAR ??!

The Planck's formula

Try to figure out Planck's face when he saw exactly his "lucky formula", coming out from calculation without any mathematical trick (constants included) !!

The Planck's formula

Planck's formula reproduces the black body spectrum in the whole ***f*** range …

$$\Psi_\lambda d\lambda = \frac{8\pi}{\lambda^4}\frac{hf}{e^{hf/kT}-1}d\lambda$$

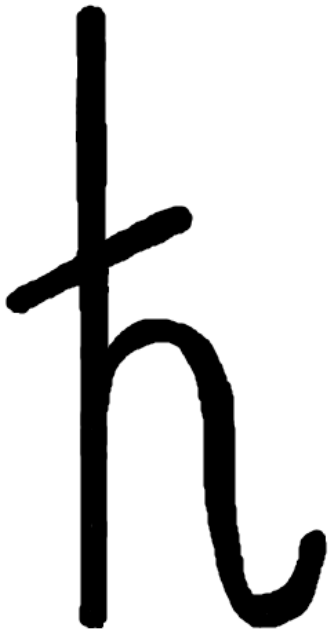
Low f (big λ) $$\Psi_\lambda = \frac{8\pi}{\lambda^5}\frac{hc}{e^{hf/kT}-1} \approx \frac{8\pi}{\lambda^5}\frac{hckT}{hf} = \frac{8\pi}{\lambda^5}\frac{k}{\frac{1}{\lambda T}} = C\frac{T}{\lambda_4}$$ Wien

High f (small λ) $$\Psi_\lambda = \frac{8\pi}{\lambda^5}\frac{hc}{e^{hf/kT}-1} \approx \frac{8\pi}{\lambda^5}\frac{hc}{e^{hf/kT}} = \frac{1}{\lambda^5}\frac{C}{e^{C'/\lambda T}}$$ RJ

Reminder (if x is small): $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$

The Planck's constant

h is a fundamental constant, playing a fundamental role in the quantum theory (when divided by 2π, it is referred to as h-bar)



- $h = 6.62606957 \times 10^{-34}\ m^2\ kg\ /\ s$ [Js]

- (When multiplied by the frequency) it defines the 'quantum of energy' (the minimum quantity of energy which can be exchanged)

- Represents the connection between the wavelength and momentum of particles (wave-particle duality!)

- It defines the scale at which the natural phenomena become "quantistic" (and classical behavior disappears)

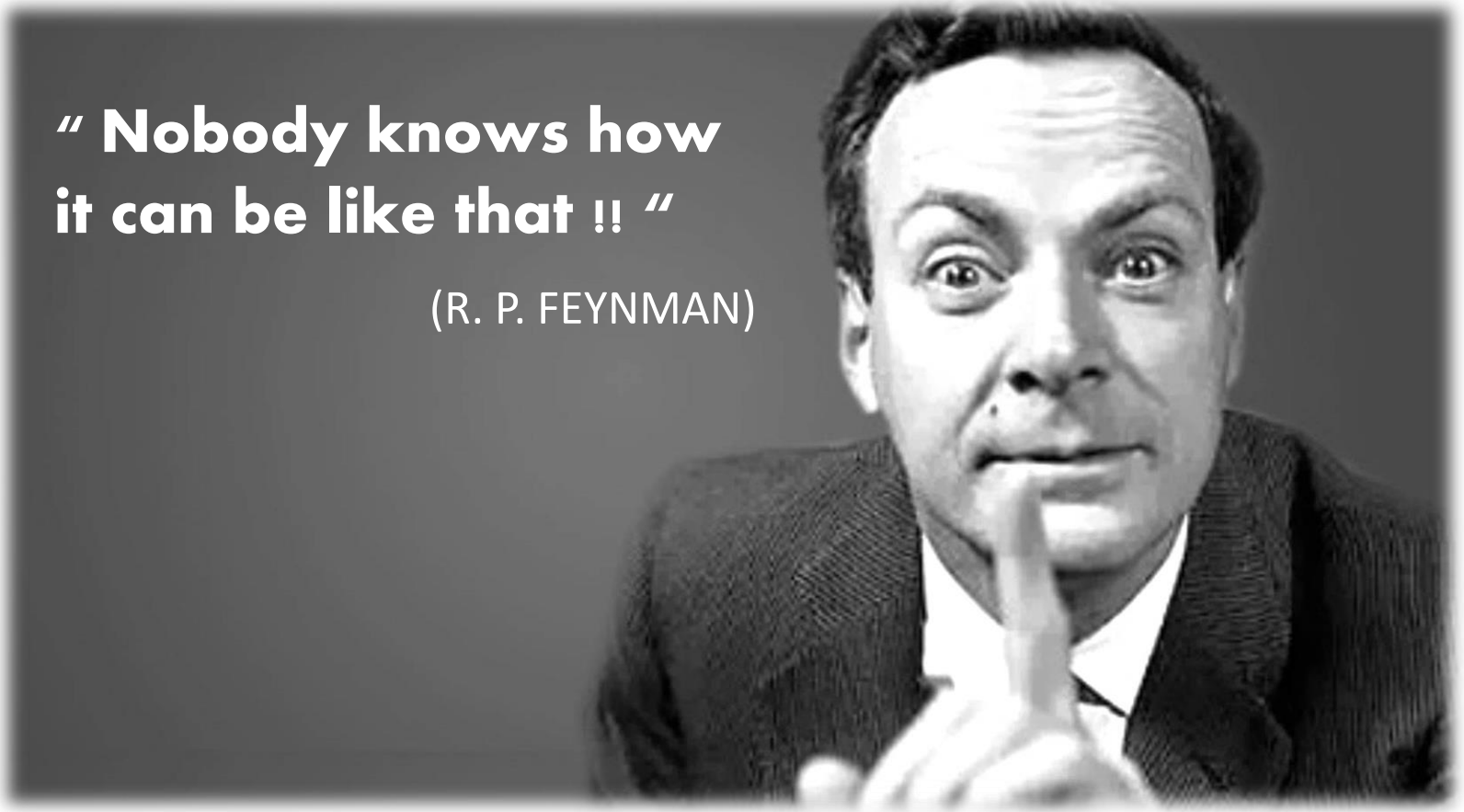
What does it mean ???

"I tried for many years to save physics from discontinuous energy levels…"

[Max Planck]

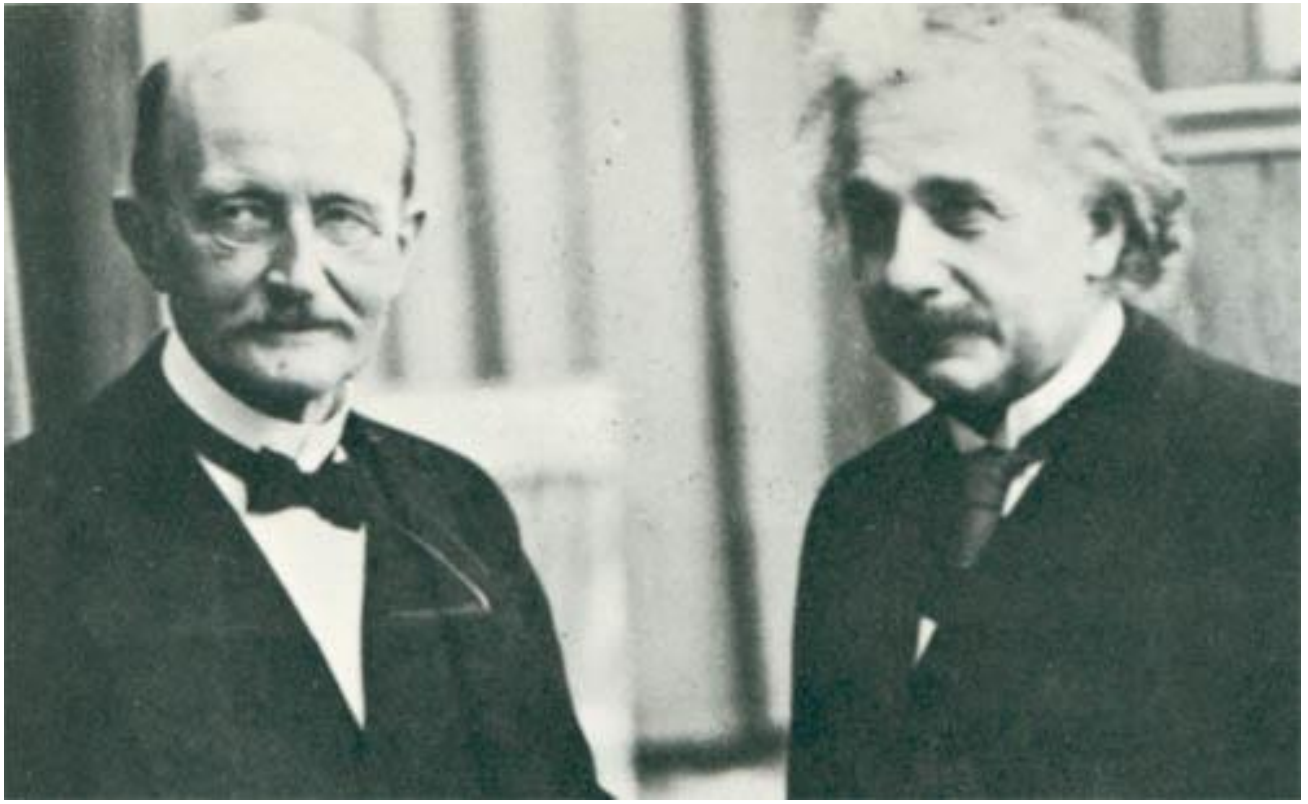
- There is no way for classical physics to explain the black body spectrum;
- Not all values of energy can be exchanged. Energy levels are discrete or **quantized**;
- Quantum theory is born. Other weird behaviors of Nature can be explained in the light of quantum mechanics (see in a while…)!

How can it be like that ?!?!



The skepticism and the genius

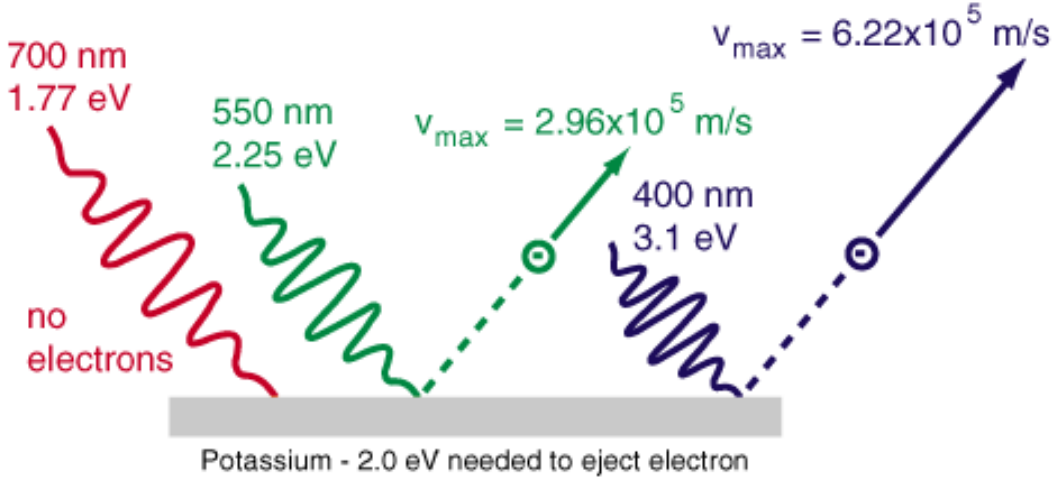
No one is able to accept the quantum hypothesis (not even Planck himself!!), except one (really outstanding) man, who immediately understood the power of the theory, absorbing and developing it in only **5 years!!**



The photoelectric effect

The **photoelectric effect** is the observation that many metals emit electrons when light shines upon them only if light reaches or exceeds a threshold frequency, below which no electrons can be emitted regardless of the amplitude and temporal length of exposure of light!

Einstein found the solution, based on Planck's hypothesis!



$$T_{\max} = h\nu - W$$

The maximum kinetic energy of e- depends only on W (fixed by materials) and v ….

The quantum mechanics was born...

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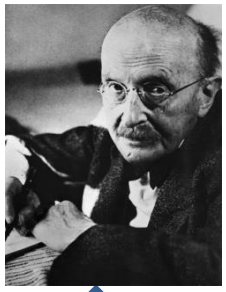
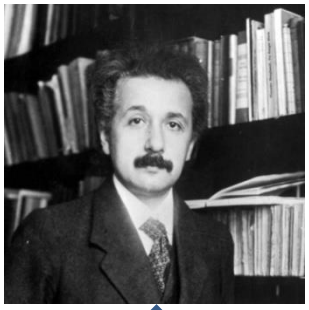
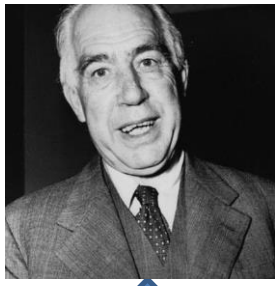
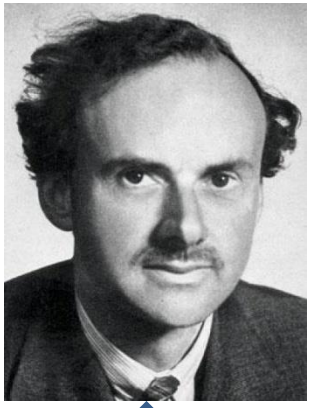
I. LANGMUIR M. PLANCK Mme CURIE H.A. LORENTZ A. EINSTEIN P. LANGEVIN Ch.E. GUYE C.T.R. WILSON O.W. RICHARDSON

Absents : Sir W.H. BRAGG, H. DESLANDRES et E. VAN AUBEL

The evolution of quantum mechanics

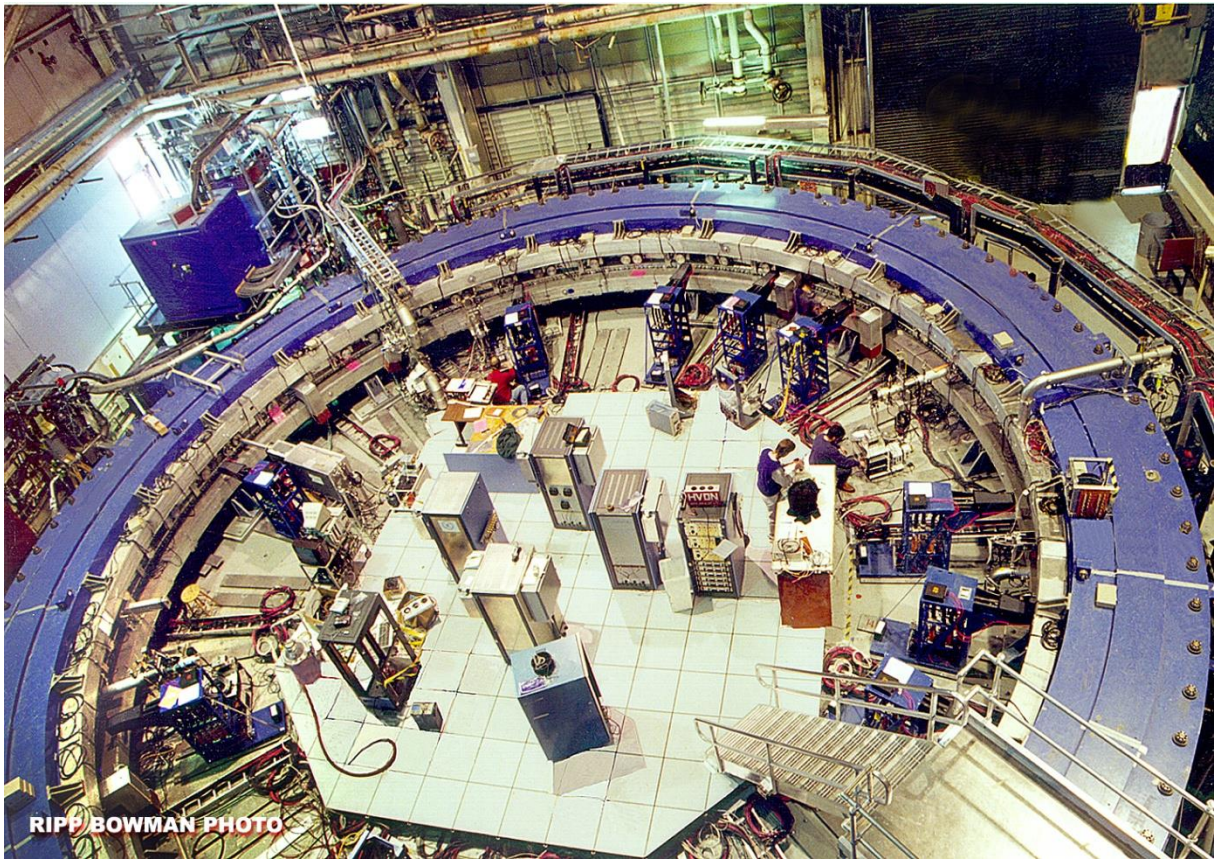


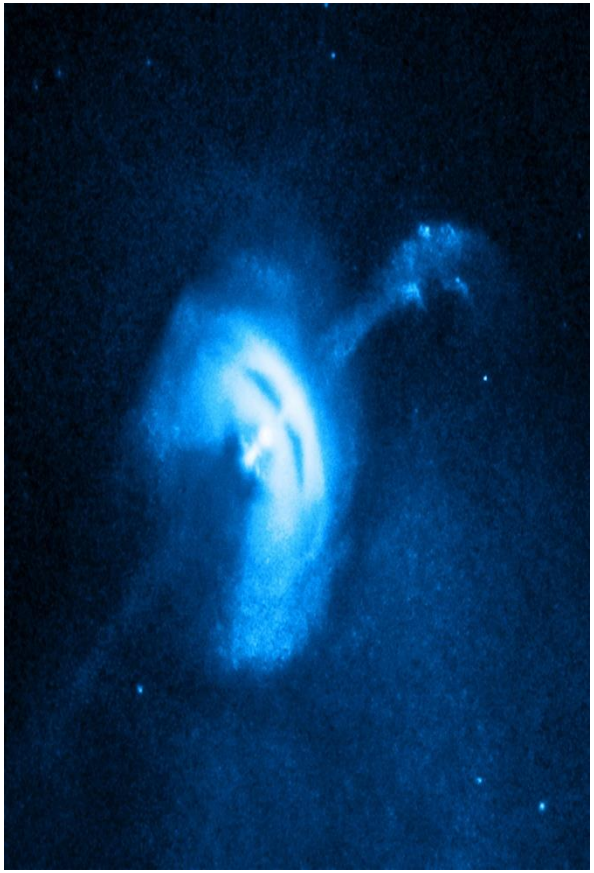




1900 1905 1913 1922 1924 1925 1926 1928 1965

The success of quantum mechanics

It is probably the most successful theory, being able to explain different phenomena observed at all scales: from elementary particle interactions (see g-2 experiment) to the evolution of stars (see the neutron star stability)!





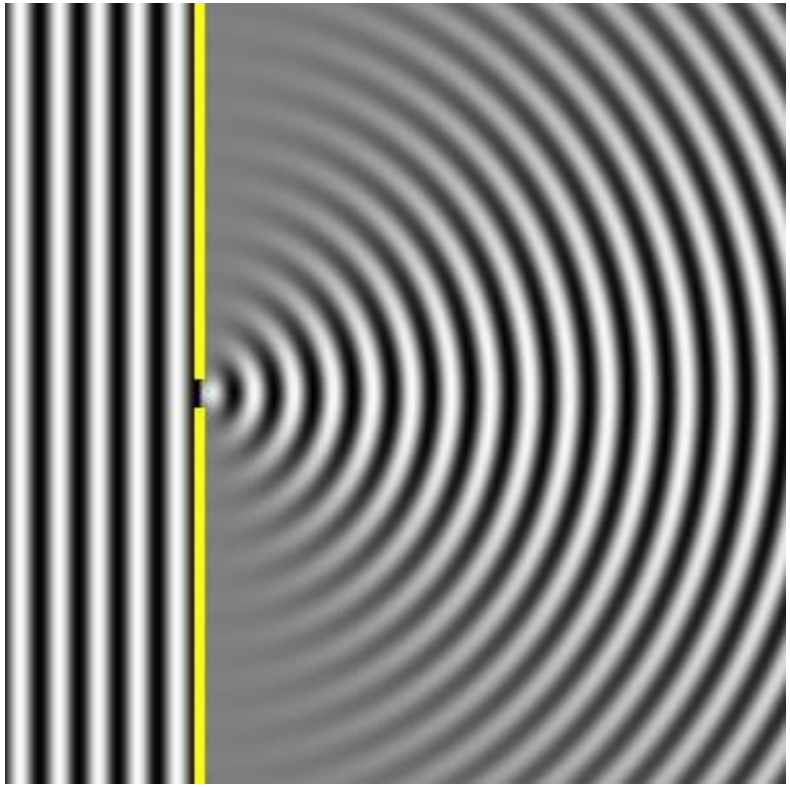
The "absurd theory"

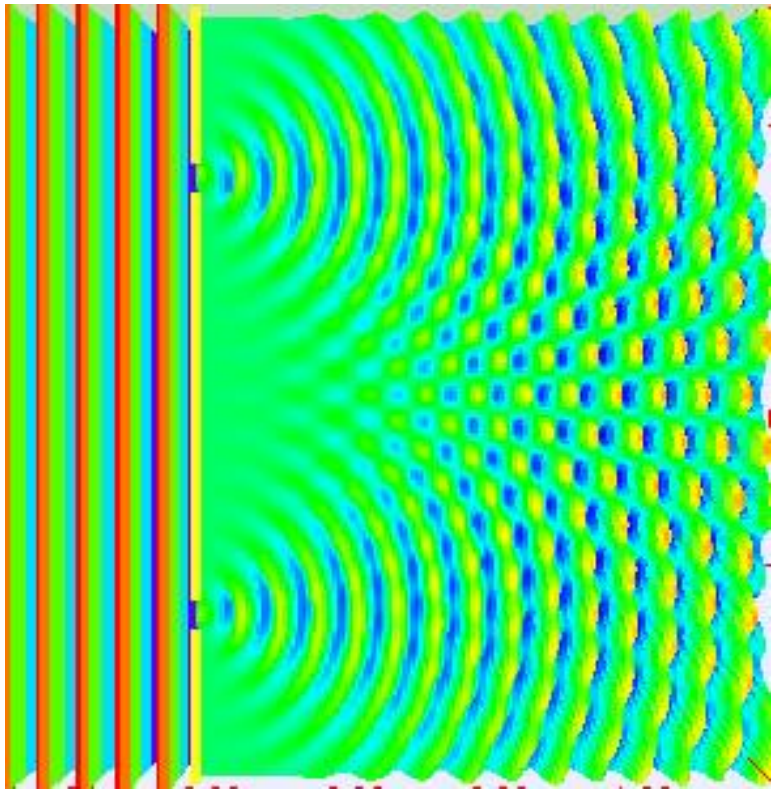
The difficulty in understanding quantum mechanics is that is it absolutely out of our "common sense" and "everyday experience".

The guideline **must** be 'logic', not the 'common sense'….

The double-slit experiment

In 1802, Thomas Young performed an experiment showing the wave-nature of light! Light produces an ***interference pattern*** when passing through a couple of thin slits…



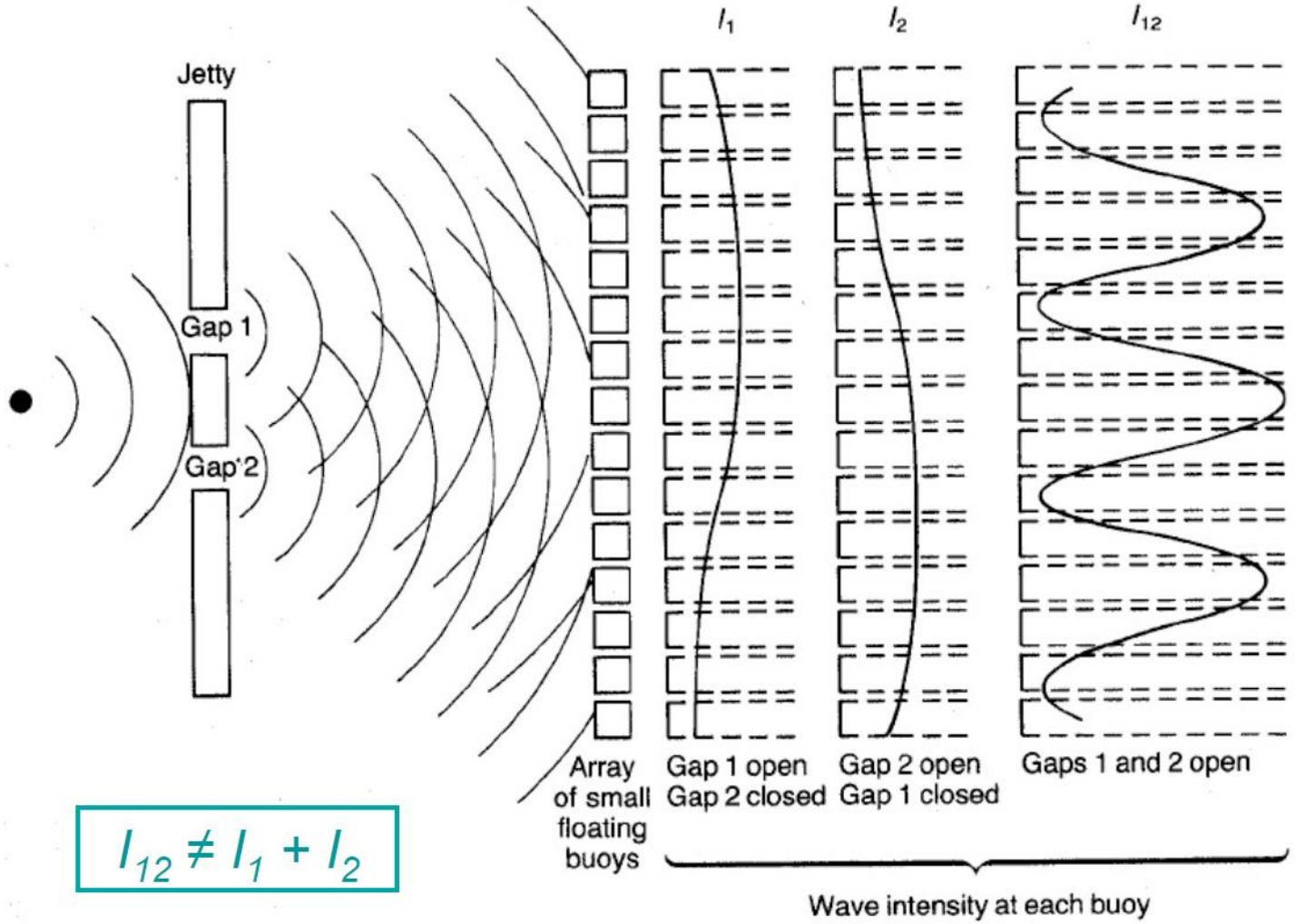


The double-slit experiment

A peculiar feature of **waves** is that they can produce **interference**

The double-slit experiment

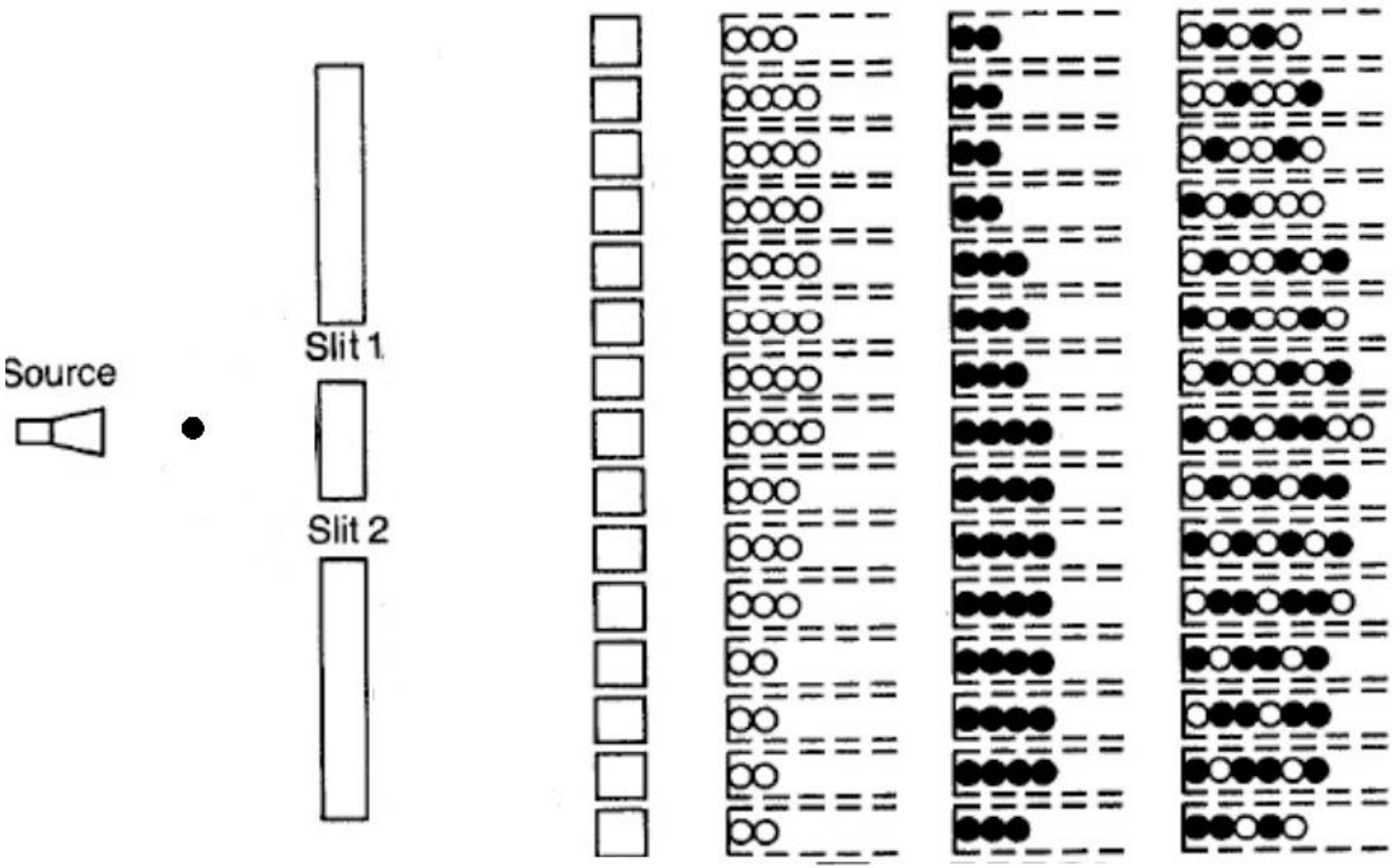
WAVES



$I_{12} \neq I_1 + I_2$

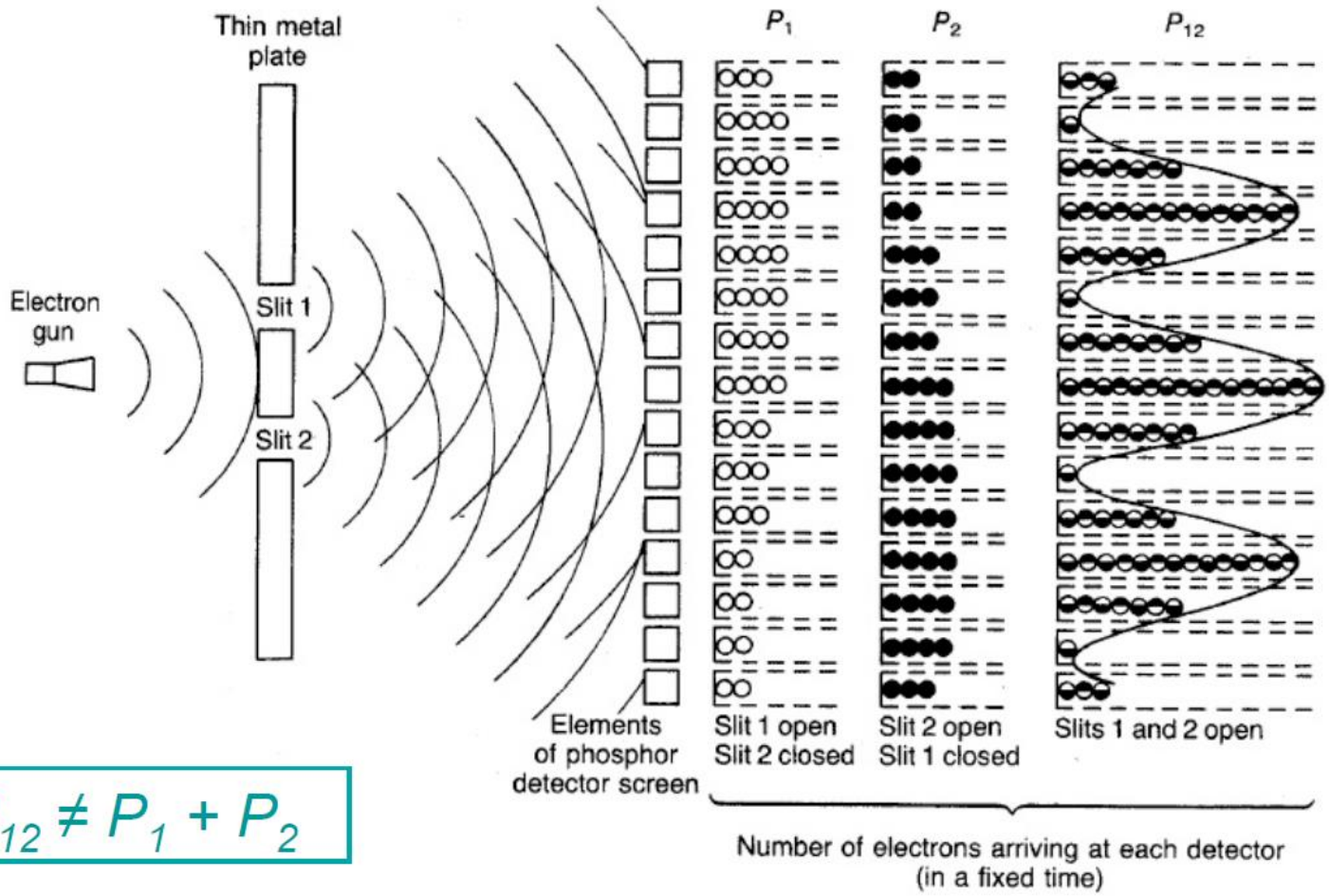
The double-slit experiment

CLASSICAL PARTICLES



The double-slit experiment

ELECTRONS

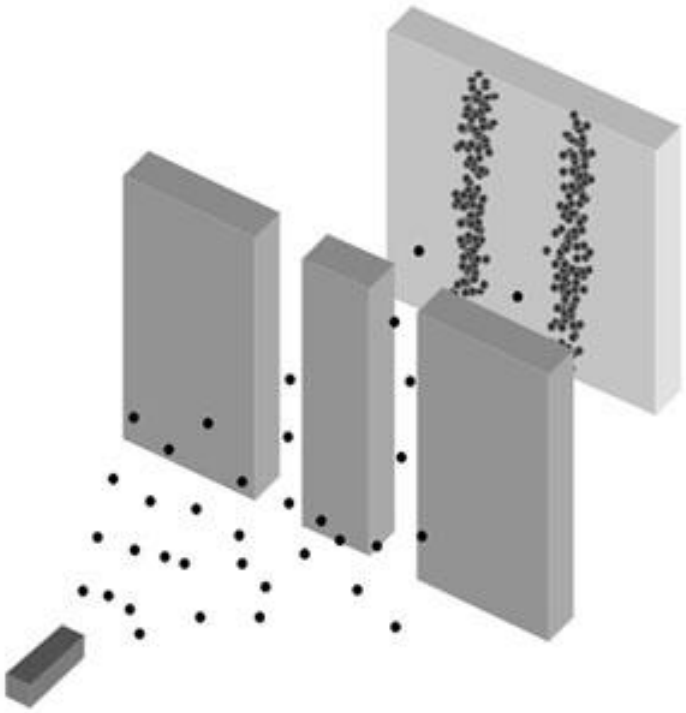


$$P_{12} \neq P_1 + P_2$$

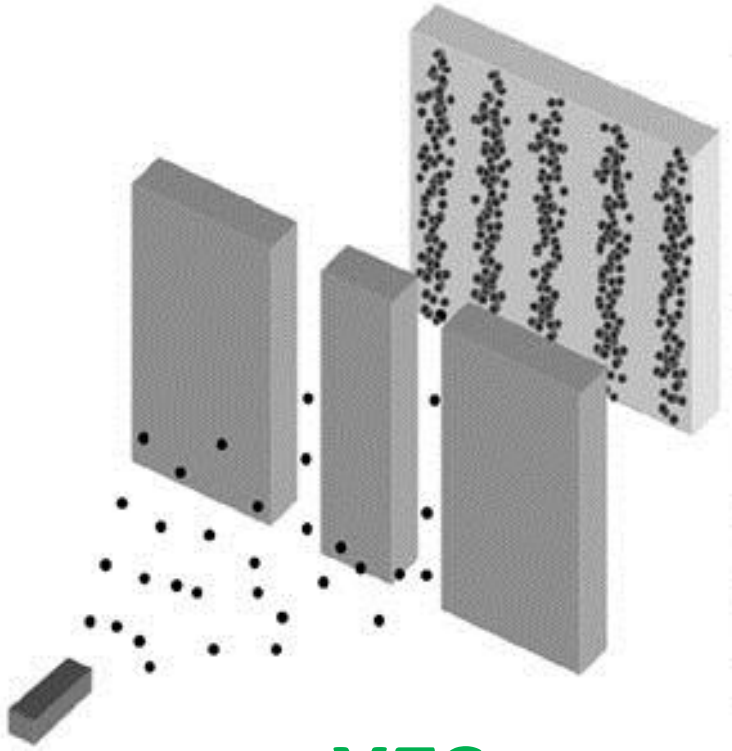
The double-slit experiment

Electrons interact with the screen as physical particles, but produce an interference pattern! They propagate with a “wave-nature”!

Based on our common sense, (the everyday experience), this is absolutely **not** expected!



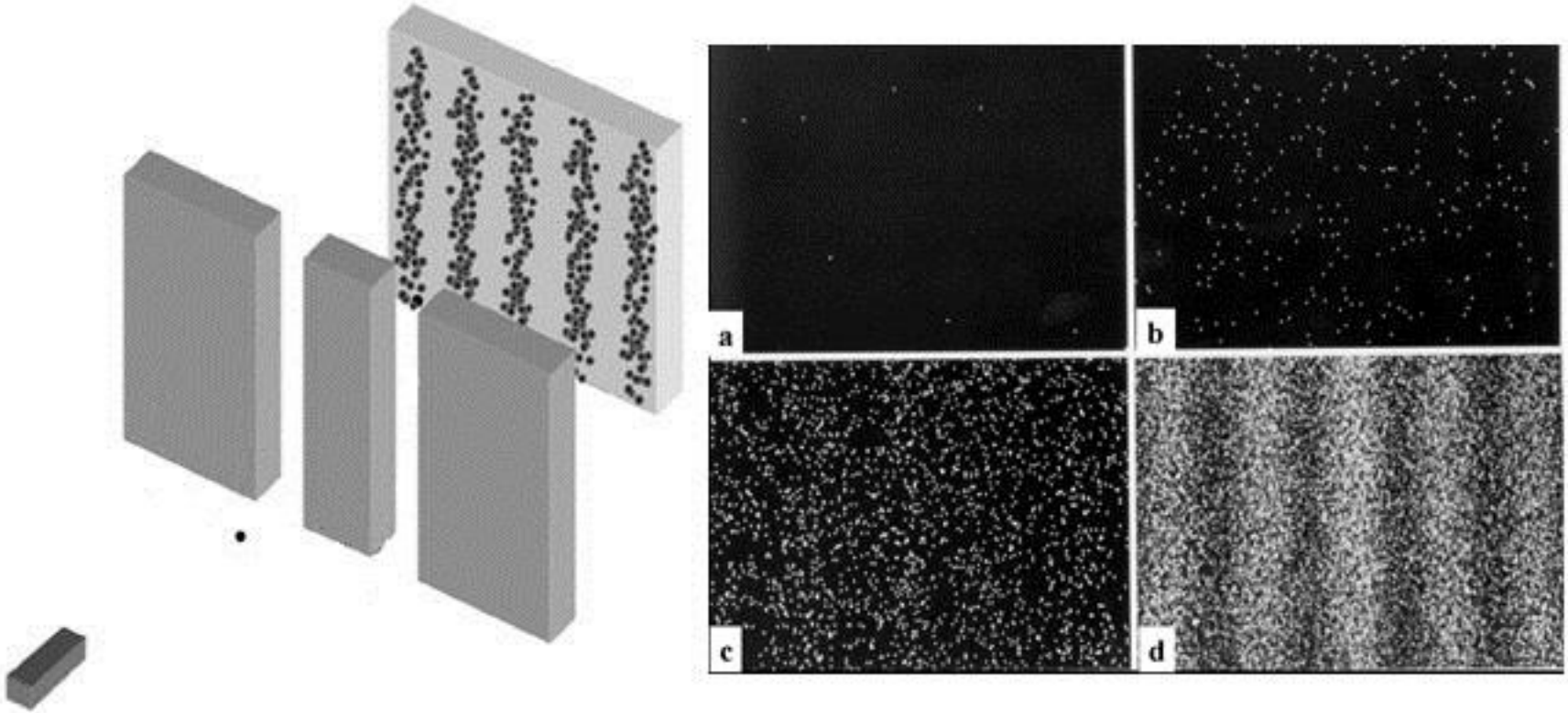
NO



YES

The double-slit experiment

Interference pattern is still there is a **SINGLE** electron is passing through the slit at a time!!!



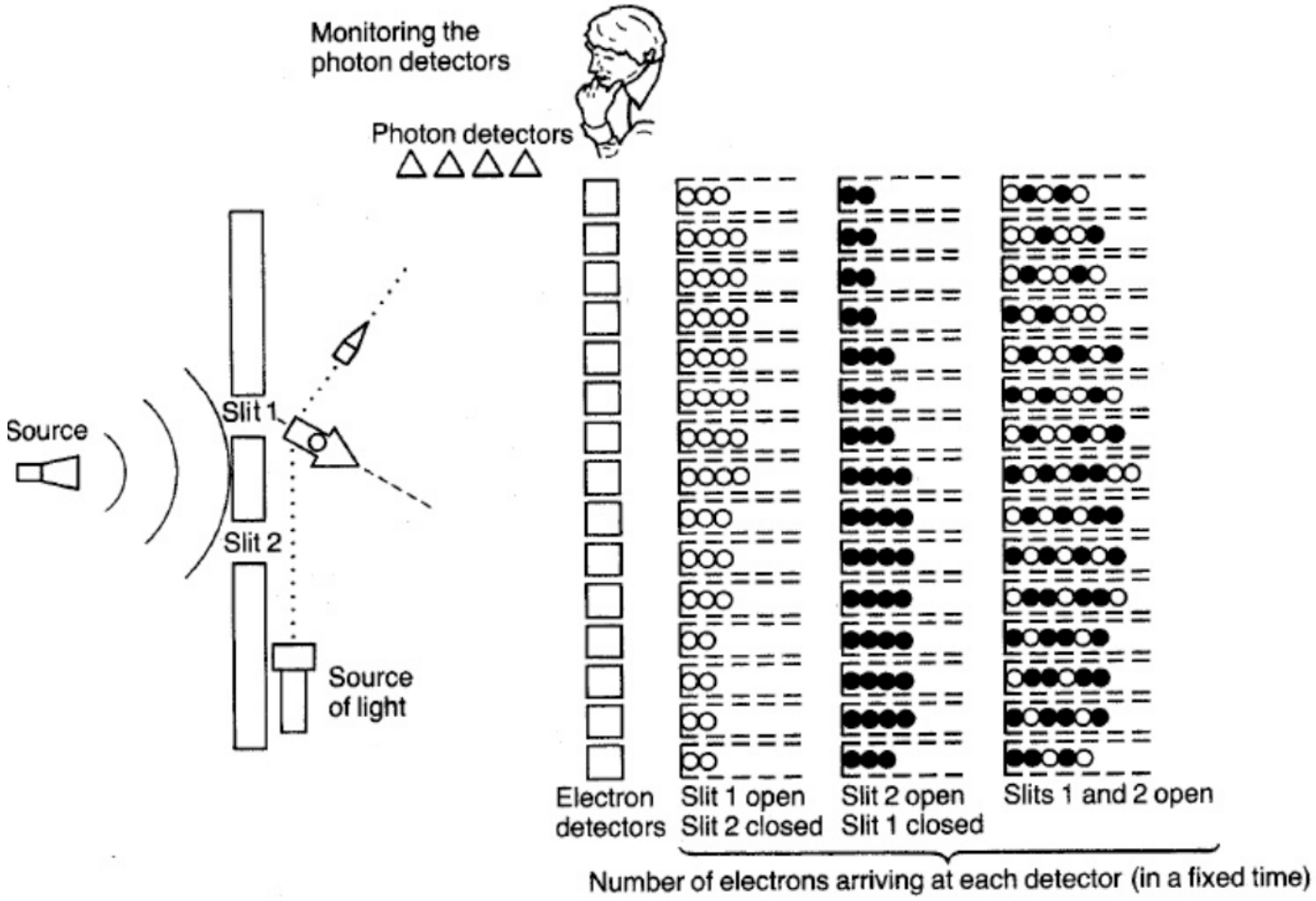
The double-slit experiment

As soon as one of the slit gets closed, the interference pattern disappears. It is as if the e^- knew whether the slits are open or not, before going through them!

How is it possible??

The double-slit experiment

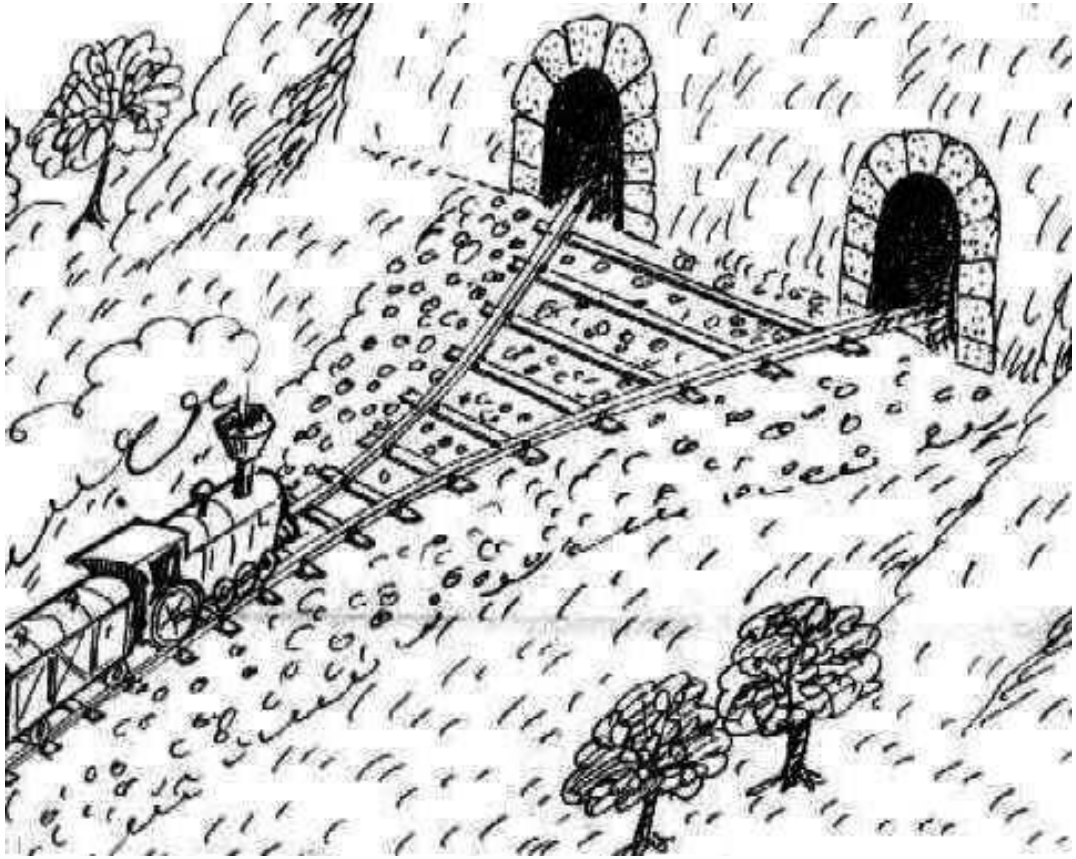
If one observe the electron while passing through the slit (i.e. measure its position to see which slit is passing) , one find it, but the interference disappear!!!



The double-slit experiment

There is no way to understand what slit the e^- passes through.

One can just conclude (based on the experimental results) that it propagates as a wave (as if it passes both slits **simultaneously**!!!) but it's detected as a particle!!!

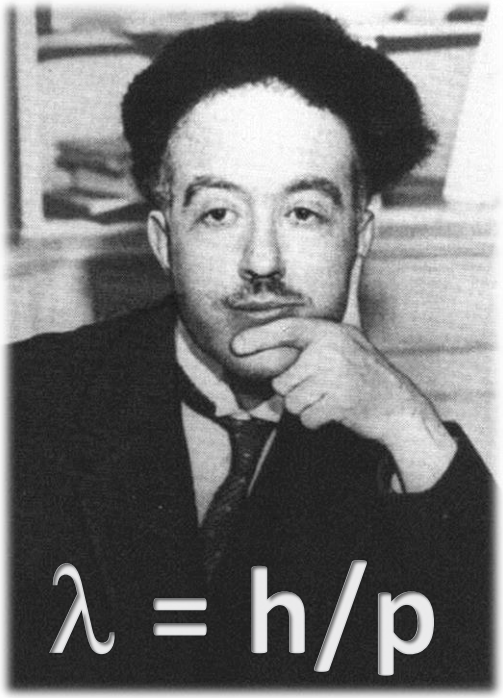


The wave-particle duality

A new conception of reality (far far away from everyday's one) is needed:

- Quantum particles behaves as classical objects (as bullets) whenever "observed"
- Quantum particles behaves as waves if "not observed"

A double nature of quantum system is required: the **wave-particle duality** is born!



"The fundamental idea of [my 1924 PhD thesis] was the following: The fact that, following Einstein's introduction of photons in light waves, one knew that light contains particles which are concentrations of energy incorporated into the wave, suggests that all particles, like the electron, must be transported by a wave into which it is incorporated... My essential idea was to extend to all particles the coexistence of waves and particles discovered by Einstein in 1905 in the case of light and photons"

Nothing is said about the "real mechanism"...

(the theory is prior to D.G. & T. experiment and Schrödinger!!)

The wave-particle duality

According to **wave-particle duality**, everything should have a wave-nature!

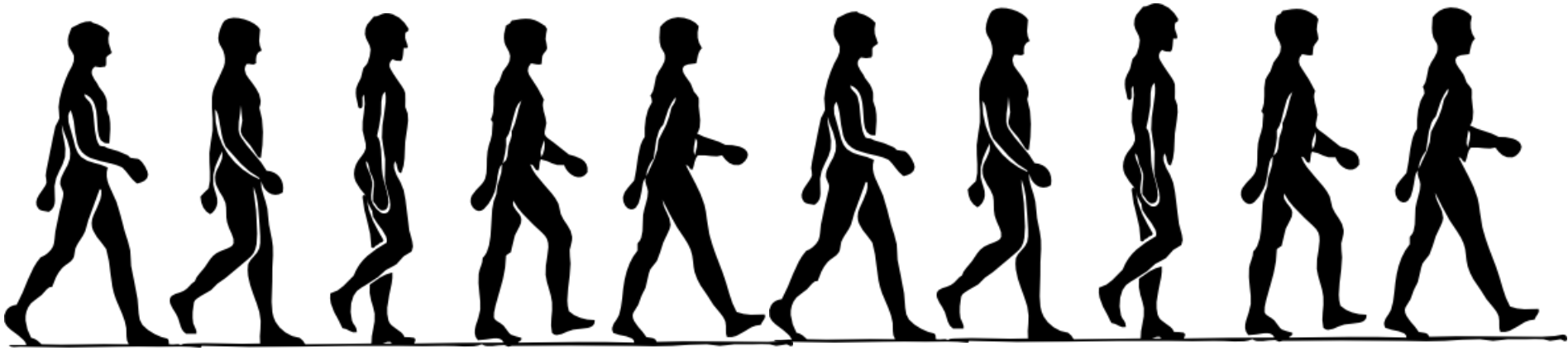
$$\lambda = h/mv$$

$m = 70\ kg$

$v = 5\ km/h$

$h = 6.626 \times 10^{-34}\ J\ s$

$\Rightarrow \quad \lambda = 6.8 \times 10^{-36}\ m$



The wave-particle duality

According to **wave-particle duality**, everything should have a wave-nature!

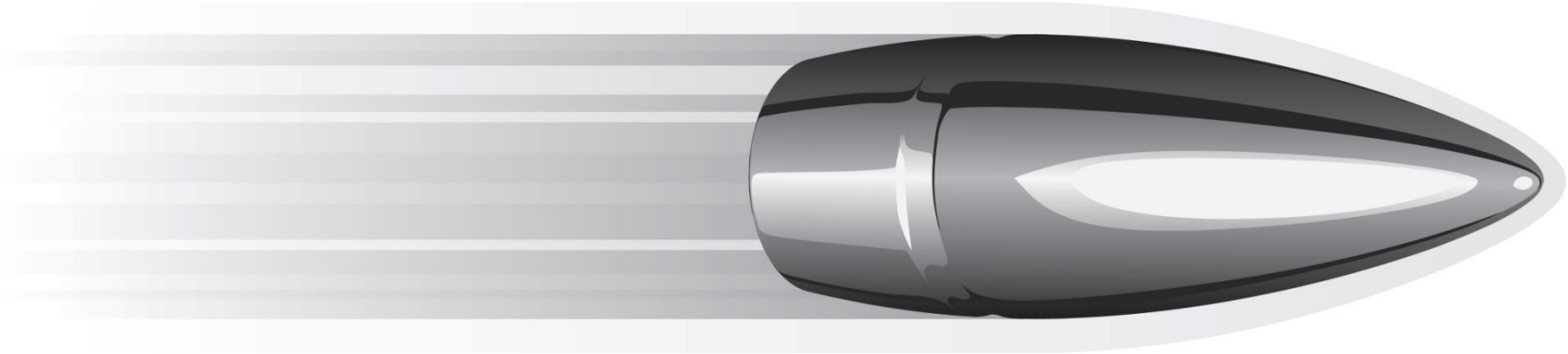
$$\lambda = h/mv$$

$m = 20\ g$

$v = 1000\ km/h$

$h = 6.626 \times 10^{-34}\ J\ s$

$\lambda = 10^{-34}\ m$



The wave-particle duality

According to **wave-particle duality,** everything should have a wave-nature!

$$\lambda = h/mv$$

$m = 9.1 \times 10^{-31}$ kg
$v = 3000$ km/s
$h = 6.626 \times 10^{-34}$ J s

$$\lambda = 2.4 \times 10^{-10} \text{ m}$$

ELECTRONS

Electrons wavelength is comparable with the atomic dimensions (that's why the interference is observed in "double slit"-like experiments!)

Atomic models should take into account the e^- wave nature! -> **BOHR's description**!

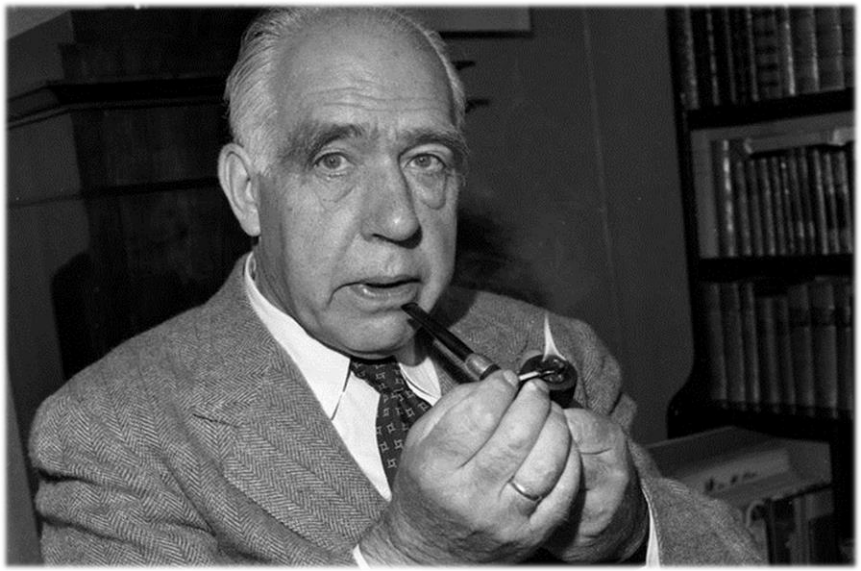
The Bohr's atom

The description of the atom provided by Rutherford has the following limitations:

- e^- is a charge in motion (should emit radiation according to Maxwell and Hertz). It should fall on the nucleus in 10^{-11} s !! -> **ATOM IS UNSTABLE!!**

- Atomic spectra!

- Observed properties of elements in the Mendeelev periodic table!

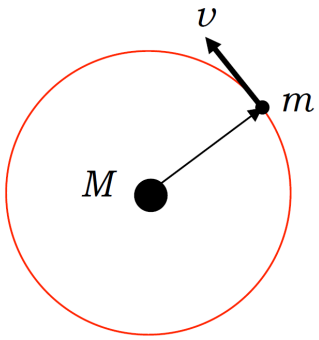


BOHR HYPOTHESIS (1913)

" Energy is not the only quantized quantity, also the angular momentum has a discrete nature! "

The Bohr's atom

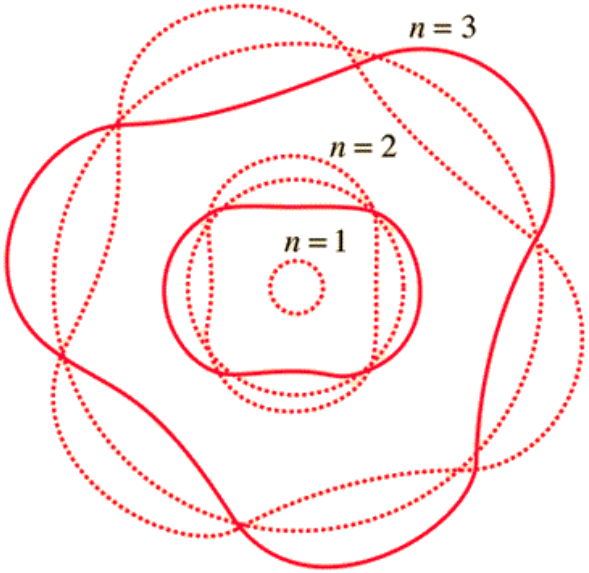
The angular momentum of a particle is defined as: **L** = **r** x **p** = **r** x m**v** -> $|L| = mvr$



$$\mathbf{mvr = n\hbar} \qquad \text{with } n \in N$$

Also r gets quantized under Bohr's hypothesis!

$$r_n = n \frac{h}{2\pi} \frac{1}{mv}$$



De Broglie hypothesis is consistent with this model:

$$r_n = n \frac{1}{2\pi} \frac{h}{mv} = \frac{n}{2\pi} \lambda \qquad \boxed{2\pi r_n = n\lambda}$$

The allowed orbits are the one for which the e^- wavelenght leads to a **stationary condition**!

The Bohr's radius

Starting from the Newton equation for the e- (Centripetal Force = Coulomb Attraction) ($k_e = 1/4\pi\varepsilon_0$)

$$\frac{mv_n^2}{r_n} = k_e \frac{e^2}{r_n^2} \qquad (mv_n)^2 = mk_e \frac{e^2}{r_n}$$

but (Bohr's hypotesis) **$mvr = n\hbar$**

$$n^2 \frac{\hbar^2}{r_n^2} = mk_e \frac{e^2}{r_n}$$

$$r_n = n^2 a_0 \qquad E_n = -\frac{1}{2n^2} \frac{k_e e^2}{a_0}$$

Bohr's Radius $\quad a_0 = \dfrac{\hbar^2}{k_e e^2 m} = 0.53$ Å

The the wave-function

According to De Broglie: the wave nature of the electron is hard to figure out.... It is associated with the particles but it is not "concrete". It's just a mathematical description!

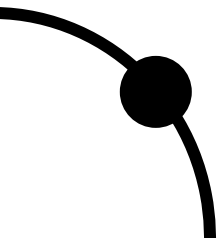
The **wave-function**, Ψ (funcition of space and time), describes the quantum system (it is also called *probability amplitude*)

The square of Ψ gives a measure of the probability of finding the particle in a given state

If an event can occur through different possibilities (for example the double-slit experiment with electrons) then the probability amplitude of the event is given by the sum of the two Ψ_1 and Ψ_2 and interference appears!

$$P_1 = |\Psi_1|^2 \qquad \Psi = \Psi_1 + \Psi_2$$

$$P_2 = |\Psi_2|^2 \qquad P = |\Psi_1 + \Psi_2|^2 \neq P_1 + P_2$$



The role of probability

In classical physics, predictability is a practical issue (theory is exact): given a position and a momentum, the future knowledge of a trajectory is fixed

In quantum mechanics, every result of a measurement is possible a priori!
Trajectory is intrinsically not predictable!

One can ONLY predict the probability of an event (with very high accuracy) but **nothing is deterministic**!

WHAT IS NOT FORBIDDEN IS COMPULSORY!

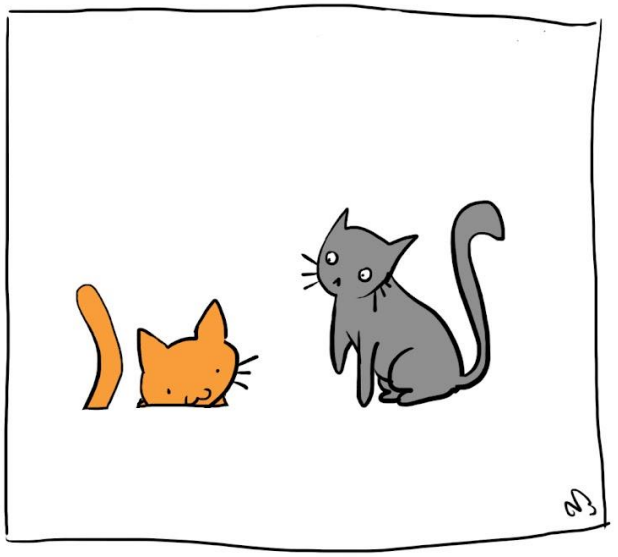
The tunnel effect

Without measuring, nothing certain can be said on the particle trajectory! So…

- One cannot say whether the e^- is passing slit #1, #2 or **both…it makes no sense at all to discuss it!**

- Particles show (fanta)scientific effects like the “tunnenling”

…it’s all a matter of probability!



The tunnel effect

The wave-function extends also beyond the potential barrier, so there is a non-null probability to find the particle in that region and if the potential barrier is thin enough the effect actually takes place!

The quantum tunneling enters in several processes:

- **Radioactive decays** (α–particles from unstable nuclei)
- **Tunnel diodes**
- **Tunnel effect microscopes** (Nobel Prize 1986)
- **Spontaneous DNA mutations**

The double-slit experiment (The Return)

In order to "see" the e^-, one has to send a photon on it –> the effect is absolutely negligible on a macroscopic object (like a bullet), but it's destructive on the electron….

To minimize this "perturbation" one should reduce the momentum of the γ, i.e. its energy, alias the frequency (or wavelength). **Warning** -> not the intensity (that is related only to the # of e^-)!!

$$E = h\nu \qquad E^2 = p^2c^2 + m^2c^4$$

(for γ m=0 -> E = pc)

$$pc = hc/\lambda \qquad p = h/\lambda$$

If one increase λ to minimize p (and hence reducing the momentum perturbation), the spatial resolution gets worse and one cannot distinguish between the two slits!!

NO WAY TO GET OUT OF THIS "TROUBLE"!!

The "uncertainty principle"

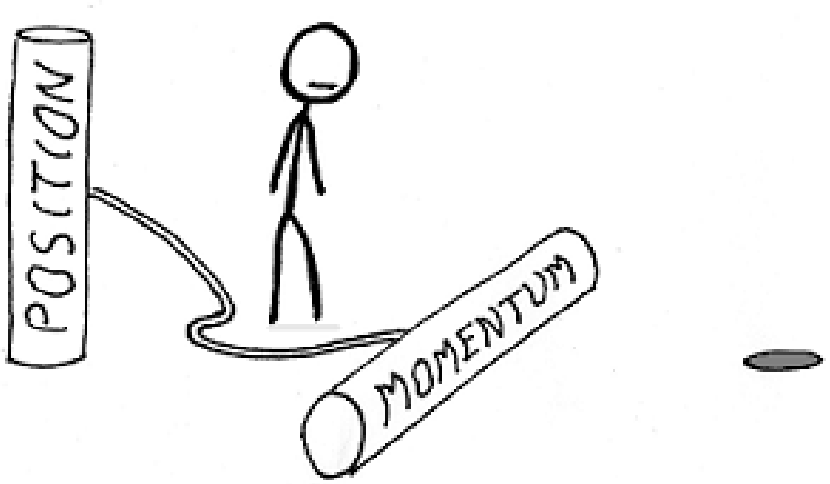
(the Heisenberg's uncertainty relation)

A general result of quantum theory (introduced by Heisenberg in 1926) is the "uncertainty principle", which is summarized by a relations like:

$$\Delta x \, \Delta p \sim h \qquad \Delta E \, \Delta t \sim h$$

Complementary variables cannot be determined simultaneously with and arbitrary precision: the limit of knowledge is fixed by the Planck's constant!

(N.B. = not all variables affect each other in couples!)



Classical vs Quantistic

The act of measuring is always introducing a perturbation in the system

CLASSICAL (ignorance!)

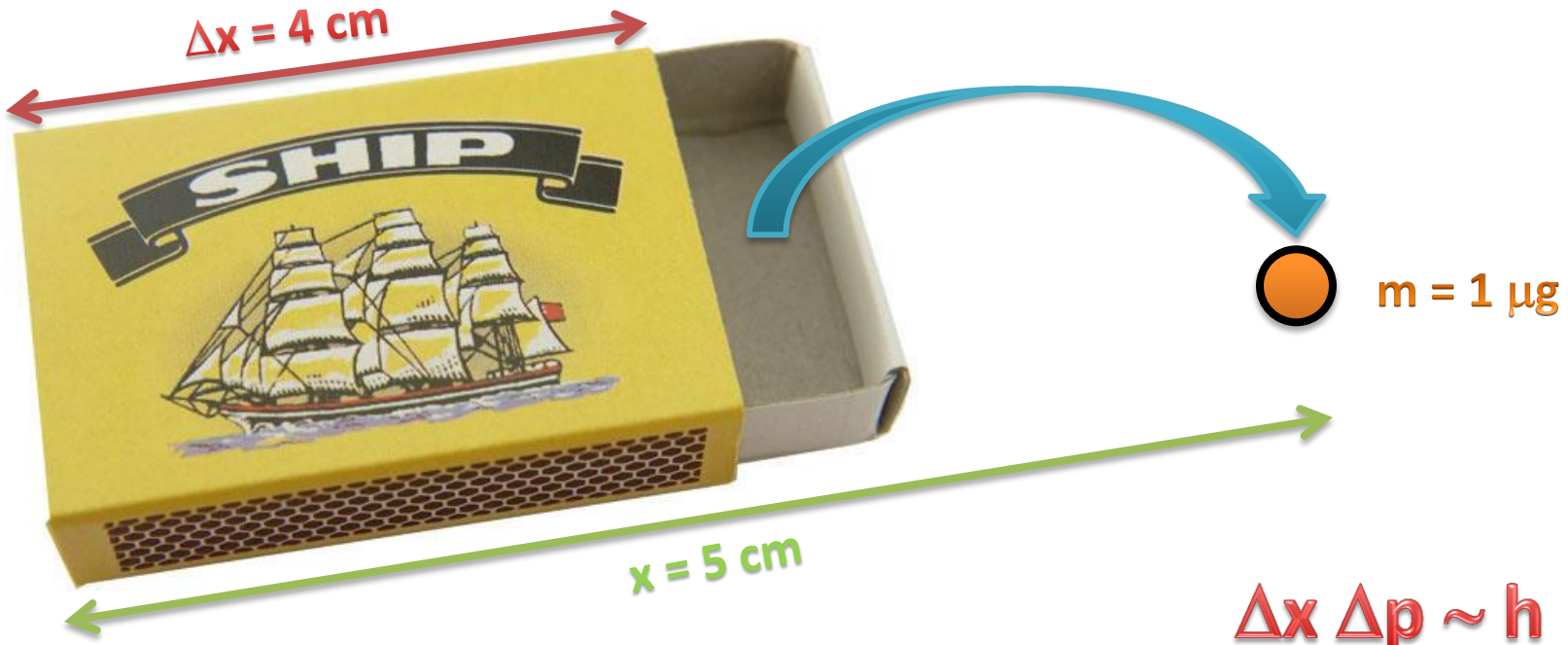
- Perturbation can be reduced at lib
- Theory is deterministic, the perturbation effect can be corrected!

QUANTISTIC (uncertainty!)

- Perturbation is ruled by h (unavoidable)
- Theory is stochastic, the perturbation cannot be corrected!

The tunnel effect ?!!?

If there exists a possibility to observe a particle “wherever” in the space…
Why we do not observe a grain of sand jumping out of a matchbox?!!?



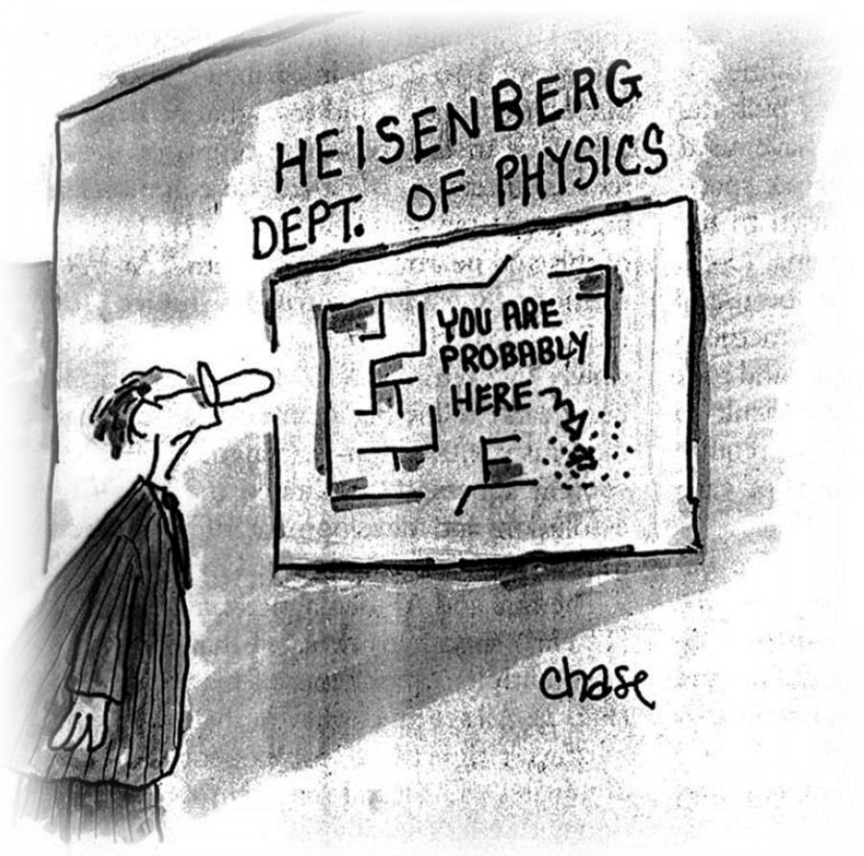
$\Delta x \, \Delta p \sim h$

$(mx/t)\,\Delta x \sim h$ $\quad t \sim (mx/h)\,\Delta x \approx 3 \times 10^{23}\,s \approx 10^{16}$ anni !!!!!

Classical vs Quantistic

Quantum mechanics has to deal with an objective casuality. We don't know the status of a system until we measure it.
In between, "everything is possible"!

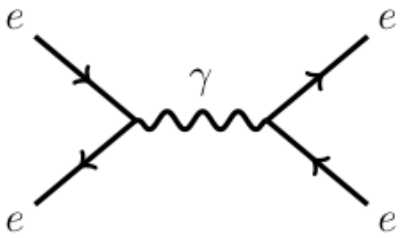
Results appear such incredible because we always try to read them in a classical way (based on everyday's experience!)

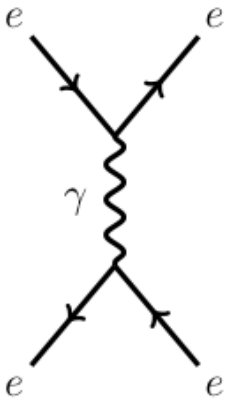


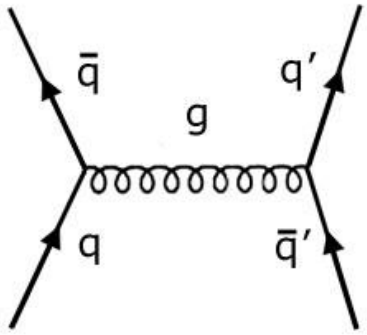
The Heisenberg's uncertainty applied

$$\Delta E \, \Delta t \geq h$$

ENERGY CONSERVATION CAN BE VIOLETED FOR A SMALL ENOUGH INTERVAL OF TIME







The time-energy uncertainty principle plays a key-role in particle interactions. Energy is violated if all particles of an electromagnetic vertex are real! Virtual particles (mass value different from the on-shell one) must intervene in the processes…

The Heisenberg's uncertainty applied

$\Delta x \, \Delta p \geq h$ (and the stability of atoms...)

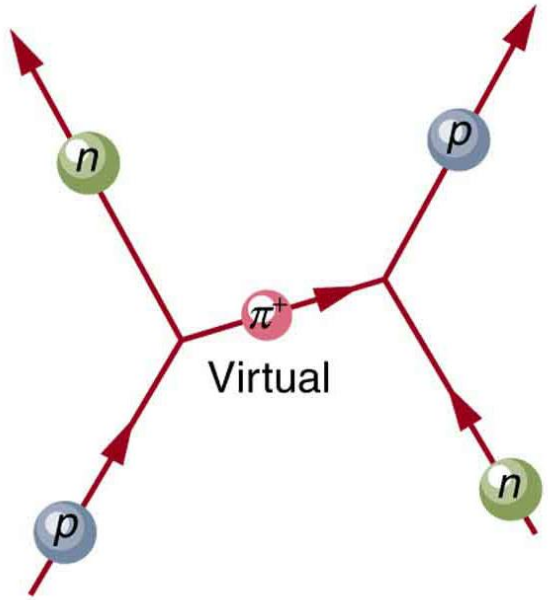
✓ If the electron fall on the in the nucleus, its position would be better determined, but the U.P. would involve a huge speed which in turn would extract e^- from the nucleus

✓ The special relativity fixes a limit on e^- velocity, and hence, on the atom dimensions (i.e. the e^- spatial delocalization!)

✓ Falling down further would require more E_k, and the E_p is not enough! Quantum fluctuations are limited by the other U.P!!! NO WAY!

The Heisenberg's uncertainty applied

$\Delta E \, \Delta t \geq h$ (and the short range of strong interaction)



n (ddu)
p (uup)
π^+ ($u\bar{d}$)

✓ At a first approximation, pions can be considered the mediators of the strong interaction (actually the gluon is)

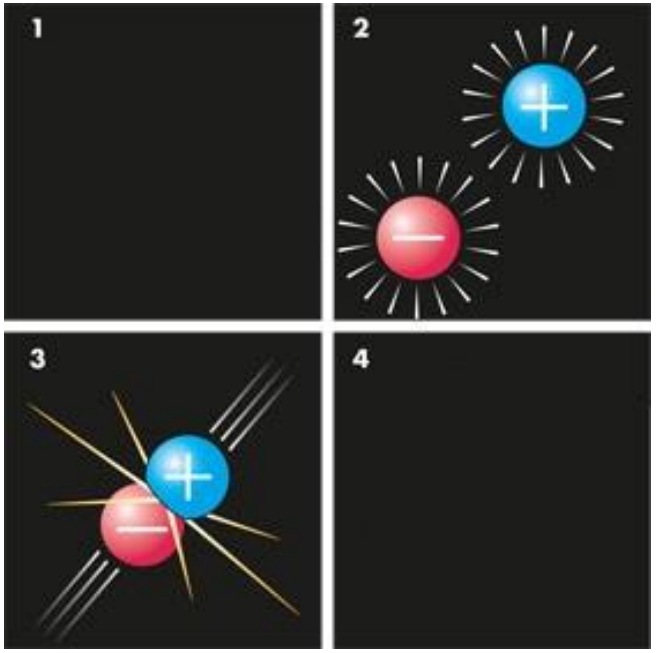
✓ Virtual pions can be created thanks to the U.P., but their energy is large (about 140 MeV, i.e. 300 times larger than an e^- !) They can live only for a very small amount of time!

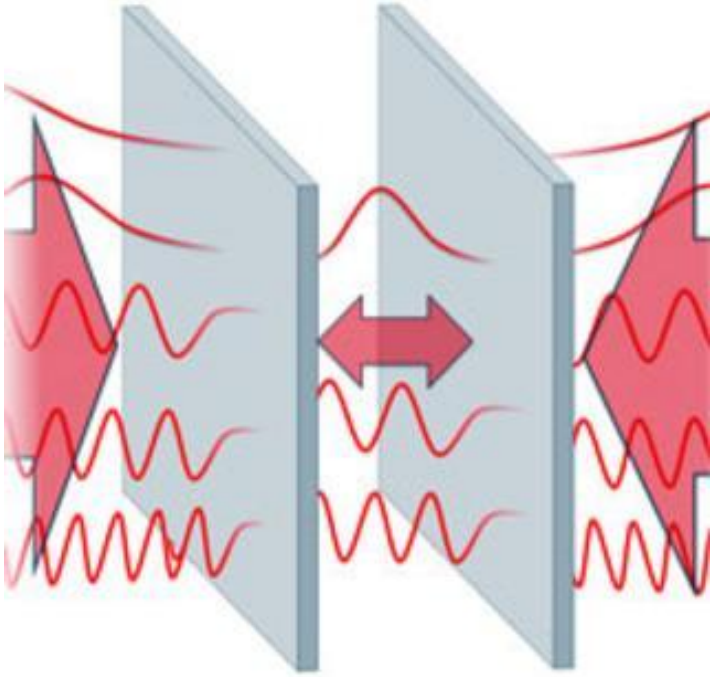
✓ Since they are short-lived, they can travel in a very short range, and hence, the nuclear force is a "short-range interaction"!

The Casimir effect

"Vacuum is not really void"

According to QM the vacuum state is not truly empty but instead contains fleeting electromagnetic waves and particles that pop into and out of existence





Virtual particles from quantum fluctuations of vacuum can interact with the plates, but the smaller space in between them, results in a lower number of allowed modes and hence, in a smaller pressure towards the outside. As a result: the plates curve toward inside!!

The Heisenberg approach

Heisenberg and Born proposed the first fully consistent quantum theory: the **matrix mechanics.** The theory is strongly "mathematichal", and the meaning of observable properties as opertors and of the state of the system as a vector is not obvious!

BUT IT PERFECTLY WORKS!!

"[…] the smallest units of matter are not physical objects in the ordinary sense; they are forms, ideas which can be expressed unambiguously only in mathematical language."

[W. Heisenberg]

MATRIX ALBEGRA

PROBABILITY

UNCERTAINTY PRINCIPLE

The Schrodinger approach

The Schrodinger equation provides the evolution of a quantum state Y, given the Hamiltonian of the system under study (it is the 'quantum equivalent' of Newton's law)

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi$$

Born reconciled Schrodinger approach to the matrix one by means of a **probabilistic interpretation of the wave-function** (funny Nobel prize story...:P)

"[…] the already […] mentioned psi-function […] is now the means for predicting probability of measurement results. In it is embodied the momentarily attained sum of theoretically based future expectation, somewhat as laid down in a catalog."

[E. Schrodinger]

The quantum theory and ...

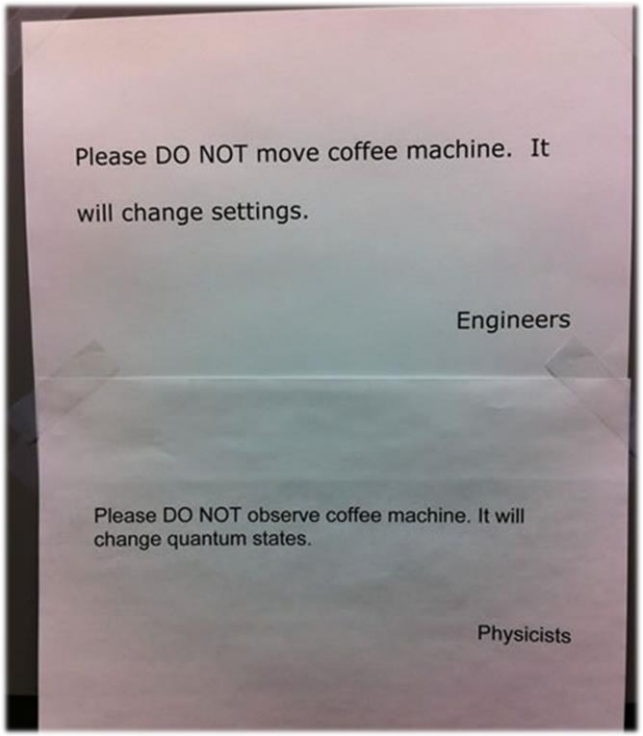
A quantum system (both in Heisenberg and in the Schrodinger approach, after the Born probabilistic interpretation of the wave function) may conists in the superimposition of different quantum states.

$$|\Psi> = c_1|\Psi_1> + c_2|\Psi_2> + c_1|\Psi_3> + \ldots$$

Each of these states is associated with a probability to be measured (calculable with arbitrary precision in both representations, although in a different way!)

The measurement process gets the system "collapse" in a CLASSICAL states (which can be identified by the state of a classical instrument).

After the measurement, the system has well defined properties



How does the collapse work?

What is the status of the system before the measurement ?

Few comments ...

"If that were the case, I'd rather do the gambler or the cobbler rather than the physical"

[A. Einstein]

"Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the "Old One." I, at any rate, am convinced that He does not throw dice"

[A. Einstein]

"God knows I am no friend of probability theory, I have hated it from the first moment when our dear friend Max Born gave it birth. For it could be seen how easy and simple it made everything, in principle, everything ironed and the true problems concealed. Everybody must jump on the bandwagon"

[E. Schrodinger]

"So it's not a situation independent from the experiment that is observed, but we ourselves call forth the facts, that then become an observation"

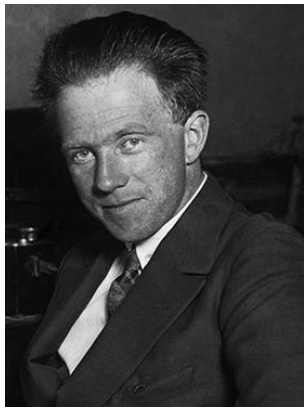
[P. Jordan]

"When I hear of Schrodinger's cat, I reach for my gun"

[S. Hawking]

Ready .... "fight"!

M. Born



W. Heisenberg

LVPR. De Broglie

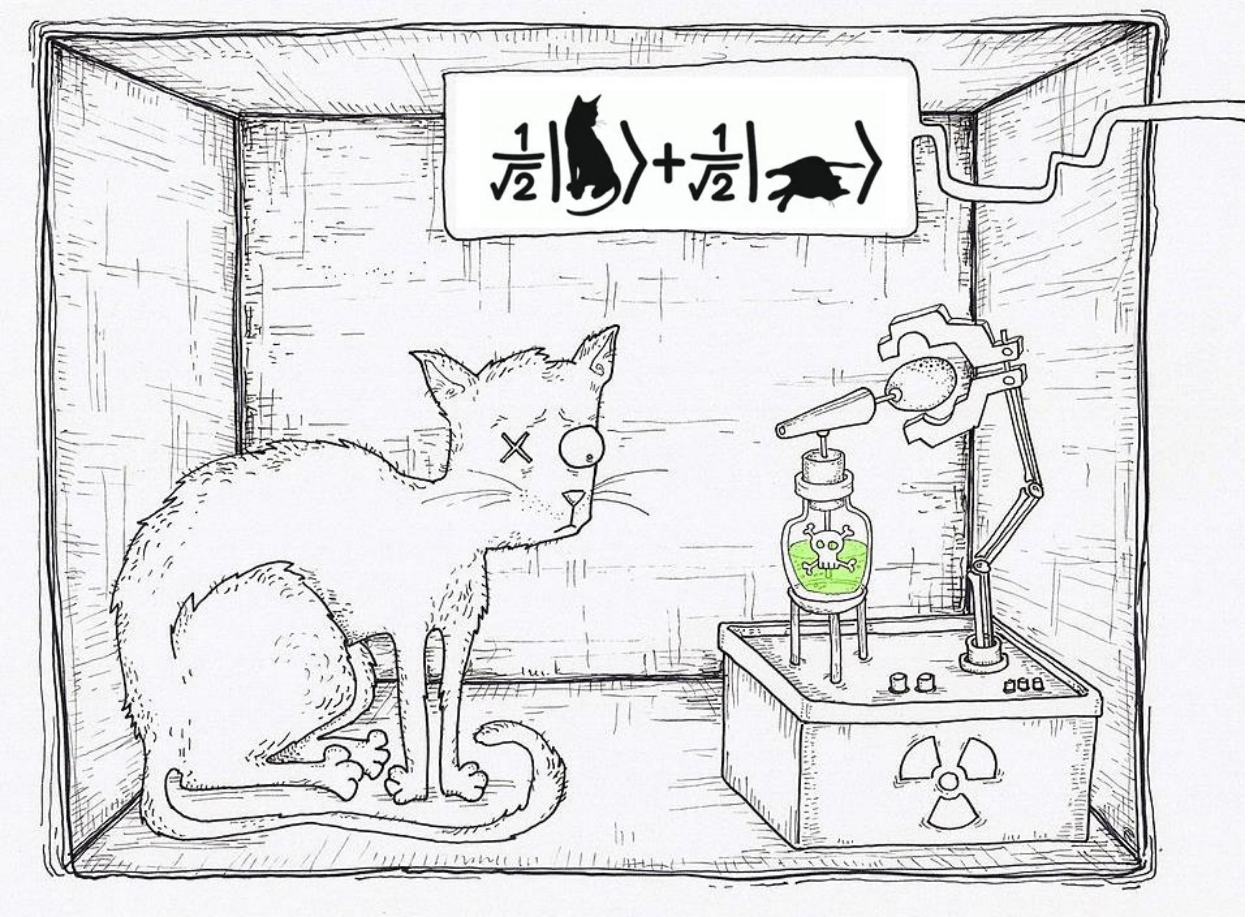
E. Schrodinger

The Bohr – Einstein debate

Schrodinger's cat

Basically, everything should be quantistic, because everything is made of atoms!
(if common sense fails when dealing with the microscopic, why it doesn't fail with classical objects??)

Schrodinger objection: **is it possible to divide the classical world from the quantum one?**



Decoherence and Many-Worlds

"A Psi-Function which involves a cat being in a 'dead' and 'alive' state at the same time, cannot be absolutely considered as a satisfactory description of real world"

[A. Einstein]

H. Dieter Zeh (1970)

Decoherence occurs when a system interacts with its environment in a themodynamically irreversible way.

Superimposition states are extremely delicate: the simple interaction of the system with an external photon, dissolves the spectrum of the Ψ function, accelerating its irreversible evolution towards a state of classic appearance.

<u>Problem is not the scale, but the impossibility to get a perfect insulation</u>

Decoherence and Many-Worlds

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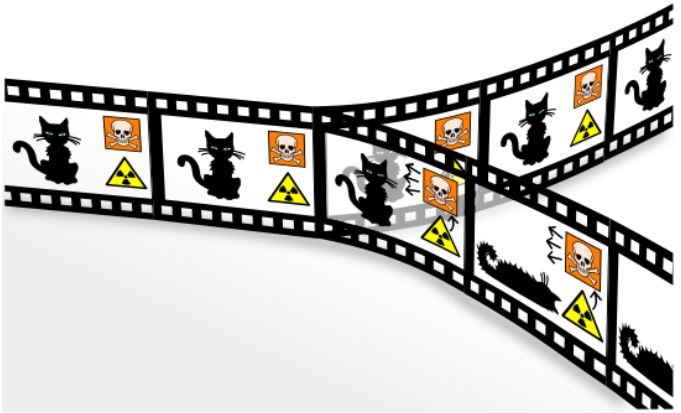
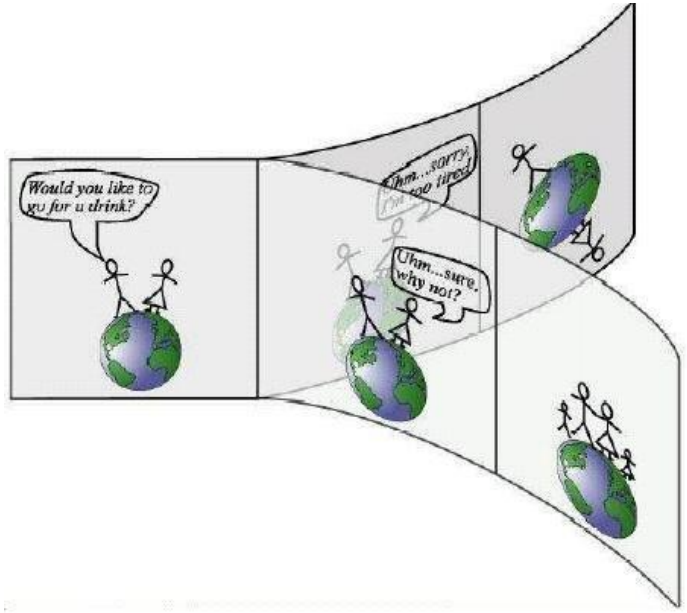


Everett (2003)

Every possibility is actually realized, but in different copies of the Universe. Each Universe-version is
itself subject to continuous multiplication and branching at each measurement process

There is no wave-function collapse at all. All possibilities realize in a Universe branch!



The Copenhagen interpretation

It holds that quantum mechanics **does not yield a description of an objective reality but deals only with probabilities** of observing various aspects of energy quanta, **entities that fit neither the classical idea of particles nor the classical idea of waves**

QM included the new theoretical models of phenomena which cannot be predicted on the basis of classical physics, **but it's completely counter-intuitive and almost "disturbing"!**

The CI is a set of rules for interpreting the mathematical formalism of quantum mechanics…

"There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how Nature is. Physics concerns what we can say about Nature..."

[N. Bohr]

The Copenhagen interpretation

Nonetheless, there are several basic principles that are accepted as being part of the interpretation **(and more in general, can be considered as the founding principles of QM!)**

1. A system is completely described by a wave-function, representing the state of the system, which evolves smoothly in time (through Schrodinger equation), except when a measurement is made, at which point it instantaneously collapses to an eigenstate of the observable that is measured. **The wave-function is nothing more than a theoretical concept**;

2. **The description of Nature is probabilistic**, with the probability of a given outcome of a measurement given by the square of the modulus of the amplitude of the wave-function (Born's rule)

3. **It is not possible to know the value of all the properties of the system at the same time** (Heisenberg's uncertainty principle)

4. **Matter exhibits a wave-particle duality**. An experiment can show the particle-like properties of matter, or the wave-like properties; in some experiments both of these complementary viewpoints must be invoked to explain the results (Bohr's complementarity principle)

5. **The quantum mechanical description of large systems will closely approximate the classical description** (correspondance principle of Bohr and Heisenberg)

The EPR paradox

"Do you really think the moon isn't there if you aren't looking at it"

[A. Einstein]

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

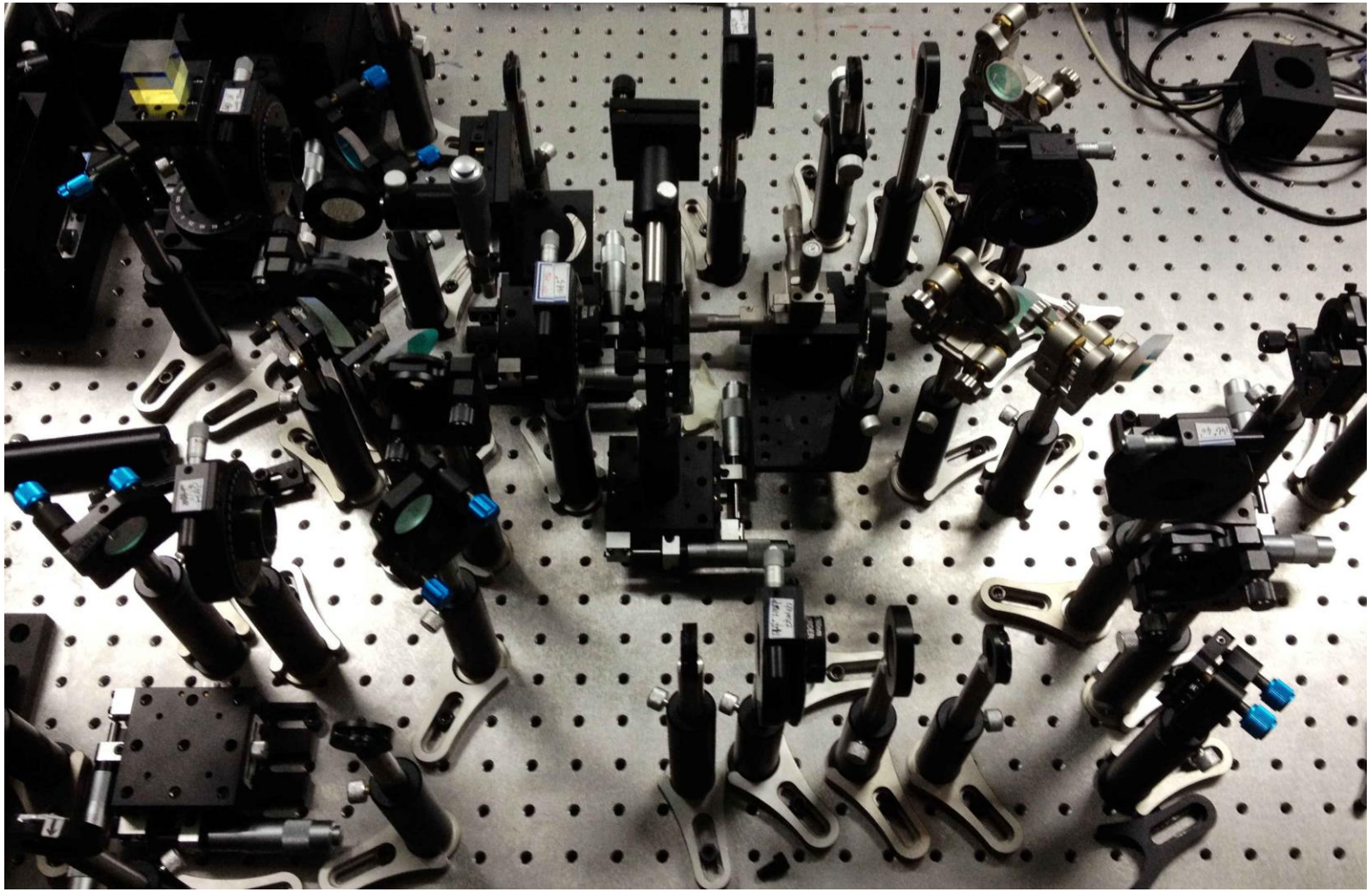
A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

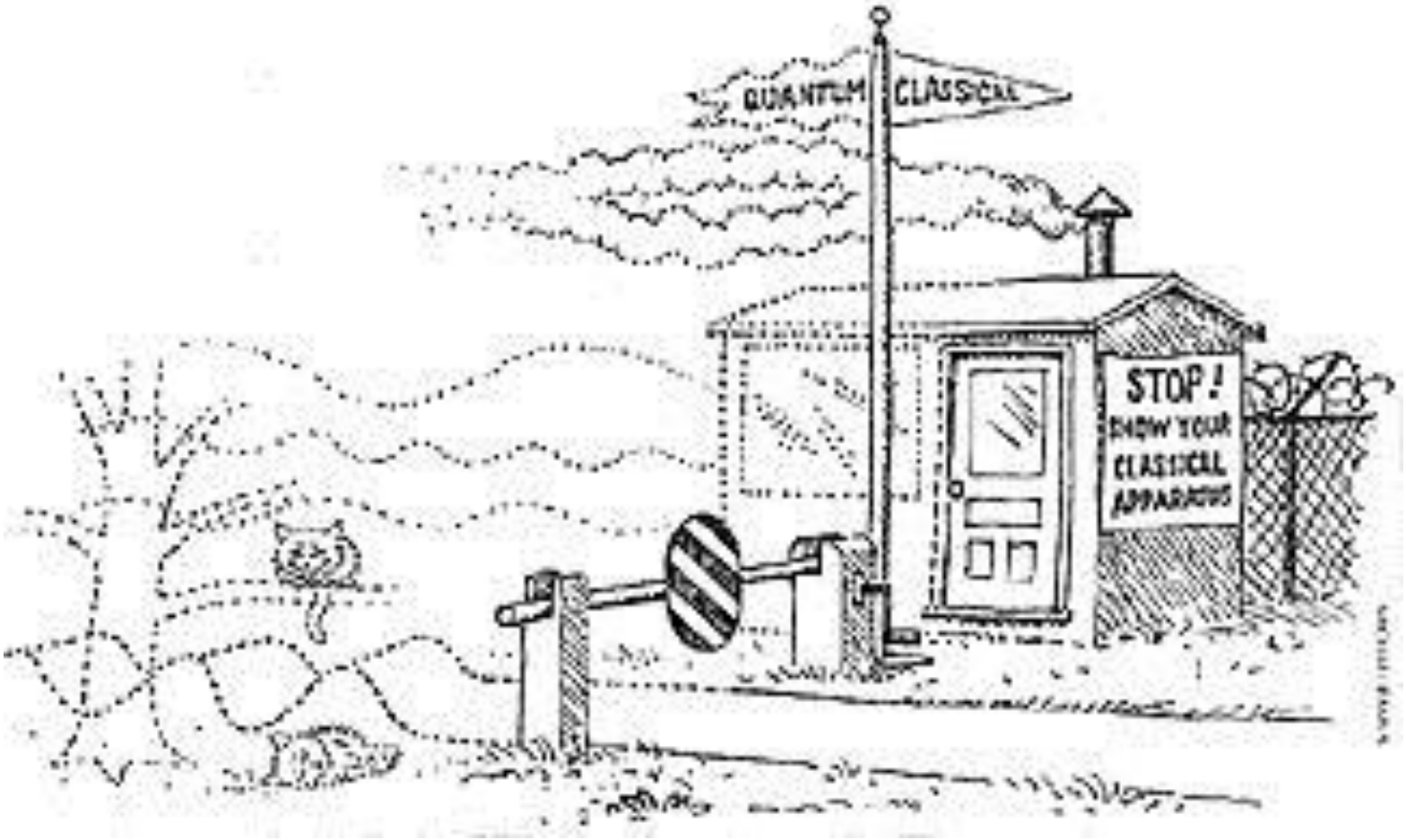
In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

Which side are you?

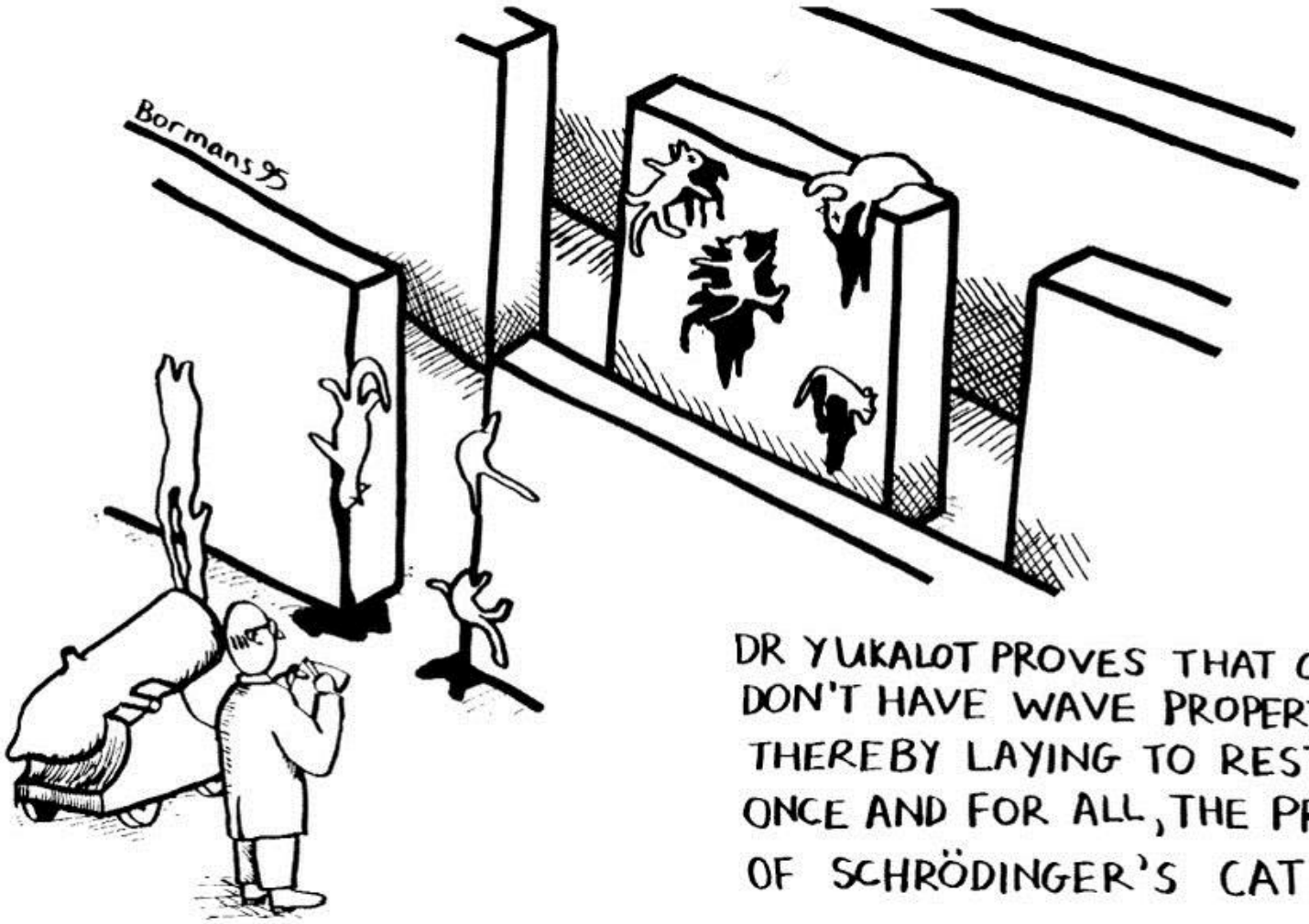
The quantum world



Where is the frontier ?!?!



In conclusion ...




That's all Folks!
(not really)