

LA GEOMETRIA DELLO SPAZIO-TEMPO

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UNIVERSITÀ DEGLI STUDI
DELL'AQUILA — LNGS 2009

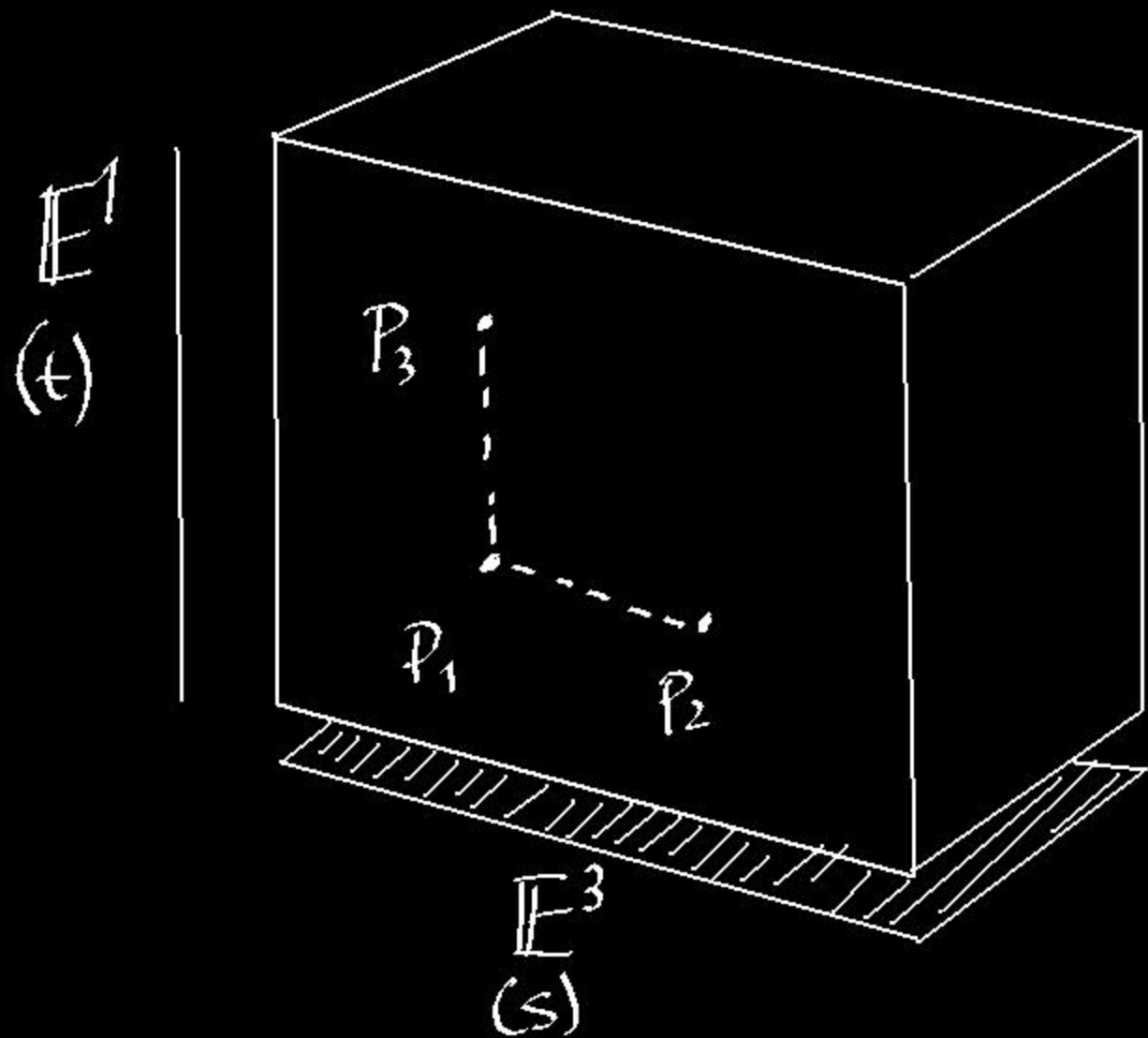
SOMMARIO

- ④ Evoluzione dei concetti di spazio e tempo da Aristotele a Einstein
- ④ Invarianza delle leggi fisiche
- ④ L'Universo in espansione
- ④ Dopo Einstein

ARISTOTELE

"SPAZIO ASSOLUTO"

$$\mathcal{A} = \mathbb{E}^1 \otimes \mathbb{E}^3$$

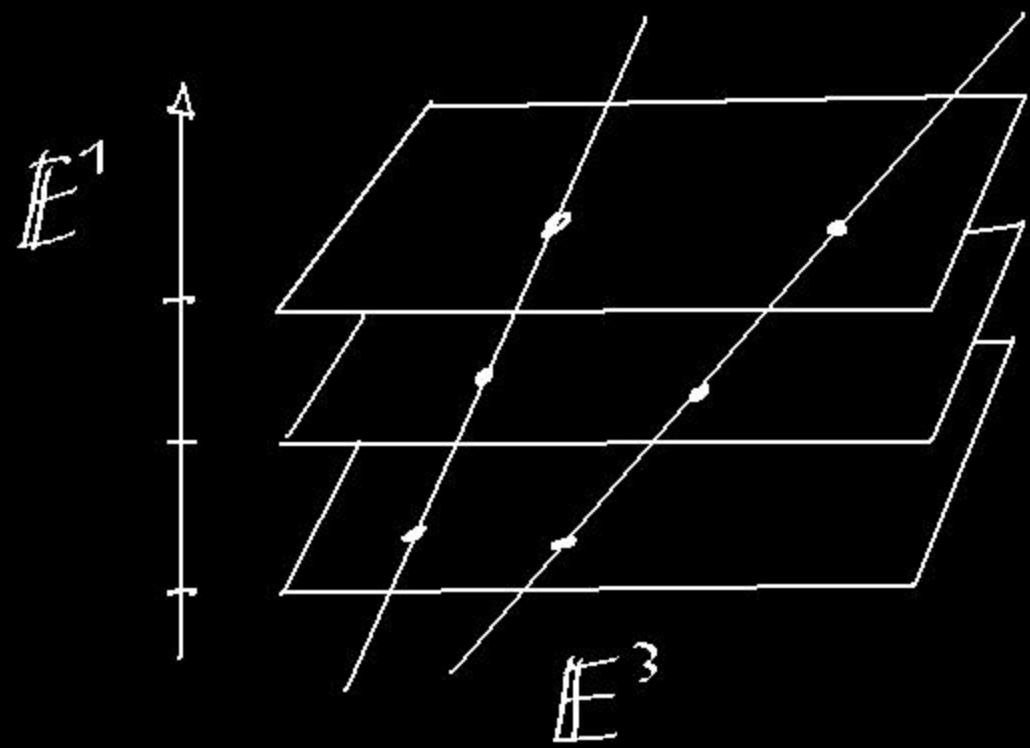


$P_1 - P_2 =$ Eventi Simultanei

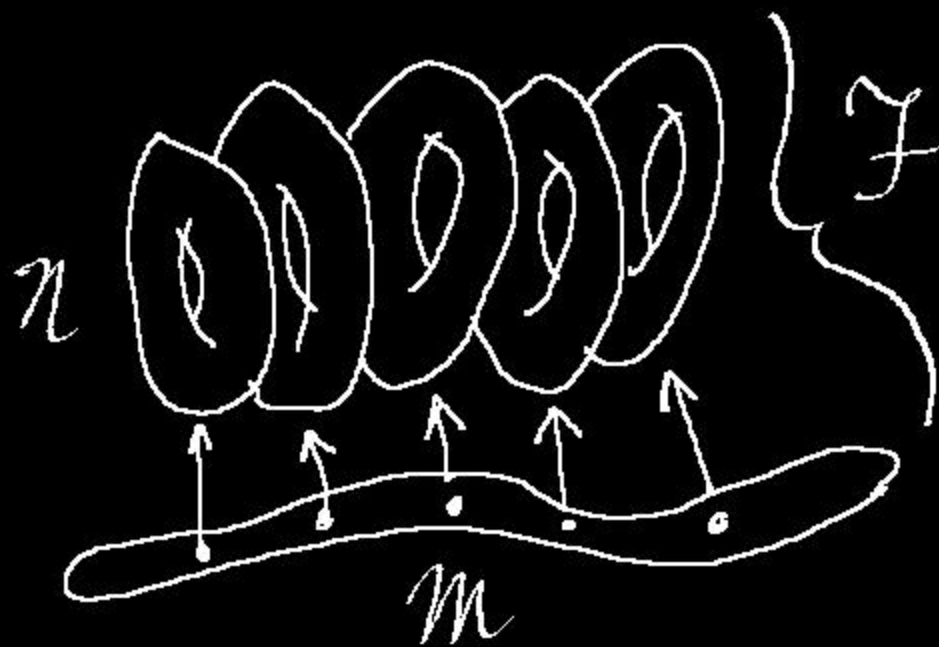
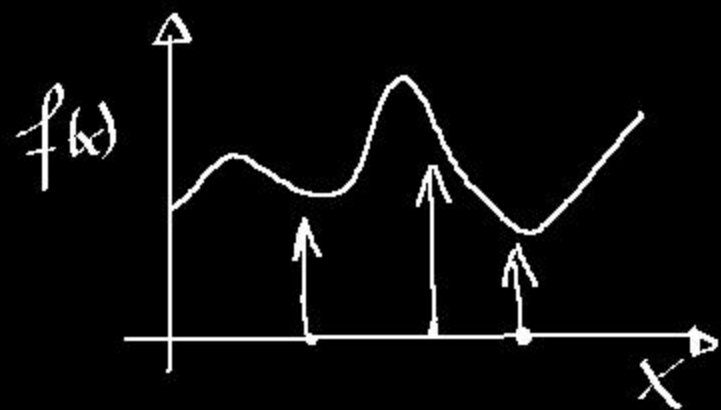
$P_2 - P_3 =$ Eventi nello stesso posto ma a tempi diversi

GALILEO

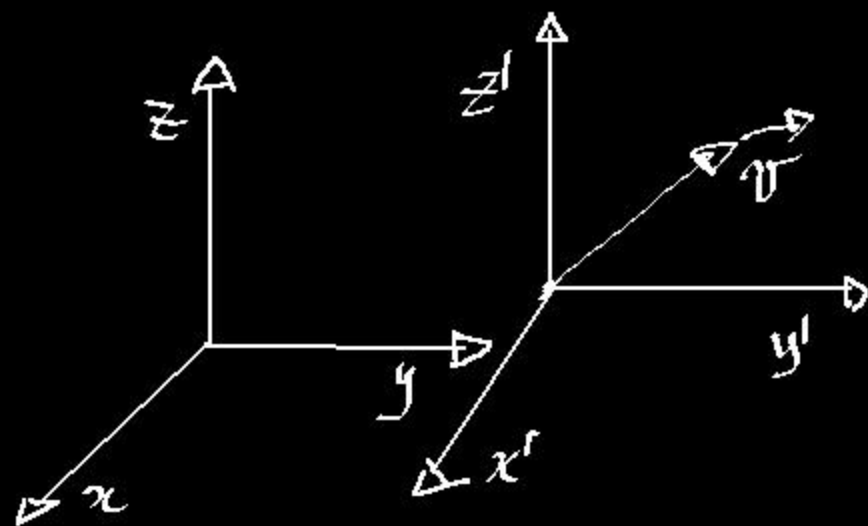
"Principio di Relatività" (sistemi inerziali)



\mathcal{G} = "Fibrato" di
base E^1 e
Fibra E^3



TENSORI



Trasformazioni di Galileo

$$\begin{cases} x' = x - v_x t \\ y' = y - v_y t \\ z' = z - v_z t \end{cases}$$

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$

Invariante

In generale: ($i=1,2,3$)

$$\begin{cases} x'_i = \sum_{j=1}^3 A_{ij} x_j + B_i \Rightarrow \\ \Delta x'_i = \sum_{j=1}^3 A_{ij} \Delta x_j \end{cases}$$

Tensori: Invariante, $v'_i = \sum_j A_{ij} v_j$, $T'_{ij} = \sum_k \sum_l A_{ik} A_{jl} T_{kl}$

LEGGI FISICHE

Leggi tensorialmente corrette = Rel. Galilei

- Legge di Newton : $m \frac{d^2 x_i}{dt^2} = m \frac{dv_i}{dt} = F_i$

- Campo conservativo : $F_i = -\frac{\partial \phi}{\partial x_i}$

- Conservazione dell'energia:

$$m \frac{dv_i}{dt} v_i dt = F_i v_i dt \Leftrightarrow d\left(\frac{1}{2} m v^2\right) = \sum_{i=1}^3 F_i dx_i$$

- II Equazione Cardinale

$$\frac{d}{dt} \left[m \left(z_j \frac{dx_i}{dt} - z_i \frac{dx_j}{dt} \right) \right] = z_j F_i - z_i F_j \Leftrightarrow \frac{dL_{ij}}{dt} = M_{ij}$$

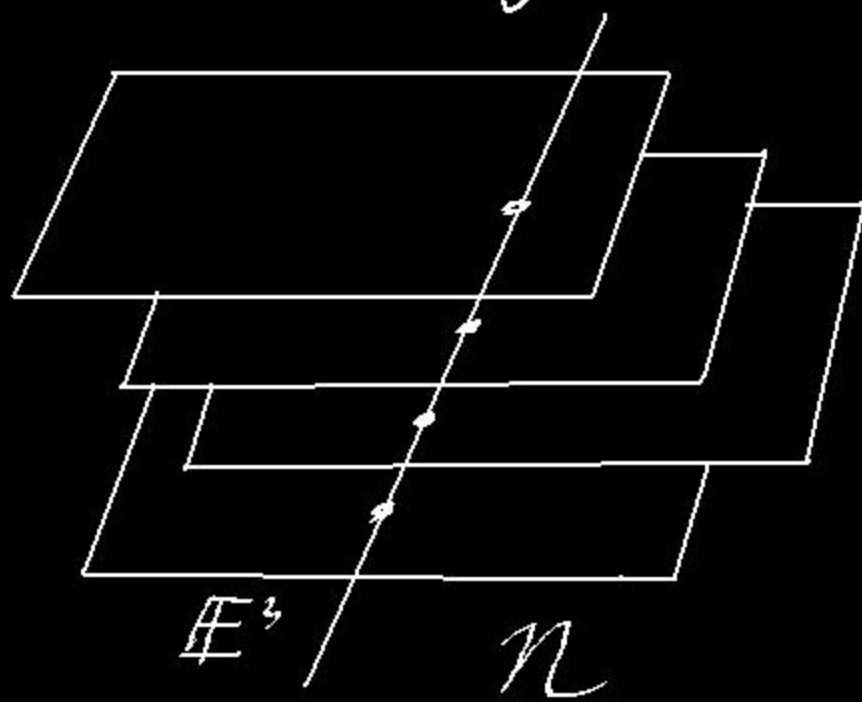
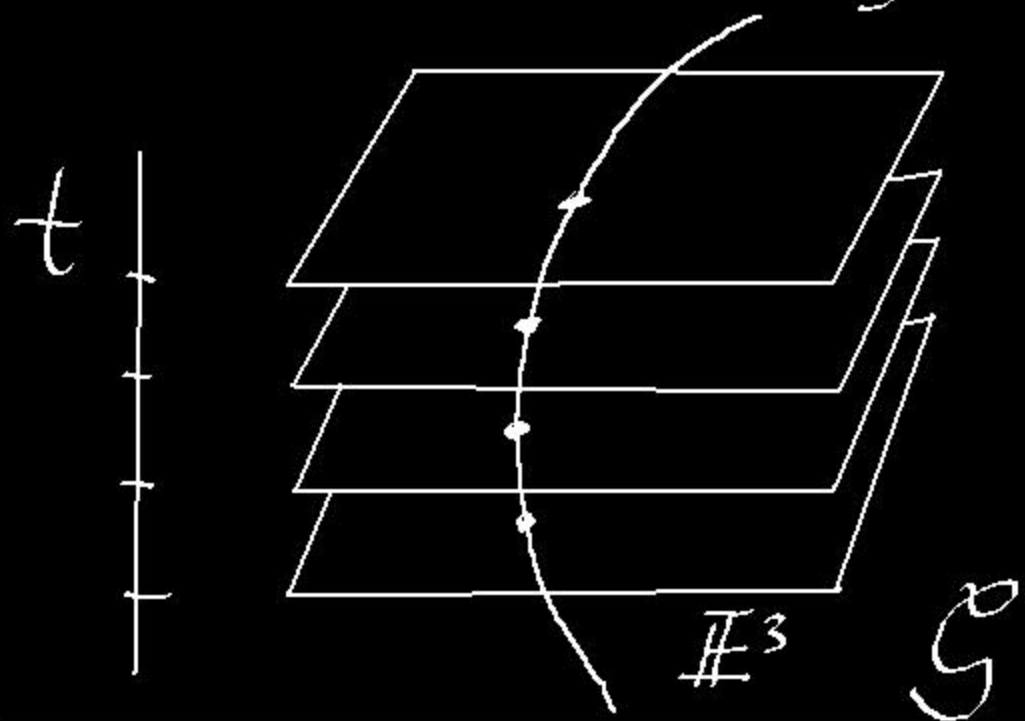
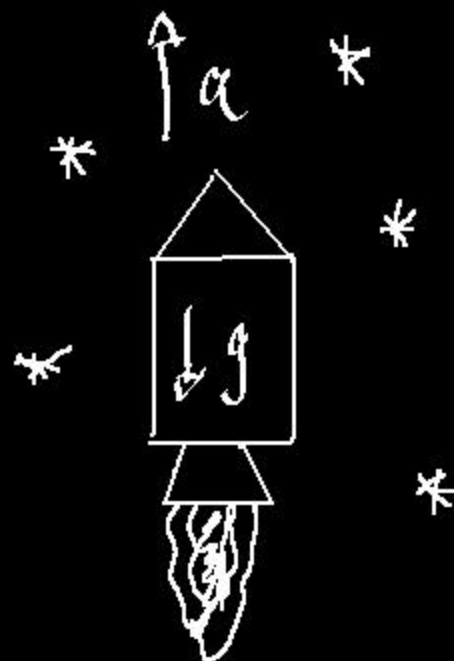
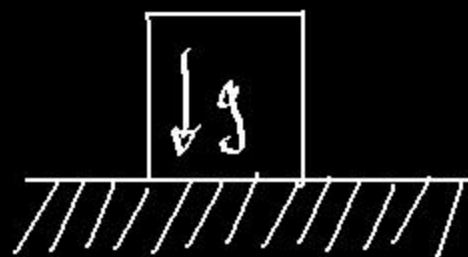
CARTAN

$$F = m^{(i)} a$$

$$F = G_N \frac{m_1^{(g)} m_2^{(g)}}{r^2}$$

"Principio di Equivalenza"

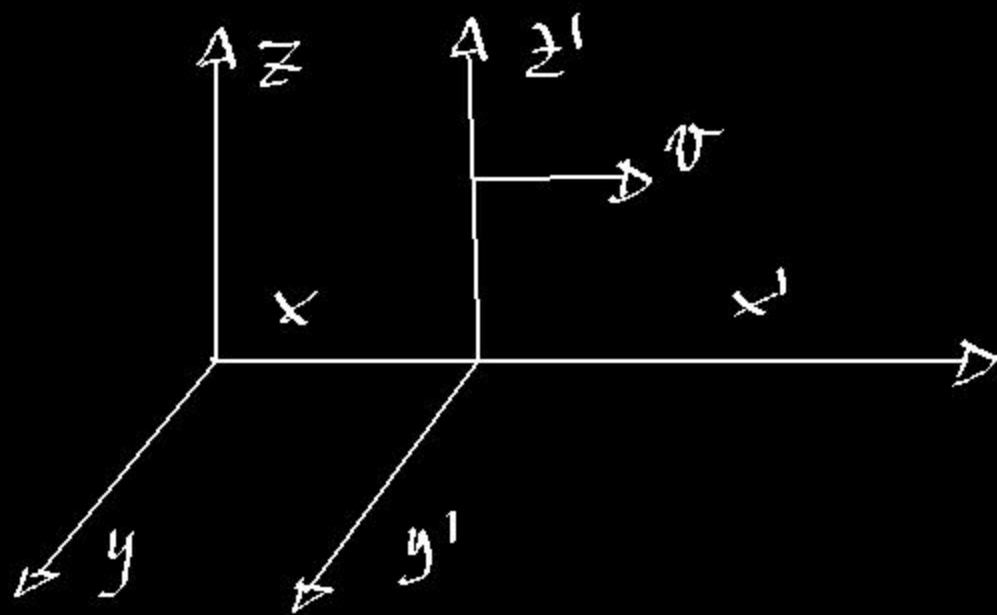
$$m^{(i)} = m^{(g)}$$



C - COSTANTE

"Trasformazioni di Lorentz" ($\beta = \frac{v}{c}$)

$$\begin{cases} x = \frac{x' + vt'}{\sqrt{1-\beta^2}} \\ y = y' \\ z = z' \\ t = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1-\beta^2}} \end{cases}$$



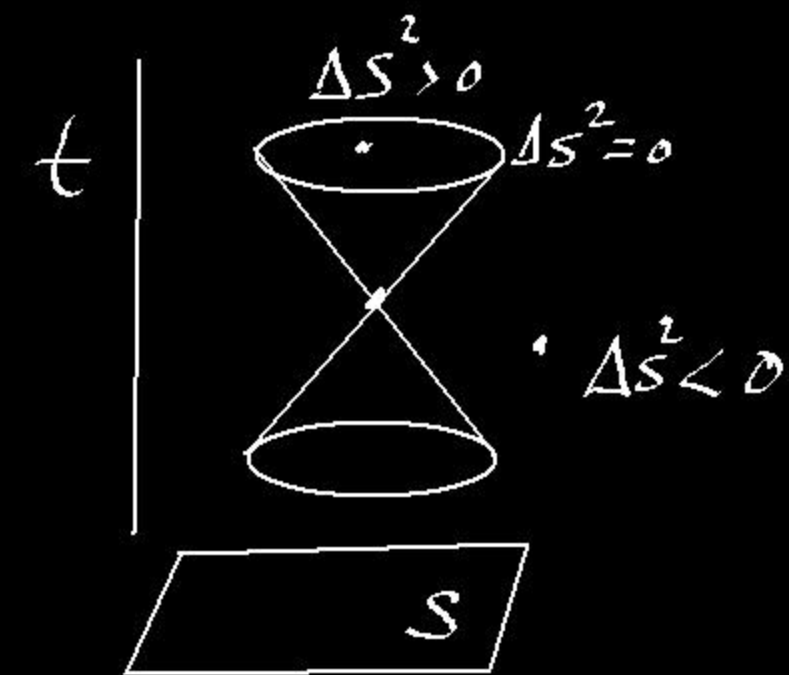
Invariante: $\Delta s^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$

P_1, t_1 P_2, t_2

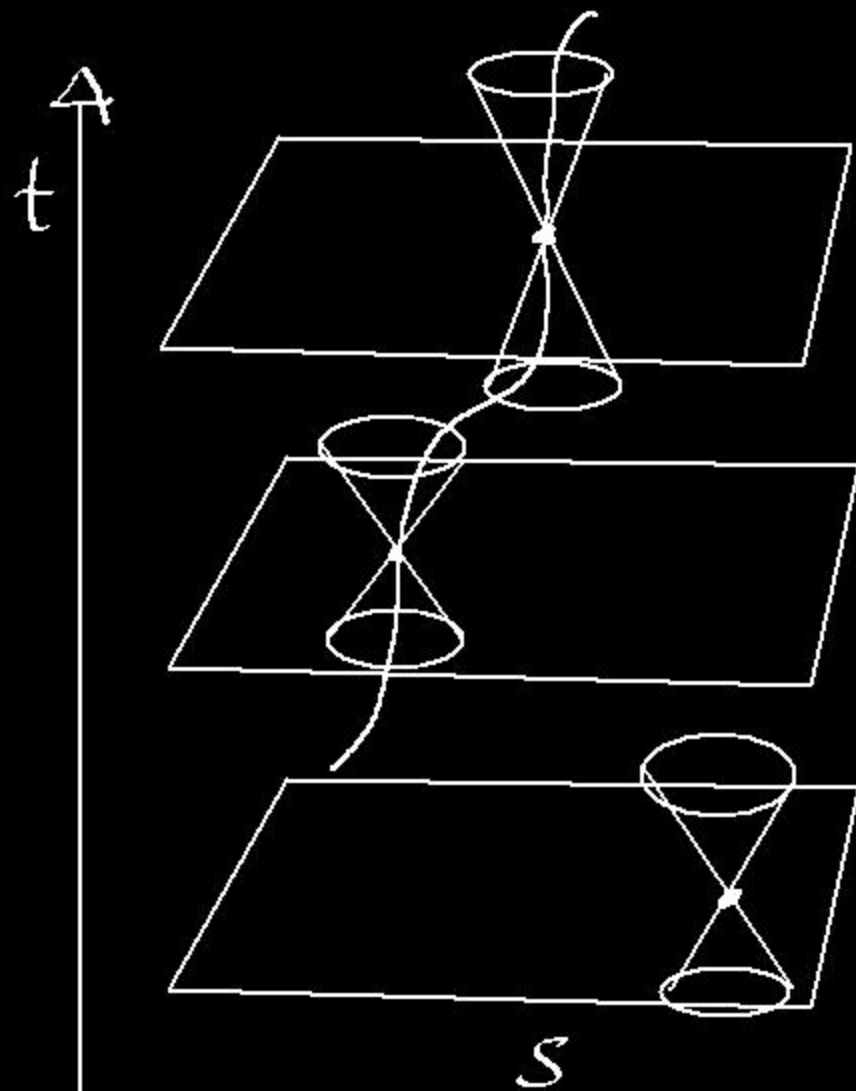
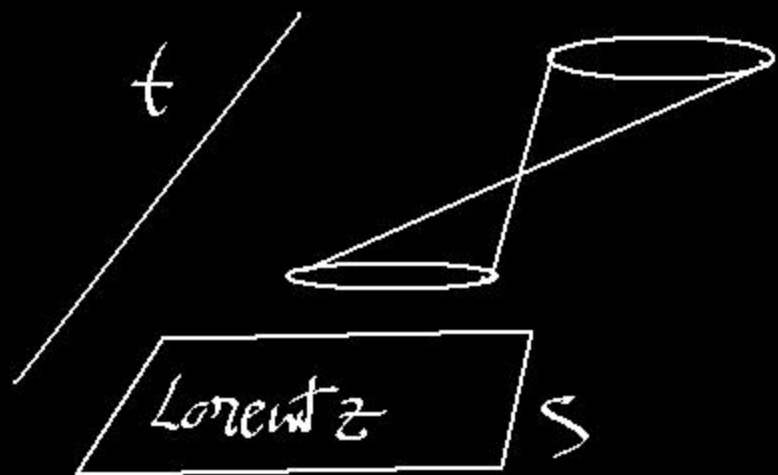
$$\Delta s'^2 = c^2 \Delta t'^2 - \Delta x'^2 - \Delta y'^2 - \Delta z'^2;$$

$$\Delta s^2 \propto \Delta s'^2; \quad \Delta s^2 = \Delta s'^2$$

EINSTEIN - RS



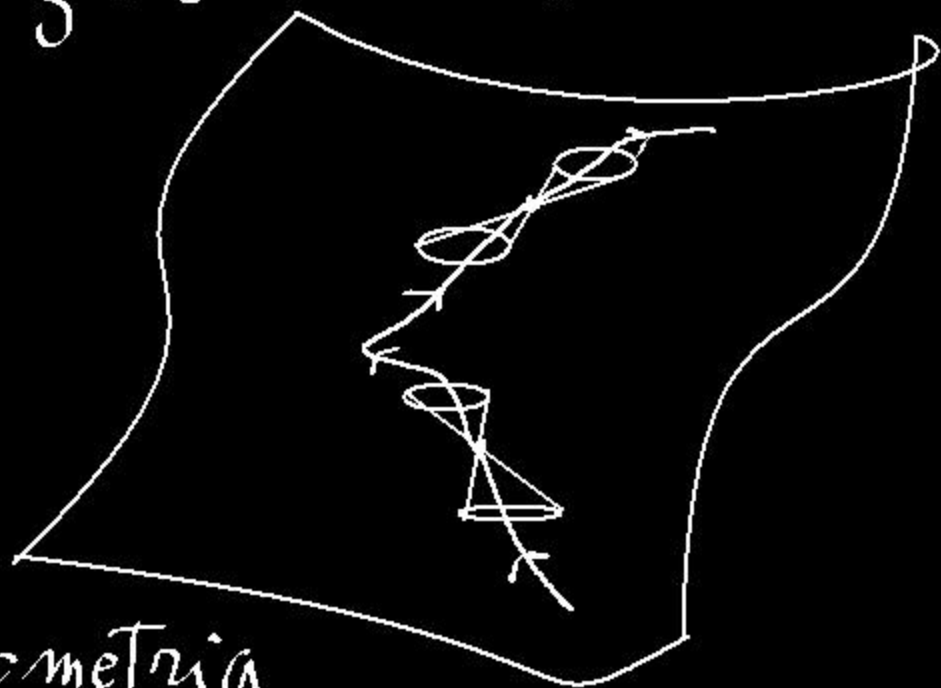
"Cone di Luce"



\mathcal{M} = Spaziotempo
(Minkowski)

EINSTEIN - RG

Spaziotempo curvo



Invariante

$$ds^2 = g_{\mu\nu}(x^\alpha) dx^\mu dx^\nu$$

$$x^\mu = (ct, x, y, z)$$

Geometria
non Euclidea

$r \ll r$

$$g_{00} \approx 1 + \frac{2\phi}{c^2}$$

$$\phi : \nabla^2 \phi = -4\pi G_N \rho_m \quad (\text{Poisson})$$

$$g(x^\alpha) : R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{4\pi G_N}{c^4} T_{\mu\nu} \quad (\text{Einstein})$$

NUOVI FENOMENI

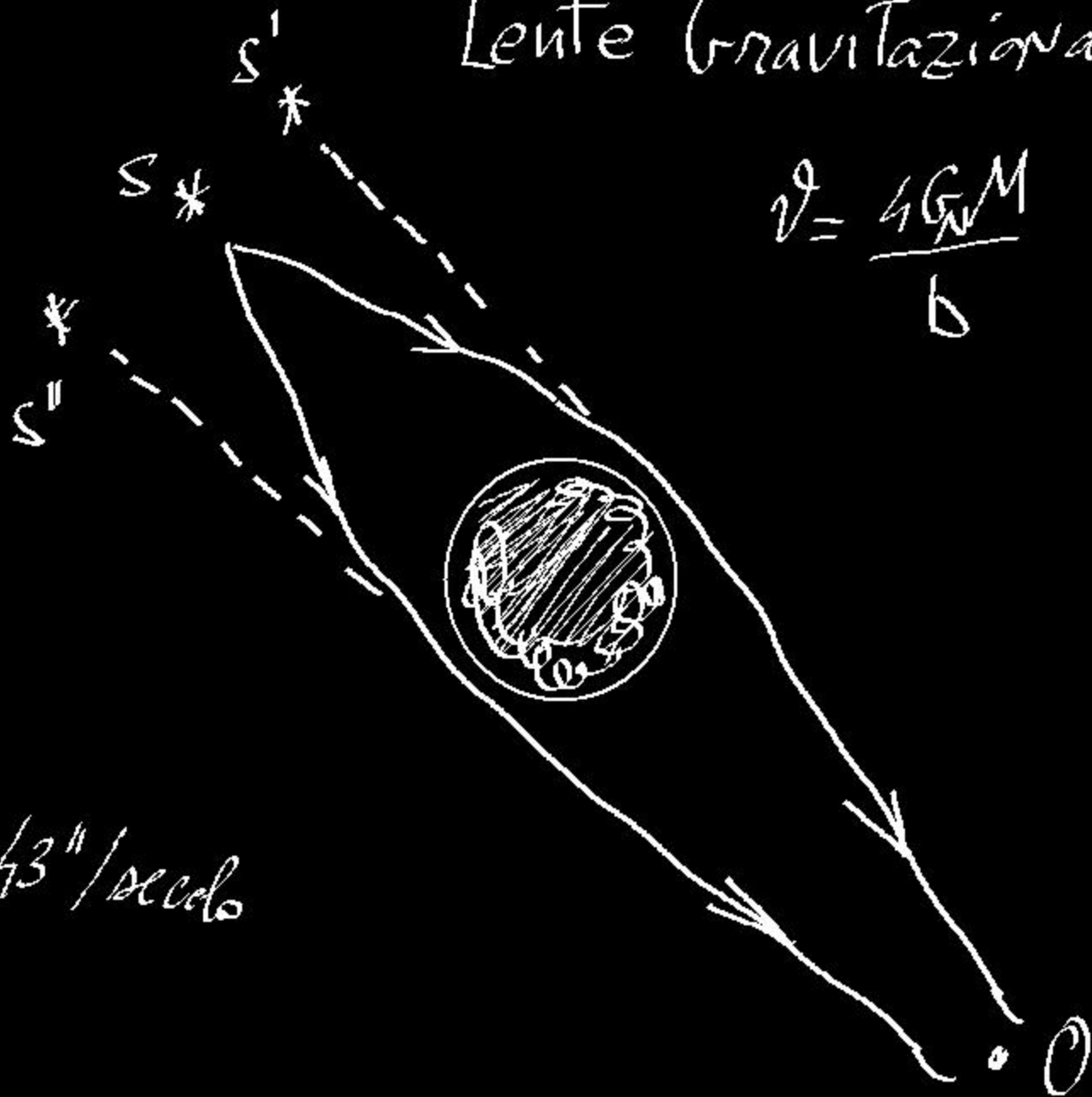
Precessione del
Perielio di
Mercurio



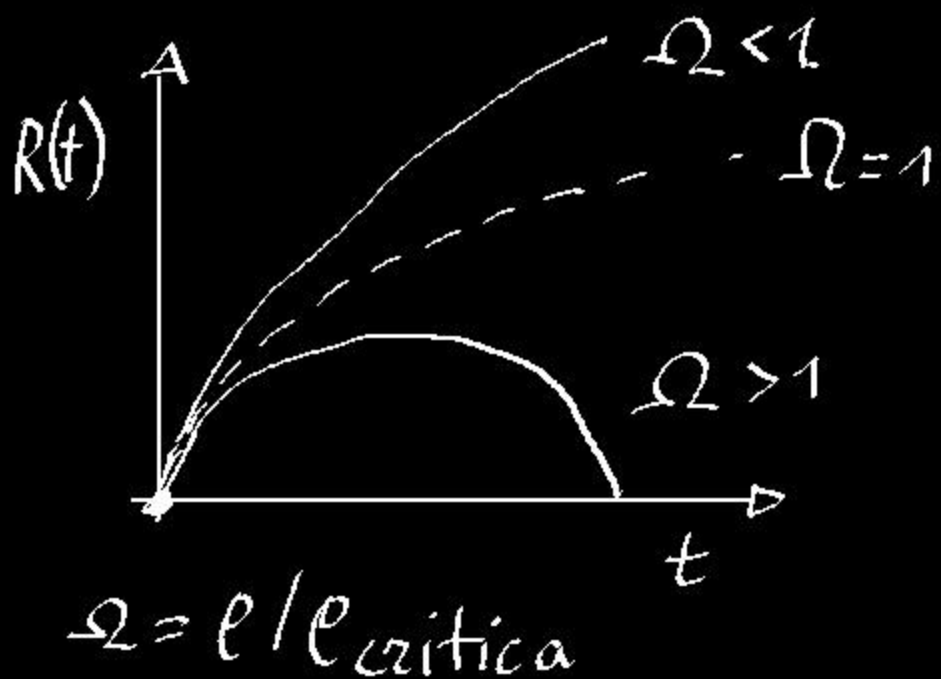
$$\Delta\phi \approx 43''/\text{secolo}$$

Lente Gravitazionale

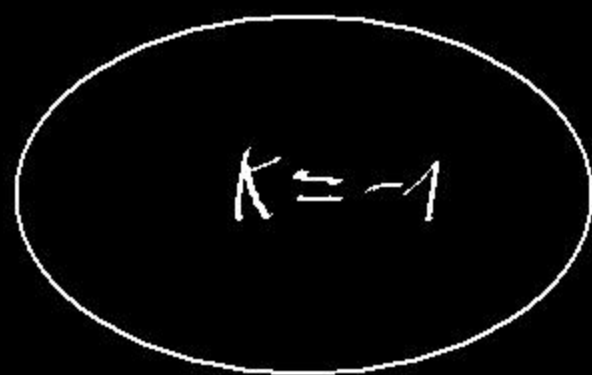
$$\vartheta = \frac{4GM}{b}$$



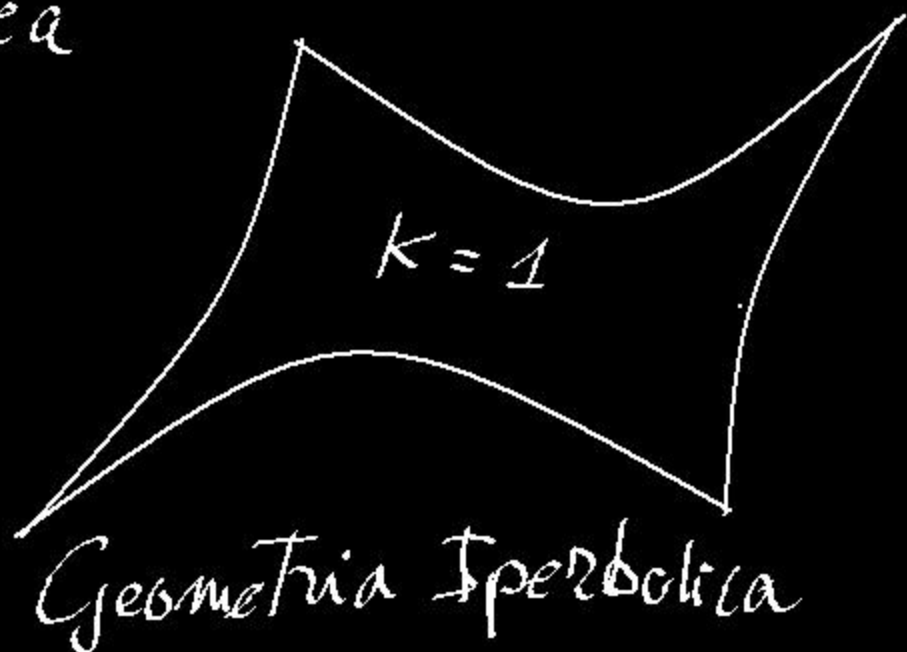
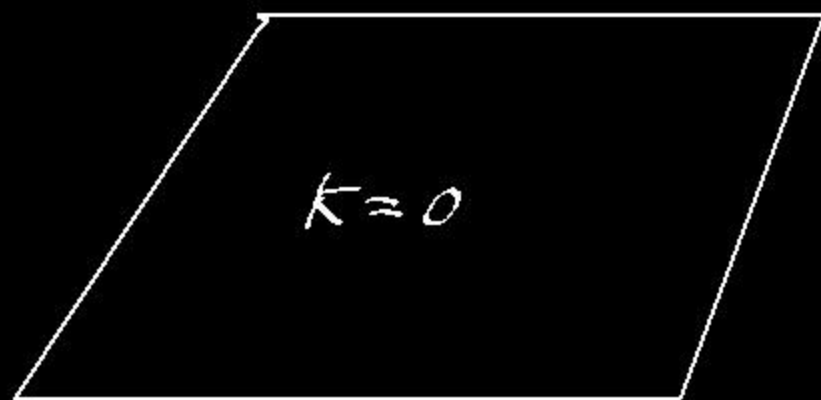
L'UNIVERSO



Geometria Ellittica



Geometria Euclidea



Geometria Iperbolica

DIMENSIONI EXTRA

$$D=1 \quad F = \text{cost.}$$

$$D=2 \quad F \propto \frac{1}{r}$$

$$D=3 \quad F \propto \frac{1}{r^2}$$

N -Extra dimensioni
spaziali

$$F \propto \frac{1}{r^{2+N}}$$

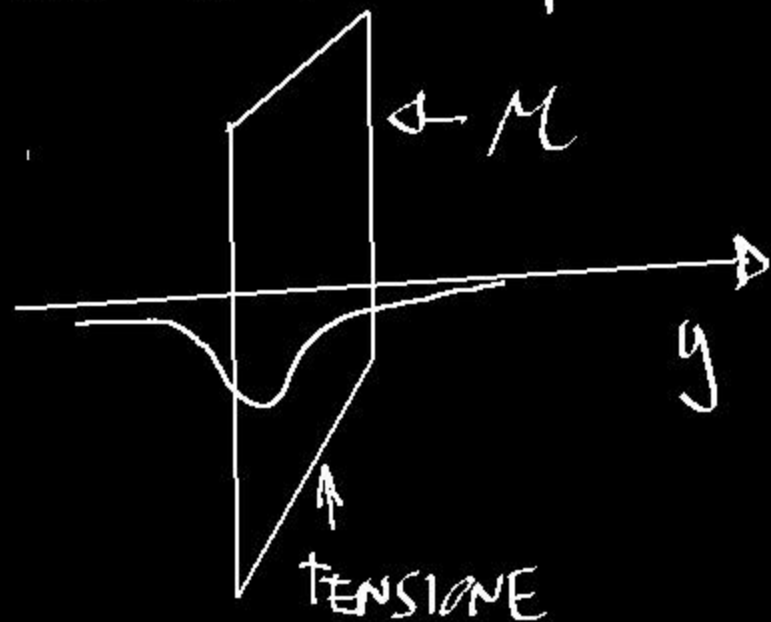
Perché non si vedono:

- Sono piccole ed
arrotate (KALUZA-KLEIN)

$$G = 0 \Rightarrow G_{\mu\nu} = T_{\mu\nu}^{EM}$$

$$x^M = (t, x_1, x_2, x_3, y)$$

Teoria delle p -brane

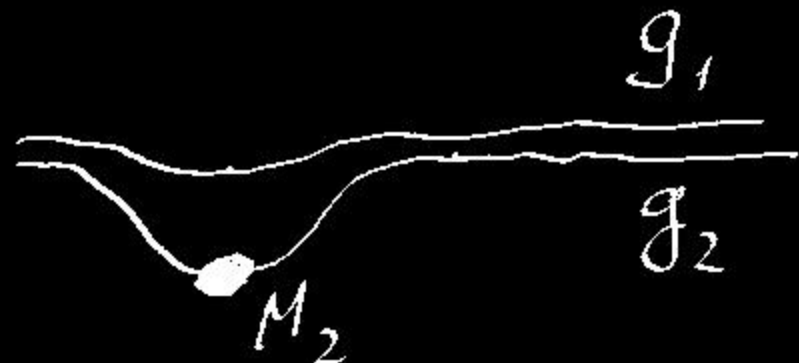


... E VARIE ...

MULTIGRAVITÀ

N-Universi
ciascuno con la
sua metrica g_i^μ

Es. N=2 (BIGRAVITÀ)

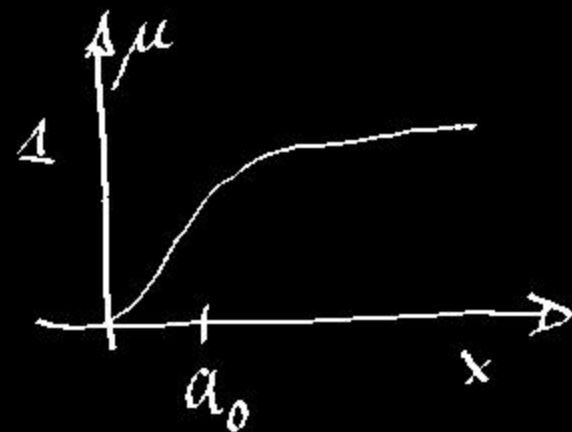


MOND

Modificazione della
legge di Newton

$$g = \frac{g_N}{\mu\left(\frac{g}{a}\right)}$$

Es:



CONCLUSIONI

"Ma Egli ha messo nel cuore degli uomini la nozione dell'eternità senza che però essi possano comprendere l'opera compiuta da Dio dall'inizio (A) alla fine (Ω)"

[Qo 3, 11]