

# SPAZIO, TEMPO E GEOMETRIA

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UNIVERSITÀ DEGLI STUDI  
DELL'AQUILA — LNGS 2008

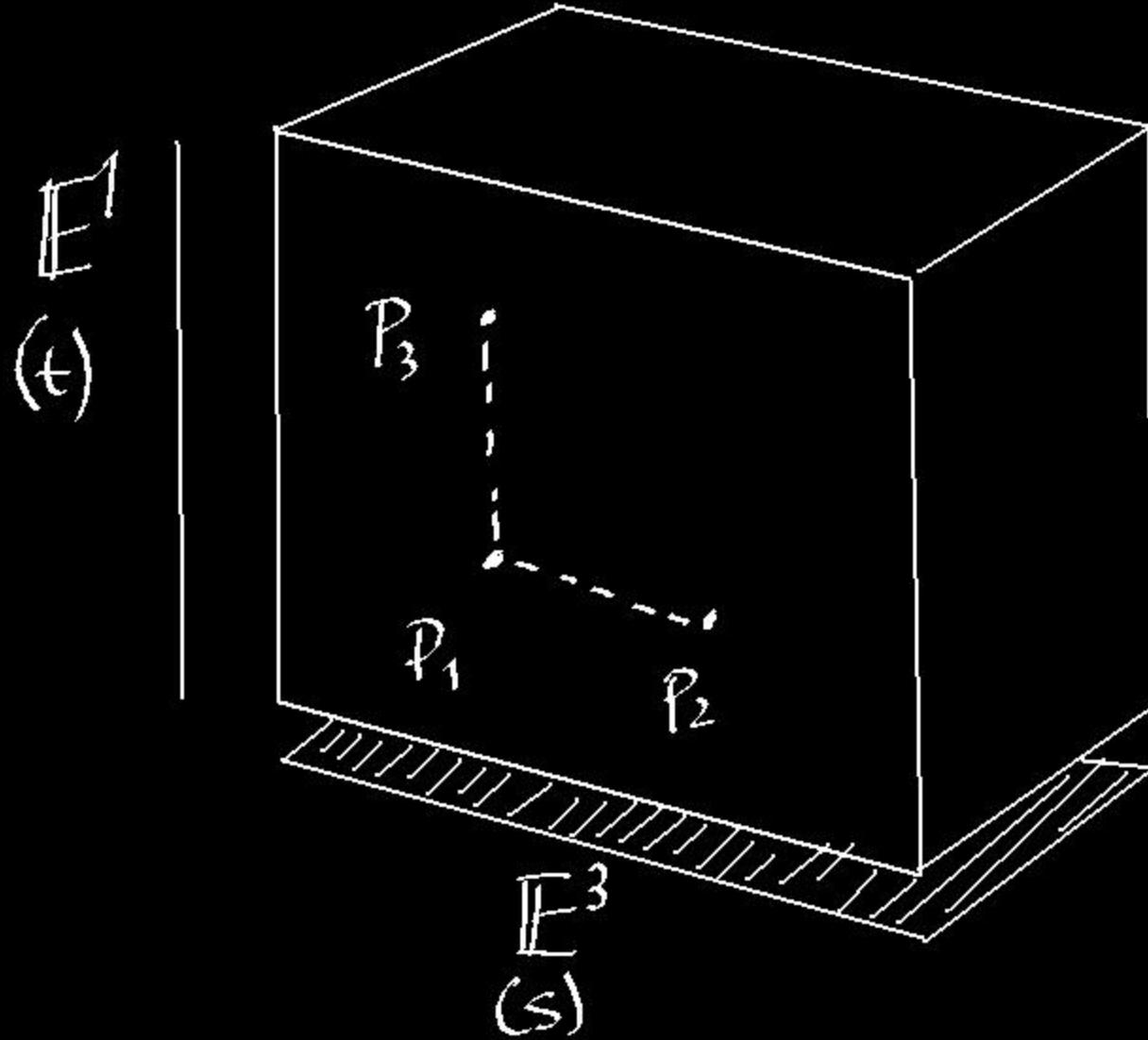
# SOMMARIO

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- ① Evoluzione dei concetti di spazio e tempo da Aristotele a Einstein
- ② Invarianza delle leggi fisiche
- ③ L'Universo in espansione
- ④ Dopo Einstein

# ARISTOTELE

"SPAZIO ASSOLUTO"



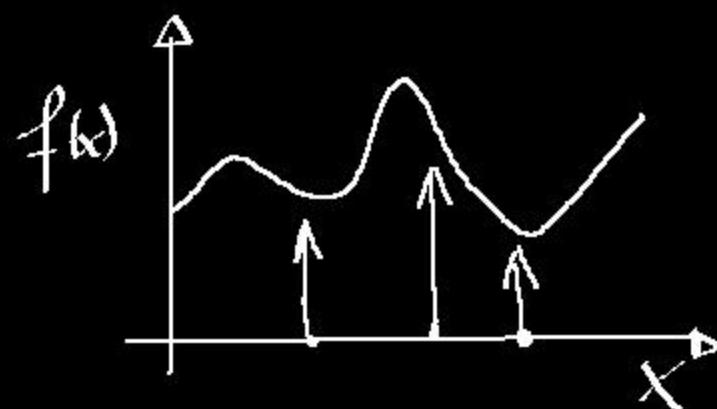
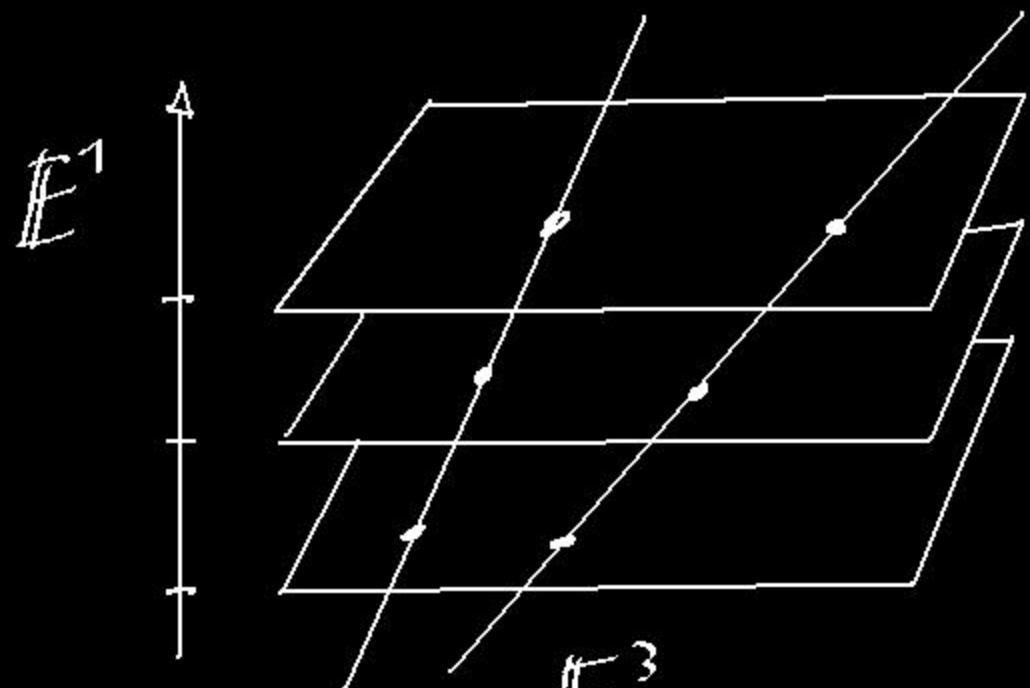
$$\mathcal{A} = \mathbb{E}^1 \otimes \mathbb{E}^3$$

$P_1 - P_2$  = Eventi  
Simultanei

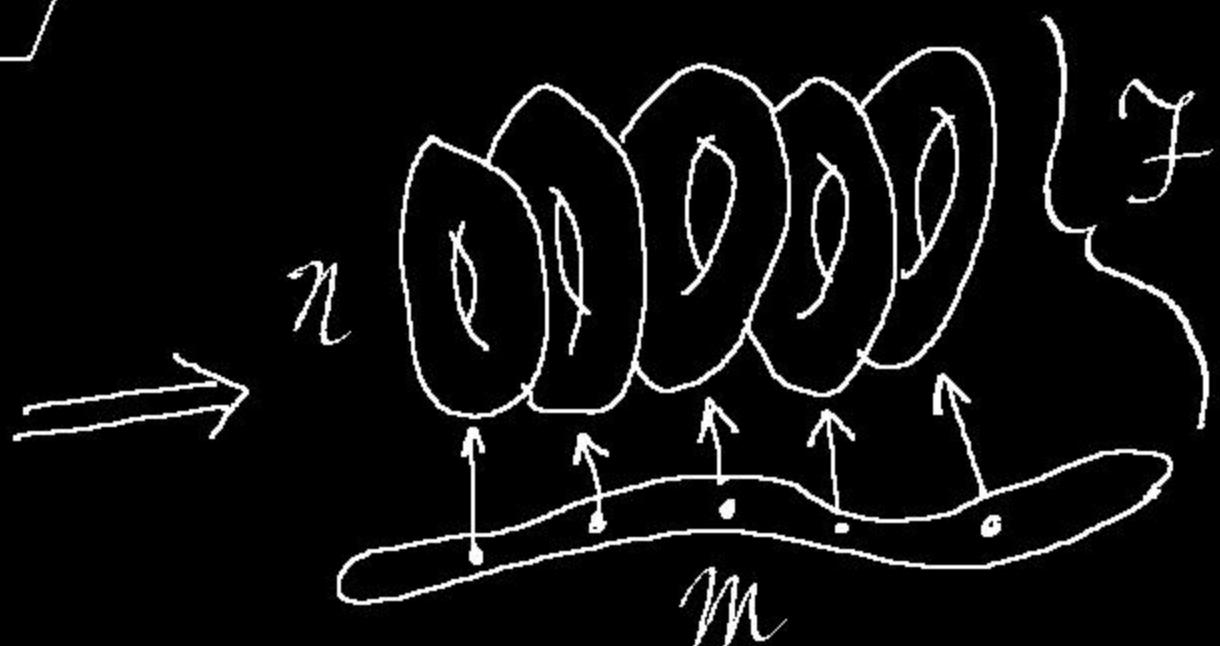
$P_2 - P_3$  = Eventi nello  
stesso posto  
ma a tempi  
diversi

# GALILEO

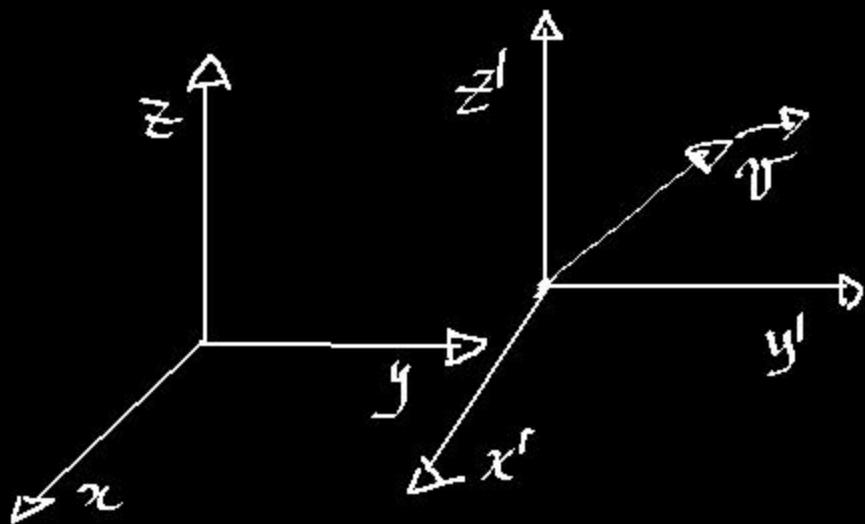
"Principio di Relatività" (sistemi inerziali)



$G$  = "Fibra" di  
base  $E^1$  e  
Fibra  $E^3$



# TENSORI



Trasformazioni di Galileo

$$\begin{cases} x' = x - v_x t \\ y' = y - v_y t \\ z' = z - v_z t \end{cases}$$

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$

Invariante

In generale: ( $i=1, 2, 3$ )

$$\begin{cases} x'_i = \sum_{j=1}^3 A_{ij} x_j + B_i \Rightarrow \end{cases}$$

$$\Delta x_i = \sum_{j=1}^3 A_{ij} \Delta x_j$$

Tensori: Invariante,  $v'_i = \sum_j A_{ij} v_j$ ,  $T'_{ij} = \sum_k \sum_\ell A_{ik} A_{j\ell} T_{k\ell}$

# LEGGI FISICHE

Leggi tensorialmente corrette = Rel. Galilei

- Legge di Newton :  $m \frac{d^2x_i}{dt^2} = m \frac{dv_i}{dt} = F_i$

- Campo conservativo :  $F_i = -\frac{\partial \phi}{\partial x_i}$

Conservazione dell'energia:

$$m \frac{dv_i}{dt} v_i dt = F_i v_i dt \Leftrightarrow d\left(\frac{1}{2}mv^2\right) = \sum_{i=1}^3 F_i dx_i$$

- II Equazione Cardinale

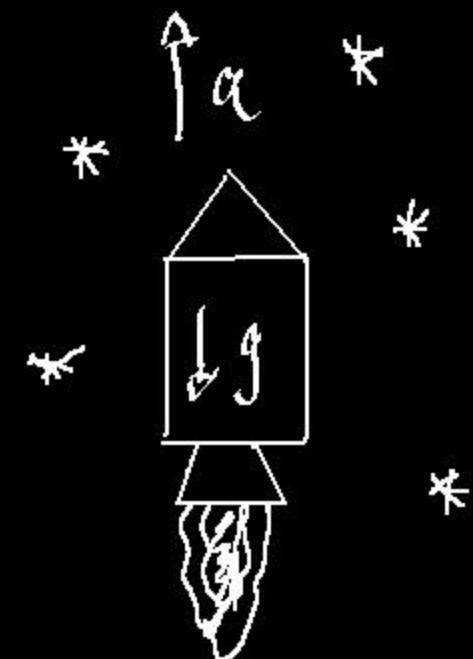
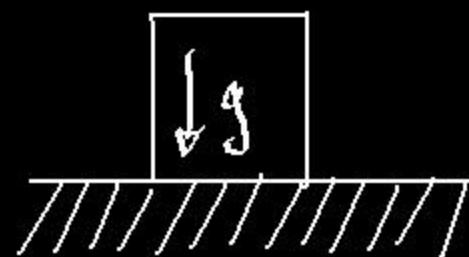
$$\frac{d}{dt} \left[ m \left( r_i \frac{dx_i}{dt} - r_i \frac{dx_i}{dt} \right) \right] = r_j F_i - r_i F_j \Leftrightarrow \frac{dL_{ij}}{dt} = M_{ij}$$

# CARTAN

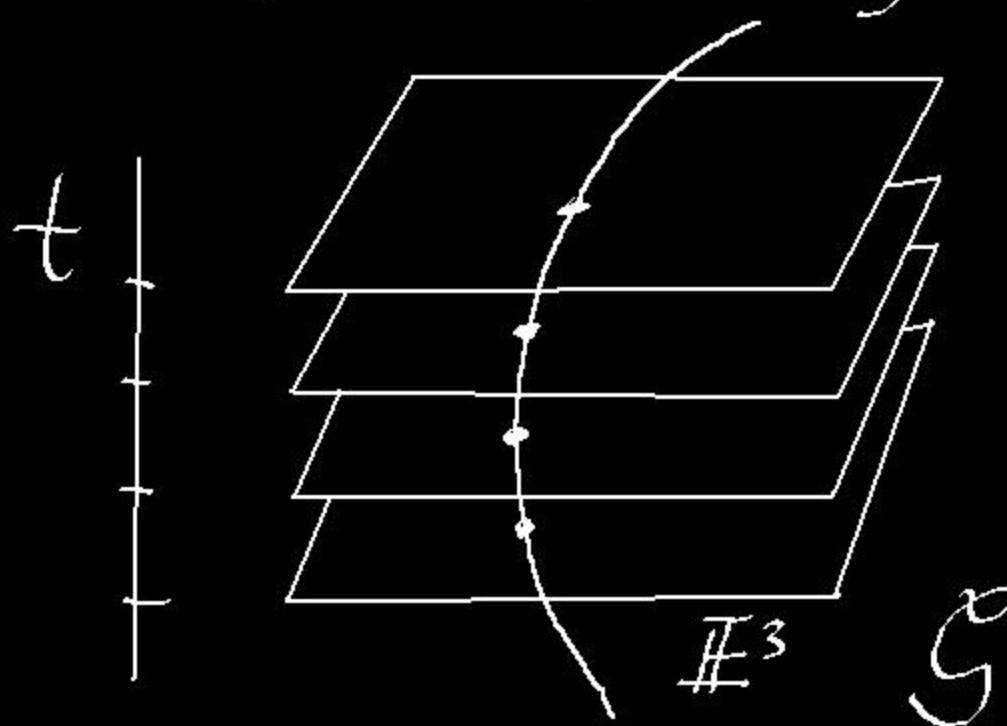
$$F = m^{(i)} a$$

$$F = G_N \frac{m_1^{(g)} m_2^{(g)}}{r^2}$$

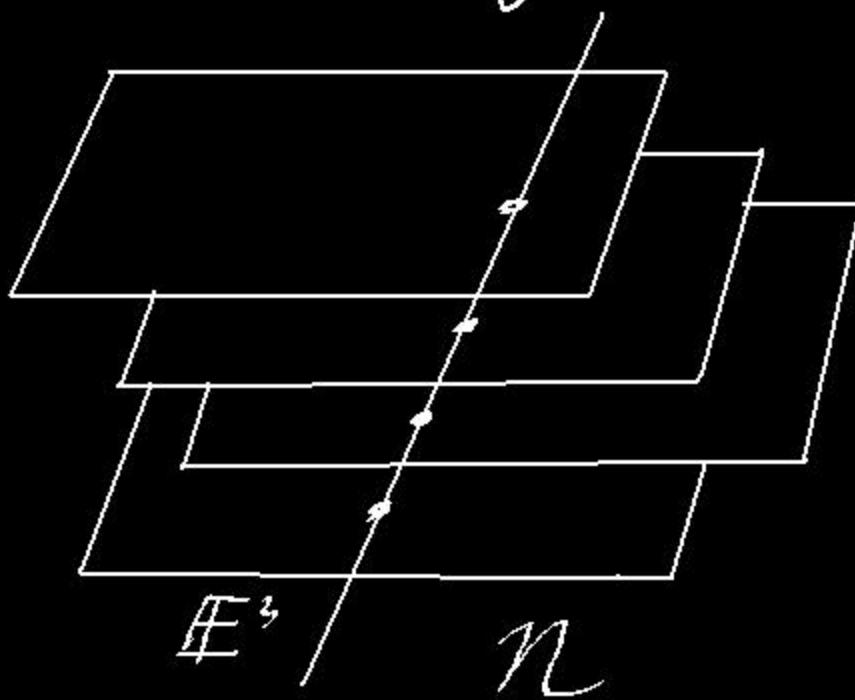
$$m^{(i)} = m^{(g)}$$



"Principio di Equivalenza"



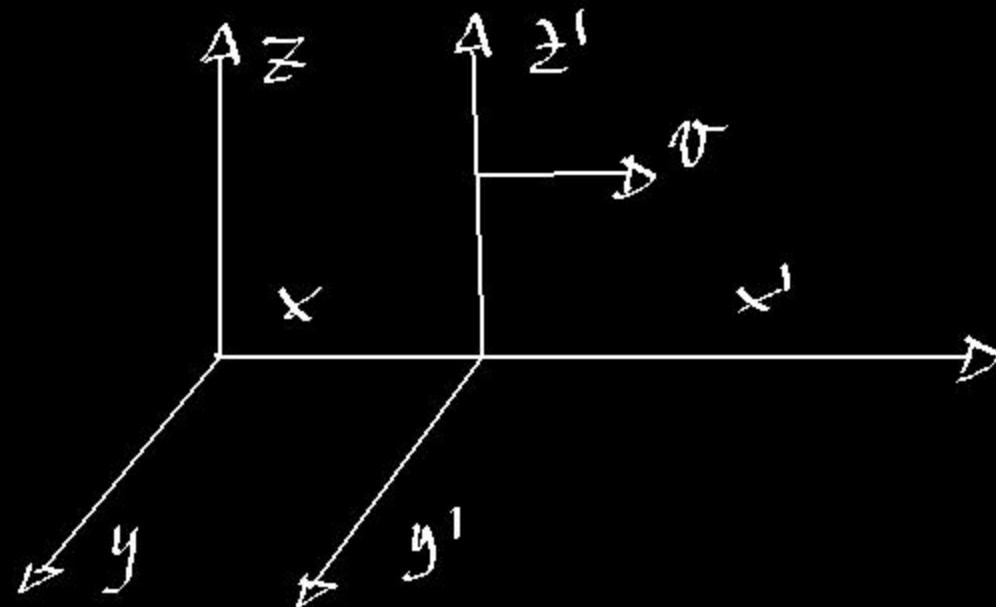
$\mathcal{G}$



# C - COSTANTE

"Trasformazioni di Lorentz" ( $\beta = \frac{v}{c}$ )

$$\left\{ \begin{array}{l} x' = \frac{x + vt'}{\sqrt{1-\beta^2}} \\ y = y' \\ z = z' \\ t = \frac{t' + \frac{\beta}{c}x'}{\sqrt{1-\beta^2}} \end{array} \right.$$



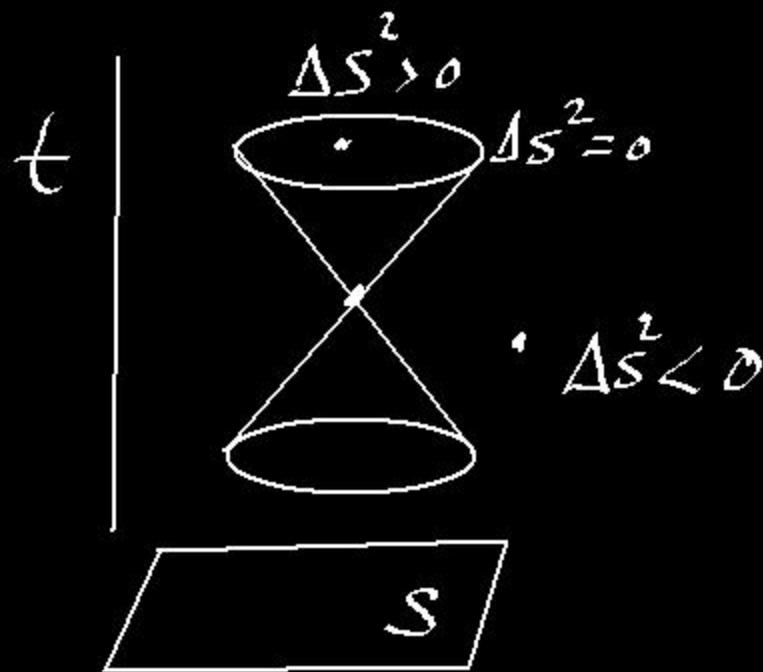
Invariante:  $\Delta s^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$

$\vec{r}_1, t_1$        $\vec{r}_2, t_2$

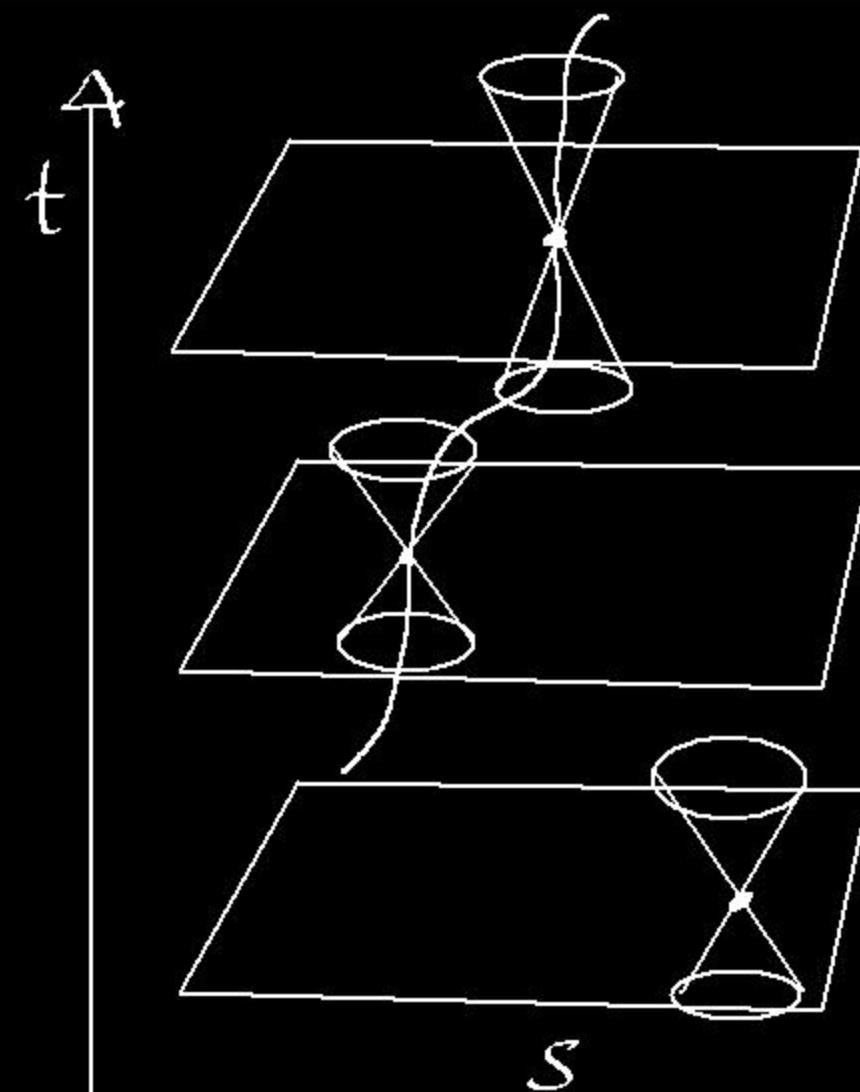
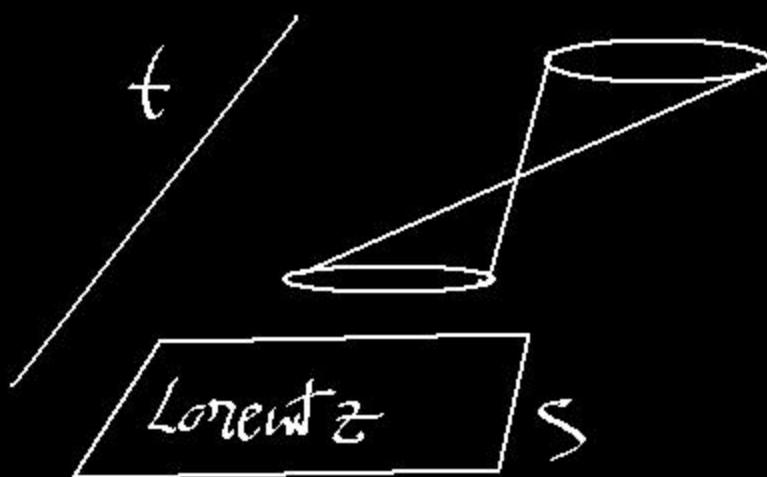
$$\Delta s'^2 = c^2 \Delta t'^2 - \Delta x'^2 - \Delta y'^2 - \Delta z'^2;$$

$$\Delta s^2 \propto \Delta s'^2; \quad \Delta s^2 = \Delta s'^2$$

# EINSTEIN - RS



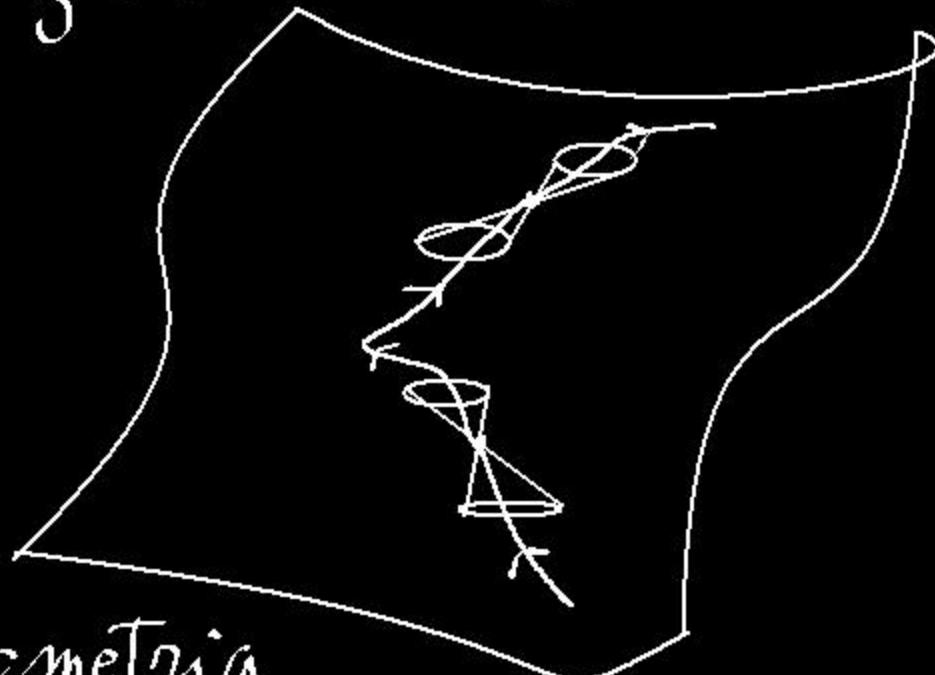
"Cono di Luce"



$M$  = Spaziotempo  
(Minkowski)

# EINSTEIN - RG

Spazio-tempo curvo



Geometria  
non Euclidea

$t \ll c$

$$g_{\mu\nu} \approx 1 + \frac{2\phi}{c^2}$$

Invariante

$$ds^2 = g_{\mu\nu}(x^\alpha) dx^\mu dx^\nu$$

$$X^\mu = (ct, x, y, z)$$

$$\left. \begin{array}{l} \phi : \nabla^2 \phi = -4\pi G_N \rho_m \\ \text{(Poisson)} \end{array} \right\}$$

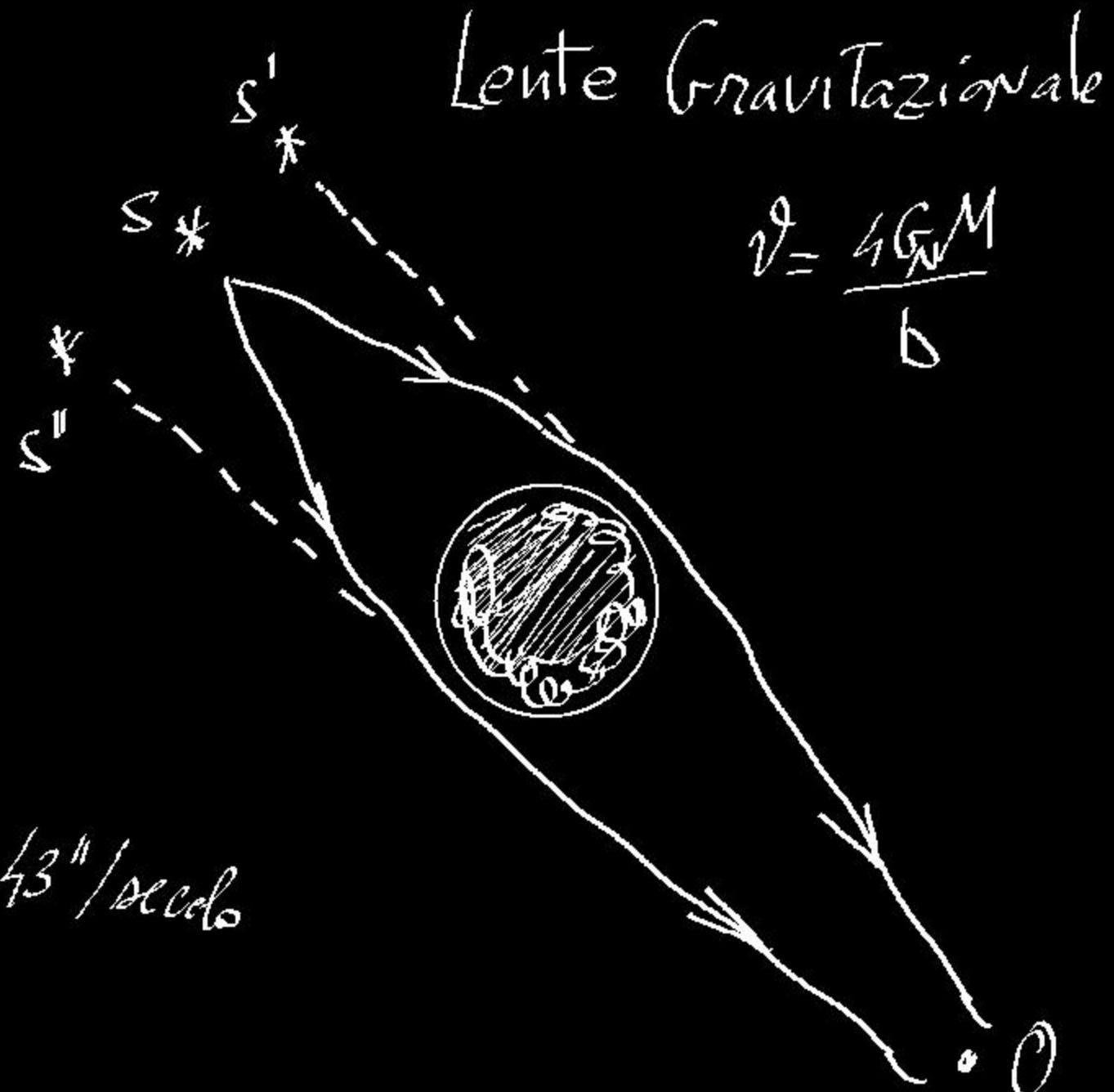
$$\left. \begin{array}{l} g_{\mu\nu}(x^\alpha) : R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G_N}{c^5} T_{\mu\nu} \\ \text{(Einstein)} \end{array} \right\}$$

# NUOVI FENOMENI

Precessione del  
Perielio di  
Mercurio



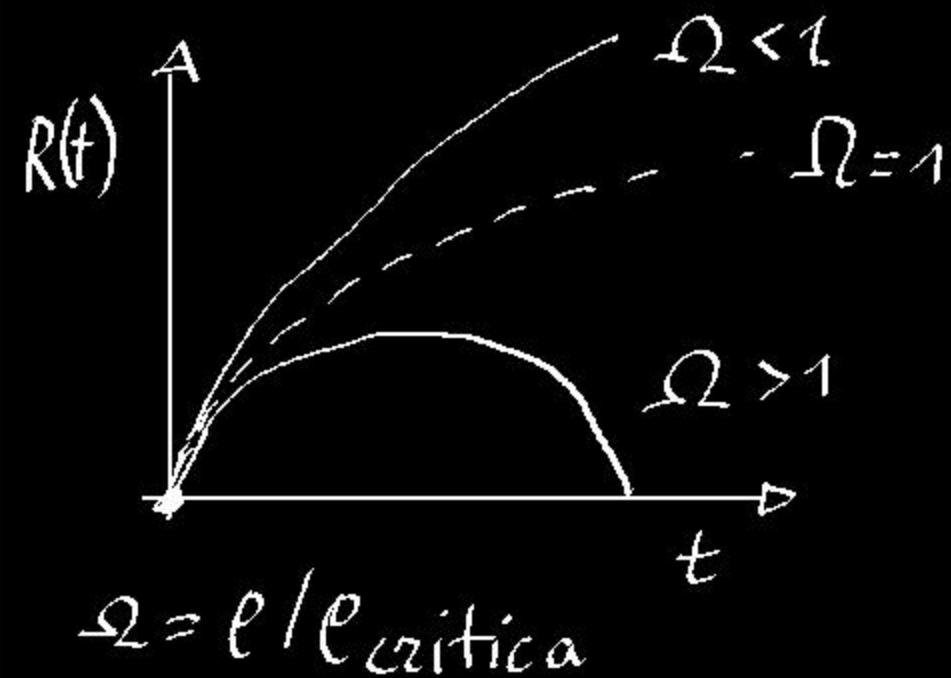
$$\Delta\phi \approx 43''/\text{secolo}$$



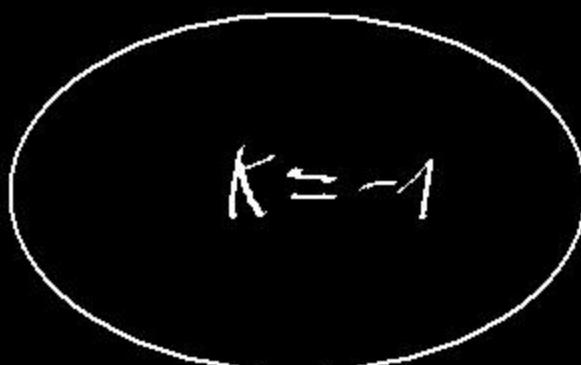
Lente gravitazionale

$$\vartheta = \frac{4GM}{b}$$

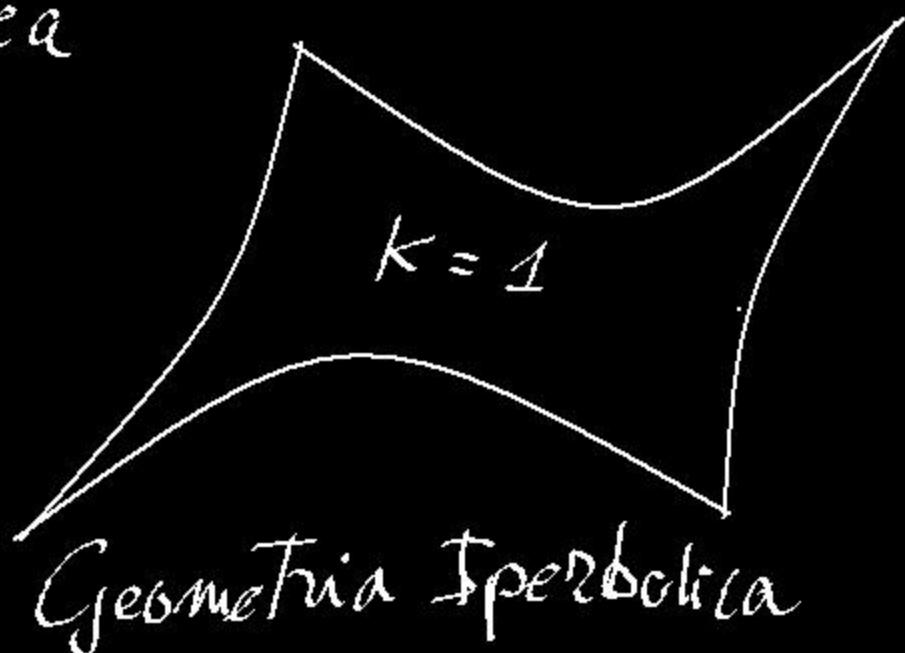
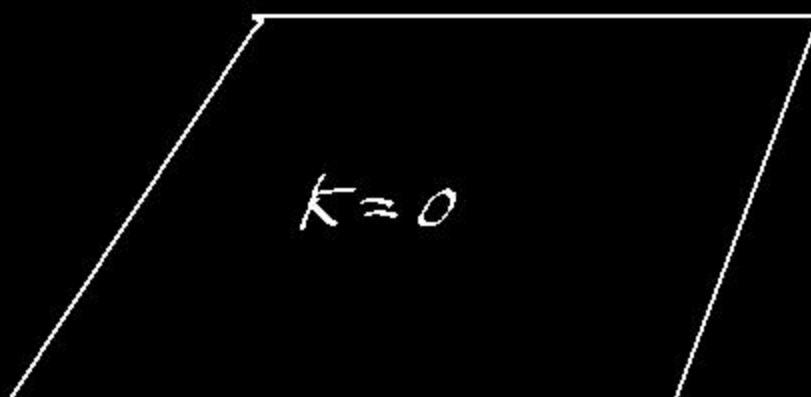
# L'UNIVERSO



Geometria Ellittica



Geometria Euclidea



Geometria Sperimentale

# DIMENSIONI EXTRA

$$D=1 \quad F = \text{cost.}$$

*N-Extra dimensioni  
spaziali*

$$D=2 \quad F \propto \frac{1}{z^2}$$

$$D=3 \quad F \propto \frac{1}{z^2}$$

$$F \propto \frac{1}{z^{2+N}}$$

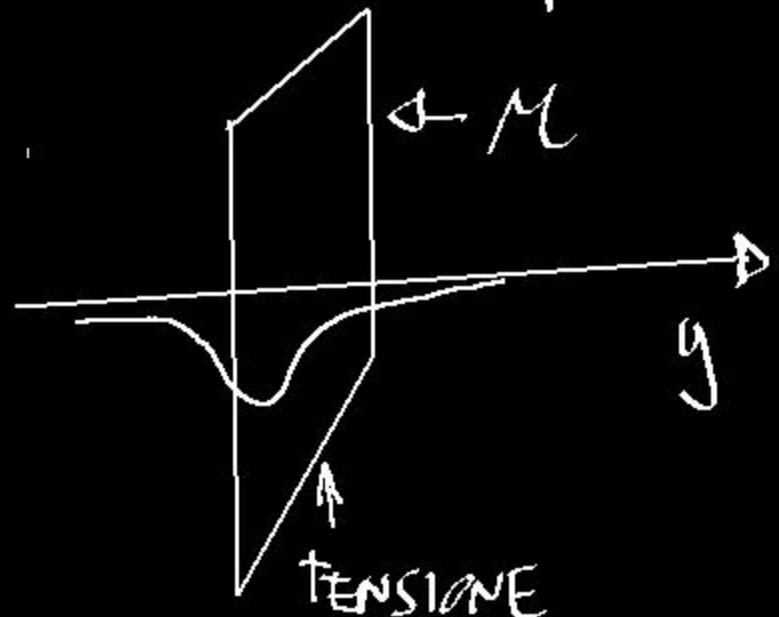
Perché non si vedono:

- Sono piccole ed  
arrotolate (KALBUCK-KLEIN)

$$G_{MN} = 0 \Rightarrow G_{\mu\nu} = T_{\mu\nu}^{EM}$$

$$x^M = (t, x_1, x_2, x_3, y)$$

Teoria delle p-brane



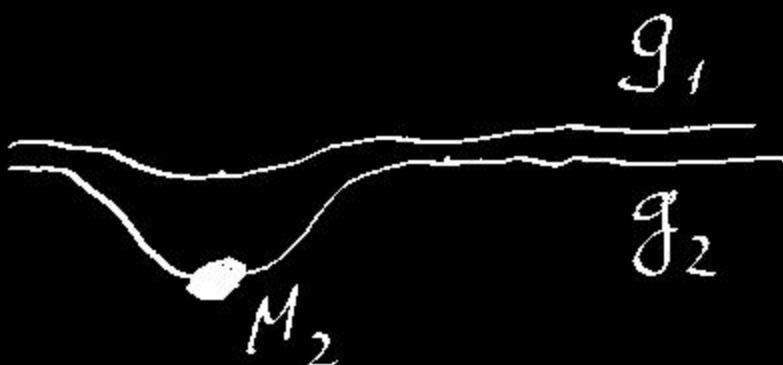
# ... E VARIE...

MULTIGRAVITÀ

N-Universi

o ciascuno con la  
sua metrifica  $g_i^{\mu}$

Esempio: N=2 (BIGRAVITÀ)

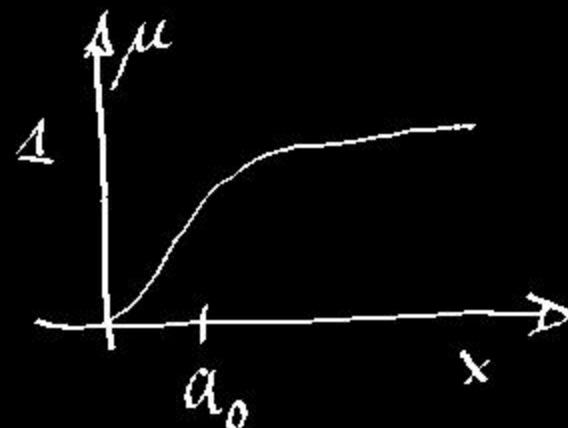


MOND

Modificazione della  
legge di Newton

$$g = \frac{g_N}{\mu(\frac{g}{a})}$$

Esempio:



# CONCLUSIONI

"Ma Egli ha messo nel cuore degli uomini la voglione dell'eternità senza che però essi possano comprendere l'opera compiuta da Dio dall'inizio (A) alla fine (Ω)"  
[Qo 3, 11]