# COSMOLOGIA
## In che Universo viviamo?

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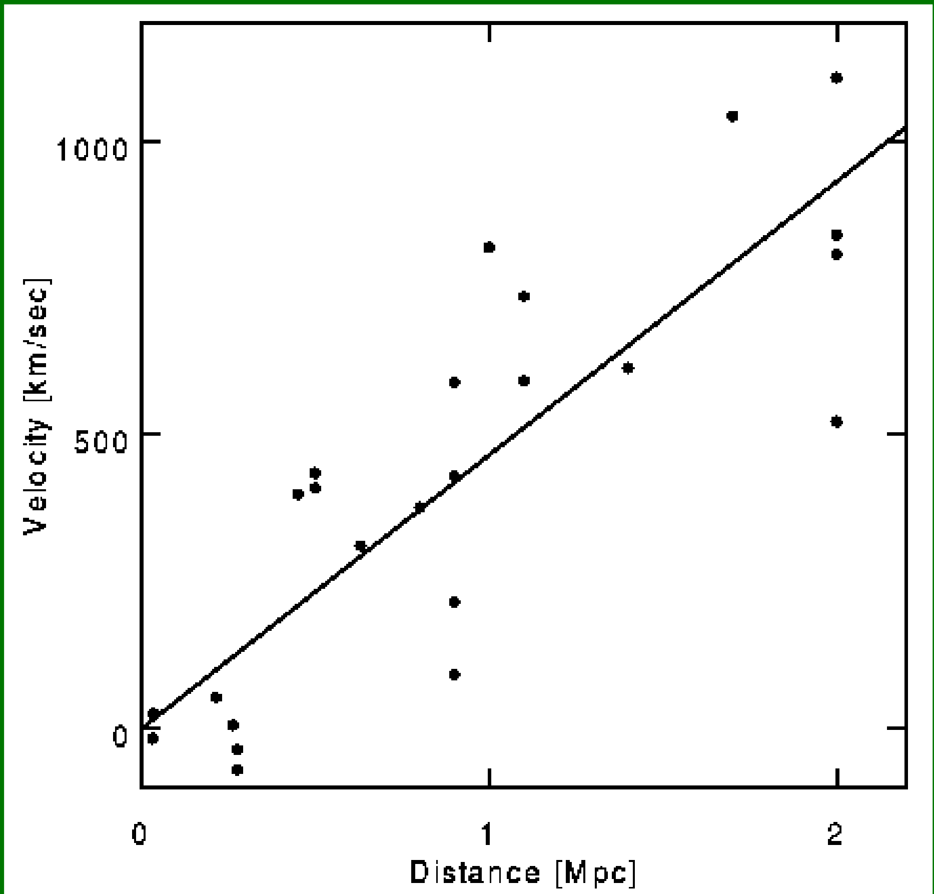
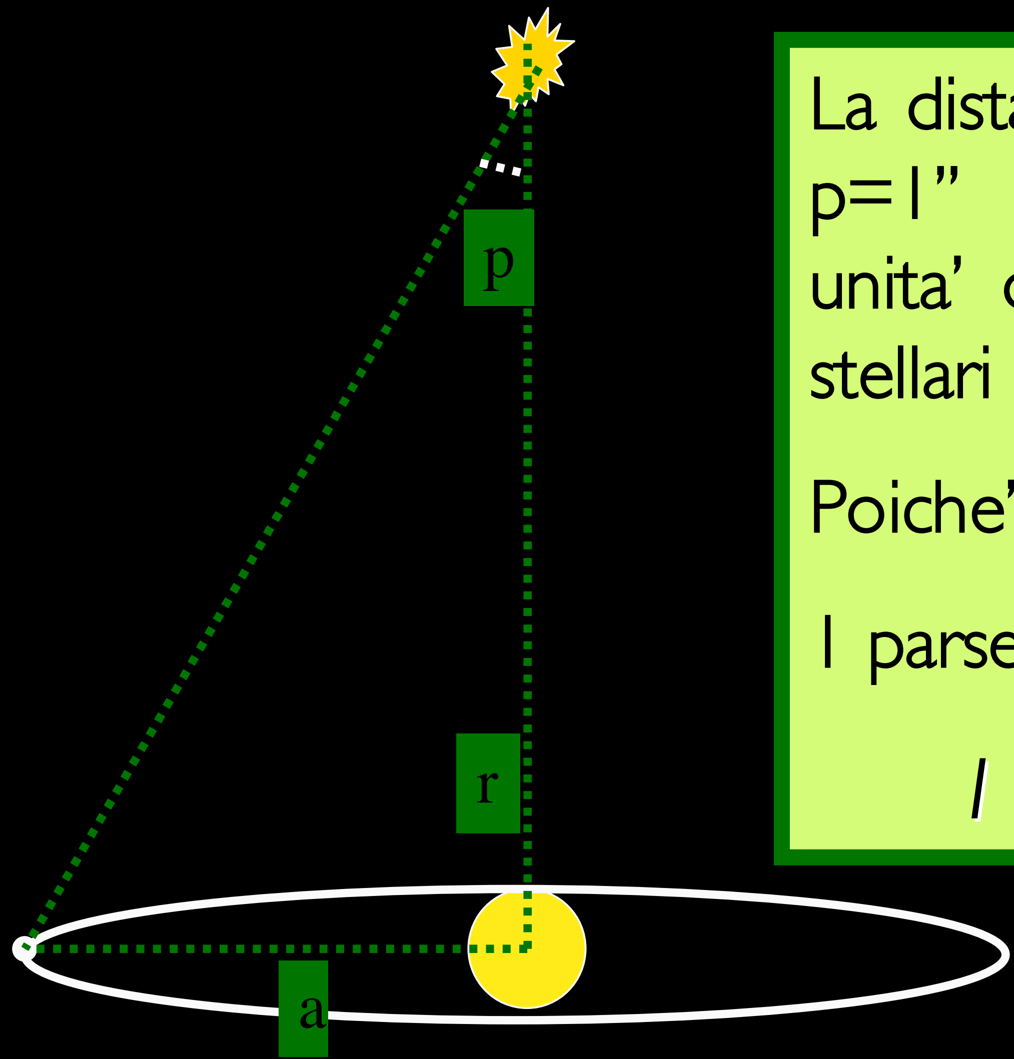
# La legge di Hubble

$$V = H_0 r$$




$$H_0 = 500 \frac{km/s}{Mpc}$$

# Il Megaparsec



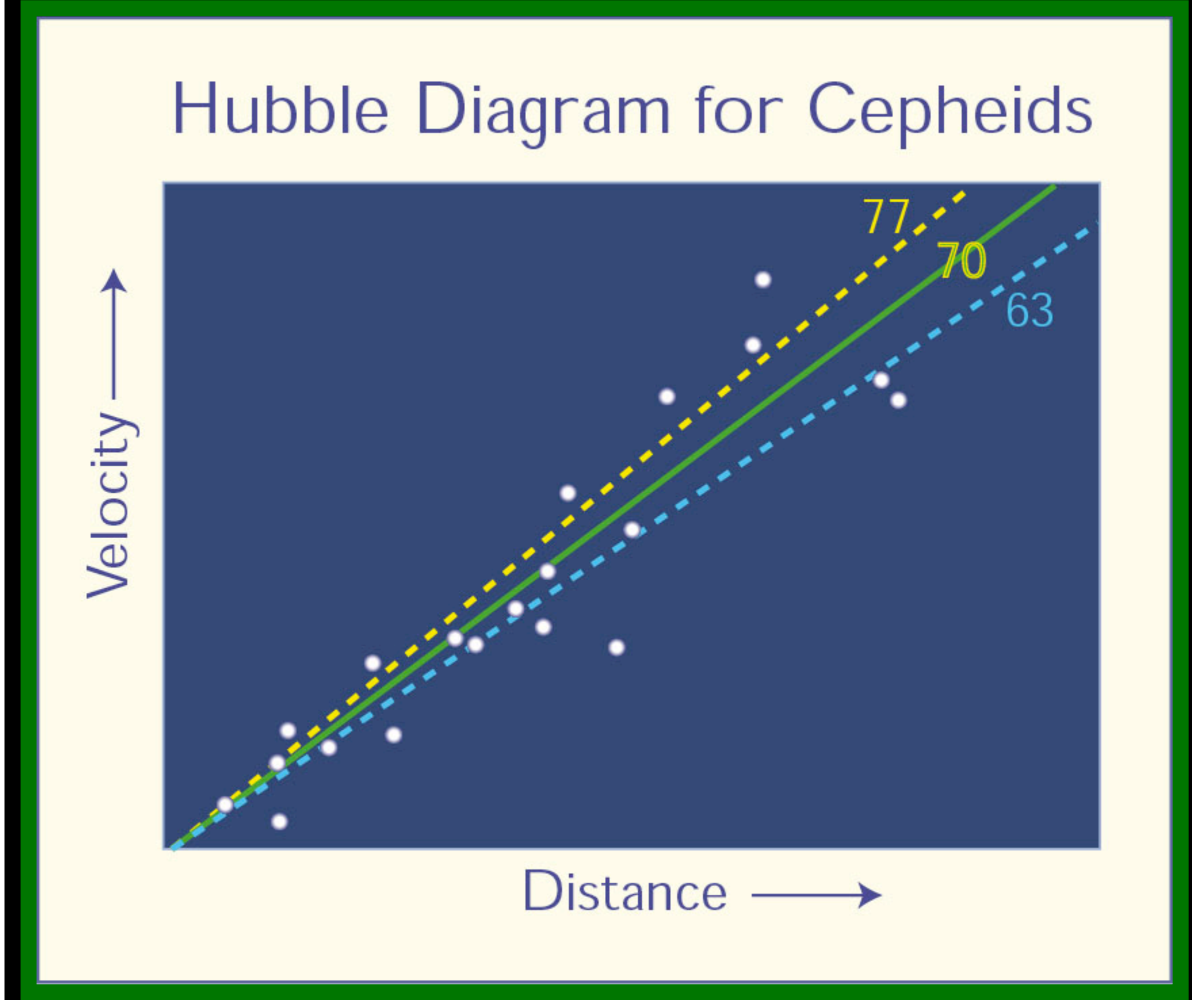

La distanza r corrispondente a p=1" e' stata assunta come unita' di misura delle distanze stellari

Poiche' a = 1.5 $10^{13}$ cm

1 parsec=3 $10^{18}$ cm

*1 Mpc =3 $10^{24}$ cm*

# The Hubble Key Project



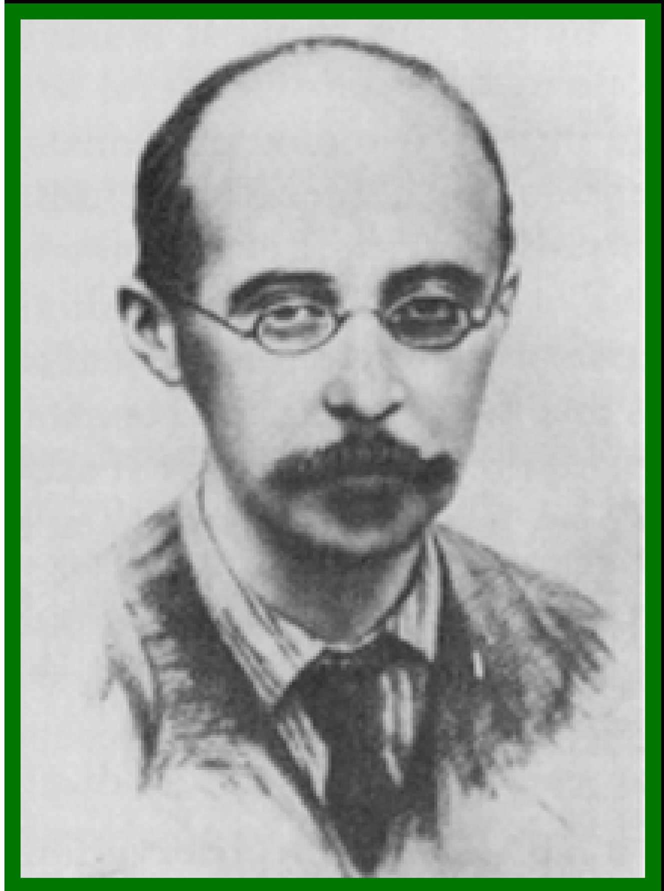

…to measure $H_o$ to 10% accuracy……..

## HUBBLE CONSTANT

$$H_0 = (70 \pm 7)\frac{\text{km/s}}{\text{Mpc}}$$

W. Freedman *et al.* (1999)

# Un universo in espansione



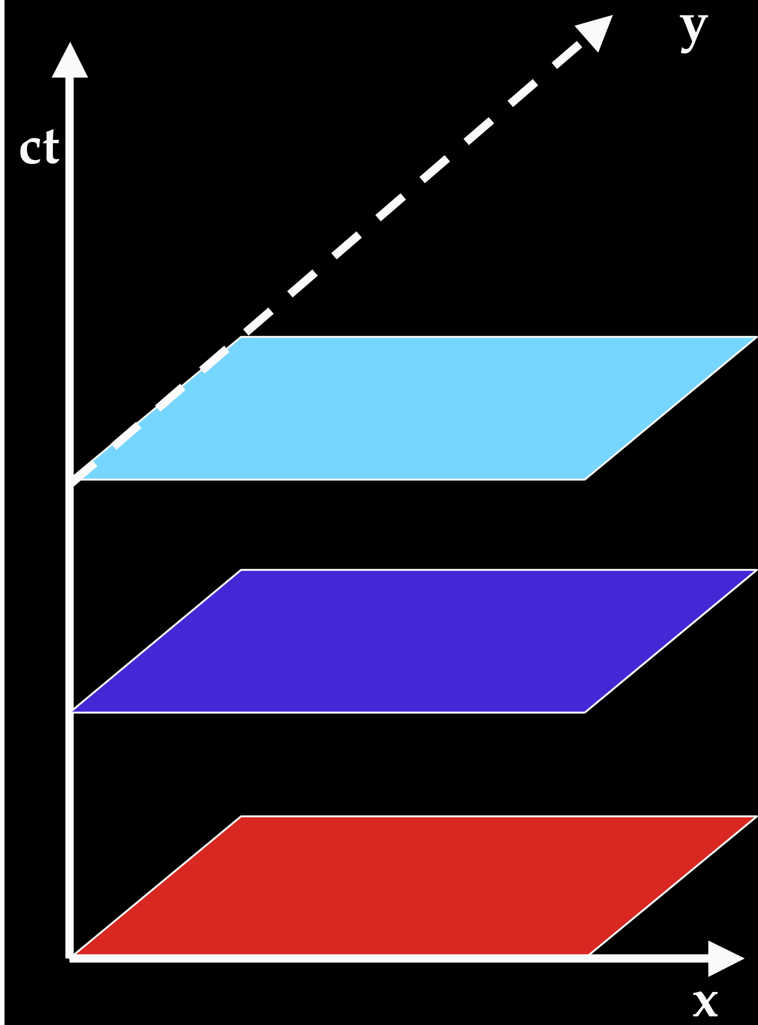
On the curvature of space

*A.Friedmann, 1922*

"The purpose of this note is … to demonstrate the possibility of a world in which the curvature of space is independent of the three spatial coordinates, but does depend on time"

**Aleksandr Aleksandrovich Friedman: 1888- 1925**

# Modelli cosmologici




**Coordinate gaussiane**

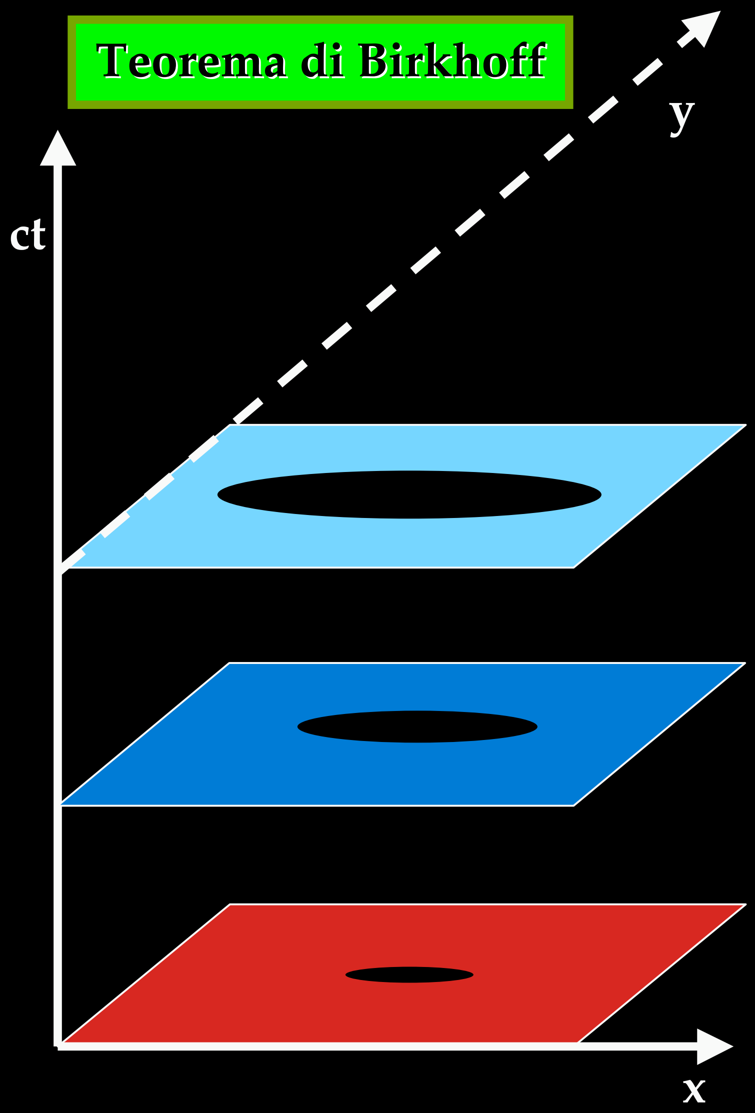
**Gauge sincrona**

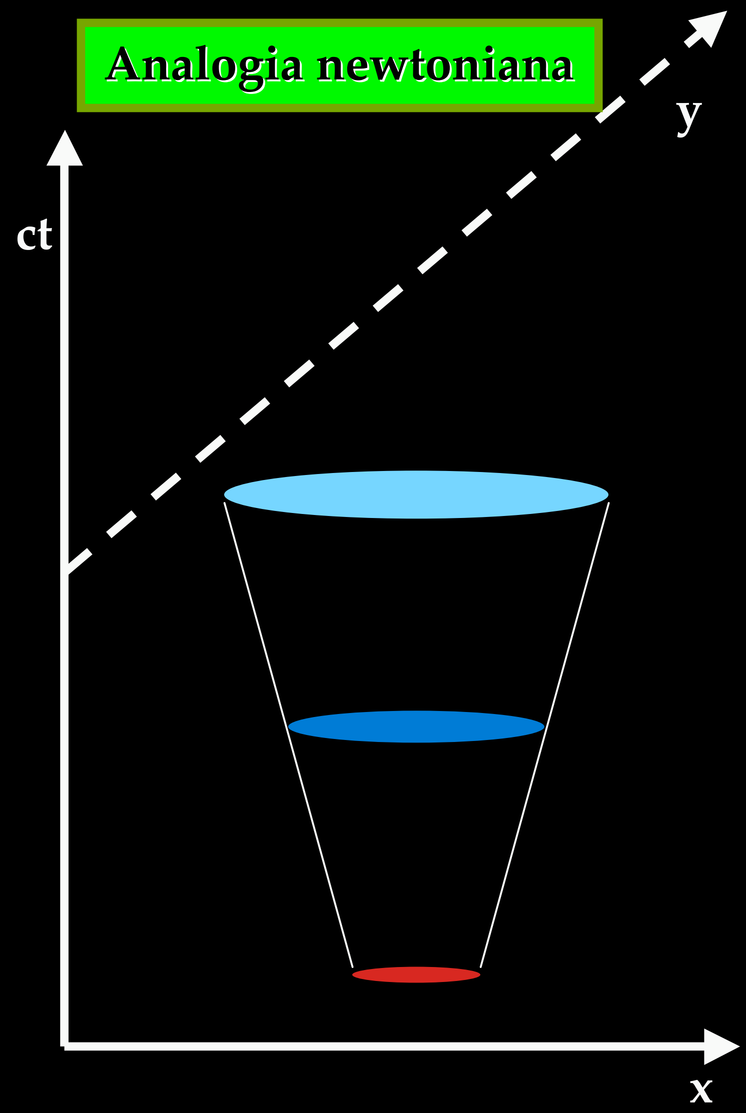
**Coordinate comobili**

$$g_{00} = 1; g_{0k} = 0;$$

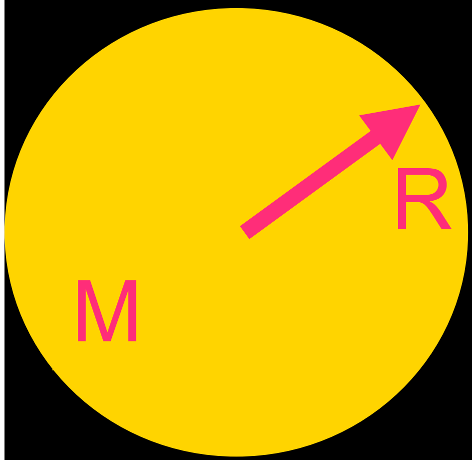
$$ds^2 = dx^{02} - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \qquad k = \begin{cases} +1 \\ 0 \\ -1 \end{cases}$$

# Modelli Cosmologici



# Modelli cosmologici



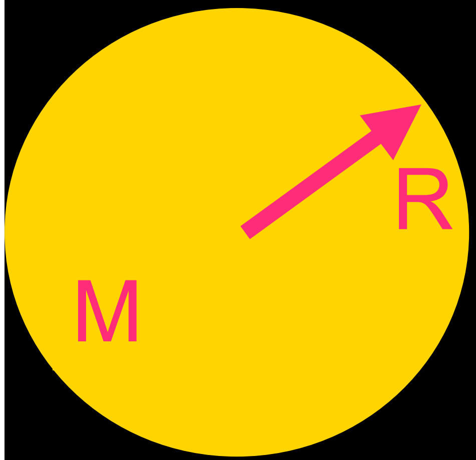

$$\ddot{R} = -\frac{GM}{R^2} \quad \Rightarrow \quad \frac{1}{2}\dot{R}^2 - \frac{GM}{R} = e$$

$$M = \frac{4}{3}\pi\rho R^3$$

$$\frac{\ddot{R}}{R} = -\frac{4}{3}\pi G\rho \quad \Rightarrow \quad \left(\frac{\dot{R}}{R}\right)^2 - \frac{2e}{R^2} = \frac{8}{3}\pi G\rho$$

$$\frac{\ddot{R}}{R} = -\frac{4}{3}\pi G\rho \quad \Rightarrow \quad \left(\frac{\dot{R}}{R}\right)^2 + \frac{kc^2}{R^2} = \frac{8}{3}\pi G\rho$$

# Modelli cosmologici



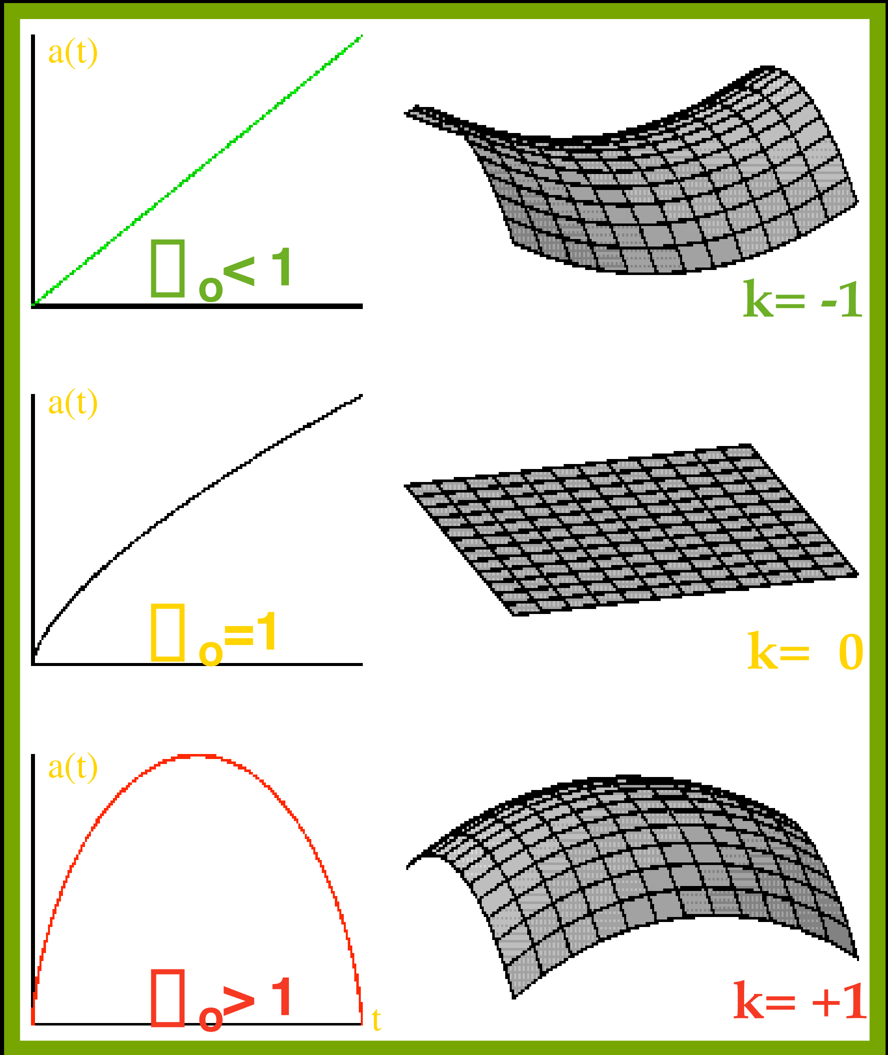

$$\left.\frac{|U|}{T}\right|_0 = \left.\frac{GM/R}{\dot{R}^2/2}\right|_0 = \left.\frac{8\pi G\rho}{3}\left(\frac{R}{\dot{R}}\right)^2\right|_0 = \frac{8\pi G\rho_0}{3H_0^2} = \frac{\rho_0}{\rho_{crit}} = \Omega_0$$

$$\rho_{crit} = 2\times10^{-29} h^2 \frac{g}{cm^3}; \qquad h = H_0 \Big/ 100\frac{km\ s^{-1}}{Mpc}$$

$$-2e = kc^2$$

| k | $\Omega_0$ | E |
|---|---|---|
| +1 | >1 | <0 |
| 0 | =1 | =0 |
| -1 | <1 | >0 |

# Cosmologia relativistica




$$a(t) = \frac{\Omega_o/2}{1-\Omega_o}[\cosh\theta - 1]$$

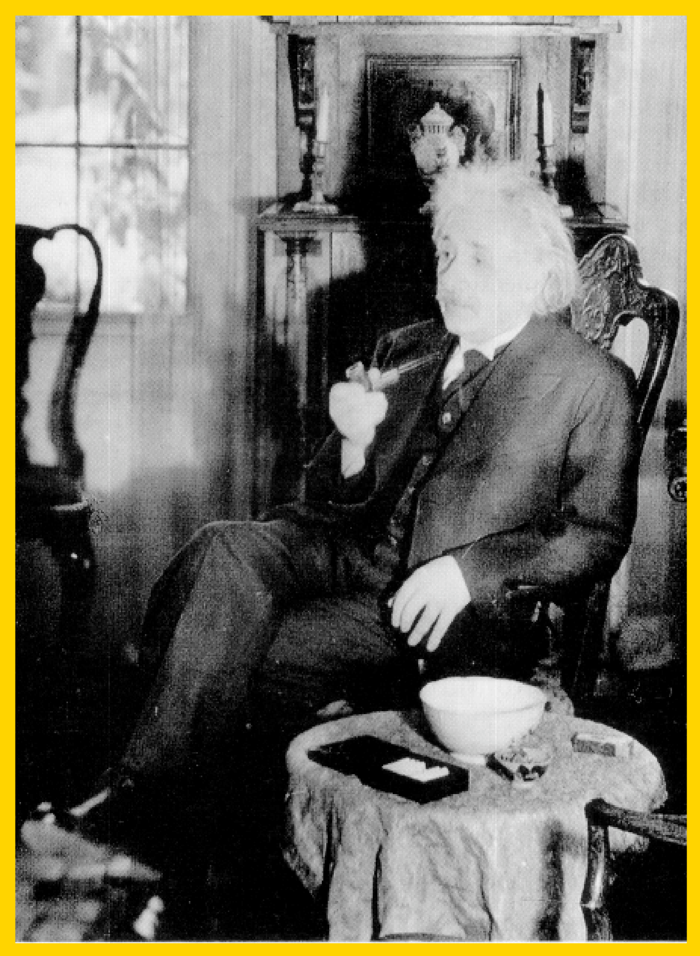
$$H_o t = \frac{\Omega_o/2}{(1-\Omega_o)^{3/2}}[\sinh\theta - \theta]$$

$$a(t) = \left(\frac{t}{t_o}\right)^{2/3}$$

$$a(t) = \frac{\Omega_o/2}{1-\Omega_o}[1 - \cos\theta]$$

$$H_o t = \frac{\Omega_o/2}{(1-\Omega_o)^{3/2}}[\theta - \sin\theta]$$

# Costante cosmologica: □



"Cosmological considerations on the general theory of relativity

A.Einstein, 1917

"In order to arrive at this consistent view, we admittedly had to introduce an extension of the field equations of gravitation which is not justified"

# Costante cosmologica: $\Lambda$

- Modifica delle equazioni di campo della relativita' generale

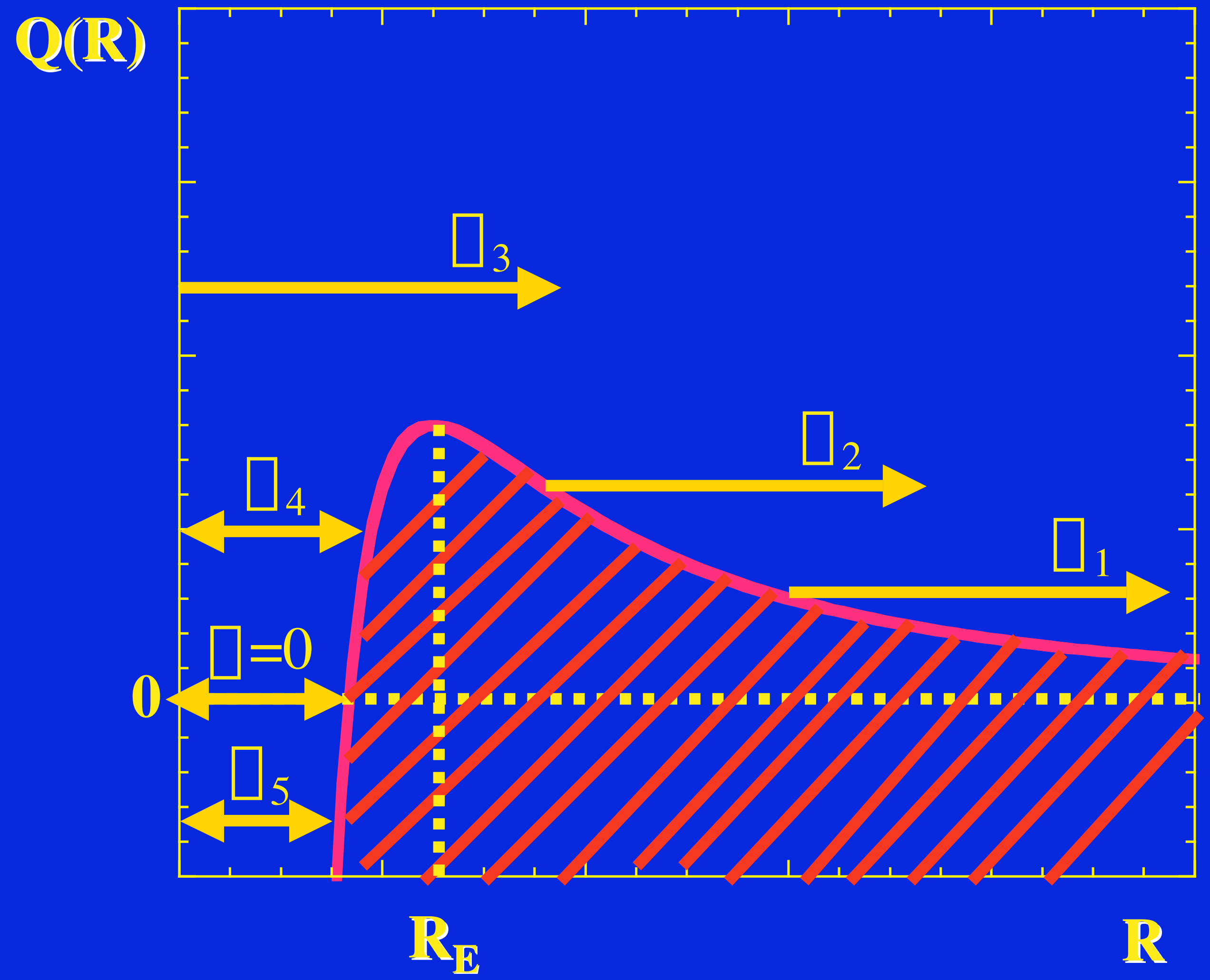
$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{kc^2}{R^2} = \frac{8}{3}\pi G\rho + \frac{1}{3}\Lambda c^2 \Rightarrow \left(\frac{\dot{R}}{R}\right)^2 = \frac{c^2}{3}\left[\Lambda - Q(R)\right]$$

$$\Lambda \geq Q(R) \equiv \frac{3k}{R^2} - \frac{8\pi G}{c^2}\rho +$$

- E' come avere un fluido con

$$\tilde{\rho} = \rho + \frac{c^2\Lambda}{8\pi G}; \qquad \tilde{p} = p - \frac{c^2\Lambda}{8\pi G};$$

# Costante Cosmologica: k=+1



# Λ & k=+1

Consideriamo un universo di polvere, p = 0. Abbiamo visto che

$$Q(R) = \frac{3}{R^2} - \frac{8\pi G}{c^2}\rho$$

ha un massimo in corrispondenza di

$$\frac{1}{\mathrm{R_E^2}} = \frac{4\pi G}{c^2}\rho_0$$

Scegliamo

$$\Lambda_E = Q(R_E) = \frac{3}{R_E^2} - \frac{8\pi G}{c^2}\rho_0 = \frac{1}{R_E^2}$$

Otteniamo la soluzione statica di Einstein

$$\left(\frac{\dot{R}}{R}\right)_E^2 = \frac{c^2}{3}\left[\Lambda_E - Q(R_E)\right] = 0;$$

$$\left(\frac{\ddot{R}}{R}\right)_E^2 = -\frac{4\pi G}{3}\tilde{\rho}_0 = -\frac{4\pi G}{3}\rho_0 + \frac{1}{3}\Lambda_E c^2 = 0$$

# Λ & k=+1

Assumiamo come ordine di grandezza $\rho_0 \approx 10^{-30} g cm^{-3}$; allora

$R_E^2 = \frac{c^2}{4\pi G \rho_0} \approx 3 \times 10^{28} cm$ $\qquad$ $\Lambda_E = \frac{1}{R_E^2} \approx 10^{-57} cm^{-2}$

La massa dell'universo di Einstein e'

$M = 2\pi^2 \rho_0 R_E^3 \approx 5 \times 10^{56} g$

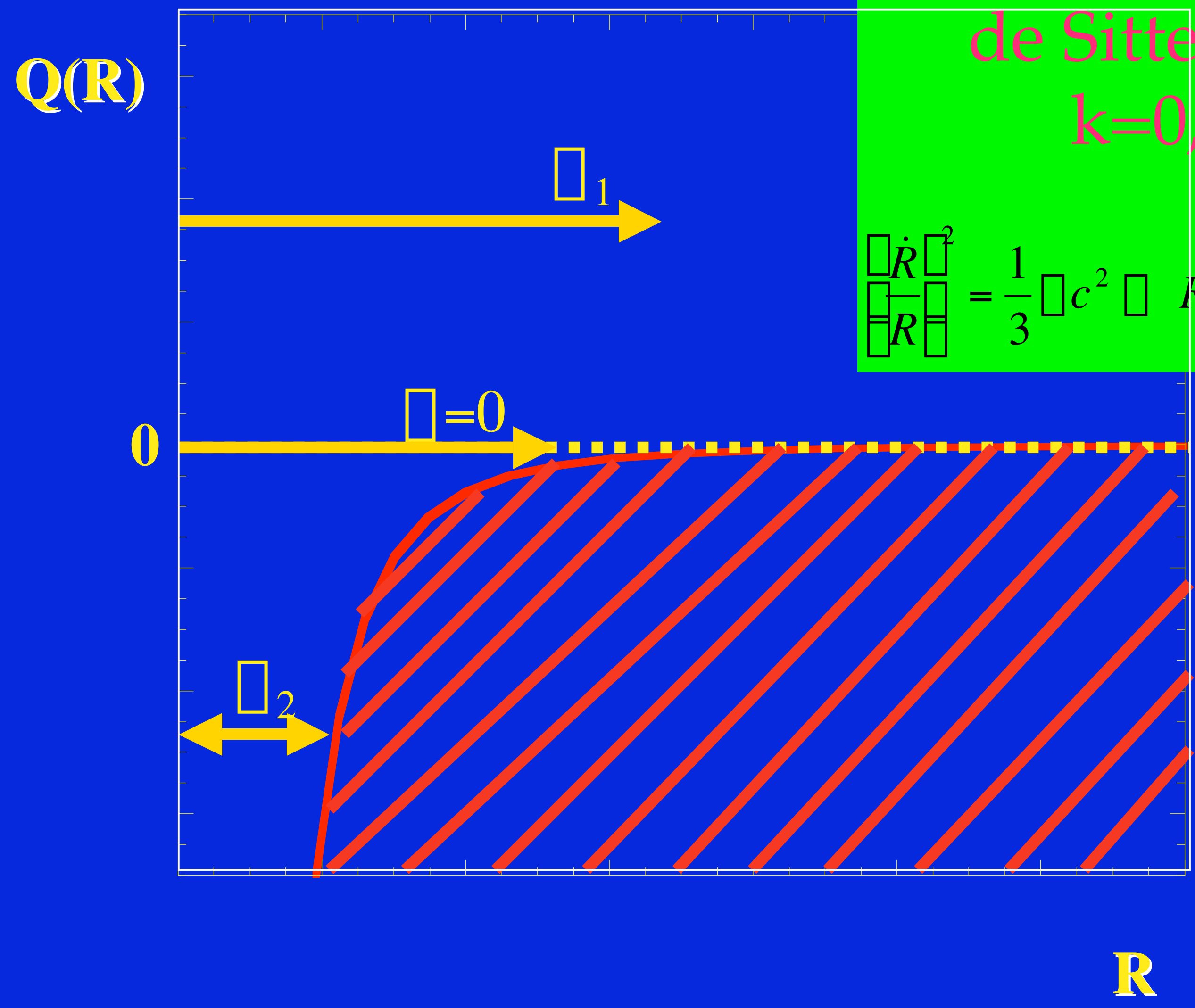
La densita' media del sistema solare (SS) e'

$\rho_{SS} \approx \frac{3 M_{Sole}}{4\pi R^3_{\text{Plutone}}} = \frac{3}{4\pi} \frac{2 \cdot 10^{33} g}{\left(40 \times 1.5 \cdot 10^{13} cm\right)} \approx 2.2 \cdot 10^{-12} g/cm^3$

L'influenza di Λ sulla dinamica del sistema solare e' trascurabile:

$\frac{\rho_0}{\rho_{ss}} \approx 5 \cdot 10^{-19}$

# Costante Cosmologica: k=0, -1




de Sitter model

k=0, ρ=0

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{1}{3}\Lambda c^2 \quad \propto \quad R \quad \exp\left(\sqrt{\frac{\Lambda}{3}}\,ct\right)$$

# Equazione di Friedman

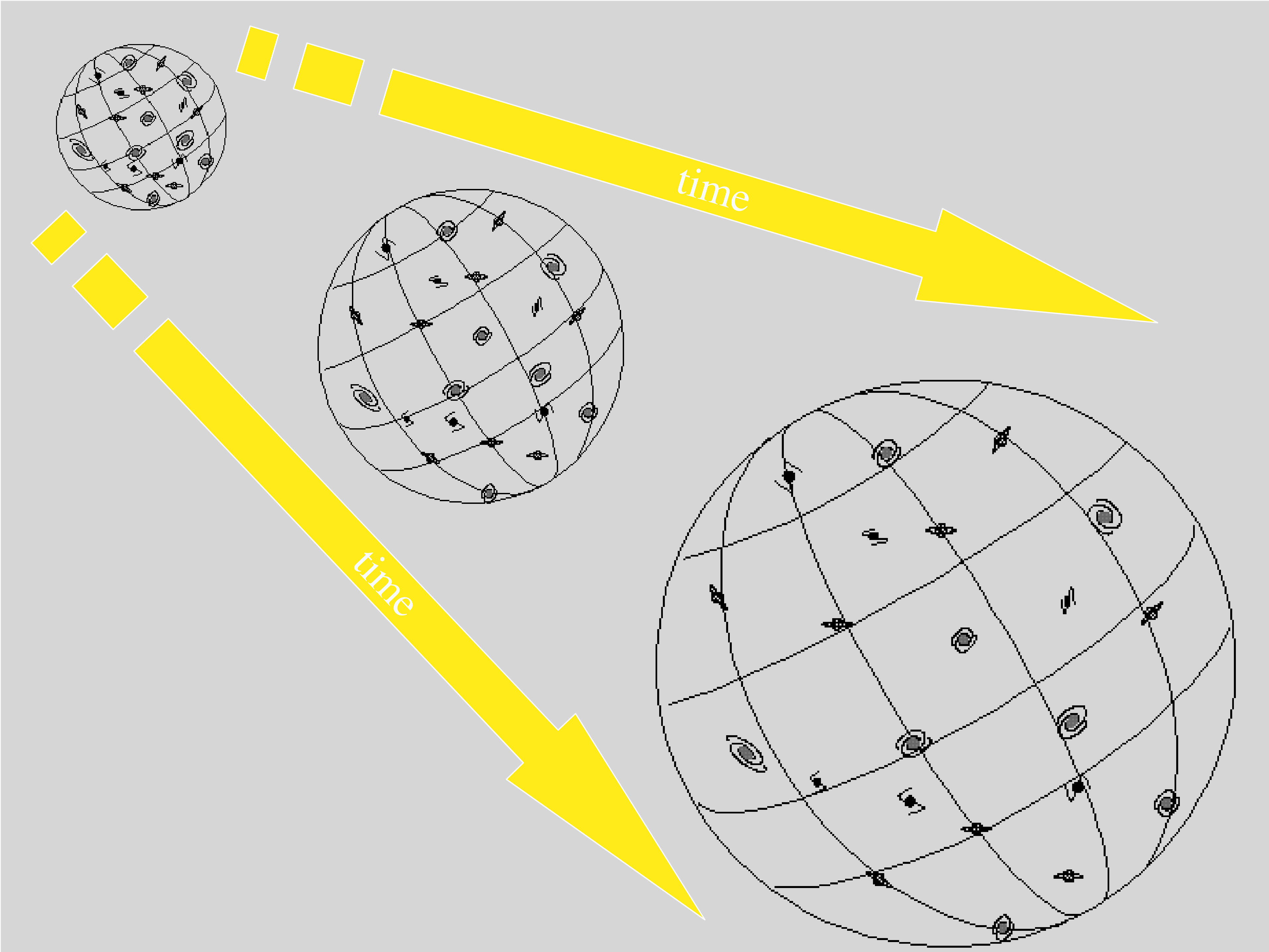
$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{kc^2}{R^2} = \frac{8\pi G}{3}\frac{\rho_0}{R^3} + \frac{1}{3}\Lambda c^2$$

Fase accelerata: domina la costante cosmologica

Fase cinematica: domina la curvatura

Tasso di espansione

Fase decelerata: domina la materia


time
time

# Problema della piattezza

Per definizione

$$\Omega(t) \equiv \frac{8\pi G}{3H^2(t)}\rho(t)$$

Dall'equazione di Friedman, con $\Lambda = 0$ e $a(t) = R(t)/R(t_0)$, si ottiene

$$H^2(t) \equiv \left(\frac{\dot{R}}{R}\right)^2 = H_0^2\left(\frac{\Omega_0}{a^3(t)} + \frac{1-\Omega_0}{a^2(t)}\right)$$

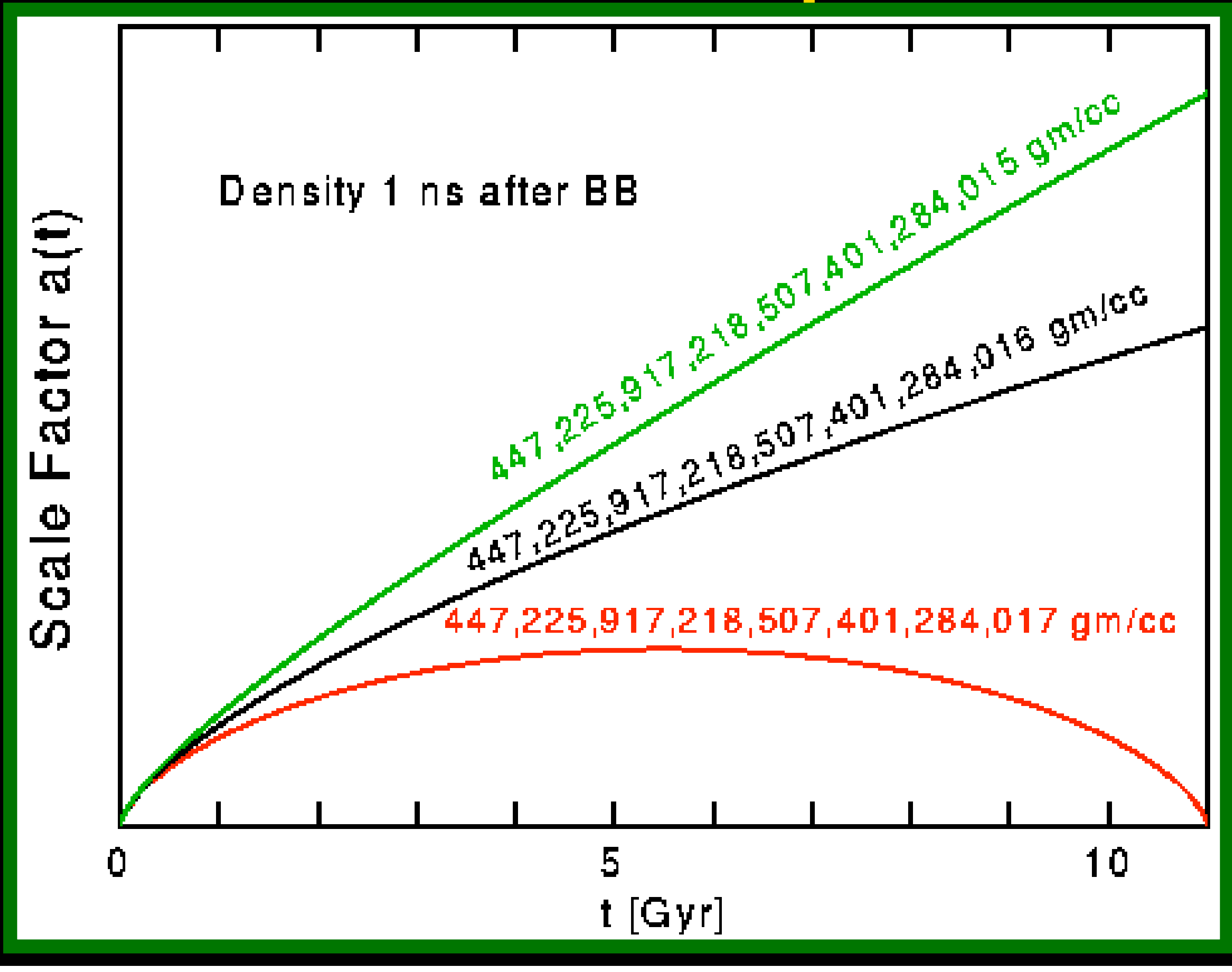
si ricava

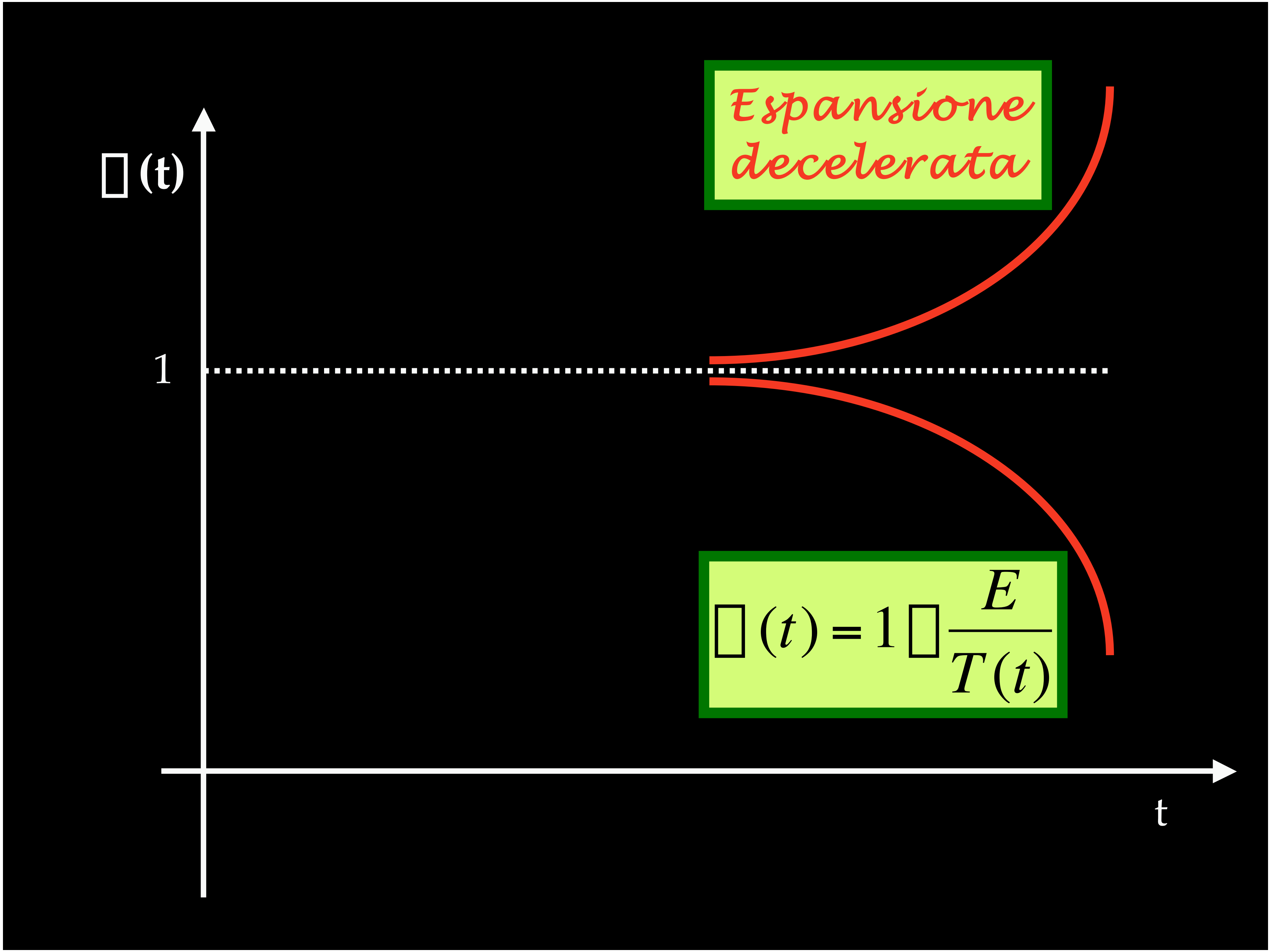
$$\frac{1}{\Omega(t)} - 1 = \left(\frac{1}{\Omega_0} - 1\right) a(t)$$

Quindi in un universo di Friedman

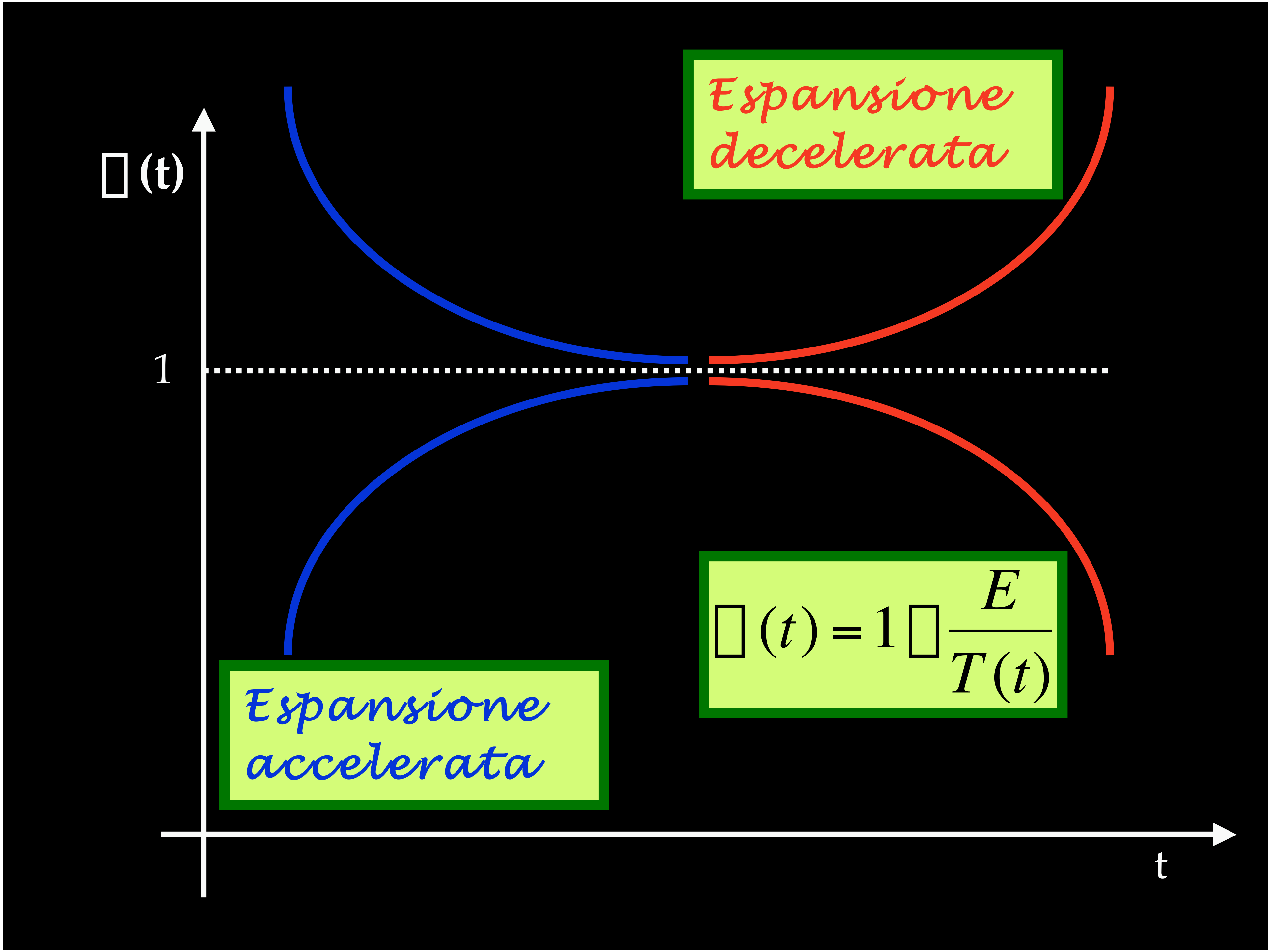
$$\lim_{t \to 0} \Omega(t) = 1$$

# Problema della piattezza





□(t)
1
Espansione decelerata
□(t) = 1 □ E / T(t)
t



(t)
1
t
Espansione decelerata
Espansione accelerata
(t) = 1 E/T(t)