Dynamics of heavy quarks in charged $\mathcal{N} = 4$ SYM plasma

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C. Herzog and AV, arXiv:0708:0609 [hep-th]

Outline

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Why bother?

- Heavy ion experiments $\Rightarrow$ Need quantitative understanding of non-Abelian plasmas at
  - High T and small/moderate μ
  - Strong coupling
  - In and (especially) out-of equilibrium
- Combination notoriously problematic
  - Perturbative methods of limited use
  - Dynamical quantities hard for lattice QCD
- Any new insights into dynamics of strongly coupled non-Abelian plasmas welcome!

$\mathcal{N} = 4$ SYM and gauge/gravity duality

- Unique feature of $\mathcal{N} = 4$ SYM: It has known string dual
  - $\mathcal{N} = 4$ SYM $\Leftrightarrow$ IIB string theory on $AdS_5 \times S_5$

$$ds^2 = R^2\left\{\frac{dr^2}{r^2} + r^2\left(-dt^2 + d\mathbf{x}^2\right) + d\Omega_5^2\right\}$$

  - $R =$ (curvature) radius of S_5 and AdS_5
  - Parameters string coupling g_s and length scale $\ell_s = \sqrt{\alpha'}$
- AdS/CFT dictionary:
  - Radial coordinate in AdS $\sim$ energy scale in CFT
  - $(R/\ell_s)^4 = \lambda$, $g_s = \lambda/(4\pi N_c)$
- Beautiful aspect:
  - Classical sugra limit: string coupling $g_s \ll 1$ and $R/\ell_s \gg 1$
  - Gauge theory at $\lambda \gg 1$, $N_c \gg 1 \Leftrightarrow$ Classical GR!

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AdS/CFT dictionary at finite temperature

- Turning on finite temperature on gauge theory side $\Leftrightarrow$ Adding a black hole in AdS_5, with horizon at radial coordinate value $r_h = \pi T$
- Chemical potentials for SO(6) R-symmetry $\Leftrightarrow$ Black hole spinning in S_5 directions

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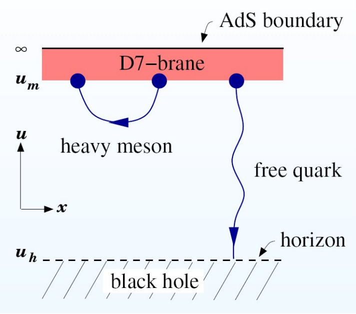
- Chemical potentials for SO(6) R-symmetry ⇔ Black hole spinning in S_5 directions
- Adding massive $\mathcal{N} = 2$ hypermultiplet ⇔ Adding a D7-brane wrapping AdS_5 down to some r_0
- Quarks and mesons ⇔ Classical strings hanging from D7 brane



What is $\mathcal{N} = 4$ SYM?

- Maximally supersymmetric Yang-Mills theory with 4 SUSY generators
  - Field content: 1 gauge field, 3 complex scalars, 4 Majorana fermions — all massless, as theory conformal
  - Relevant symmetries: local SU(N_c) gauge symmetry (all fields in adj. repr.) and global SO(6) R symmetry

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$\mathcal{N} = 4$ SYM vs. QCD: $T = 0$

- Conformally invariant: No Λ_{QCD}, fixed coupling
- No S-matrix, no particles
- Conclusion: No real similarity

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$\mathcal{N} = 4$ SYM vs. QCD: $T \neq 0$

- Deconfined non-Abelian plasma
- Similar IR physics: Debye screening, finite magnetic mass
- Similar hydro properties
- Conclusion: Near perfect *qualitative* match!

Plan of attack

- Challenge 1: Obtain as much insight into strongly coupled $\mathcal{N} = 4$ SYM as possible
  - Especially dynamical quantities inaccessible in strongly coupled QCD interesting
  - An enormous activity during past few years
- Challenge 2: Interpolate to moderate couplings
  - Compute NLO $1/\lambda$ corrections at strong coupling
  - Investigate weak coupling limit of $\mathcal{N} = 4$ SYM
- Challenge 3: Attempt to infer information on QCD
  - Couplings relevant for heavy ion physics in 'no man's land'
  - No clear prescription, but a lot of room to play around...
- Holy Grail: Find string/gravity dual of QCD

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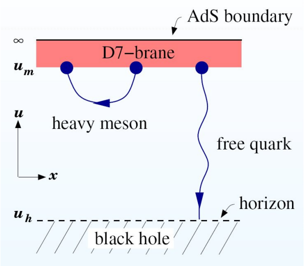
Heavy quarks in strongly coupled $\mathcal{N} = 4$ SYM

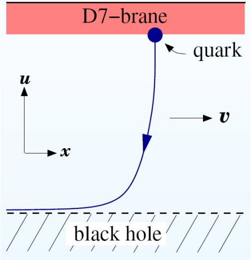
Herzog *et al*, hep-th/0605158;...

- Consider dragging a heavy quark of a fundamental $\mathcal{N} = 2$ hypermultiplet through strongly interacting $\mathcal{N} = 4$ SYM plasma
- Want to compute friction coefficient μ in eom

$$\frac{dp}{dt} = -\mu p + f$$

  - Dragging the quark at constant velocity, $f = Mv\mu$
  - Letting the quark slow down after initial kick, $p(t) = p(0)e^{-\mu t}$





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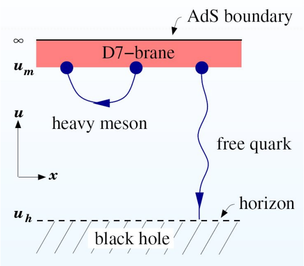

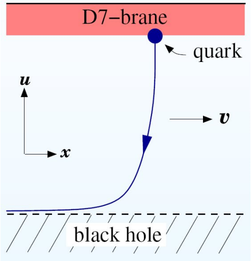

Gravity computation a'la Herzog *et* al.

- Start from classical Nambu-Goto action

$$S = -\frac{1}{2\pi\alpha'}\int d\sigma\, d\tau\, \sqrt{-\det G}$$

- And solve for classical string profile with correct bc's

$$x(r,t) = x_0 + vt + \frac{v}{2r_h}\left\{\frac{\pi}{2} - \arctan\frac{r}{r_h} - \mathrm{arccoth}\frac{r}{r_h}\right\}$$

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- Result for friction and diffusion coefficients

$$\frac{dp}{dt} = -\frac{\pi}{2}\sqrt{\lambda}T^2\frac{v}{\sqrt{1-v^2}},$$
$$\mu = \frac{\pi}{2}\frac{\sqrt{\lambda}T^2}{M_{kin}},$$
$$D = \frac{T}{\mu M_{kin}}\frac{2}{\pi\sqrt{\lambda}T}$$

- This may be compared to weak coupling expansion

$$D = \frac{12\pi}{d_A g^4 T}\left\{\ln\frac{2T}{m_D} + \frac{13}{12} - \gamma_E + \frac{1}{3}\ln 2 + \frac{\zeta'(2)}{\zeta(2)}\right\}$$

P. Chesler and AV, hep-ph/0607148

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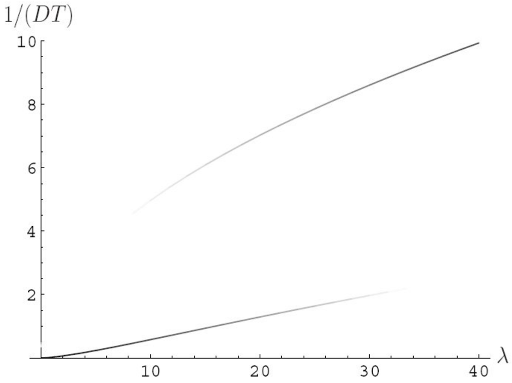
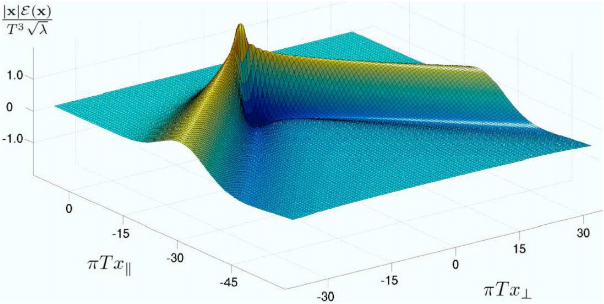

$1/(DT)$
10
8
6
4
2
10
20
30
40
λ



- Can also compute energy perturbation in the plasma created by the heavy quark; P. Chesler and L. Yaffe, arXiv:0706.0368 [hep-th]

Puzzle with R charges

- To model finite *u* and *d* quark chemical potentials in QCD, want to investigate finite R charge μ in $\mathcal{N} = 4$ SYM
- In the literature, there exist two mutually incompatible solutions
  - Herzog's (H; hep-th/0605191) 5*d* solution with velocity dependent μ
  - Caceres and Guijosa's (CG; hep-th/0605235) simplified 10*d* solution with velocity independent μ
- Obvious questions:
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  - If yes, what is its relation to CG on the SYM side?
- Would be interesting to find all relevant 10*d* solutions...

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The spinning black D3-brane background

$$
\begin{aligned}
ds_{10}^2 &= \sqrt{\Delta}\, ds_5^2 + \frac{R^2}{\sqrt{\Delta}} \sum_{i=1}^{3} X_i^{-1} \left(d\mu_i^2 + \mu_i^2 (d\psi_i + A^i/R)^2 \right) \ , \\
ds_5^2 &= -(H_1 H_2 H_3)^{-2/3} h\, dt^2 + (H_1 H_2 H_3)^{1/3} (h^{-1} dr^2 + \frac{r^2}{R^2} d\mathbf{x}^2) \ ; \\
X_i &= H_i^{-1} (H_1 H_2 H_3)^{1/3} \ ; \quad A^i = \frac{(1 - H_i^{-1})\sqrt{m}}{\ell_i} dt \ ; \\
h &= -\frac{m}{r^2} + \frac{r^2}{R^2} H_1 H_2 H_3 \ ; \quad \Delta = \sum_{i=1}^{3} X_i \mu_i^2 \ ; \quad H_i = 1 + \frac{\ell_i^2}{r^2} \ ; \\
\mu_1 &= \sin\theta_1 \ ; \quad \mu_2 = \cos\theta_1 \sin\theta_2 \ ; \quad \mu_3 = \cos\theta_1 \cos\theta_2 \ .
\end{aligned}
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- Field theory temperature and chemical potentials:

$$T = \frac{1}{2\pi r_h^2 R^2} \frac{2r_h^6 + (\ell_1^2 + \ell_2^2 + \ell_3^2) r_h^4 - \ell_1^2 \ell_2^2 \ell_3^2}{\prod_{i=1}^3 \sqrt{r_h^2 + \ell_i^2}},$$

$$\Phi_i = \left.\frac{A^i}{R}\right|_{r\to\infty} - \left.\frac{A^i}{R}\right|_{r=r_h} = -\frac{\ell_i \prod_{j=1}^3 \sqrt{r_h^2 + \ell_j^2}}{R^2 r_h (r_h^2 + \ell_i^2)}$$

- Challenge again: Solve classical equations of motion for strings stretching from D7 brane to horizon
  - Boundary conditions at D7 brane and dependence on angular coordinates important for field theory interpretation
- From now on, for simplicity consider turning on only one chemical potential

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Single charge solutions

- Introduction of chemical potentials breaks SO(6) R symmetry into SO(4). Turning on μ for
  - Two elements of Cartan sub-algebra of SO(6) in unbroken SO(4) $\Rightarrow$ D7 brane and string neutral
  - One element in broken part $\Rightarrow$ D7 brane and string charged
- First case corresponds to polar, second to equatorial strings

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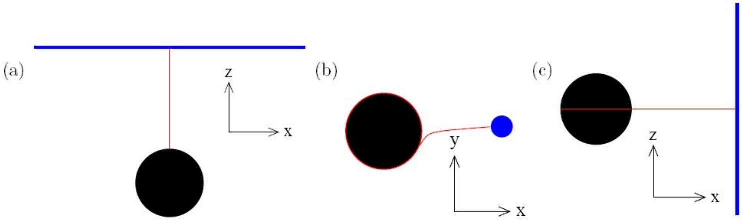

- Various possibilities for the D7 brane the string is attached to
  - Non-rotating brane not extending in ψ directions $\Rightarrow \dot{\psi} = 0$
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  - Rotating brane

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  - Rotating brane
- Simplest case with known stable D7 brane configuration; CG solution
  - On field theory side, seems to correspond to non-zero R charge with zero chiral condensate
  - Possible problem: For charged string, R charge density diverges

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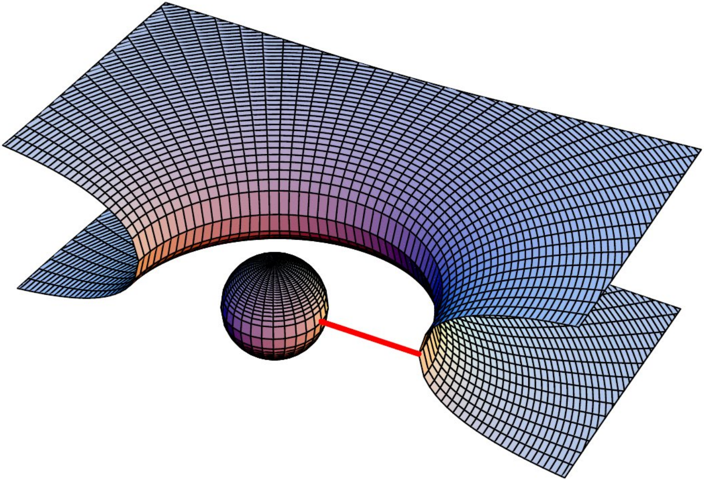
- Intuitive gravitational picture; uplift of H solution
  - On the field theory side, seems to correspond to non-zero chiral condensate
  - Problem: At $T = 0$, brane configuration unstable
  - Too cool to be physical...



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- No known examples of suitable classical brane solutions
  - Field theory interpretation includes spatial gradient in R charge chemical potential

Results I

- General result for string profile in terms of forces in x and ψ directions, $P \equiv 2\pi\alpha'\pi_x^1$ and $L \equiv 2\pi\alpha'\pi_\psi^1$

$$(x')^2 = \frac{\beta\epsilon}{\alpha\gamma}\frac{\left(\pm Lv\gamma(\phi+\omega)+P(-\alpha+v^2\gamma)\right)^2}{\gamma\epsilon(L(\phi+\omega)\pm Pv)^2-L^2f^2\alpha\gamma-P^2\alpha\epsilon-\alpha\gamma\epsilon I_1},$$

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  - CG solution with $\omega \equiv \dot{\psi} = 0$, H with $L = 0$
  - α, β, etc. complicated functions of r, r_h, ℓ, R
- P from requiring x', ψ' be well-defined from r_h to r_0
  - Both numer. and denomin. vanish at some $r = r_1(r_h, \ell, R)$

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Results II

- Final result: Drag force $\pi_x^1 = \frac{1}{2\pi\alpha'} g_{xx}(r_1)v$
- All solutions investigated (CG, H) appear stable against small perturbations
  - Full analysis awaits
  - Polar CG solutions leave door open for super-radiant instabilities
- Main lesson: Drag force depends strongly on transverse (S_5) geometry
  - Bad news for hopes to obtain information relevant for QCD
  - Is R charge chemical potential at all analogous to (light) quark μ in QCD?

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Summary and conclusions

- AdS/CFT duality unique and powerful tool for studying strongly coupled non-Abelian plasmas
  - Dynamical properties of $\mathcal{N} = 4$ SYM theory available at large N_c, λ limit
  - Recently much progress in gravity duals of other related theories
- Hope to eventually learn lessons about QGP physics
  - At finite T, SYM and QCD plasmas qualitatively similar
  - Quantitative predictions presently questionable; weak coupling calculations important step in this direction
- Discrepancy in drag force of heavy quarks in charged $\mathcal{N} = 4$ plasma solved
  - CG solution seems stable and physically simplest
  - Strong dependence of results on transverse geometry