The average phase factor from chiral perturbation theory

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with: Jac Verbaarschot

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What Analytic computation of $\langle e^{2i\theta} \rangle$ $\mu < m_\pi/2$

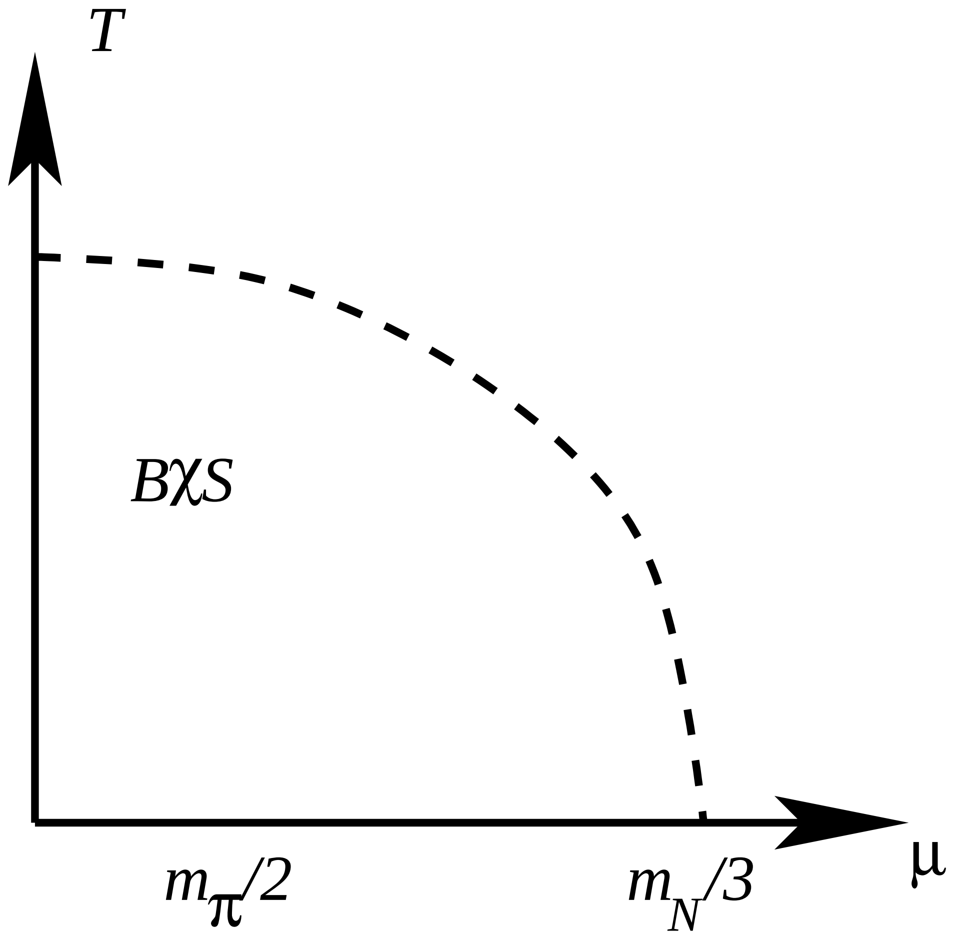
Why Understand numerical QCD at $\mu \neq 0$

How Chiral Perturbation Theory $T < T_c$

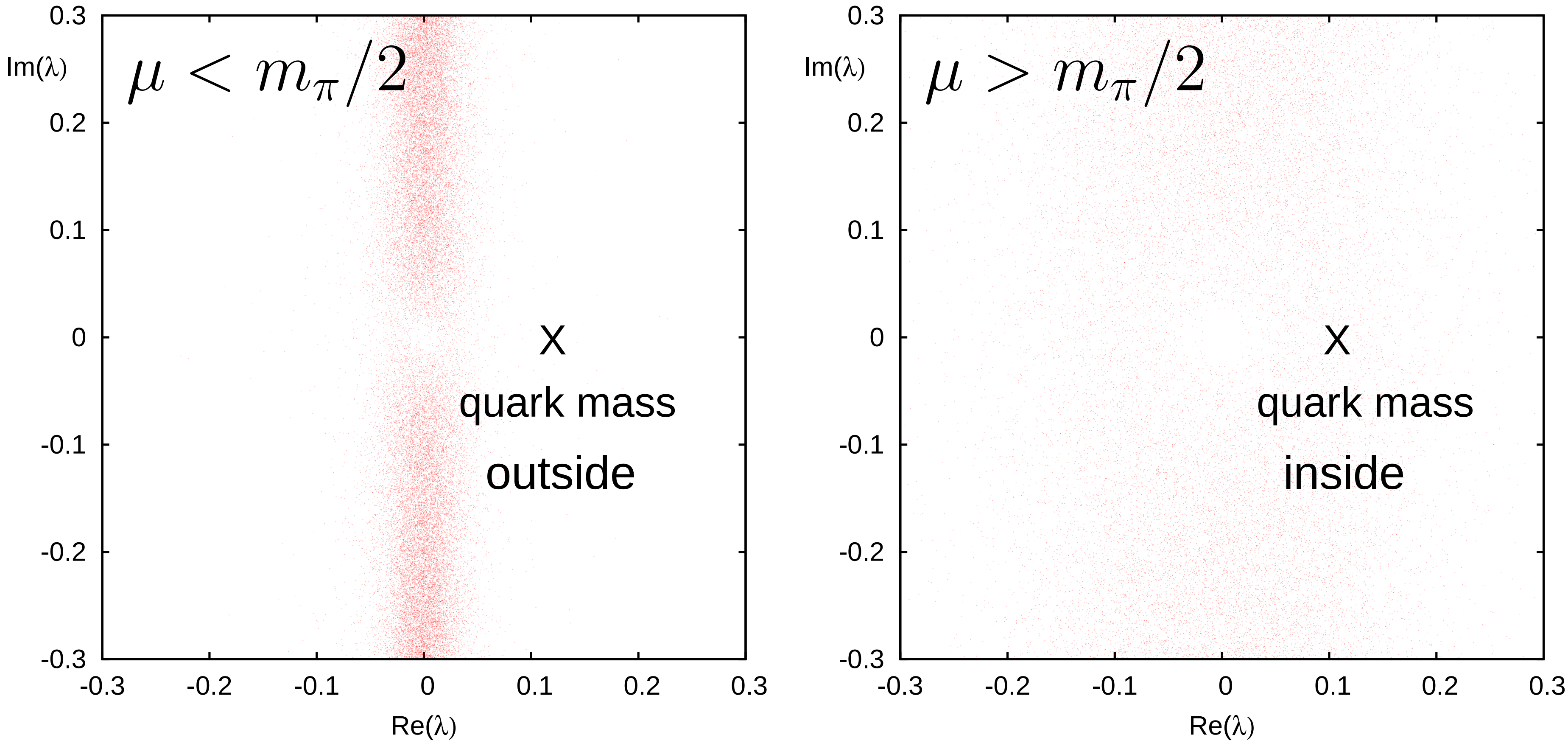
Toussaint NPPS 17 (1990) 248 de Forcrand Laliena PRD 61 (2000) 034502
Sasai Nakamura Takaishi NPPS **129** (2004) 539 Ejiri PRD 73 (2006) 054502
Allton Ejiri Hands Kaczmarek Karsch Laermann Schmidt Scorzato PRD 66 (2002) 074507
Schafer PRD57 (1998) 3950 Fodor Katz Schmidt JHEP 03 (2007) 121

D'Elia Renzo Lombardo arXiv:0705.3814 Conradi D'Elia arXiv:0707.1987

The Big Picture



Quark mass & the eigenvalue distribution

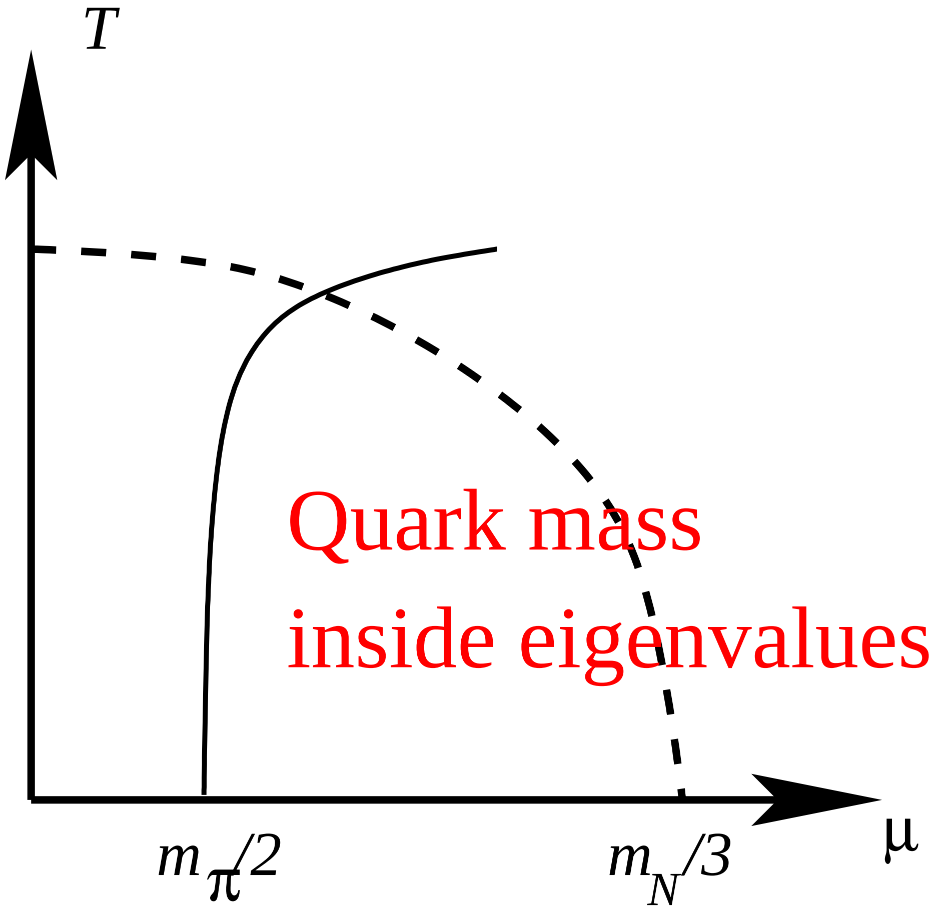


$$(D + \mu\gamma_0)\psi_k = \lambda_k\psi_k$$

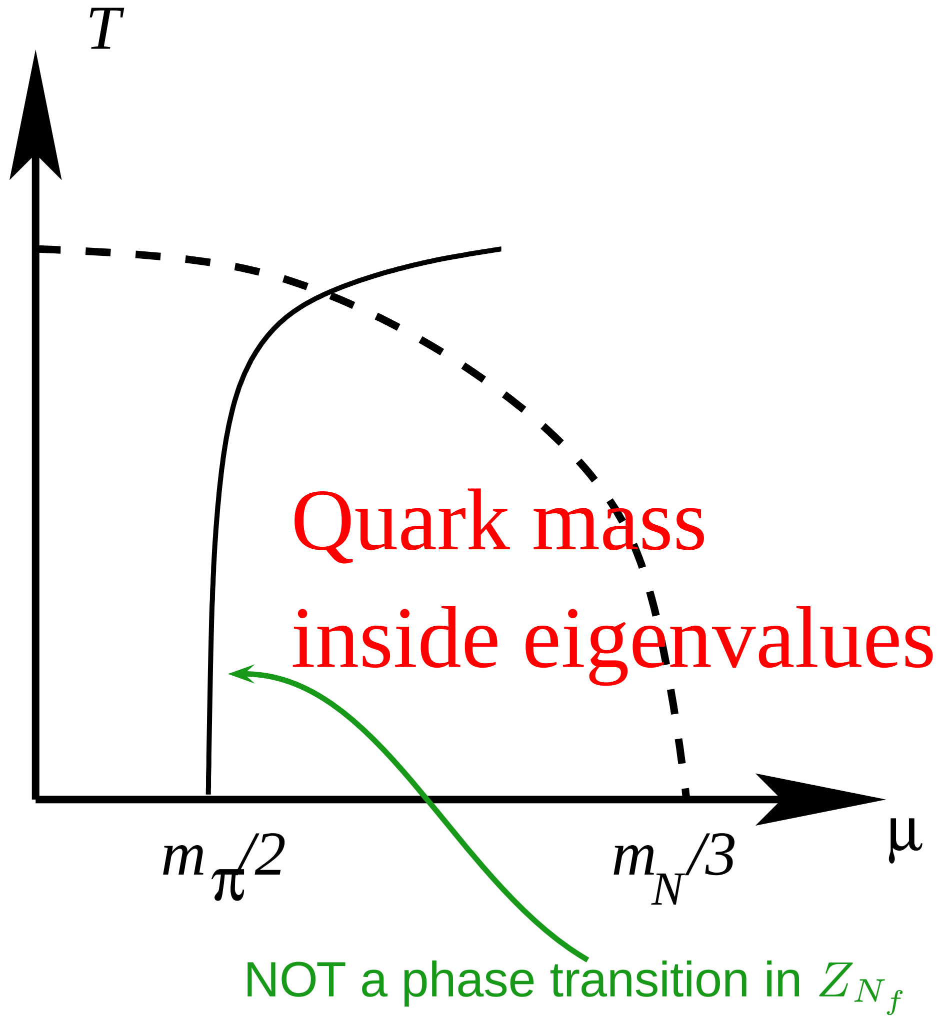
Bloch Wettig Lattice 2006 Gibbs PRINT-86-0389

Davies Klepfish PLB 256 (1991) 68 Lombardo Kogut Sinclair PRD 54 (1996) 2303

The big picture



The big picture



The average phase factor

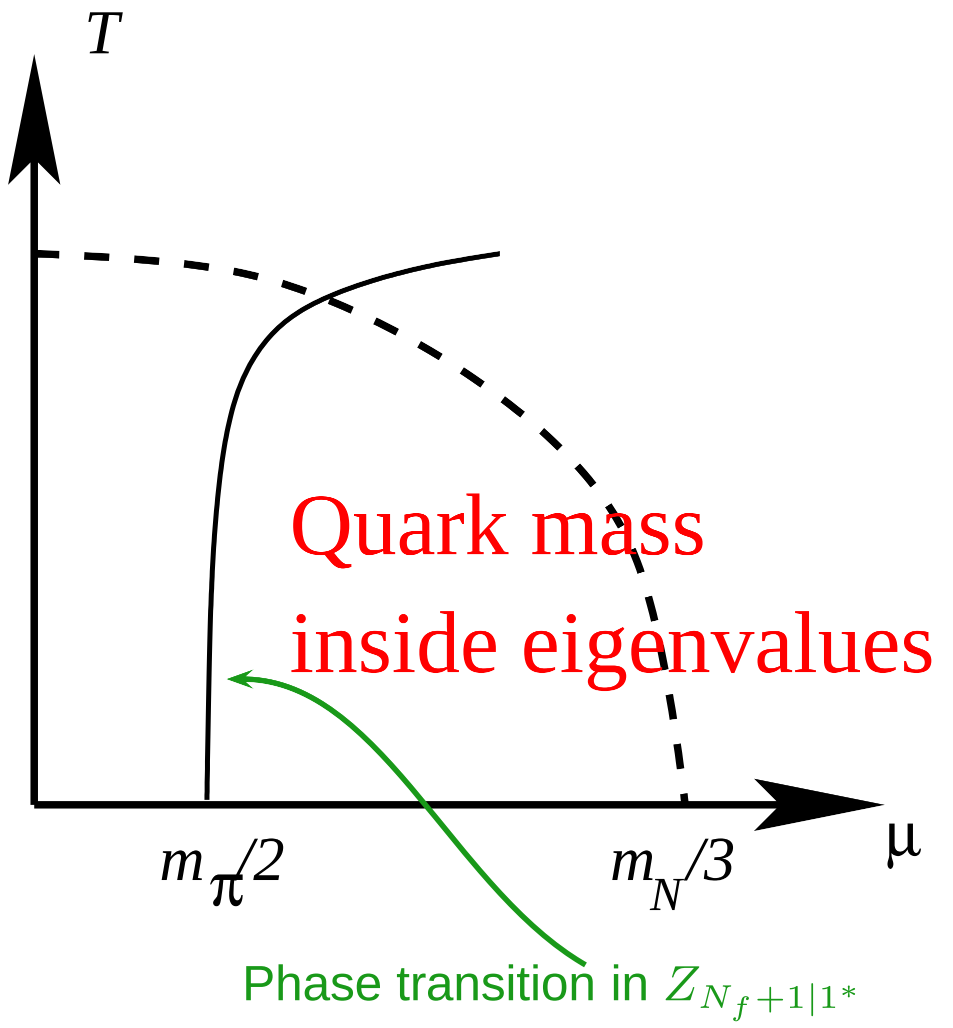
$$\langle e^{2i\theta} \rangle_{N_f} \equiv \left\langle \frac{\det(D + \mu\gamma_0 + m)}{\det(D + \mu\gamma_0 + m)^*} \right\rangle_{N_f}$$

is a ratio of two partition functions

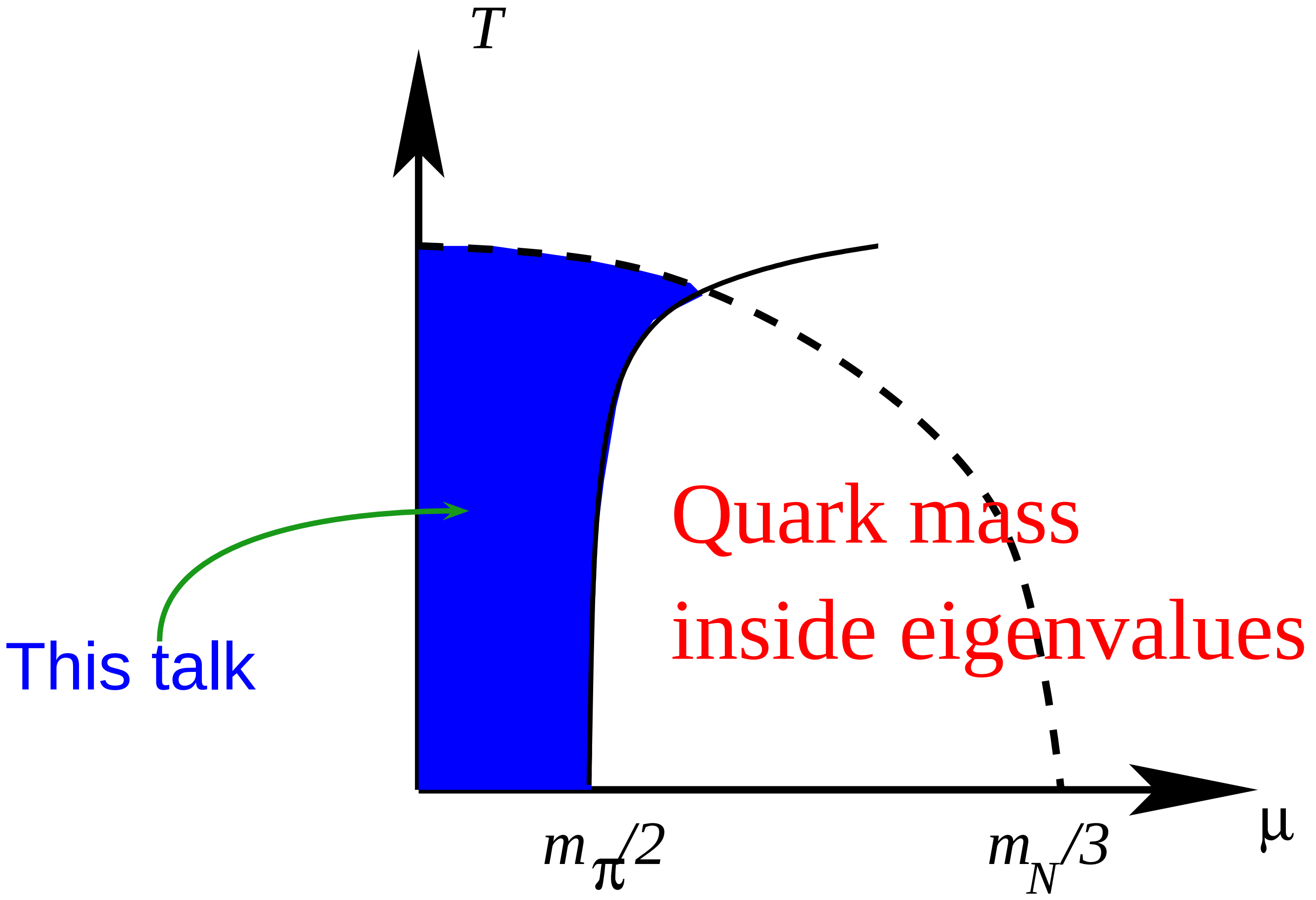
Phase transition at $\mu = m_\pi/2$

$$\langle e^{2i\theta} \rangle_{N_f} = \frac{Z_{N_f+1|1^*}}{Z_{N_f}} = e^{-V\Delta\Omega}$$

The big picture



The big picture



Why the average phase factor can be computed in CPT

In CPT:

Z_{N_f} independent of μ

(ordinary pions have baryon charge 0)

$Z_{N_f+1|1^*}$ depends on μ

(pions with a conjugate quark have baryon charge)

Kogut Stephanov Toublan PLB 464 (1999) 183

Kogut + .. NPB 582 (2000) 477

Hasenfratz Leutwyler NPB 343 (1990) 241

Splittorff Verbaarschot NPB 57 (2006) 259

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$\mu > m_\pi/2$ Bose condensate of charged pions

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Hasenfratz Leutwyler NPB 343 (1990) 241

Splittorff Verbaarschot NPB 57 (2006) 259

The average phase factor in CPT

$$\langle e^{2i\theta} \rangle_{N_f} = \frac{Z_{N_f+1|1^*}}{Z_{N_f}} = e^{-(N_f+1)V\Delta G_0}$$

ΔG_0 is the difference between charged and neutral pions

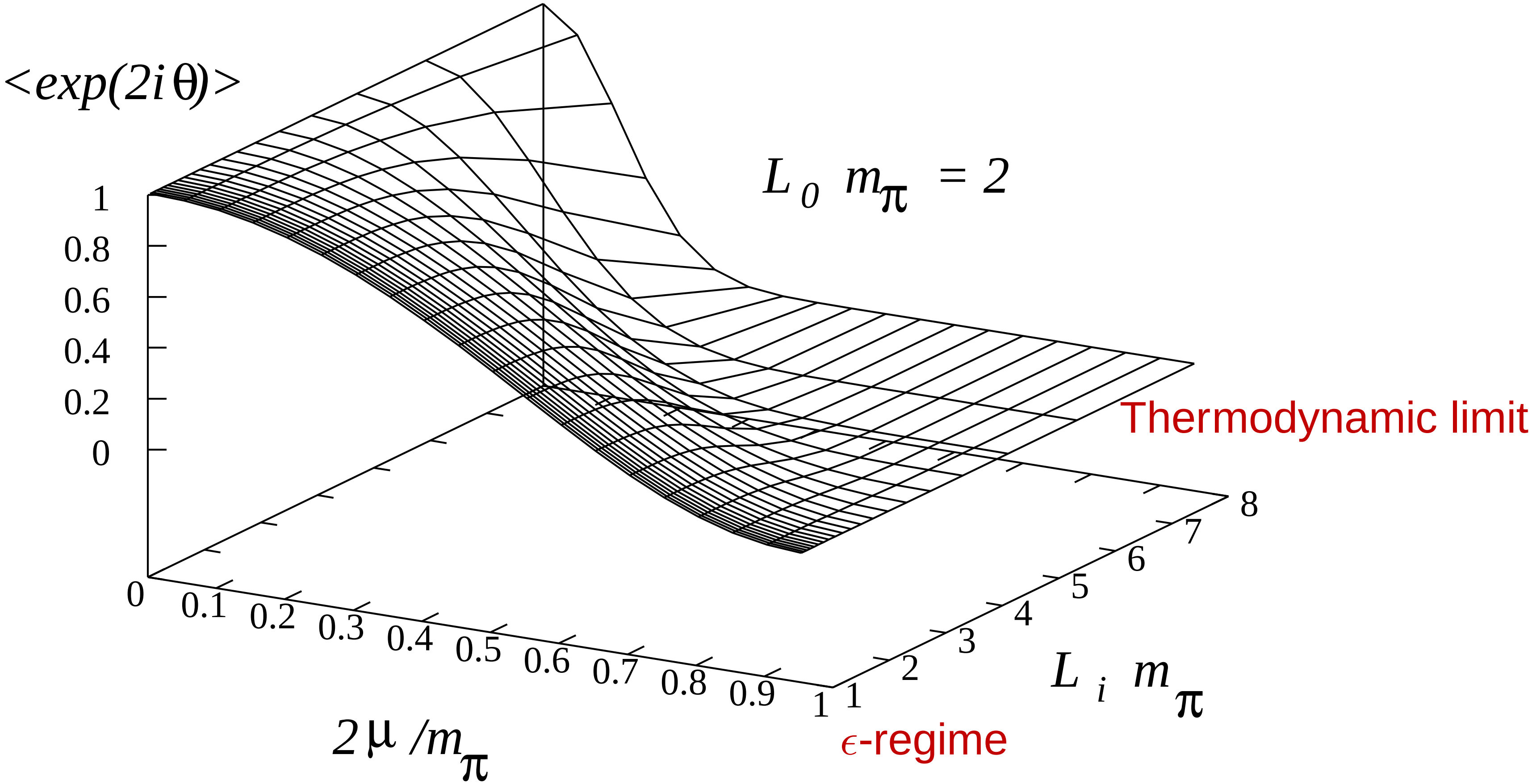
ΔG to 1-loop order in a box with dimensions $V = L_i^3 L_0$

ΔG_0 is independent of the cutoff

Splittorff Verbaarschot PRL 98 (2007) 031601 Splittorff Svetitsky hep-lat/0703004

Average phase factor $N_f = 2$ from One-Loop CPT

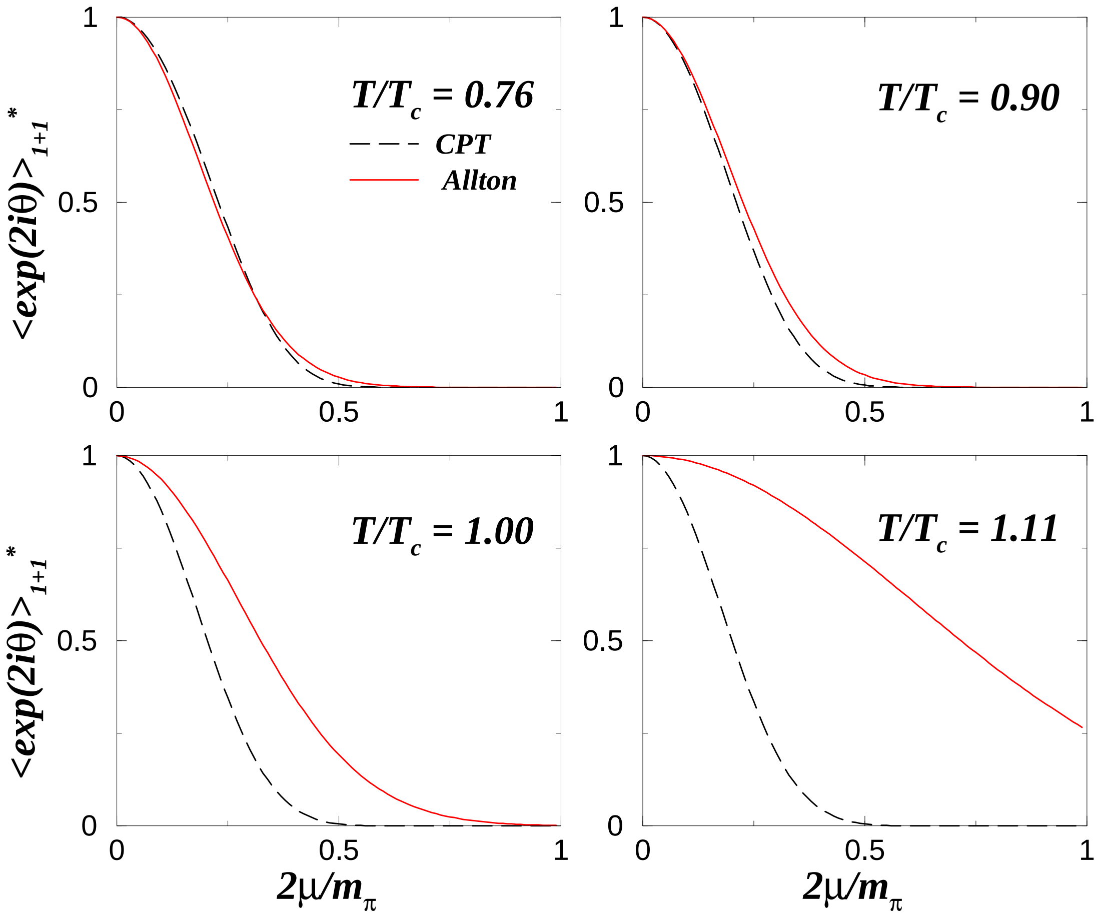
- increasing the volume for fixed temperature



Splittorff Verbaarschot arXiv:hep-lat/0702011

Average phase factor on the lattice

$$\langle e^{2i\theta} \rangle_{1+1^*} = e^{L_i^3 T (c_2 - c_2^I)\mu^2}$$



Allton+... Phys.Rev. D71 (2005) 054508 Splittorff Verbaarschot 2007

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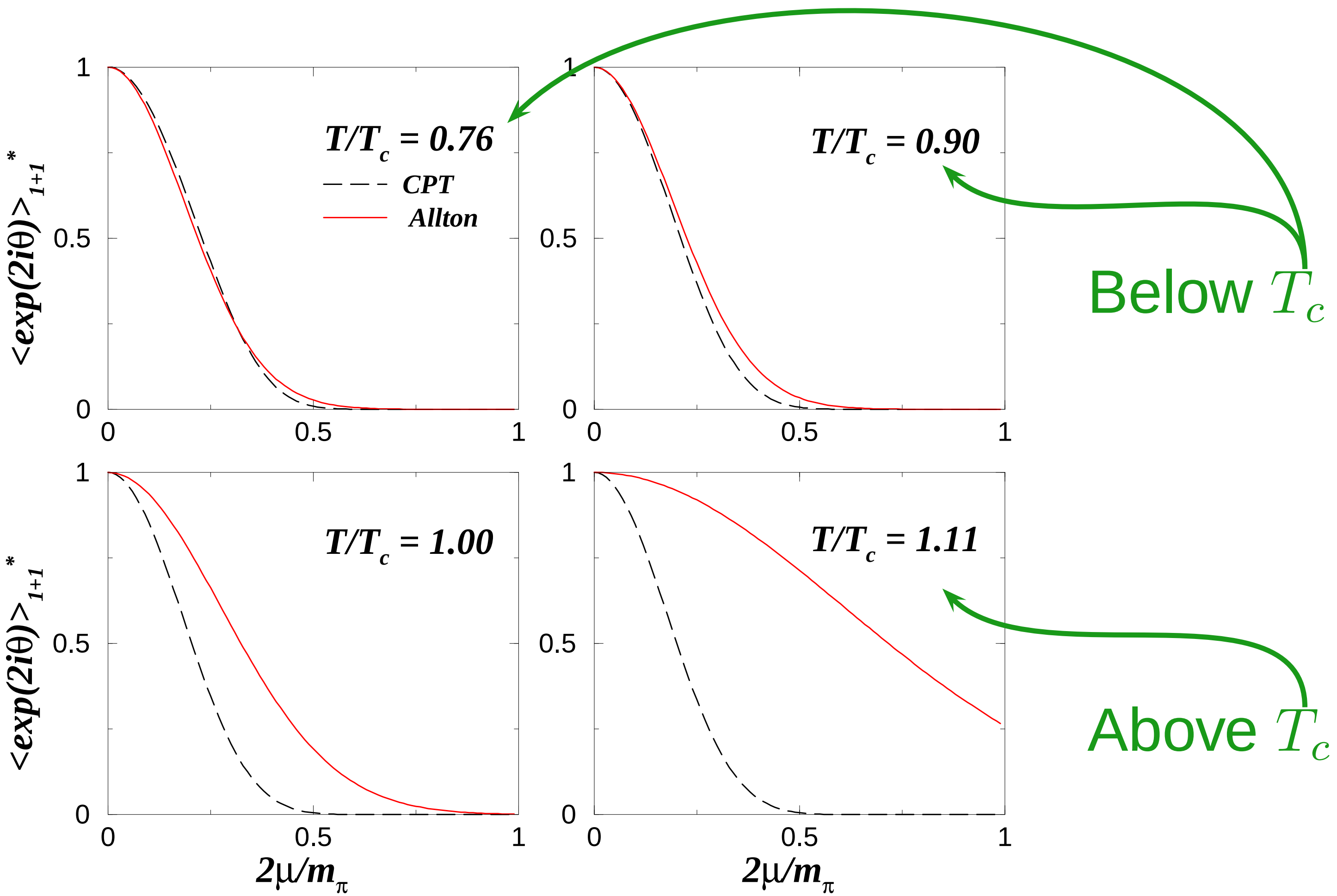

Allton+... Phys.Rev. D71 (2005) 054508

Splittorff Verbaarschot 2007

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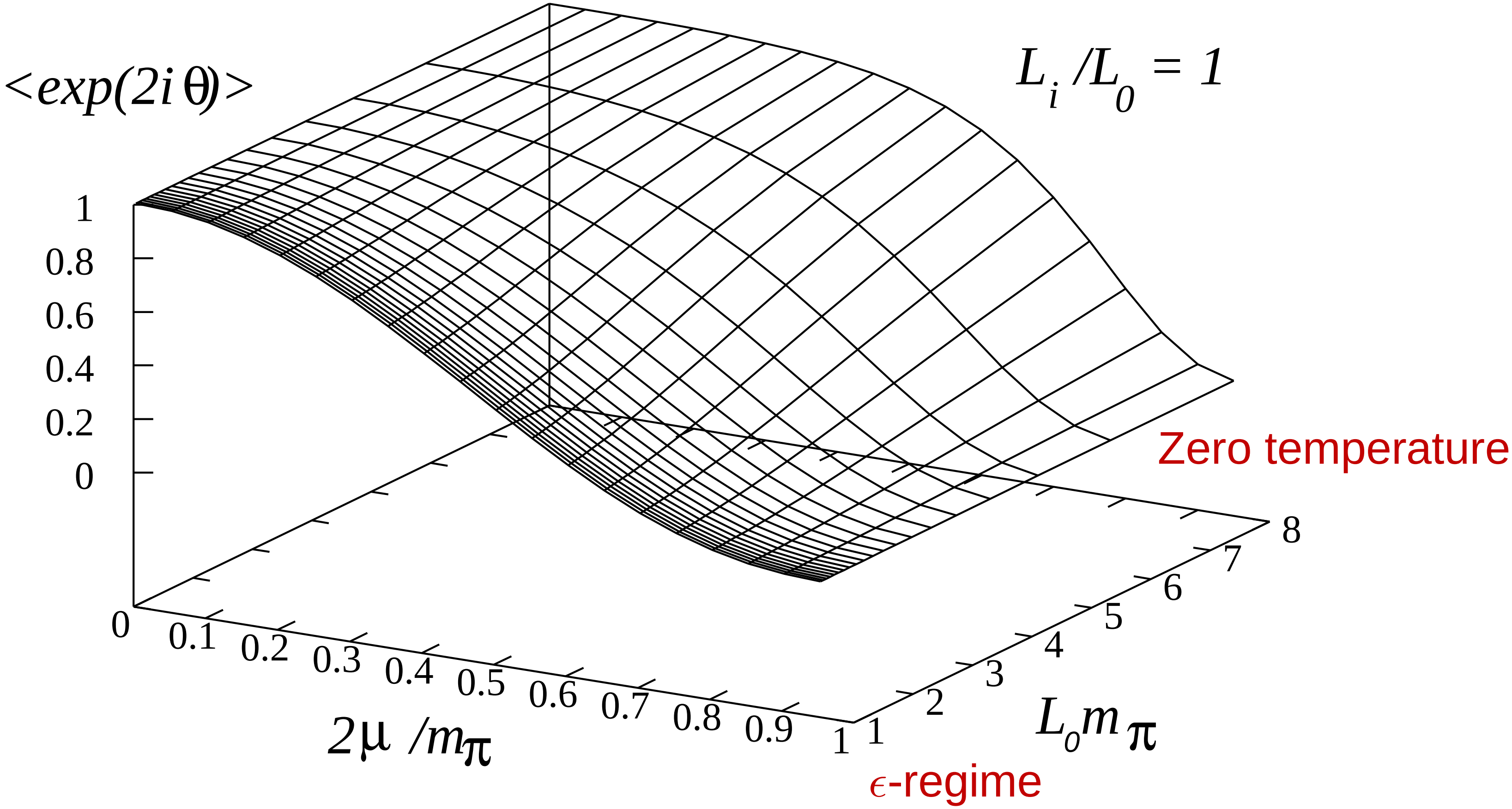
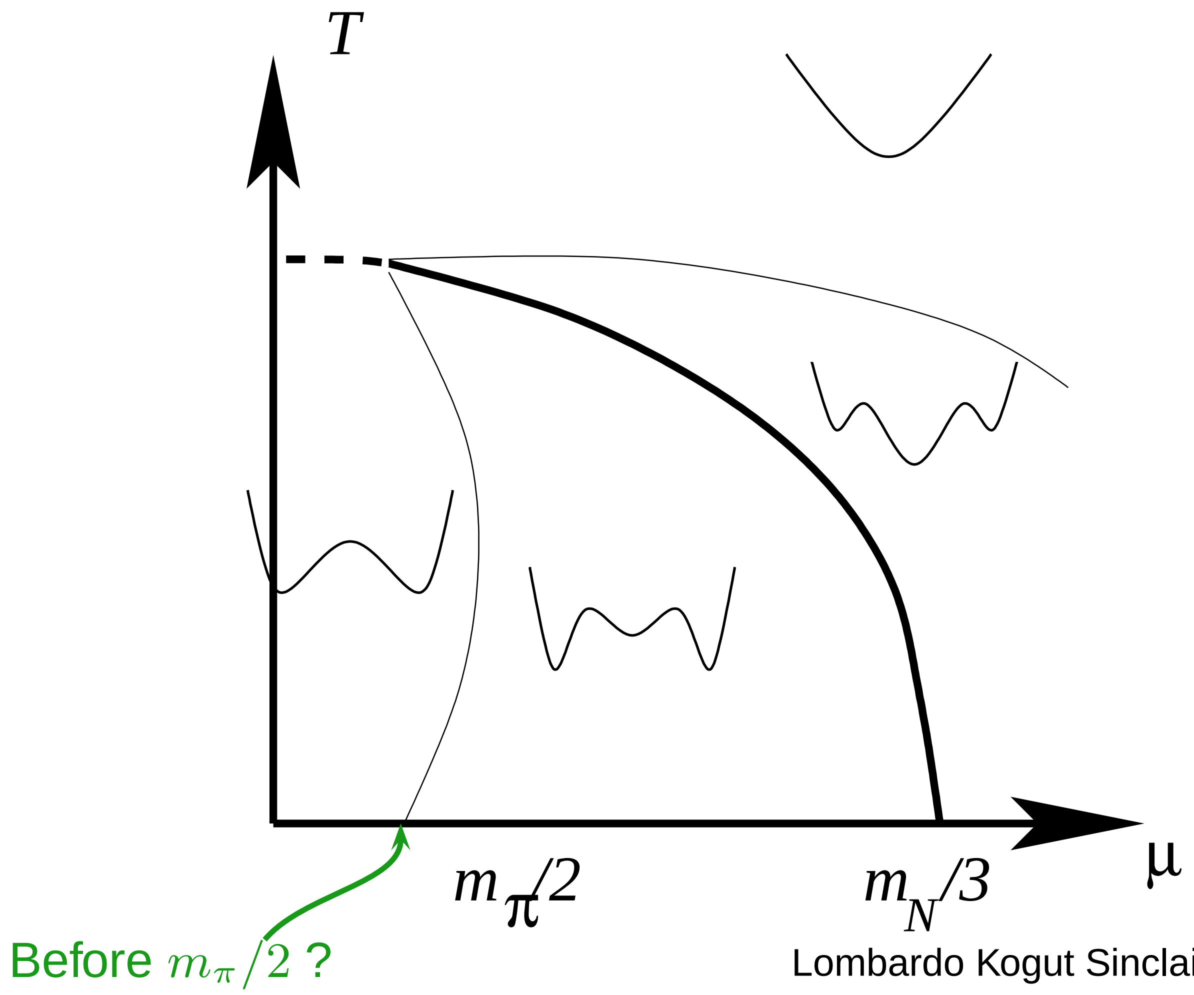


Ravagli Verbaarschot arXiv:0704.1111

No sign problem at zero temperature $\mu < m_\pi/2$ (L_0/L_i fixed)

Suggests: It is possible to look for spinodal on cold lattices ($L_0 >> L_i$)

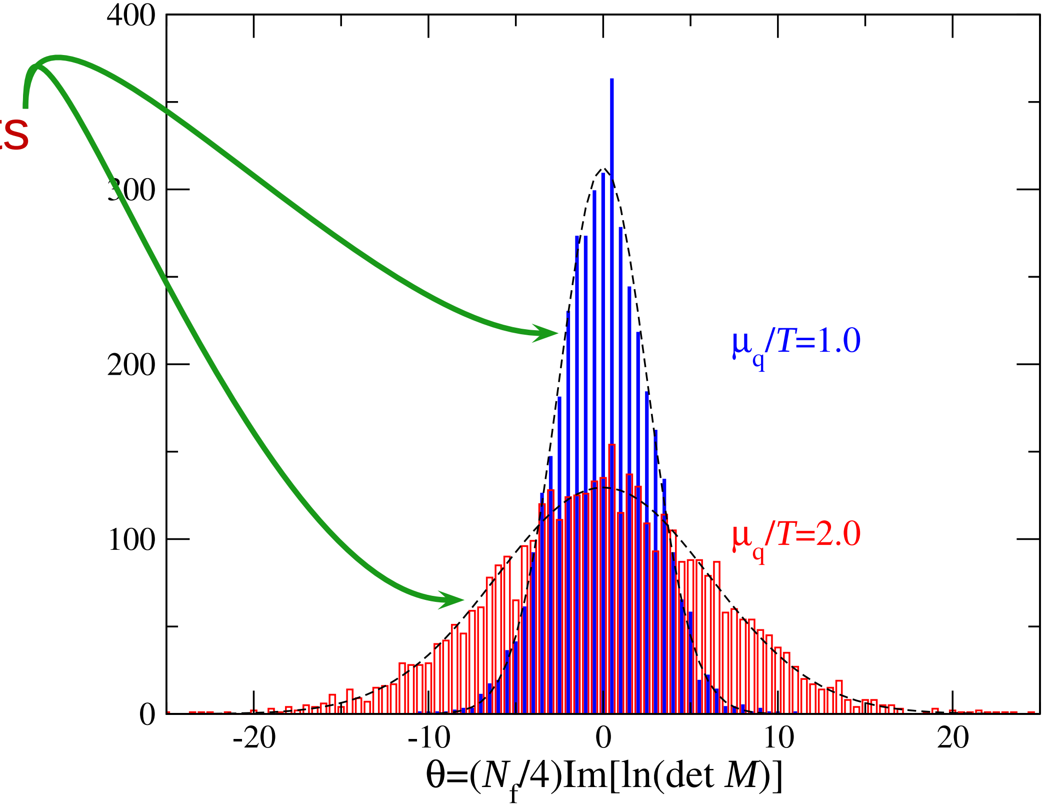


Lombardo Kogut Sinclair PRD 54 (1996) 2303

Kawamoto Miura Ohnishi Ohnuma PRD 75 (2007) 014502

The distribution of the phase

Gaussian fits



Ejiri arXiv:0706.3549

The distribution of the phase

$$\rho_{N_f}(\theta) \equiv \langle \delta(\theta - \theta') \rangle_{N_f}$$

from the definition

$$\rho_{N_f=2}(\theta) = e^{iN_f\theta} \;\; \rho_{1+1^*}(\theta) \;\; \frac{Z_{1+1^*}}{Z_{N_f=2}}$$

Splittorff Verbaarschot 2007 Ejiri arXiv:0706.3549

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1-loop CPT: Envelope is Gaussian width $\sim \sqrt{V}$

$$\rho^{CPT}_{N_f}(\theta) = e^{iN_f\theta} \;\; \frac{1}{\sqrt{V\Delta G_0 \pi}} e^{-\theta^2/(V\Delta G_0)} \;\; e^{(N_f/2)^2 V \Delta G_0}$$

Splittorff Verbaarschot 2007 Ejiri arXiv:0706.3549

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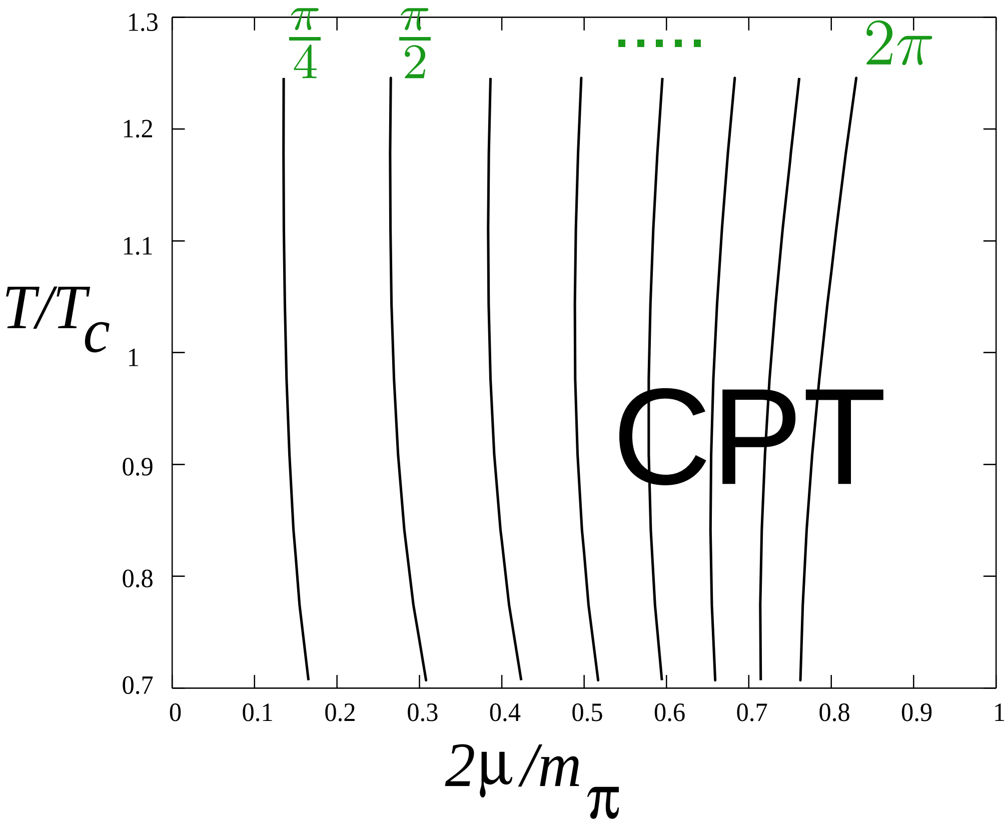
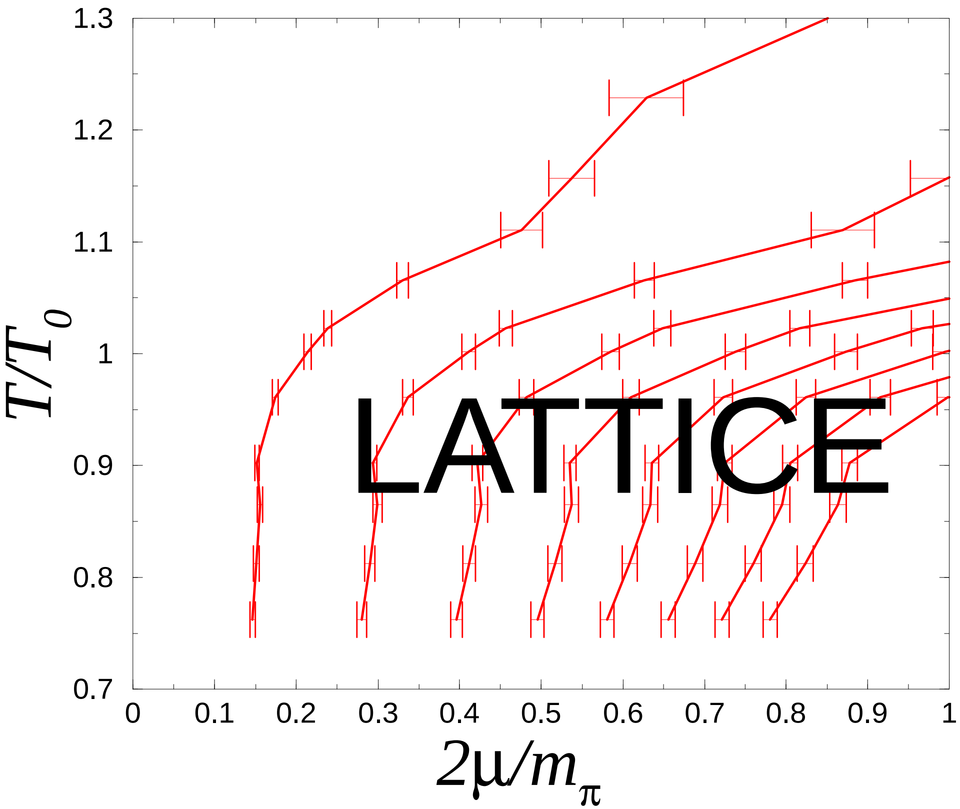
$\sim e^V$

Huge cancellations → small non Gaussian terms big effect.

Splittorff Verbaarschot 2007 Ejiri arXiv:0706.3549

The standard deviation $\sqrt{\langle\theta^2\rangle - \langle\theta\rangle^2}$





Allton+... Phys.Rev. D71 (2005) 054508

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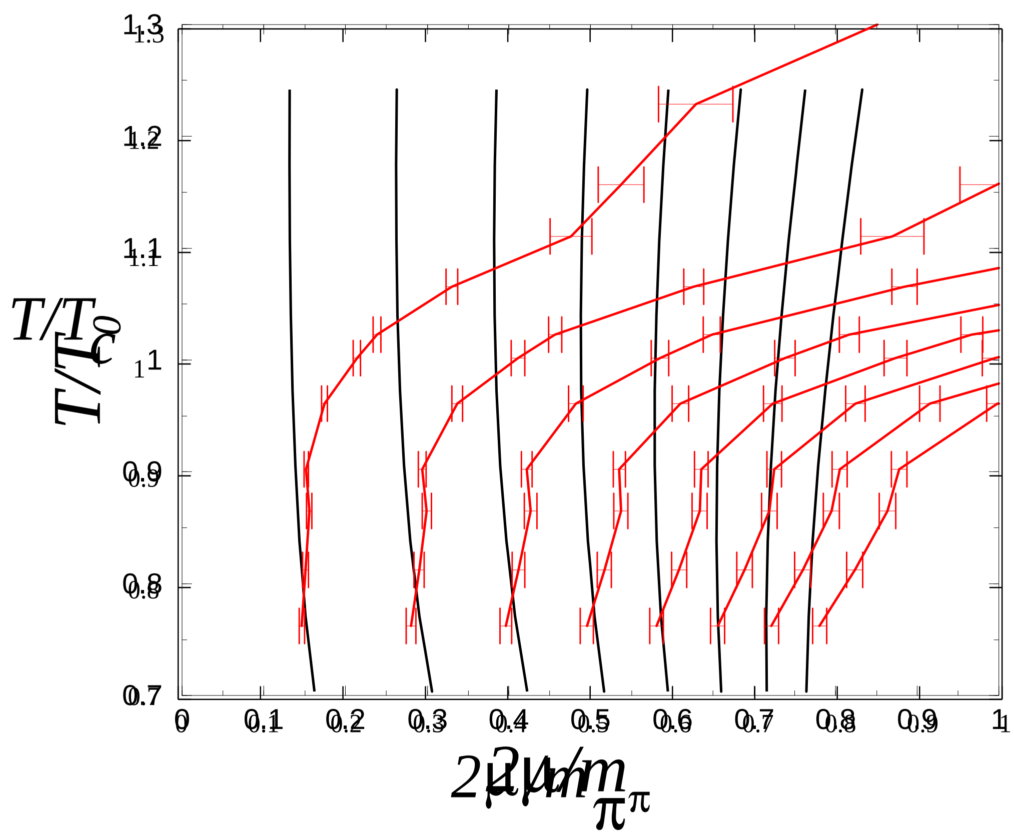

Allton+... Phys.Rev. D71 (2005) 054508

Conclusions

Analytic understanding of the strength of the sign problem

Agreement with lattice data below T_c

$< e^{2i\theta} > \sim 1$ at low T and $\mu < m_\pi/2$ Look for spinodal ?

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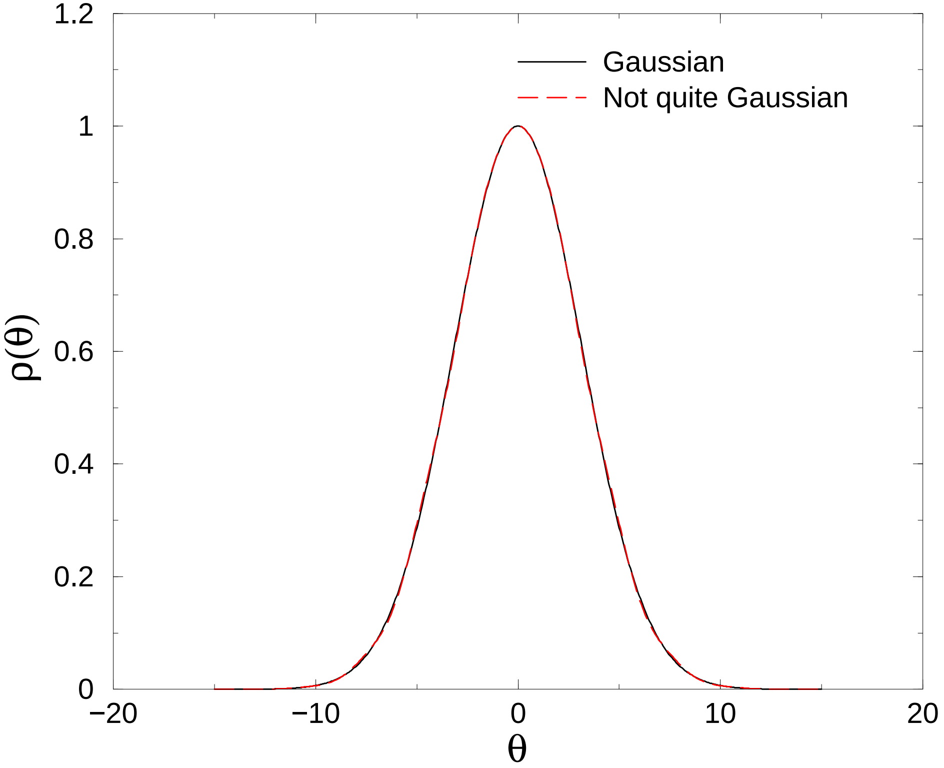
The density of angles is Gaussian at 1-loop WARNING

The standard deviation of the phase consistent with lattice

Huge cancellations $\rightarrow$ small terms big effect.

Example:



$$\frac{\int d\theta \rho_{not-quite-gauss}(\theta) e^{2i\theta}}{\int d\theta \rho_{gauss}(\theta) e^{2i\theta}} \sim 10^6$$