Charge Fluctuations and Correlations at zero and non zero Density

Christian Schmidt
for the RBC-Bielefeld Collaboration

Outline

- Introduction
- Taylor expansion in B, S, Q chemical potentials

  $\longrightarrow$ generalized susceptibilities at $\mu_{B,S,Q} = 0$

- Baryon, strangeness and charge fluctuations at $\mu_B > 0 \quad (\mu_S = \mu_Q = 0)$
- Dens matter with constraints on charges

  $\longrightarrow$ the HIC fireball $(\langle n_S \rangle = 0)$

  $\longrightarrow$ flavor symmetric matter $(\langle n_q \rangle = \langle n_s \rangle)$

- Correlations between charges at $\mu_B > 0$

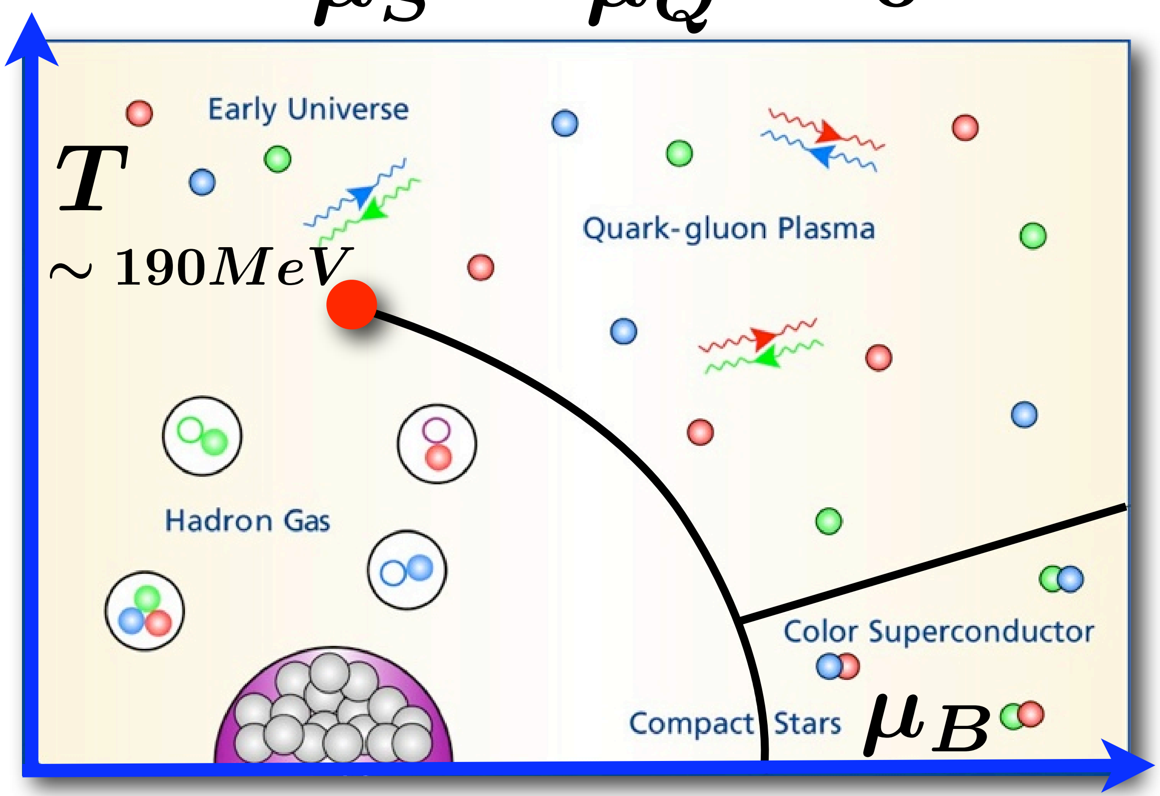
Introduction (1)

- Fluctuations of B, S, Q can be measured experimentally

- LGT at $\mu = 0$
  $\longrightarrow$ RHIC, LHC

- LGT at $\mu > 0$
  $\longrightarrow$ RHIC at low energies, FAIR@GSI

$\mu_S = \mu_Q = 0$



$\sim$ few times nuclear matter density

Introduction (2)

At $\mu = 0$ and $m_{u,d}$ small, we expect
O(4)-critical behavior

- scaling field:

$$t = \left|\frac{T-T_c}{T_c}\right| + A\left(\left(\frac{\mu_u}{T_c}\right)^2 + \left(\frac{\mu_d}{T_c}\right)^2\right) + B\frac{\mu_u}{T_c}\frac{\mu_d}{T_c}$$

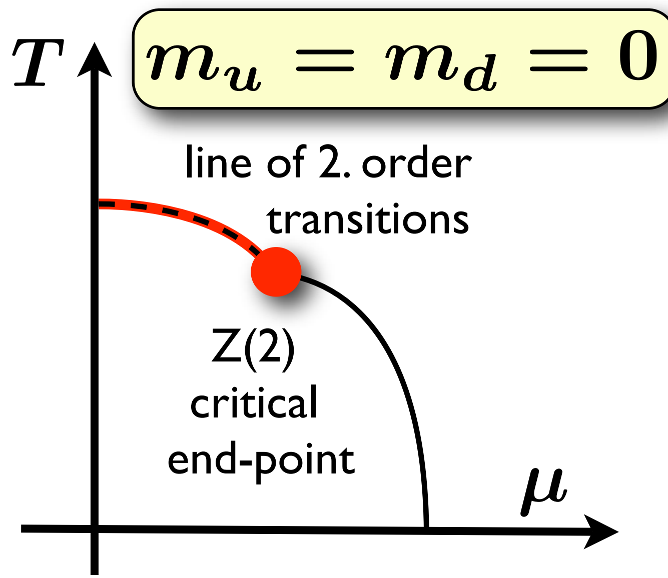
- singular part:

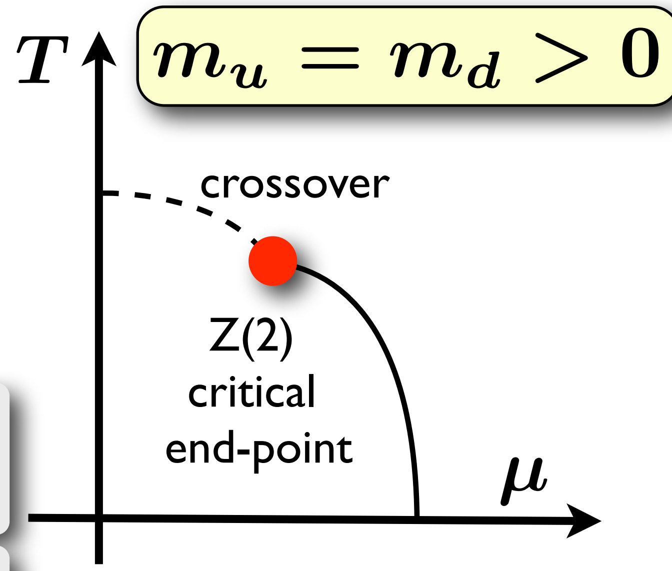
$$f_s(T,\mu_u,\mu_d) = b^{-1} f_s(t b^{1/(2-\alpha)}) \sim t^{(2-\alpha)}$$

- O(4)/O(2): $\alpha < 0$, small

$\left\langle (\delta n_{u,d})^2 \right\rangle \longrightarrow$ dominated by T dependence of regular part

$\left\langle (\delta n_{u,d})^4 \right\rangle \longrightarrow$ develops a cusp!





Taylor expansion in $\mu_{B,S,Q}$

QCD is naturally formulated with quark chemical potentials $\mu_{u,d,s}$

- we set $\mu_u \equiv \mu_d$ and start from Taylor expansion of the pressure

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_q,\mu_s) = \sum_{i,j} c_{i,j}^{q,s} \left(\frac{\mu_q}{T}\right)^i \left(\frac{\mu_s}{T}\right)^j$$

- use unbiased, noisy estimators to calculate $c_{i,j}^{q,s}$
  $\longrightarrow$ see Talk by C. Miao (Lattice 2007)

- Line of constant physics: $m_q = m_s/10$
  (physical strange quark mass)

- measure currently up to $\mathcal{O}(\mu^8) \quad \longleftrightarrow \quad (N_t = 4)$
  $\mathcal{O}(\mu^4) \quad \longleftrightarrow \quad (N_t = 6)$

Taylor expansion in general

$$c_2 = \frac{N_\tau}{2N_\sigma^3}\left(\frac{n_f}{4}\left\langle\frac{\partial^2 \ln\det M}{\partial\mu^2}\right\rangle + \left(\frac{n_f}{4}\right)^2\left\langle\left(\frac{\partial \ln\det M}{\partial\mu}\right)^2\right\rangle\right)$$

$$\begin{aligned} c_4 = \frac{1}{4!N_\sigma^3 N_\tau}\Bigg\{ & \frac{n_f}{4}\left\langle\frac{\partial^4 \ln\det M}{\partial\mu^4}\right\rangle \\ & +4\left(\frac{n_f}{4}\right)^2\left\langle\frac{\partial^3 \ln\det M}{\partial\mu^3}\frac{\partial \ln\det M}{\partial\mu}\right\rangle + 3\left(\frac{n_f}{4}\right)^2\left\langle\left(\frac{\partial^2 \ln\det M}{\partial\mu^2}\right)^2\right\rangle \\ & +6\left(\frac{n_f}{4}\right)^3\left\langle\frac{\partial^2 \ln\det M}{\partial\mu^2}\left(\frac{\partial \ln\det M}{\partial\mu}\right)^2\right\rangle + \left(\frac{n_f}{4}\right)^4\left\langle\left(\frac{\partial \ln\det M}{\partial\mu}\right)^4\right\rangle \\ & -3\left(\frac{n_f}{4}\left\langle\frac{\partial^2 \ln\det M}{\partial\mu^2}\right\rangle + \left(\frac{n_f}{4}\right)^2\left\langle\left(\frac{\partial \ln\det M}{\partial\mu}\right)^2\right\rangle\right)^2\Bigg\} \end{aligned}$$

$\longrightarrow$ non-linear in number of flavors

Taylor expansion in general

$$\frac{\partial \ln \det M}{\partial \mu} = \mathrm{Tr}\left(M^{-1} \frac{\partial M}{\partial \mu} \right)$$

$$\frac{\partial^2 \ln \det M}{\partial \mu^2} = \mathrm{Tr}\left(M^{-1} \frac{\partial^2 M}{\partial \mu^2} \right) - \mathrm{Tr}\left(M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} \right)$$

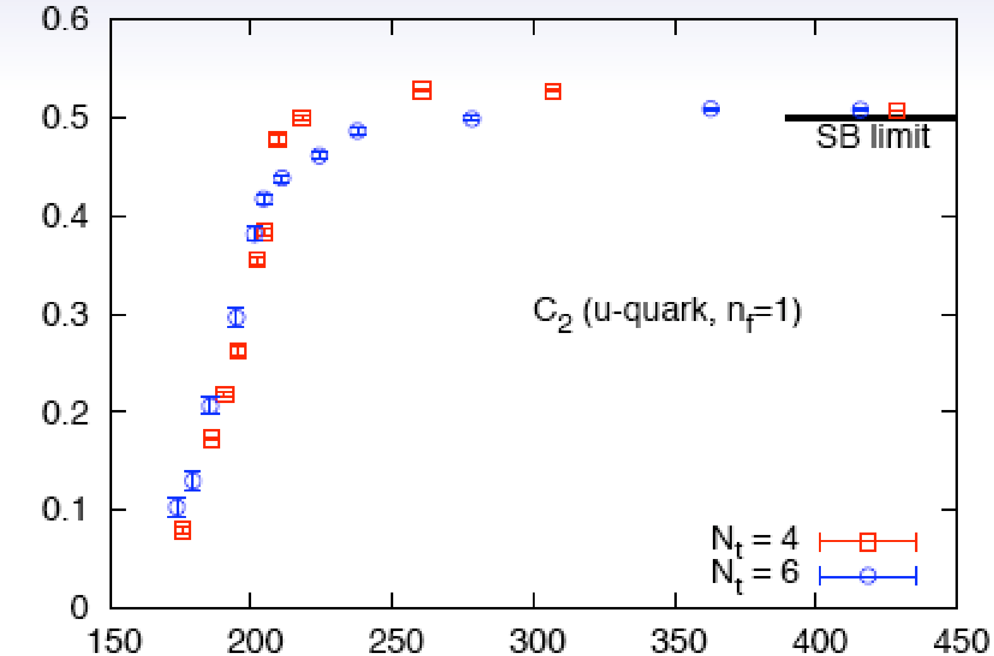
$$\frac{\partial^3 \ln \det M}{\partial \mu^3} = \mathrm{Tr}\left(M^{-1} \frac{\partial^3 M}{\partial \mu^3} \right) - 3\mathrm{Tr}\left(M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^2 M}{\partial \mu^2} \right) + 2\mathrm{Tr}\left(M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} \right)$$

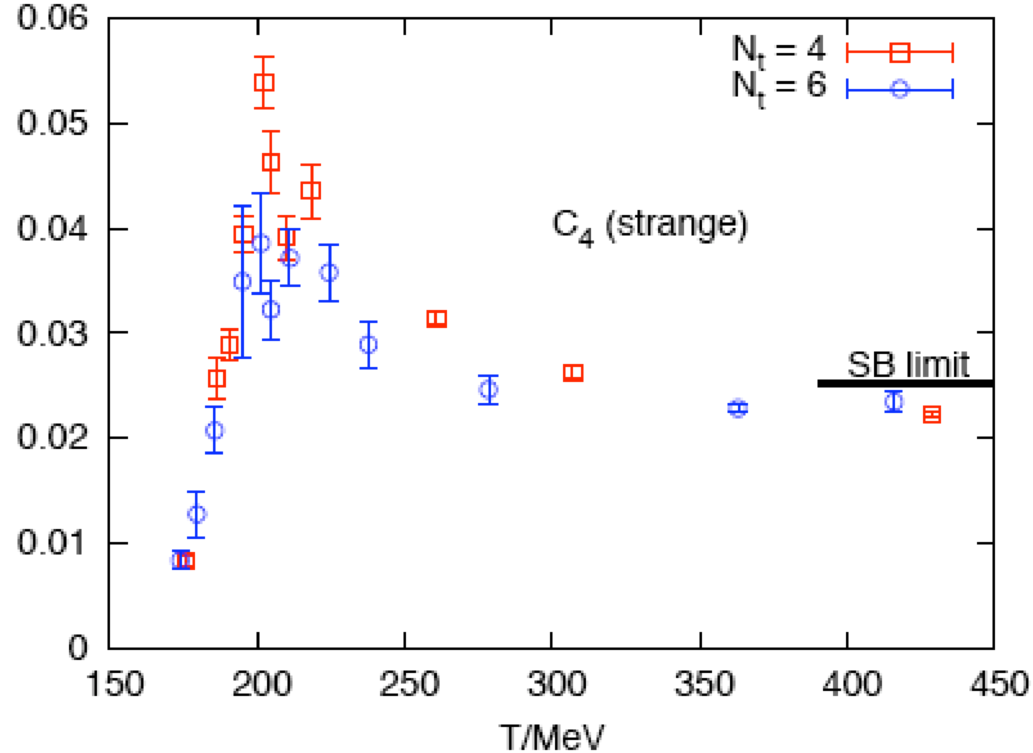
$$\frac{\partial^4 \ln \det M}{\partial \mu^4} = \mathrm{Tr}\left(M^{-1} \frac{\partial^4 M}{\partial \mu^4} \right) - 4\mathrm{Tr}\left(M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^3 M}{\partial \mu^3} \right) - 3\mathrm{Tr}\left(M^{-1} \frac{\partial^2 M}{\partial \mu^2} M^{-1} \frac{\partial^2 M}{\partial \mu^2} \right)$$
$$+12\mathrm{Tr}\left(M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^2 M}{\partial \mu^2} \right) - 6\mathrm{Tr}\left(M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} \right)$$

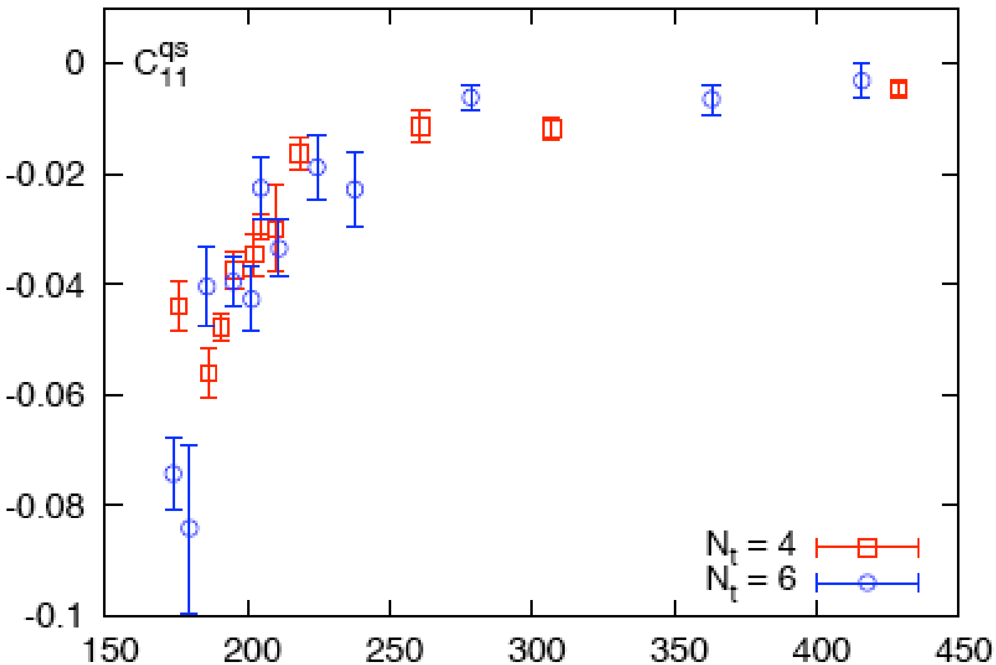
$\longrightarrow$ computational effort:

Order	Operators	Inversions
4	11	5
6	29	12
8	95	35

Results for expansion coefficients $c^{qs}_{i,j}$







$\longrightarrow$ small cut-off effects

Taylor expansion in $\mu_{B,S,Q}$

QCD is naturally formulated with quark chemical potentials $\mu_{u,d,s}$

- we set $\mu_u \equiv \mu_d$ and start from Taylor expansion of the pressure

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_q,\mu_s) = \sum_{i,j} c_{i,j}^{q,s} \left(\frac{\mu_q}{T}\right)^i \left(\frac{\mu_s}{T}\right)^j$$

- expansion coefficients $c_{i,j}^{q,s}$ are related to B,S,Q-fluctuations

$$n_B = \frac{\partial(p/T^4)}{\partial(\mu_B/T)} = \frac{1}{3}(n_u + n_d + n_s)$$

$$n_S = \frac{\partial(p/T^4)}{\partial(\mu_S/T)} = -n_s$$

$$n_Q = \frac{\partial(p/T^4)}{\partial(\mu_Q/T)} = \frac{1}{3}(2n_u - n_d - n_s)$$

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q$$

$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q$$

$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$$

- choice of $\mu_u \equiv \mu_d$ is equivalent to $\mu_Q \equiv 0$

Hadronic fluctuations $(\mu_B = 0)$

In general we have:

- $\frac{\partial^p (p/T^4)}{\partial(\mu_X/T)^p}$ related to $< n_X^p >_{\mu=0}$
- $\frac{\partial^{p+q} (p/T^4)}{\partial(\mu_X/T)\partial(\mu_Y/T)^q}$ related to $< n_X^p n_Y^q >_{\mu=0}$

to be more precise:

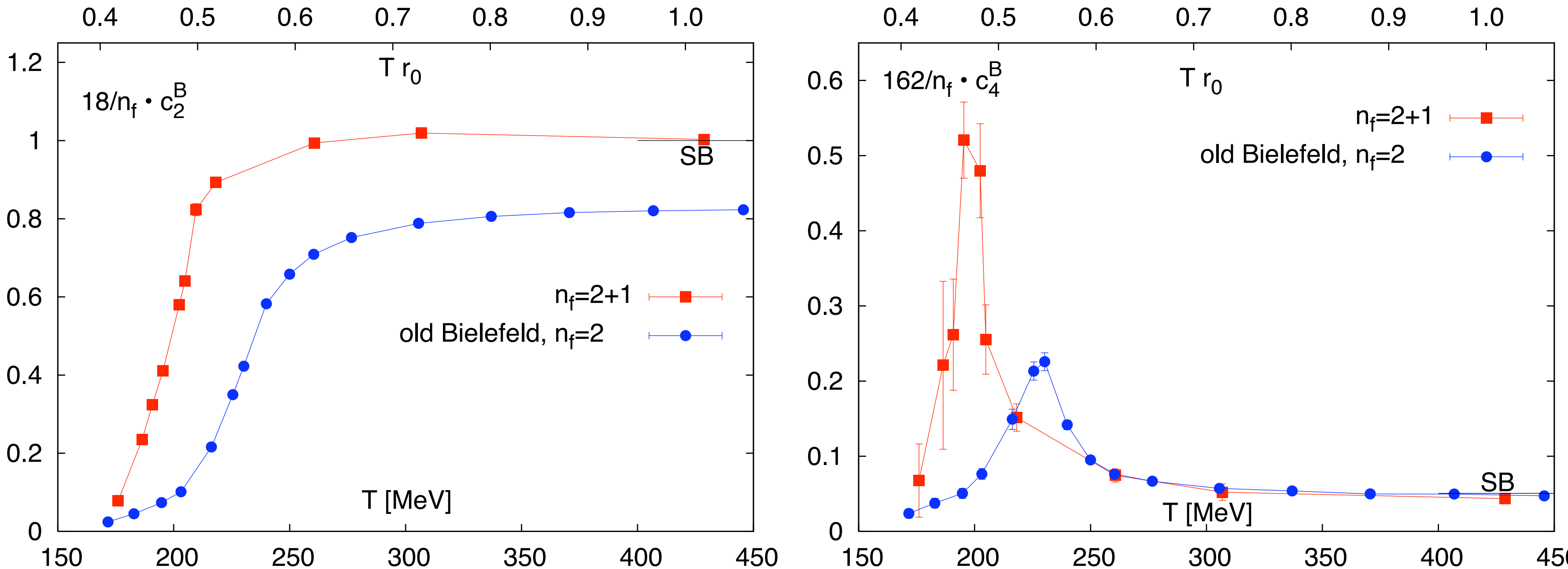
$$2c_2^X = \frac{\partial^2 (p/T^4)}{\partial(\mu_X/T)^2} = \frac{1}{VT^3}\left\langle(\delta N_X)^2\right\rangle = \frac{1}{VT}\left\langle N_X^2\right\rangle$$

$$24c_4^X = \frac{\partial^4 (p/T^4)}{\partial(\mu_X/T)^4} = \frac{1}{VT^3}\left(\left\langle(\delta N_X)^4\right\rangle - 3\left\langle(\delta N_X^2)\right\rangle^2\right)_{\mu=0}$$

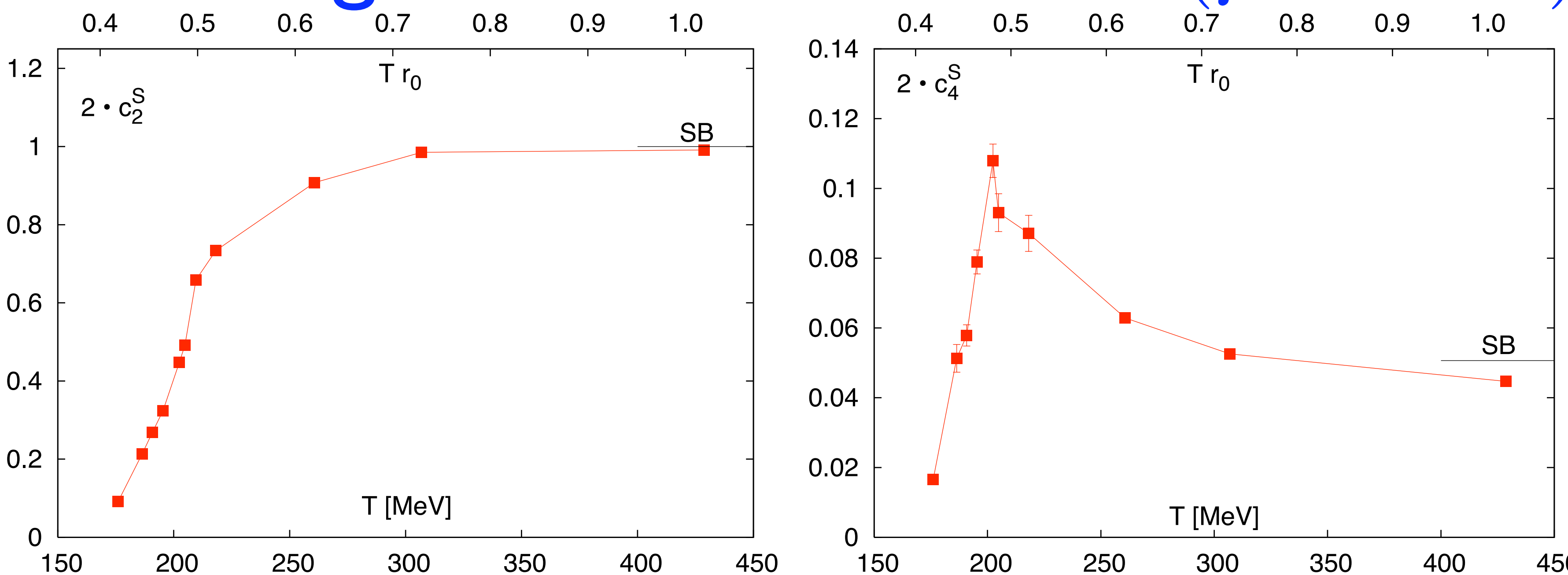
$$= \frac{1}{VT^3}\left(\left\langle N_X^4\right\rangle - 3\left\langle N_X^2\right\rangle^2\right)_{\mu=0}$$

$$c_{11}^{XY} = \frac{\partial^2 (p/T^4)}{\partial(\mu_X/T)\partial(\mu_Y/T)} = \frac{1}{VT^3}\left(\langle N_X N_Y\rangle - \langle N_X\rangle\langle N_Y\rangle\right)_{\mu=0}$$

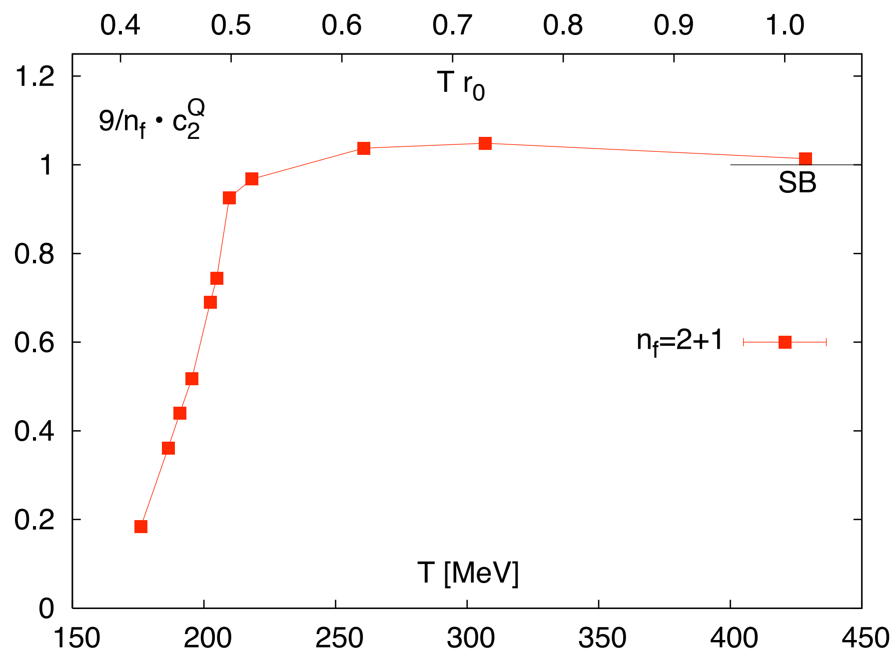
Baryon number fluctuations $(\mu_B = 0)$

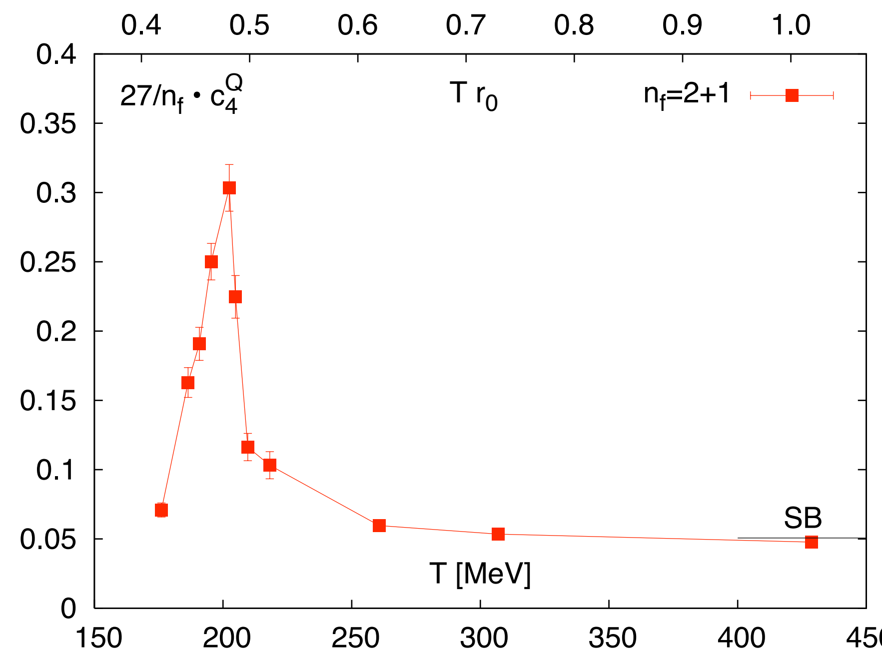


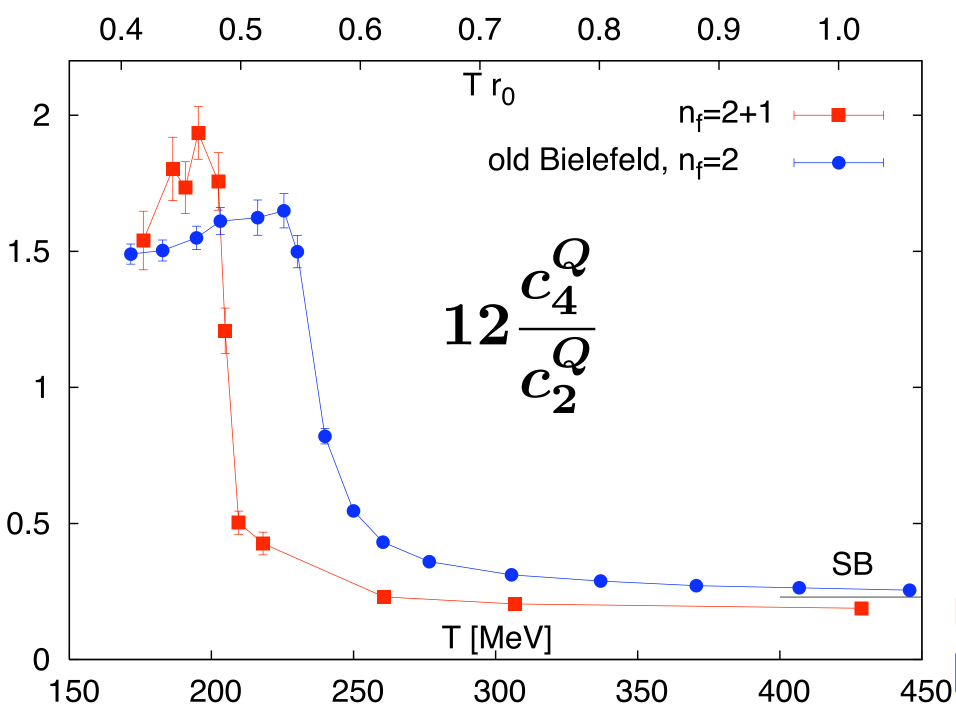
Strangeness fluctuations $(\mu_B = 0)$



Electric charge fluctuations $(\mu_B = 0)$







- Ratio is sensitive to charge quanta in the QGP

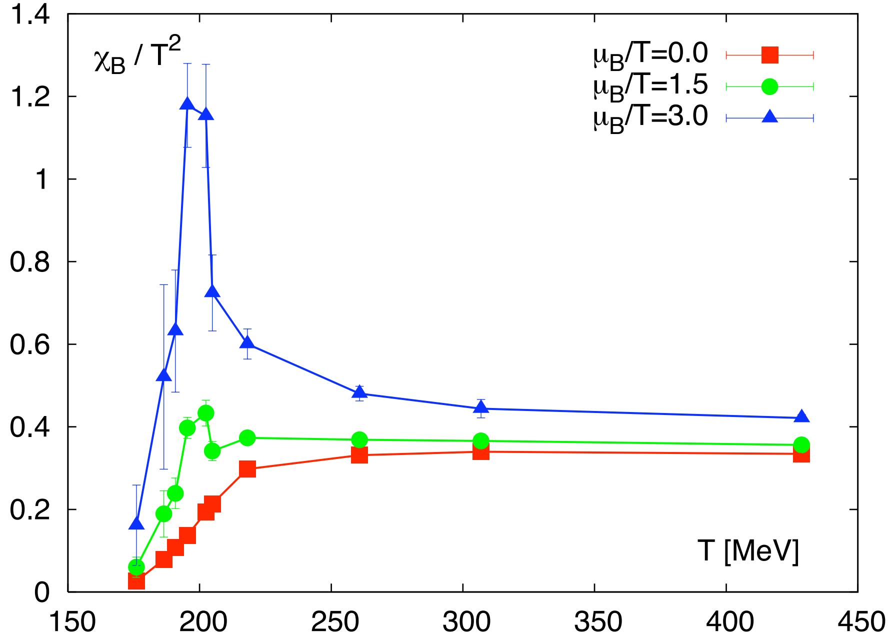
→ consistent with a gas of quasi-free quarks and gluons.

red: RBC-Bielefeld, 2+1 flavor, preliminary

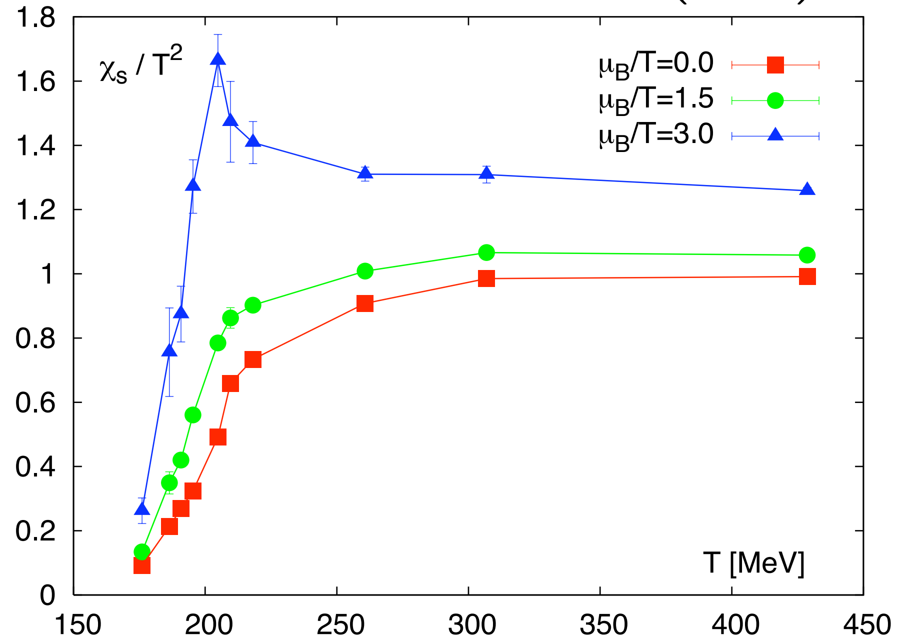
blue: Ejiri, Karsch, Redlich, PLB 633 (2006) 275

Baryon number and strangeness fluctuations at $\mu_B > 0 \quad (\mu_S = 0)$

$$\frac{\chi_B}{T^2} = 2c_2^B + 12c_4^B \left(\frac{\mu_B}{T}\right)^2$$



$$\frac{\chi_S}{T^2} = 2c_2^B + 2c_{22}^{BS} \left(\frac{\mu_B}{T}\right)^2$$



$\longrightarrow$ large baryon number fluctuations

$\longrightarrow$ enhanced strangeness fluctuations (~ factor 3 at Tc)

The HIC case: $\langle N_s \rangle \equiv 0$

- Solving order by order:

$$\begin{aligned} N_s(\mu_q,\mu_q) &= \sum_{i,j}(j+1)c^{qs}_{i(j+1)}\mu_q^i\mu_s^j \\ &= c^{qs}_{11}\mu_q + 2c^{qs}_{02}\mu_s \\ &+ c^{qs}_{31}\mu_q^3 + 2c^{qs}_{22}\mu_q^2\mu_s + 3c^{qs}_{13}\mu_q\mu_s^2 + 4c^{qs}_{04}\mu_s^3 \\ &+ \mathcal{O}\left(\mu^5\right) \\ &\equiv 0 \end{aligned}$$

- Solution:

$$\mu_s(\mu_q) = \sum_i d_i\mu_q^i = \left(-\frac{c^{qs}_{11}}{2c^{qs}_{02}}\right)\mu_q + \left(\frac{2c^{qs}_{04}{c^{qs}_{11}}^3 - 3c^{qs}_{02}{c^{qs}_{11}}^2c^{qs}_{13} + 4{c^{qs}_{02}}^2c^{qs}_{11}c^{qs}_{22} - 4{c^{qs}_{02}}^3c^{qs}_{31}}{8{c^{qs}_{02}}^4}\right)\mu_q^3 + \mathcal{O}\left(\mu_q^5\right)$$

$\longrightarrow$ new expansion Coefficients: $\hat{c}^q_i$

$$\begin{aligned} \frac{\Delta p}{T^4} &= \sum_i \hat{c}^q_i\mu_q^i \equiv \sum_{i,j}c_{ij}\mu^i\left(\sum_k d_k\mu_q^k\right)^j \\ &= \left(c^{qs}_{20} - \frac{{c^{qs}_{11}}^2}{4c^{qs}_{02}}\right)\mu_q^2 + \left(c^{qs}_{40} + \frac{c^{qs}_{04}{c^{qs}_{11}}^4}{16{c^{qs}_{02}}^4} - \frac{{c^{qs}_{11}}^3c^{qs}_{13}}{8{c^{qs}_{02}}^3} + \frac{{c^{qs}_{11}}^2c^{qs}_{22}}{4{c^{qs}_{02}}^2} - \frac{c^{qs}_{11}c^{qs}_{31}}{2c^{qs}_{02}}\right)\mu_q^4 + \mathcal{O}(\mu_q^6) \end{aligned}$$

The HIC case: $\langle N_s \rangle \equiv 0$

- Solving order by order:

$$\begin{aligned} N_s(\mu_q,\mu_q) &= \sum_{i,j} (j+1) c^{qs}_{i(j+1)} \mu_q^i \mu_s^j \\ &= c^{qs}_{11}\mu_q + 2c^{qs}_{02}\mu_s \\ &+ c^{qs}_{31}\mu_q^3 + 2c^{qs}_{22}\mu_q^2\mu_s + 3c^{qs}_{13}\mu_q\mu_s^2 + 4c^{qs}_{04}\mu_s^3 \\ &+ \mathcal{O}\left(\mu^5\right) \\ &\equiv 0 \end{aligned}$$

$\langle N_Q \rangle \equiv 0$ <u>should be constrained as well</u>

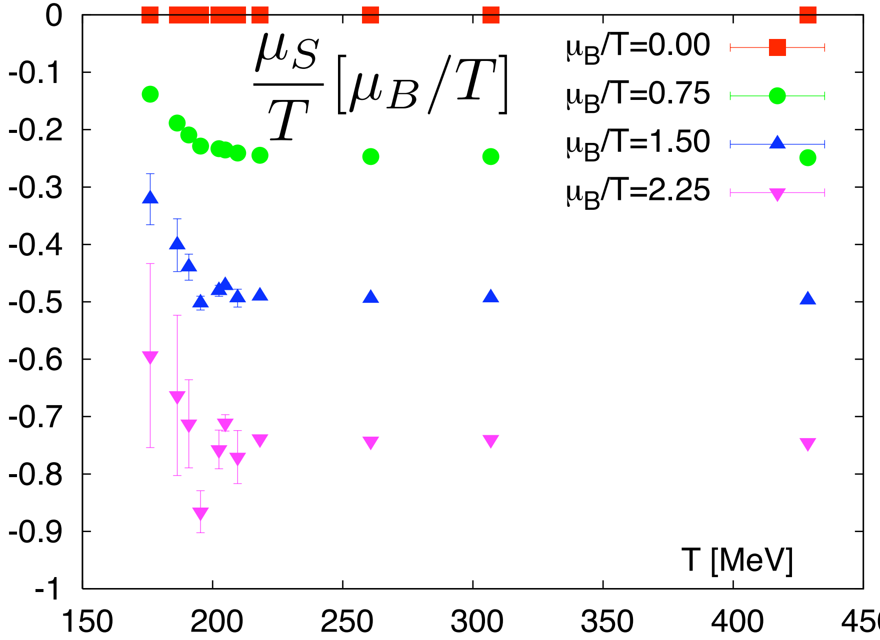
- Solution:

$$\mu_s(\mu_q) = \sum_i d_i \mu_q^i = \left(-\frac{c^{qs}_{11}}{2c^{qs}_{02}}\right)\mu_q + \left(\frac{2c^{qs}_{04}c^{qs\,3}_{11} - 3c^{qs}_{02}c^{qs\,2}_{11}c^{qs}_{13} + 4c^{qs\,2}_{02}c^{qs}_{11}c^{qs}_{22} - 4c^{qs\,3}_{02}c^{qs}_{31}}{8c^{qs\,4}_{02}}\right)\mu_q^3 + \mathcal{O}\left(\mu_q^5\right)$$

$\longrightarrow$ new expansion Coefficients: $\hat{c}^q_i$

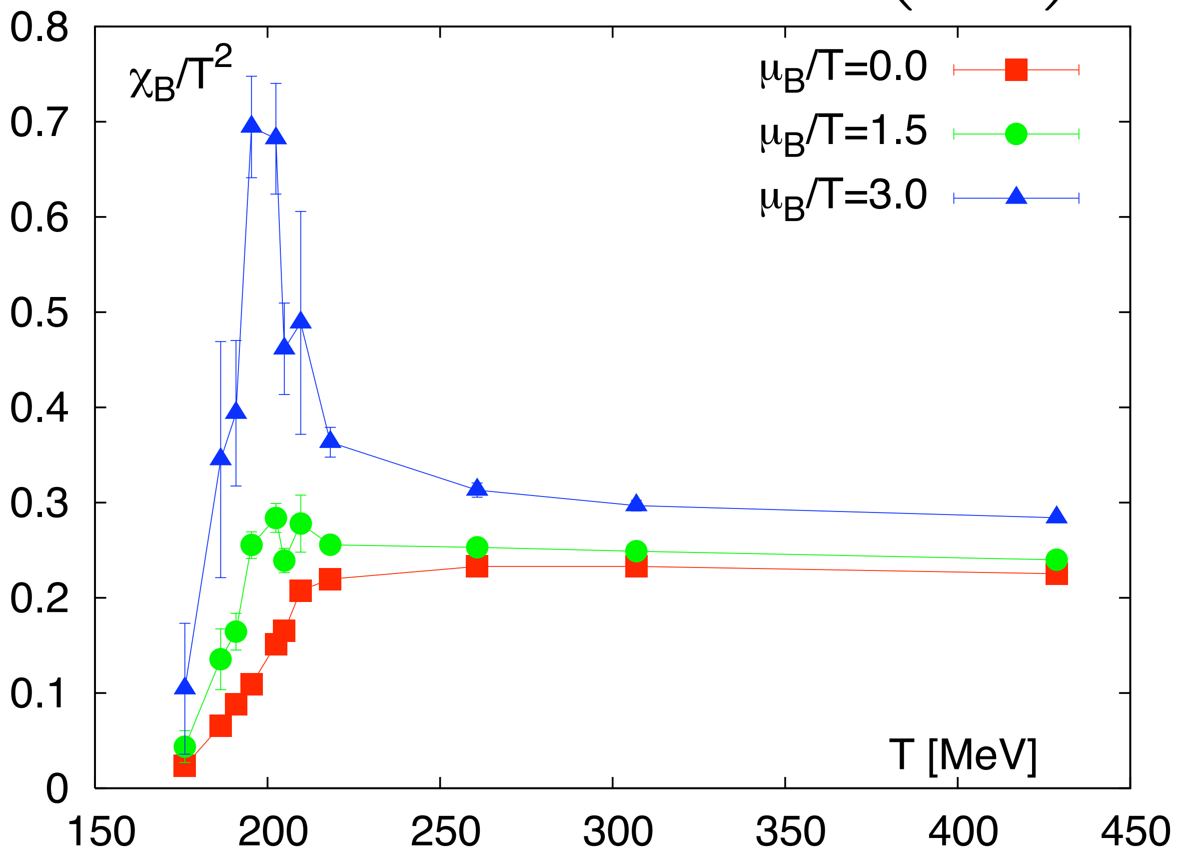
$$\begin{aligned} \frac{\Delta p}{T^4} &= \sum_i \hat{c}^q_i \mu_q^i \equiv \sum_{i,j} c_{ij}\mu^i \left(\sum_k d_k \mu_q^k\right)^j \\ &= \left(c^{qs}_{20} - \frac{c^{qs\,2}_{11}}{4c^{qs}_{02}}\right)\mu_q^2 + \left(c^{qs}_{40} + \frac{c^{qs}_{04}c^{qs\,4}_{11}}{16c^{qs\,4}_{02}} - \frac{c^{qs\,3}_{11}c^{qs}_{13}}{8c^{qs\,3}_{02}} + \frac{c^{qs\,2}_{11}c^{qs}_{22}}{4c^{qs\,2}_{02}} - \frac{c^{qs}_{11}c^{qs}_{31}}{2c^{qs}_{02}}\right)\mu_q^4 + \mathcal{O}(\mu_q^6) \end{aligned}$$

The HIC case: $\langle N_s \rangle \equiv 0$

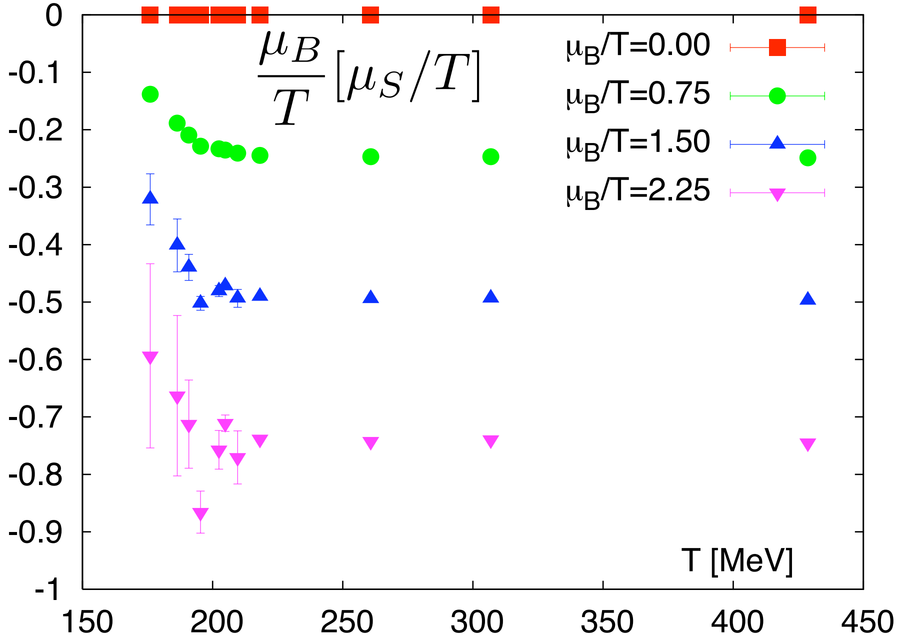


baryon fluctuations

$$\frac{\hat{\chi}_B}{T^2} = 2\hat{c}_2^B + 12\hat{c}_4^B\left(\frac{\mu_B}{T}\right)^2$$

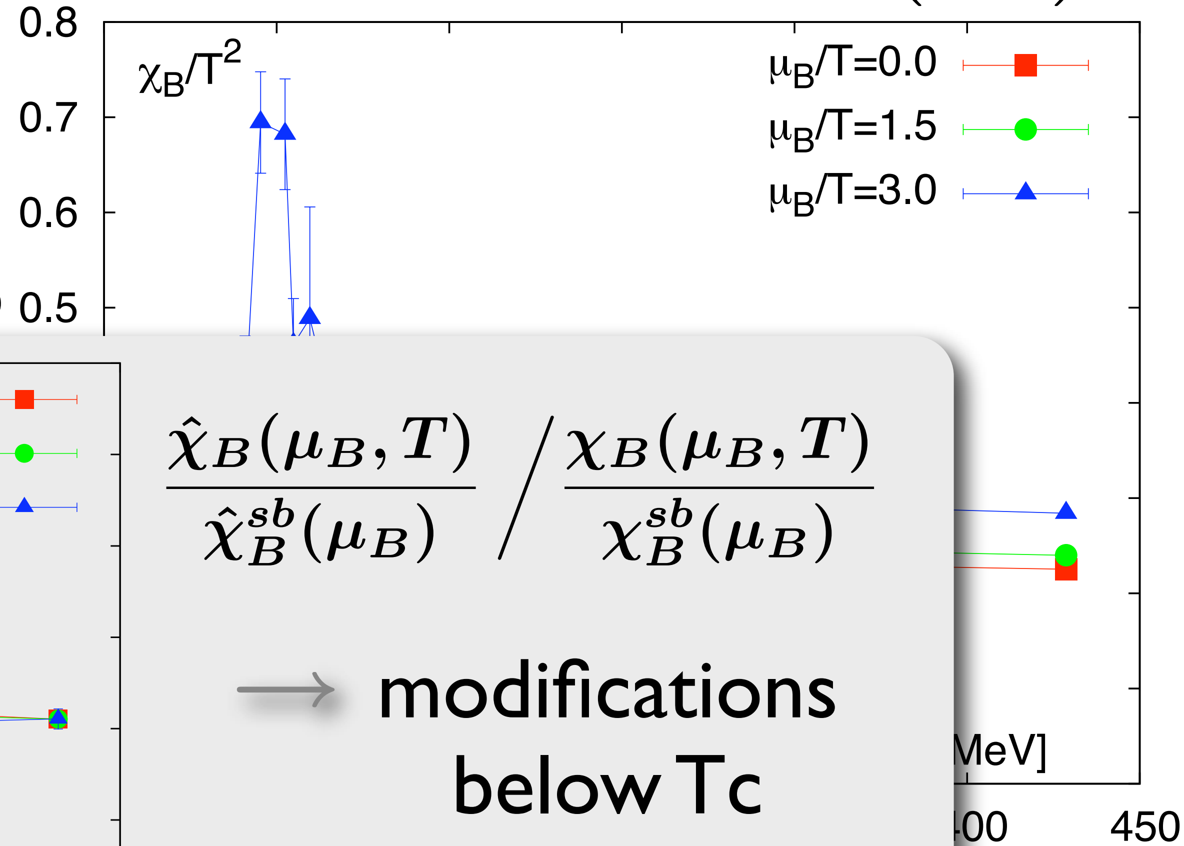


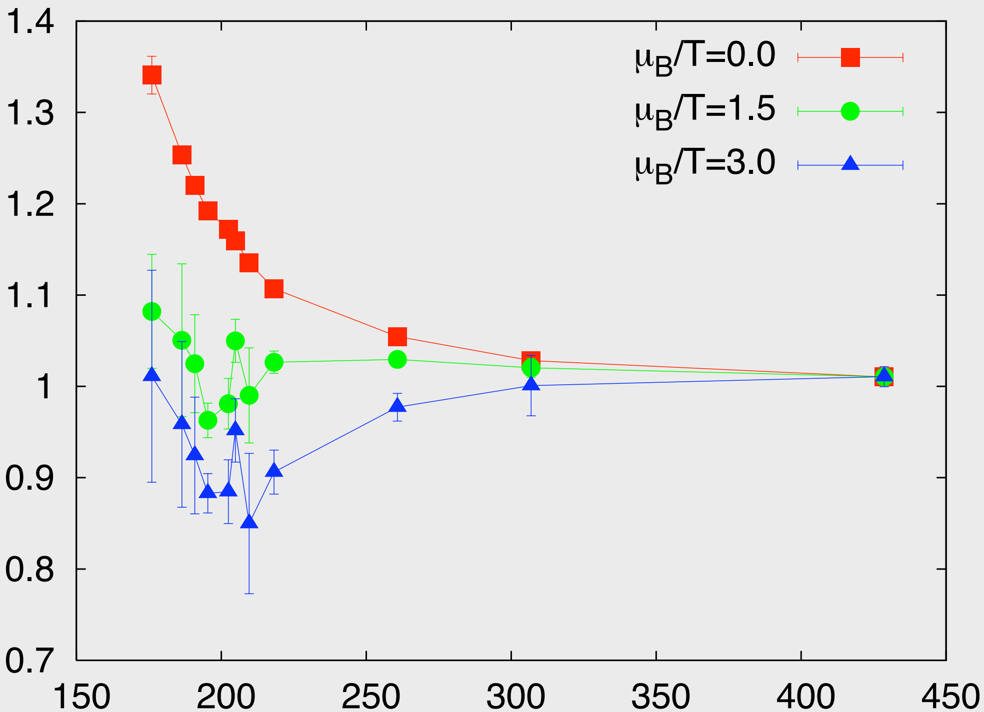
The HIC case: $\langle N_s \rangle \equiv 0$



baryon fluctuations

$$\frac{\hat{\chi}_B}{T^2} = 2\hat{c}_2^B + 12\hat{c}_4^B \left(\frac{\mu_B}{T}\right)^2$$

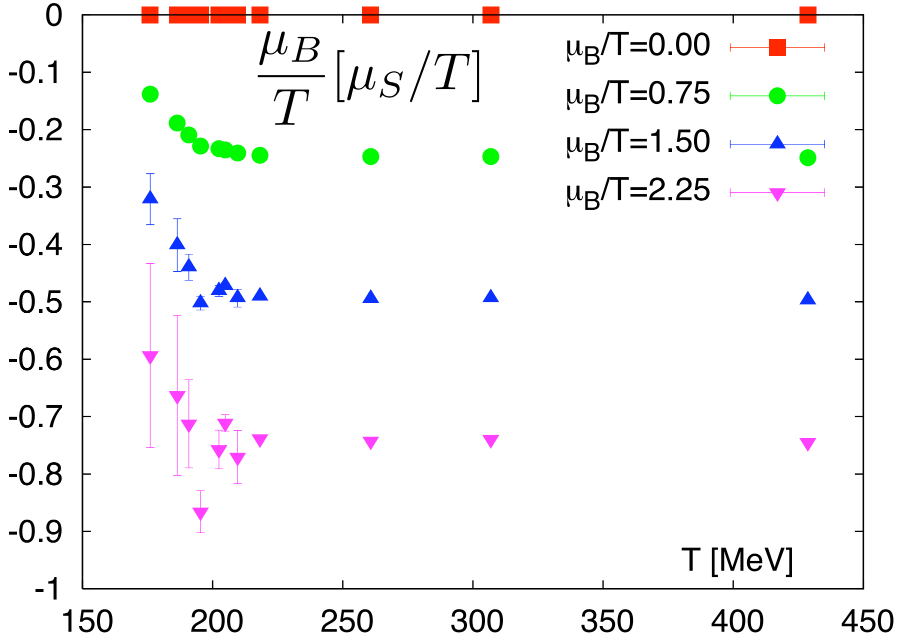




$$\frac{\hat{\chi}_B(\mu_B, T)}{\hat{\chi}_B^{sb}(\mu_B)} \Big/ \frac{\chi_B(\mu_B, T)}{\chi_B^{sb}(\mu_B)}$$

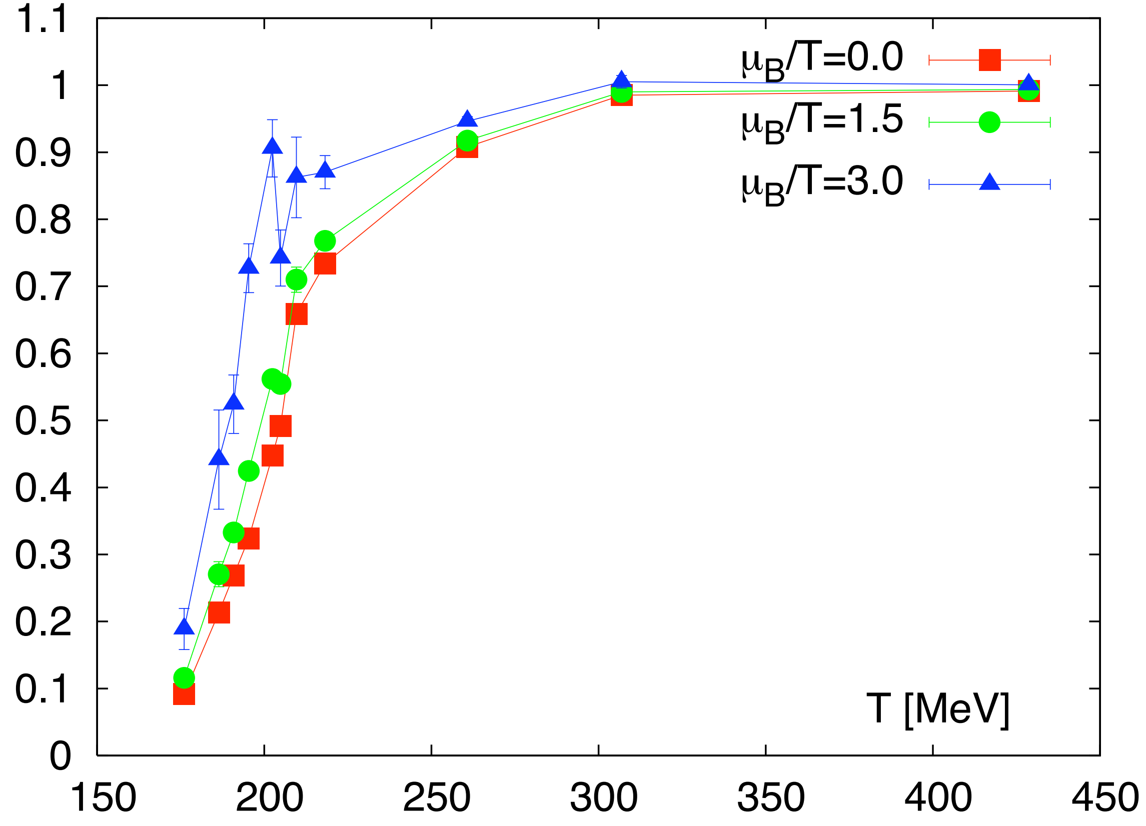
→ modifications below Tc ~ 30%

The HIC case: $\langle N_s \rangle \equiv 0$



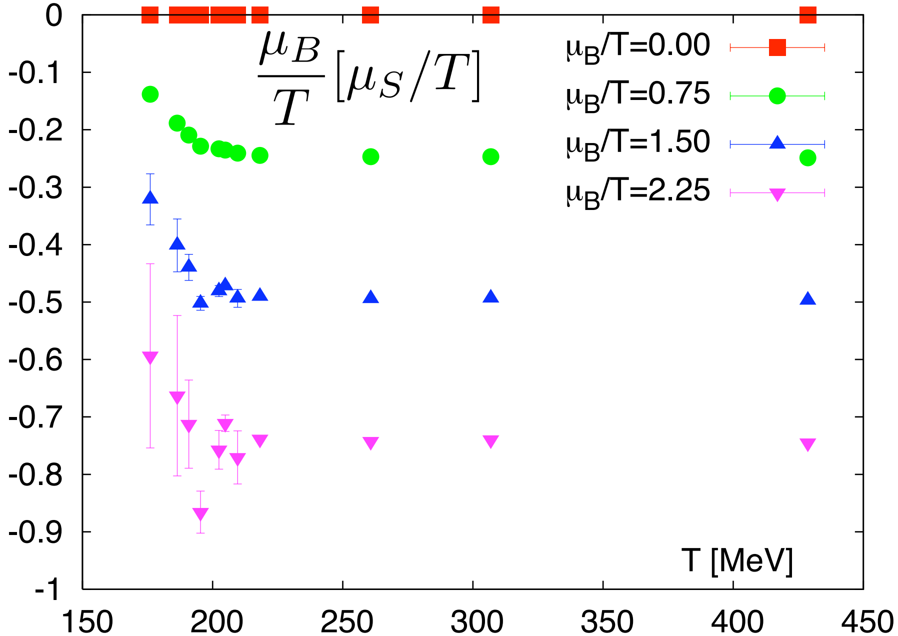
strangeness fluctuations

$$\frac{\hat{\chi}_S}{T^2} = 2\hat{c}_2^B + 2\hat{c}_{22}^{BS}\left(\frac{\mu_B}{T}\right)^2$$

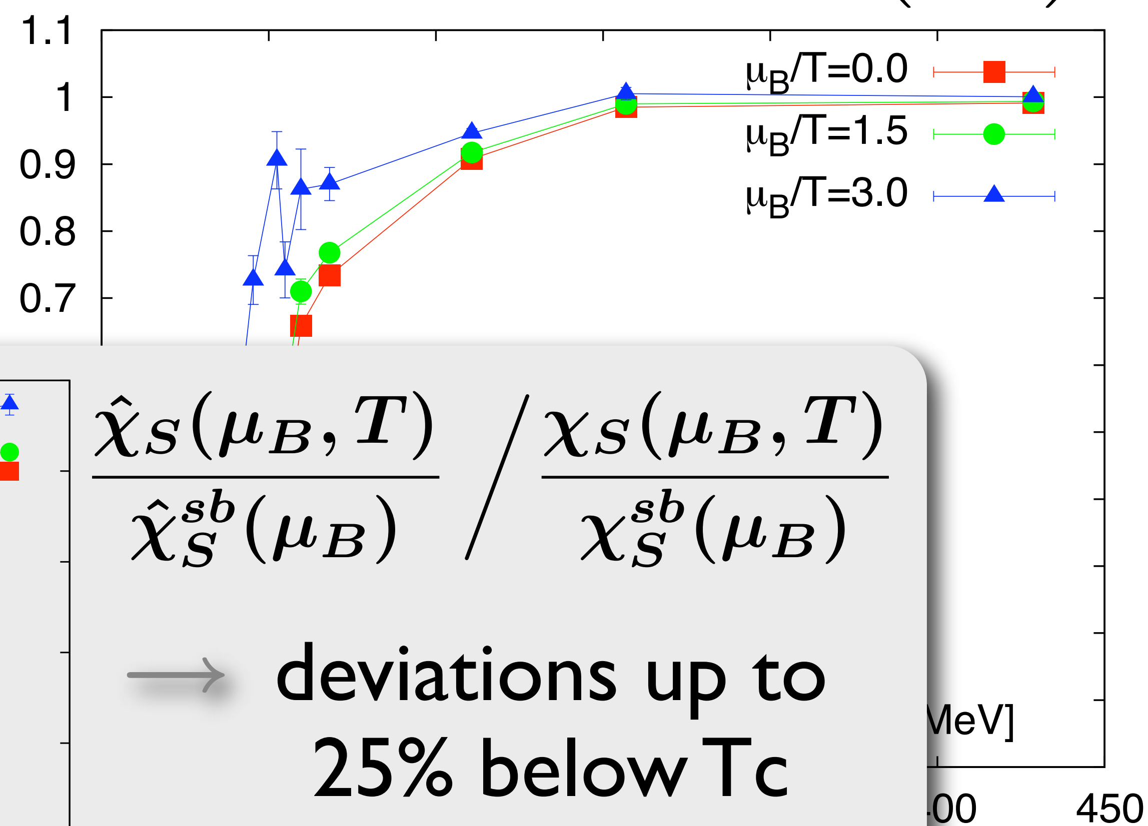


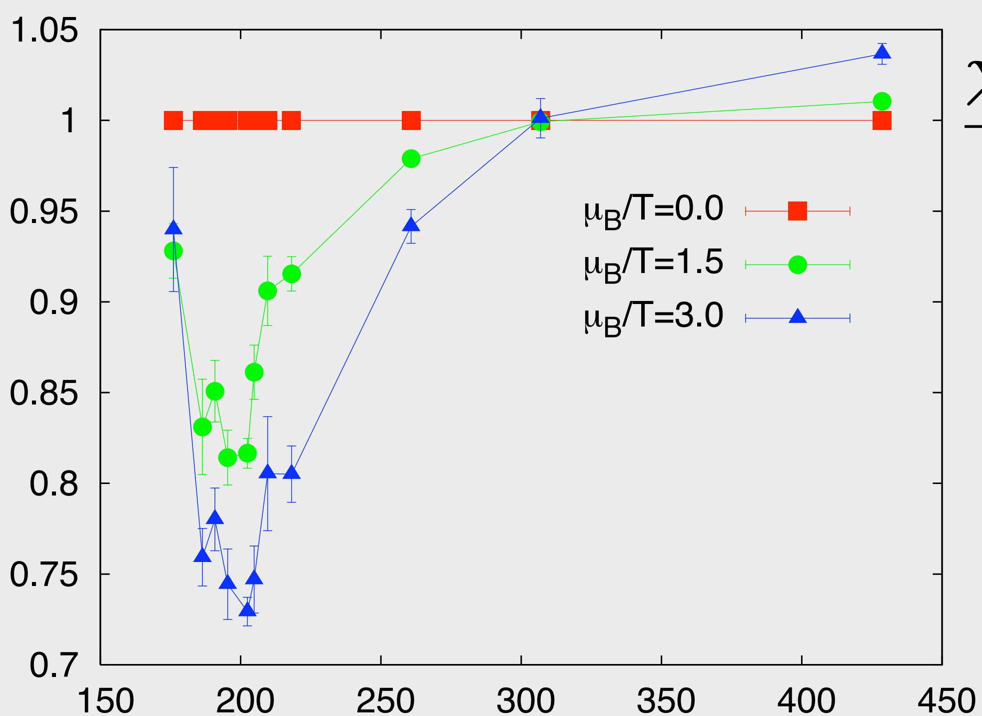
The HIC case: $\langle N_s \rangle \equiv 0$

strangeness fluctuations



$$\frac{\hat{\chi}_S}{T^2} = 2\hat{c}_2^B + 2\hat{c}_{22}^{BS}\left(\frac{\mu_B}{T}\right)^2$$





$$\frac{\hat{\chi}_S(\mu_B,T)}{\hat{\chi}_S^{sb}(\mu_B)} \Big/ \frac{\chi_S(\mu_B,T)}{\chi_S^{sb}(\mu_B)}$$

→ deviations up to 25% below Tc

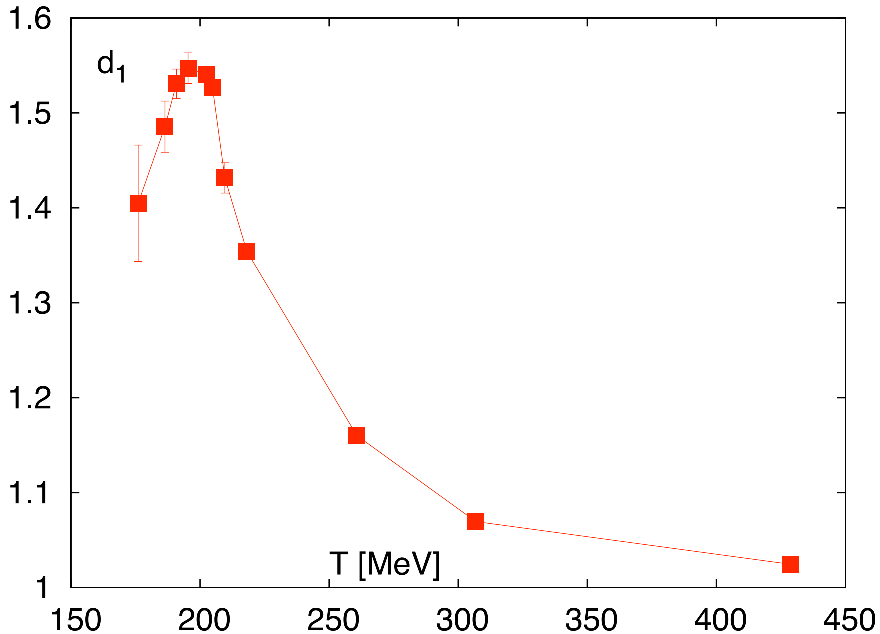
The flavor symmetric case:

- Solving order by order:

$$\langle N_s(\mu_q,\mu_s)\rangle \equiv \frac{1}{2}\langle N_q(\mu_q,\mu_s)\rangle$$

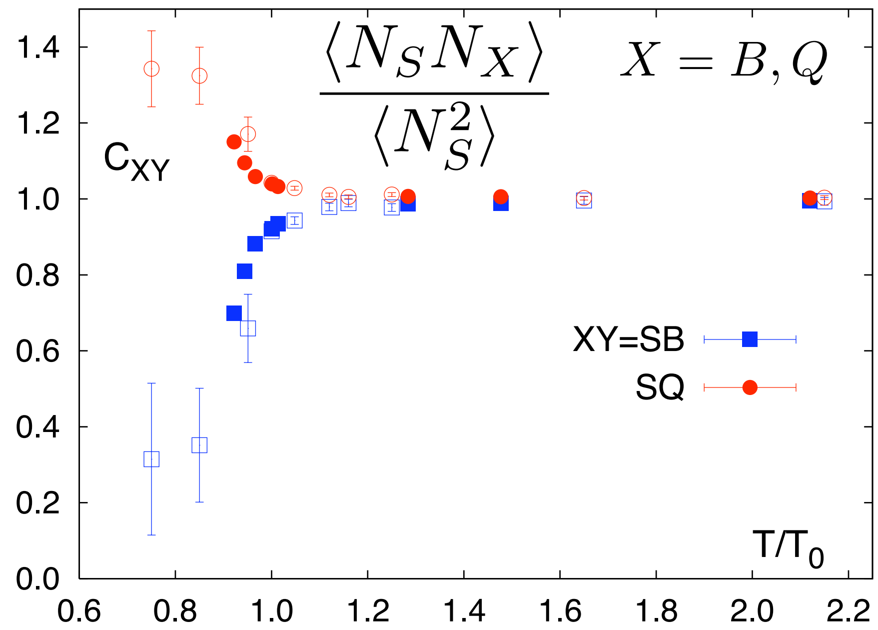
- Solution:

$$\mu_s(\mu_q) = \sum_i d_i \mu_q^i = \left(\frac{c_{20}-c_{11}}{2c_{02}-c_{11}/2}\right)\mu_q + \left(\frac{64c_{04}(c_{11}-c_{20})^3}{(-4c_{02}+c_{11})^4} + \frac{24c_{13}(c_{11}-c_{20})^2}{(-4c_{02}+c_{11})^3} + \frac{8c_{22}(c_{11}-c_{20})}{(-4c_{02}+c_{11})^2} + \frac{2c_{31}}{(-4c_{02}+c_{11})}\right)\mu_q^3 + \mathcal{O}(\mu_q^5)$$



$\longrightarrow$ Eventually of interest for cosmology and neutron / quark stars

Correlations of S,B and S,Q



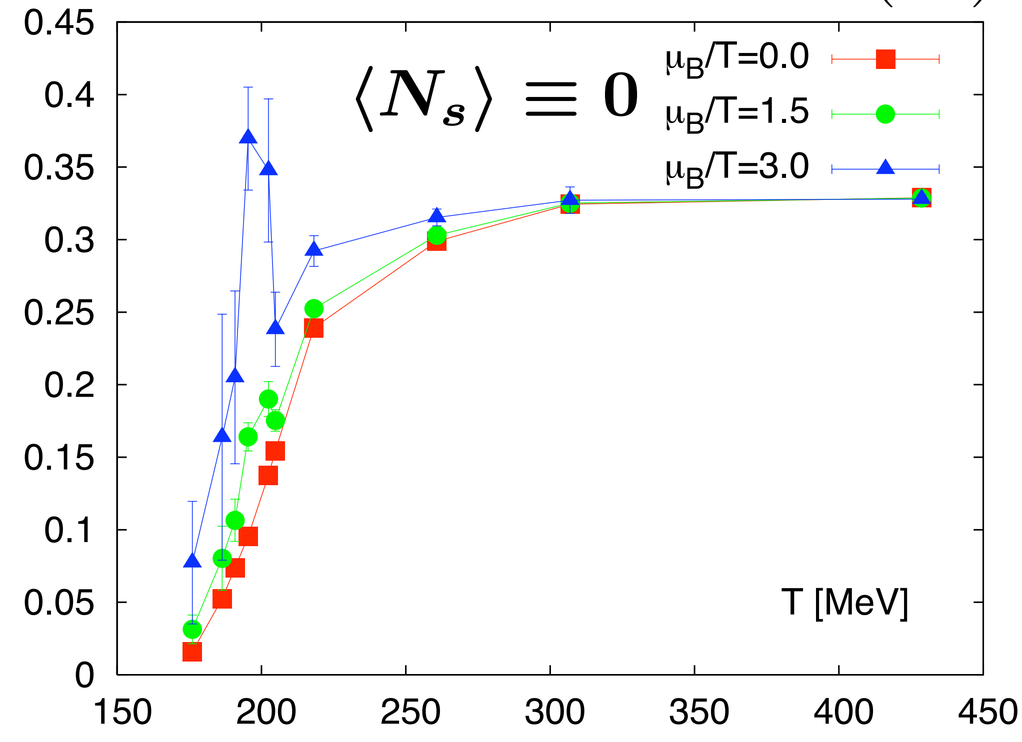
full: RBC-Bielefeld, nf=2+1, preliminary
open: Gavai, Gupta, nf=2, partially quenched

- Small quenching effect

$\longrightarrow$ Normalization effect ?

$$\frac{\langle N_S N_X \rangle}{\langle N_S^2 \rangle} = \frac{f_X c_{11}^{qs} + 2c_s^s}{2c_2^s}$$

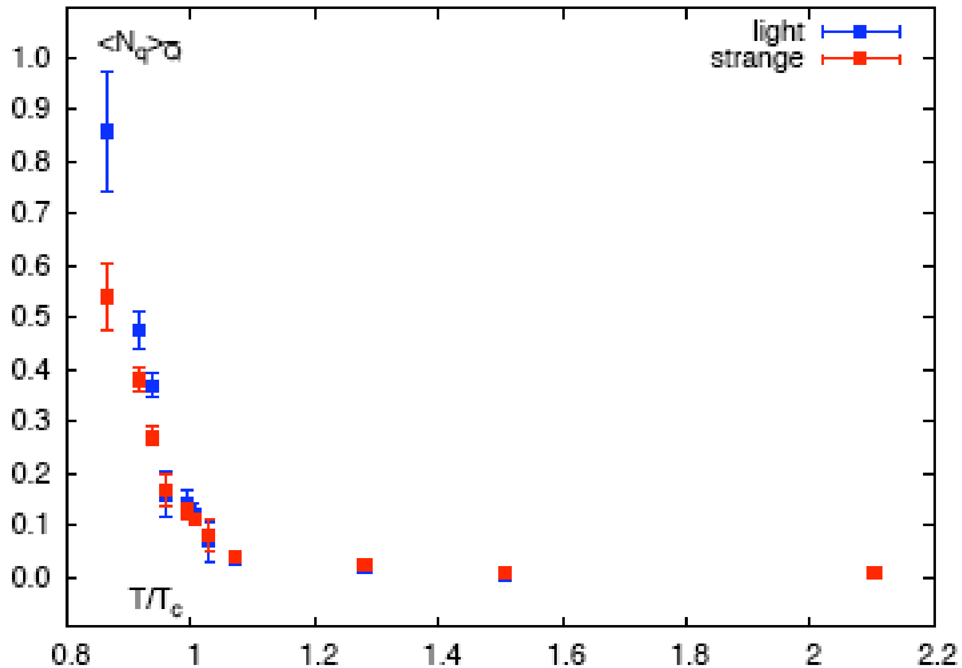
$$\frac{1}{T}\left(\langle N_B N_S \rangle - \langle N_B \rangle \langle N_S \rangle\right) = \hat{c}_{11}^{BS} + 3\hat{c}_{31}^{BS}\left(\frac{\mu_B}{T}\right)^2$$



$\longrightarrow$ Correlations increase for $\mu_B > 0$

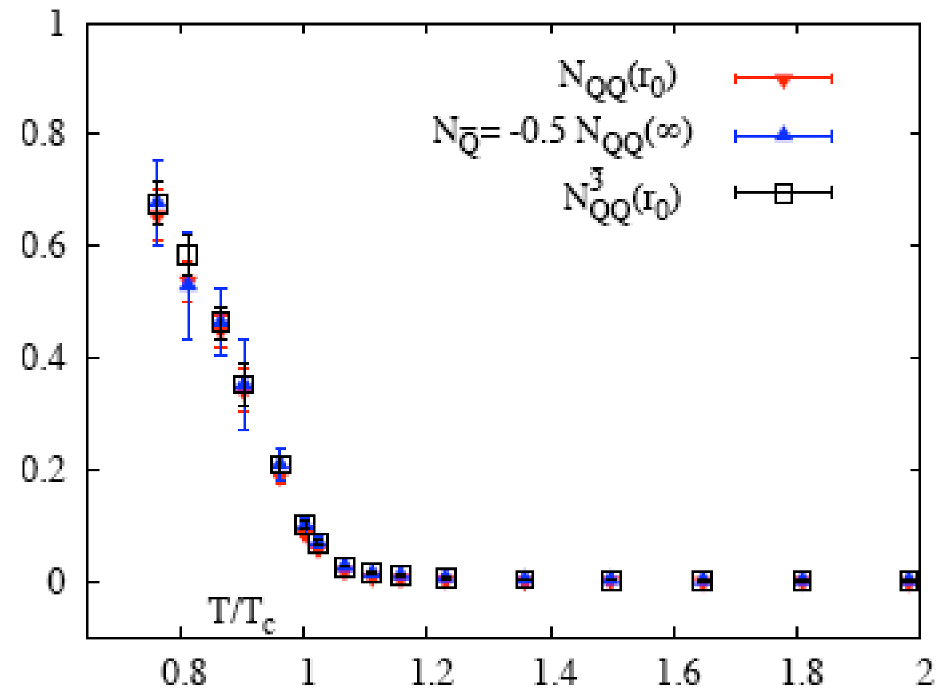
Correlations of Nq, L

- Induced (total) quark number density in presence of a static anti-quark $\langle N_q \rangle_{\bar{Q}} = \frac{\langle c_1^q L^* \rangle}{\langle L \rangle}$



$N_f = 2 + 1$

RBC-Bielefeld, preliminary

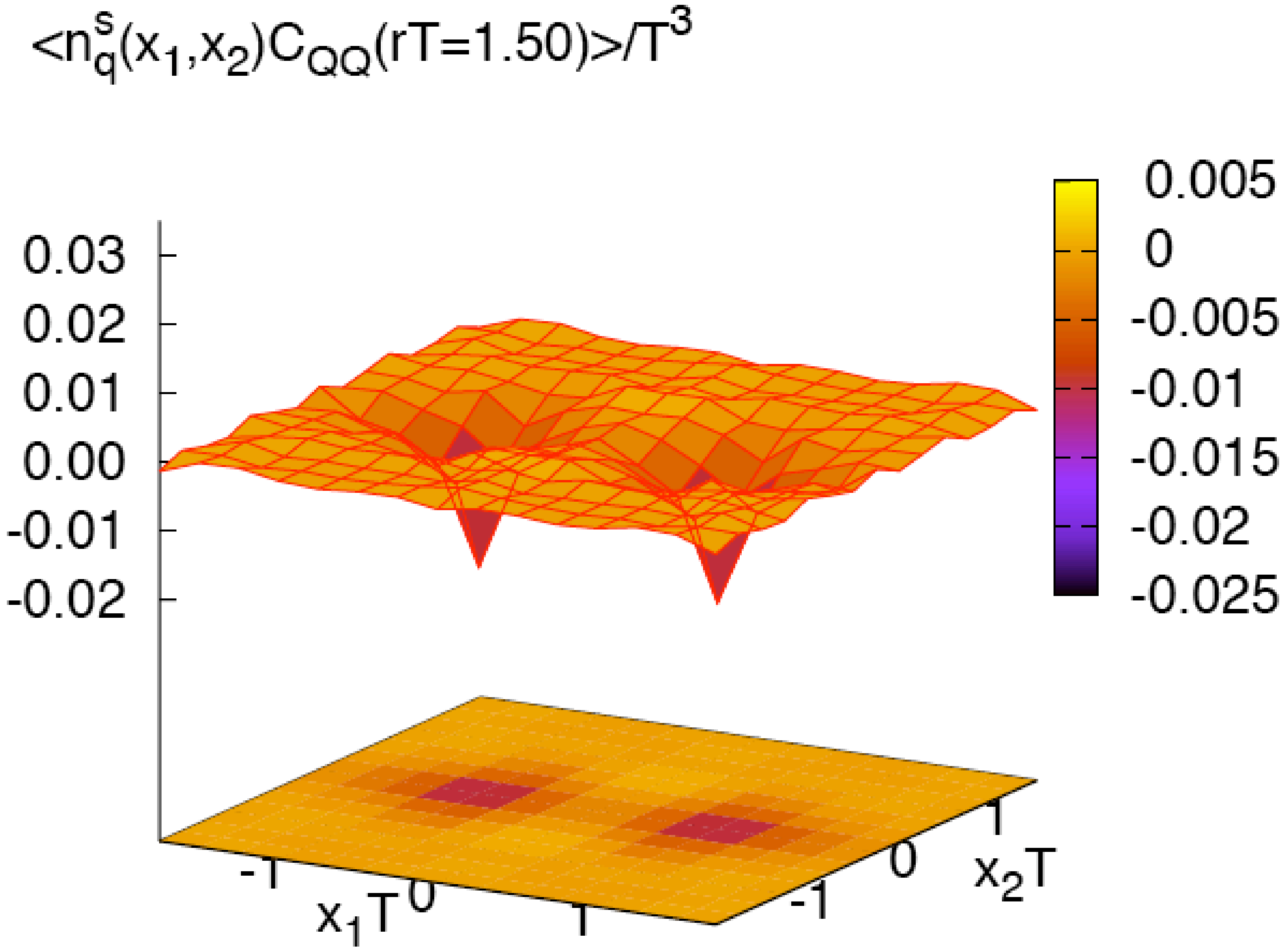


$N_f = 2, m/T = 0.4$

Doering, Huebner, Kaczmarek, Karsch, PRD 75, 0504504 (2007)

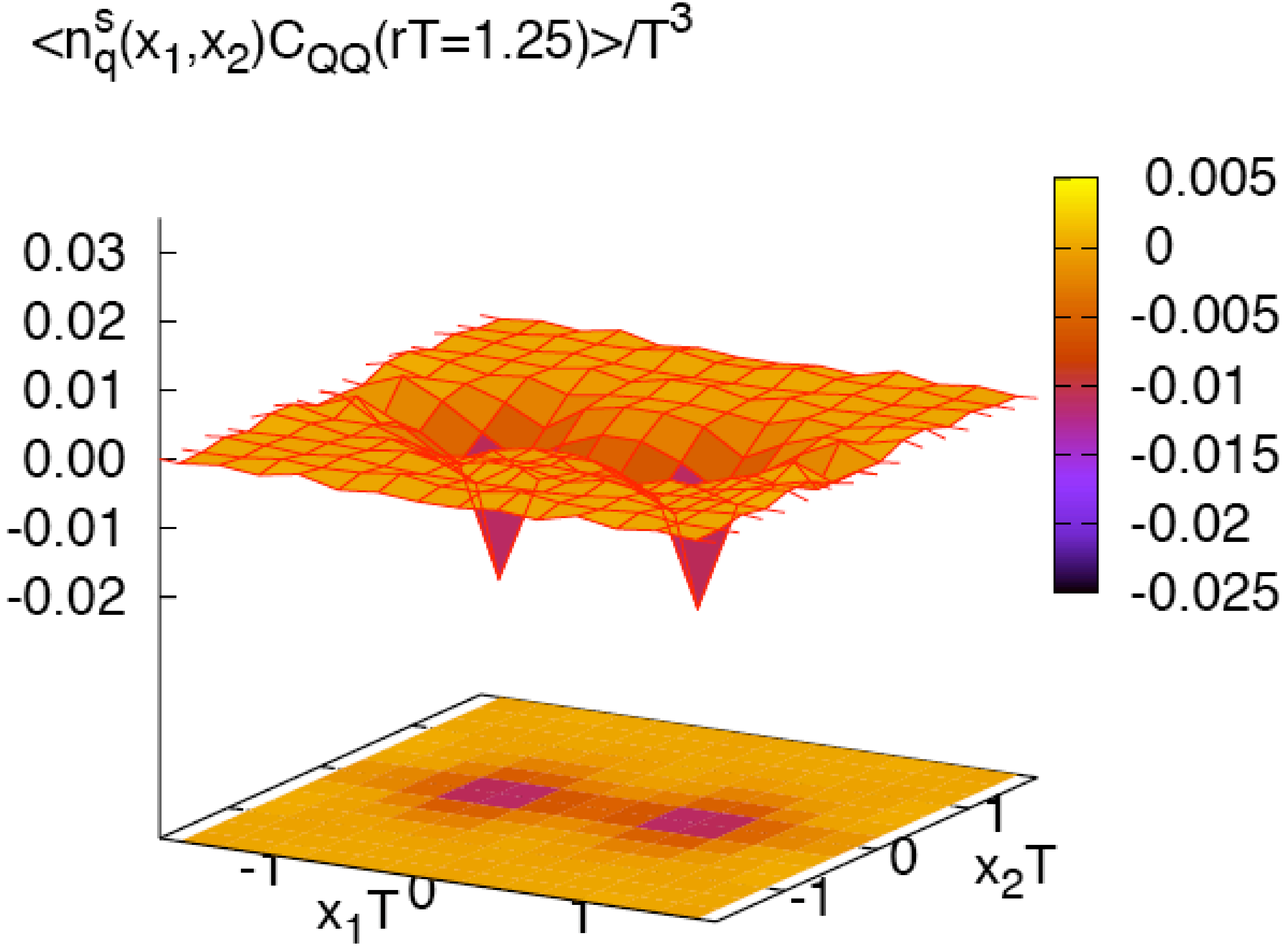
⟶ Talk by K. Huebner (Lattice 2007)

Correlations of Nq, L, L



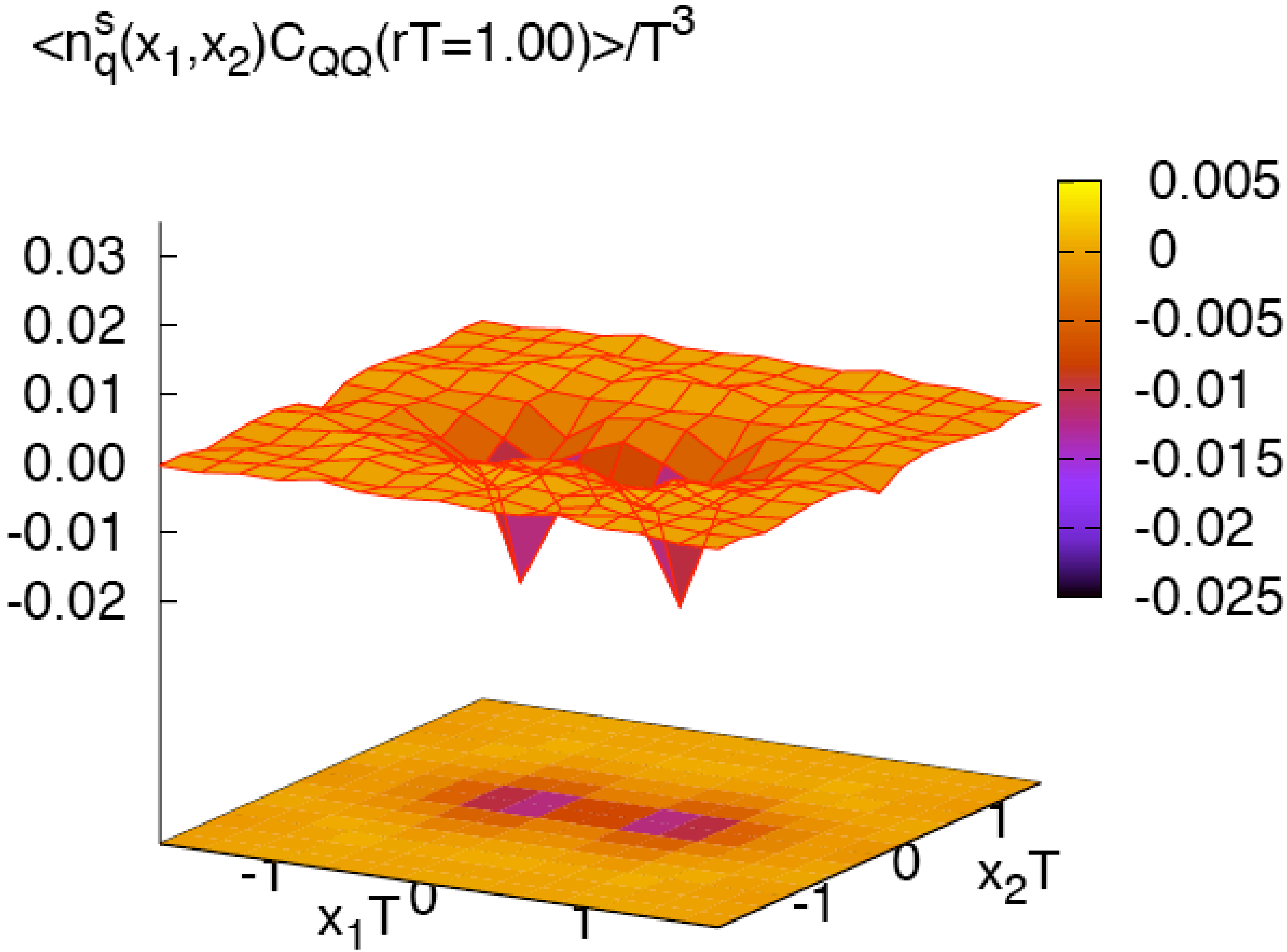
→ Mesonic screening

Correlations of Nq, L, L



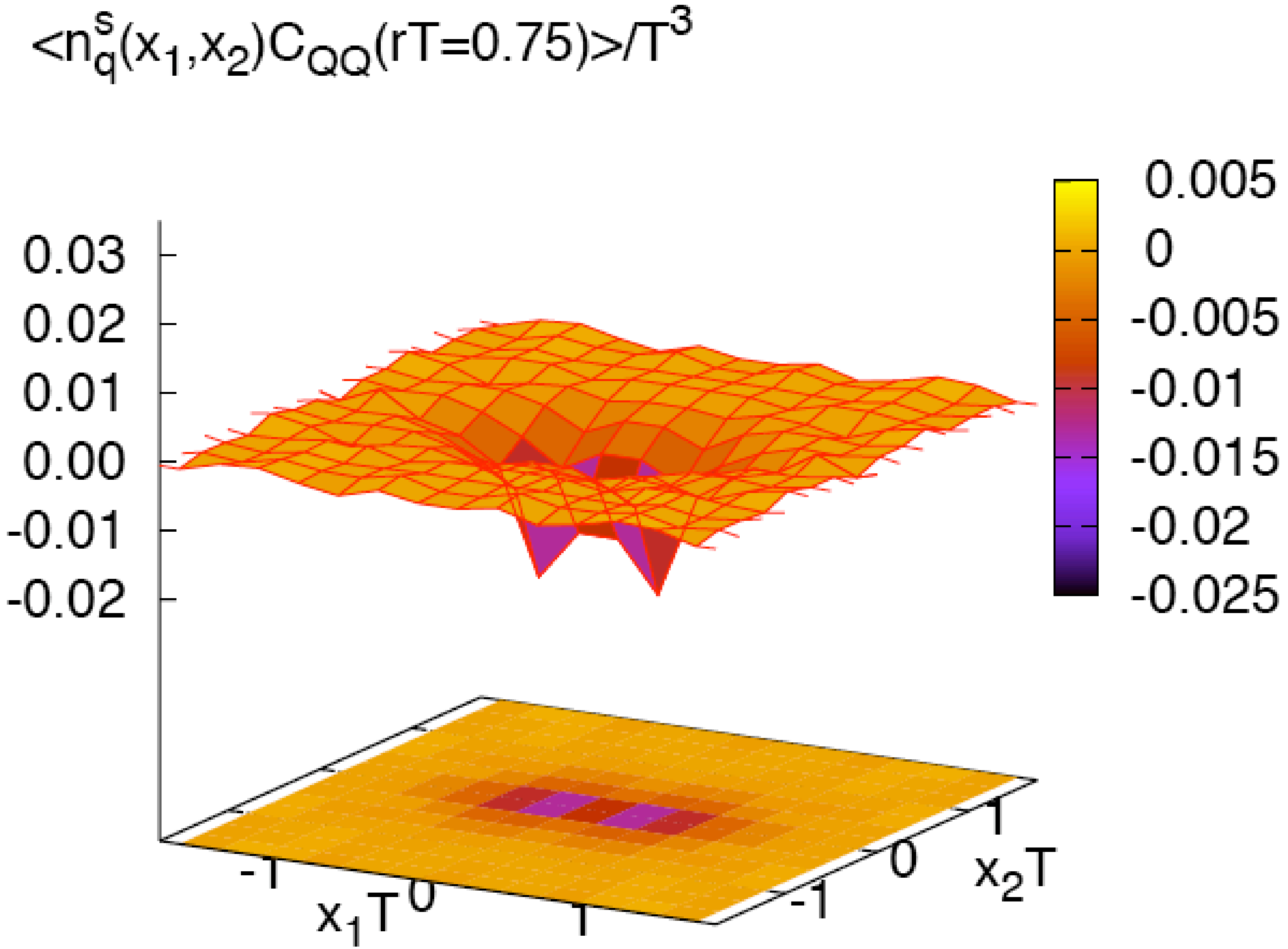
→ Mesonic screening

Correlations of Nq, L, L



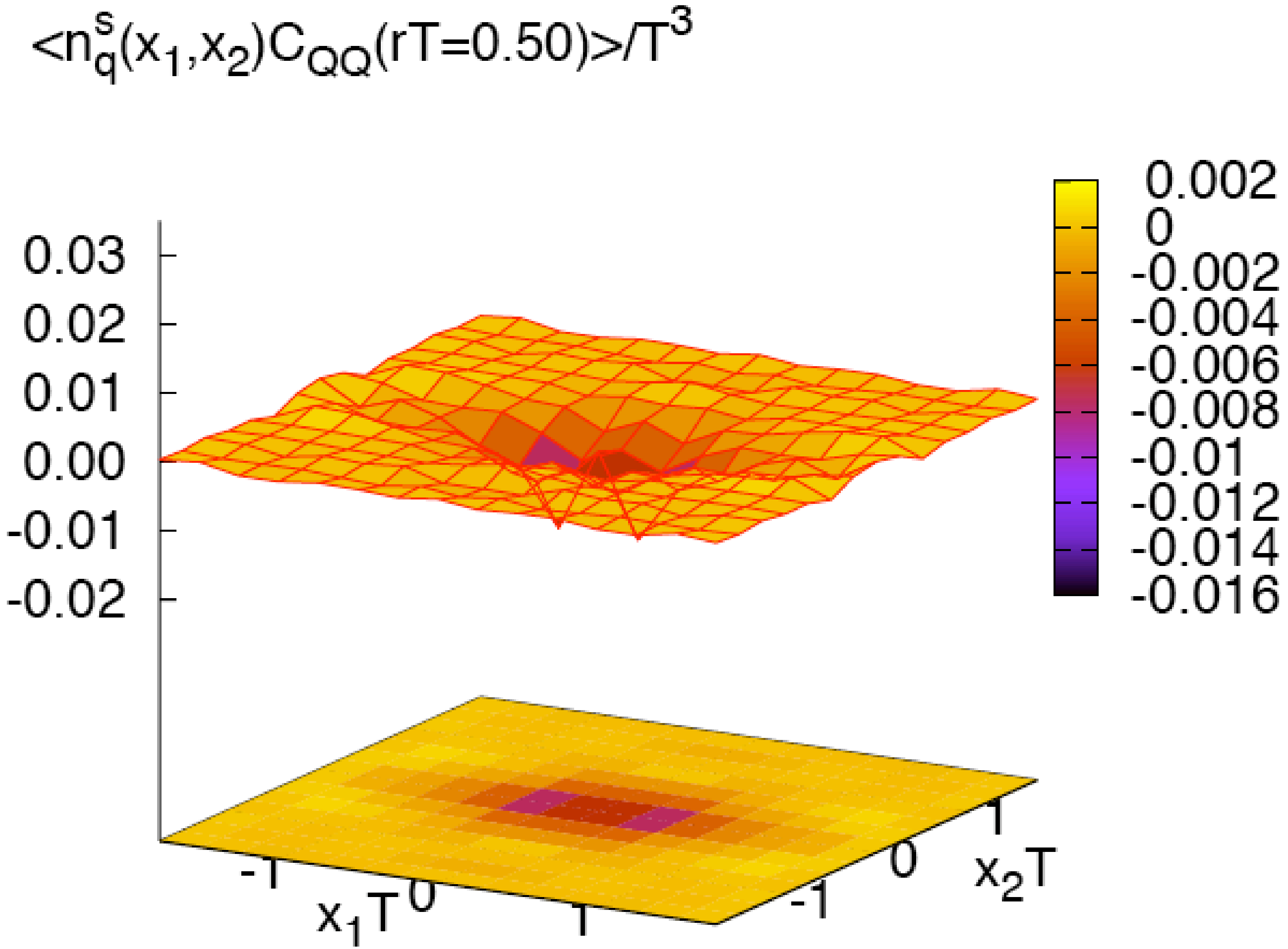
→ Mesonic screening

Correlations of Nq, L, L



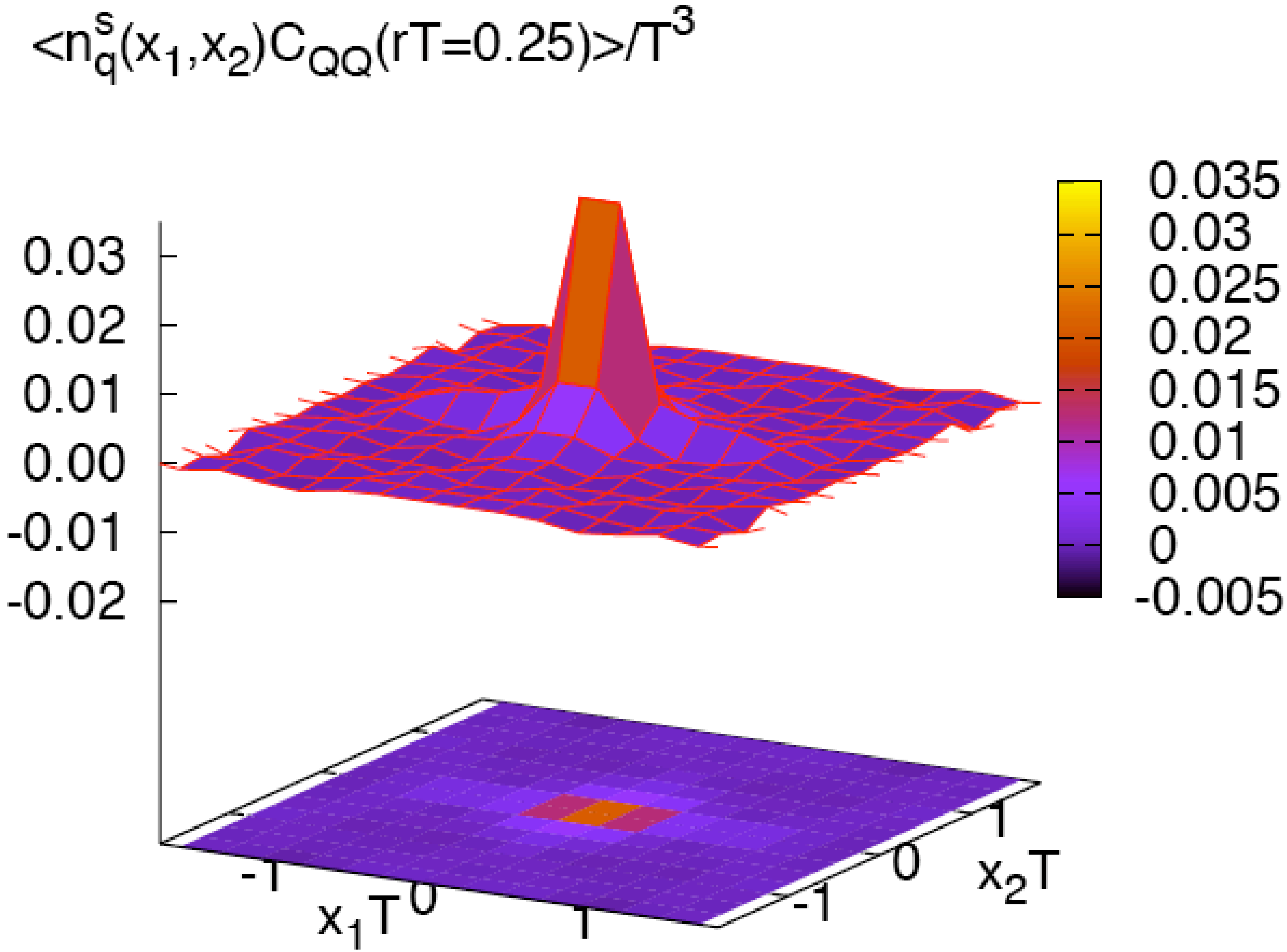
→ Mesonic screening

Correlations of Nq, L, L



→ Mesonic screening

Correlations of Nq, L, L



→ Baryonic screening

Conclusions / Summary

- Taylor expansion method at 2+1 flavor have rich HIC phenomenology.
- All results on fluctuations and correlations develop a peak with increasing chemical potential and are consistent with a gas of quasi-free quarks, already at 1.2-1.5 Tc.
- The strangeness can be easily constrained to zero, which is more relevant for HIC.
- In future: the electric charge density should be constrained to zero as well (for the HIC case).
- The density of all quarks can be constrained to be equal (interesting for cosmology, stars?)
- The local quark number density can be calculated in presence of static test charges.