More on the phase diagram of QCD

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   - $N_f = 2$
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3. Summary

Thermodynamic variables

Definition

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Phase diagram: space of intensive variables. Each point belongs to exactly one thermodynamic state, except: 2-phase coexistence imposes one constraint among the intensive variables, 3-phase coexistence imposes two constraints, etc.

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In a D-dimensional phase diagram, there are $D - 1$ dimensional hypersurfaces of two-phase coexistence, $D - 2$ dimensional hypersrufaces of three-phase coexistence etc. Each surface of n-phase coexistence either has no boundary or ends in a n-critical point ($n = 2$: critical; $n = 3$ tri-critical, *etc.*).

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Example: pure water

Gibbs space is 3 dimensional, $U(S, V)$; the phase diagram is two dimensional (T, P); there are lines of first-order phase transitions, ending in critical points, and isolated points of three-phase coexistence (triple points).

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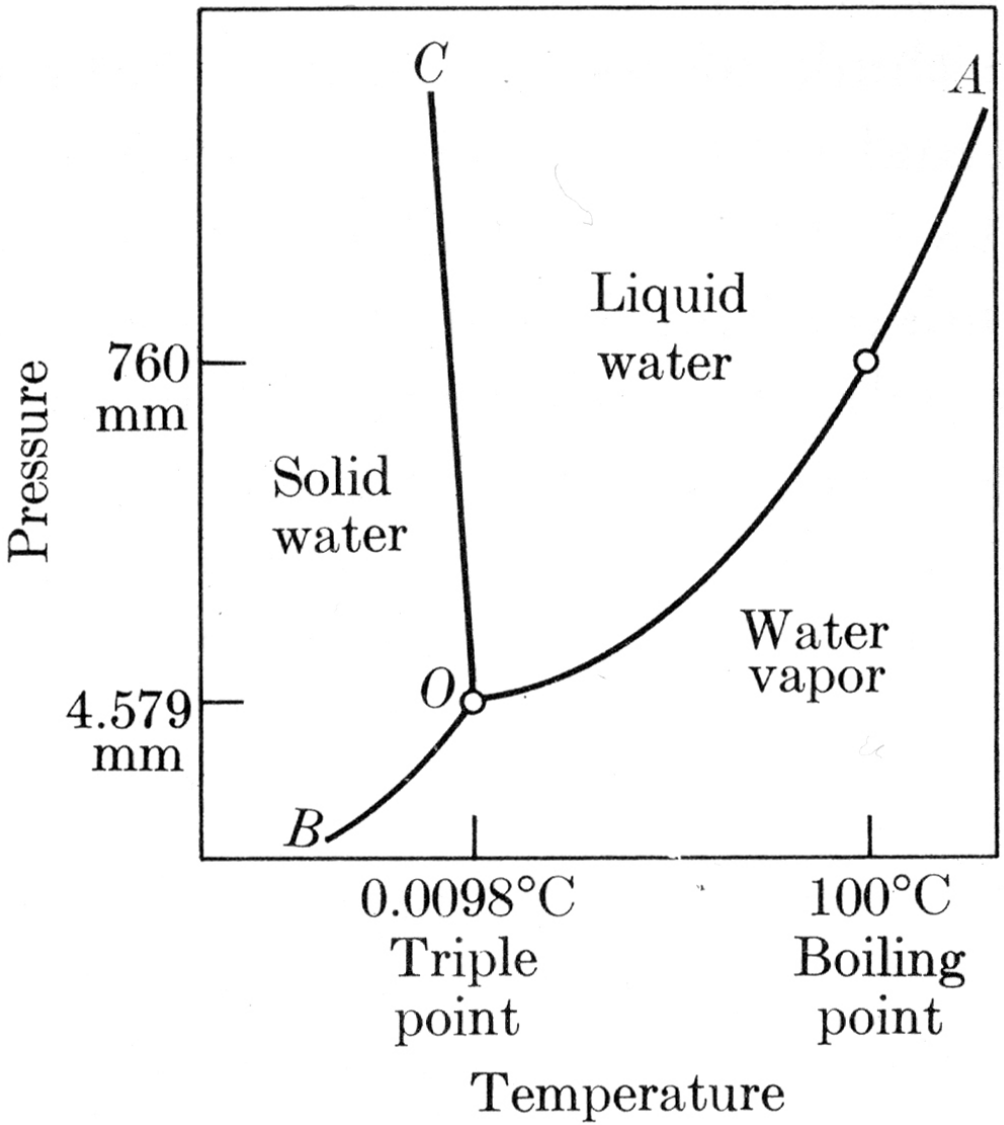

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Many dimensions

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Popular slices

- The classic phase diagram: T, μ_B and m. Single order parameter: $s = \langle\overline{\psi}\psi\rangle$. Looks at chiral symmetry. (Berges and Rajagopal; Halasz, *et al.*; 1998)
- The real-world phase diagram: T, μ_B and μ_I. Two order parameters: $s = \langle\overline{\psi}\psi\rangle$ and $p = \langle\overline{\psi}\tau_2\psi\rangle$. Chiral symmetry restoration as well as pion condensation. (Son and Stephanov 2000; Klein *et al.*2003; Nishida 2003; Barducci *et al.*2004)

The classic phase diagram

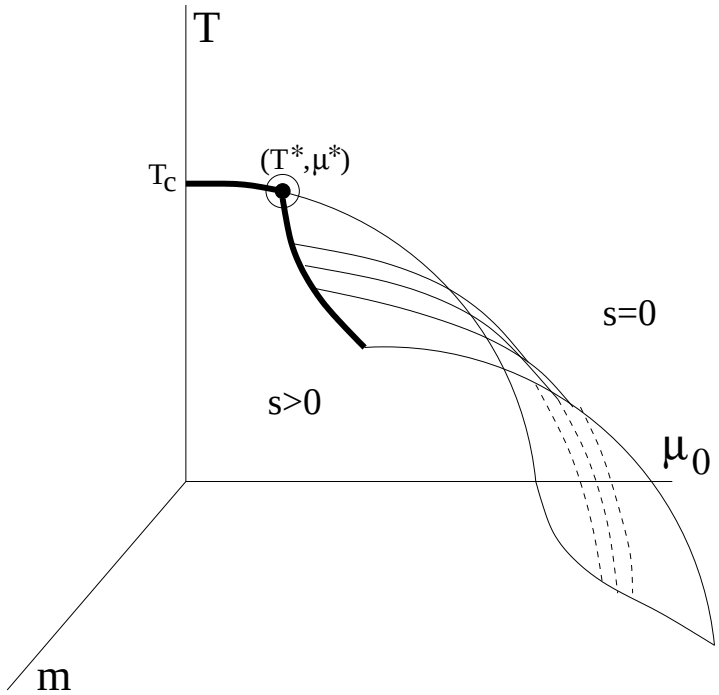

Figure: 2 critical lines: one in the $O(4)$ universality class, one Ising; meet at a tricritical point. For $m_\pi/m_\rho = 0.3$, $\mu_E \simeq T_c \simeq T_E$. (Gavai and Gupta, 2004). Parts of the coexistence surface observed. (Ejiri, 2006).

The classic phase diagram explored

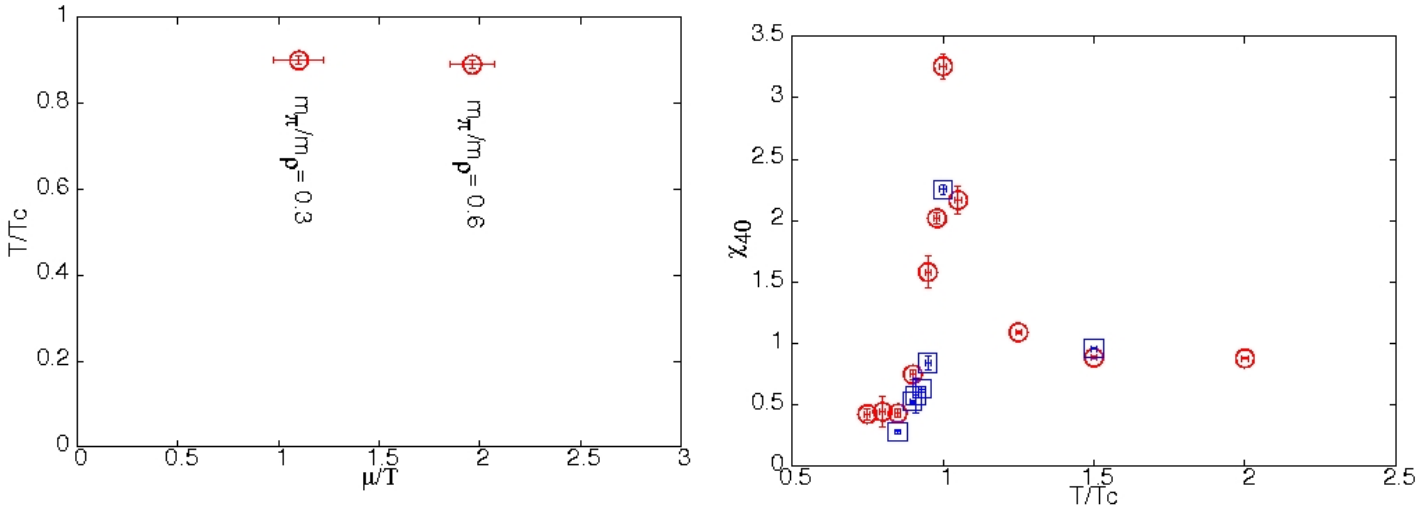

Figure: (Gavai and Gupta, in preparation).

The real-world phase diagram

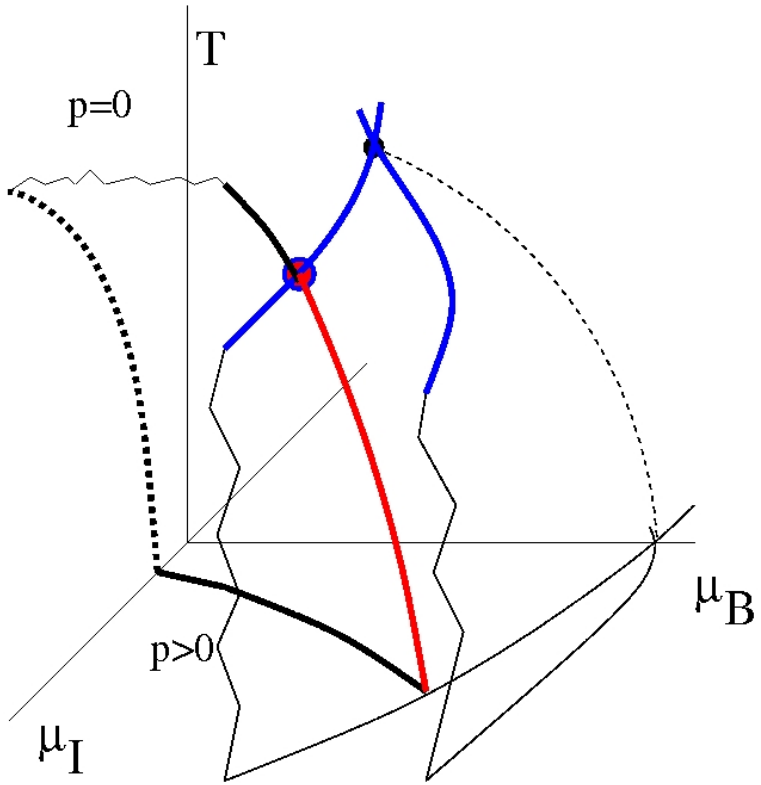

Figure: Surfaces of two phase coexistence; meet along a line of three phase coexitence, end point of which is a new tricritical point: **the observable tricritical point of QCD**.

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- If yes, then: does the first order surface of pion condensation meet surfaces of both chiral transitions? Assumed that chirally restored phase has no pions. May not be true if superconducting phases are taken into account.
- If so, then two or more "observable tri-critical points" can exist.

$N_f = 2 + 1$

7 dimensional phase diagram

Uses T, μ_B, μ_I, μ_Y, $m = (m_u + m_s)/2$, m_s and $\Delta m = m_d - m_u$.

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The story till now

- Finite temperature phase diagram explored: 3-d slice (varying T, m and m_s) obtained in the hyperplane $\mu_B = \mu_I = \mu_Y = \Delta m = 0$. (Columbia, Bielefeld)
- First study of 4-d slice (varying T, m, m_s and μ_B) obtained in the hyperplane $\mu_I = \mu_Y = \Delta m = 0$. (de Forcrand and Philipsen 2006)

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7 dimensional phase diagram

Uses T, μ_B, μ_I, μ_Y, $m = (m_u + m_s)/2$, m_s and $\Delta m = m_d - m_u$. In high dimensions need an easier visualization tool: **the flag diagram**.

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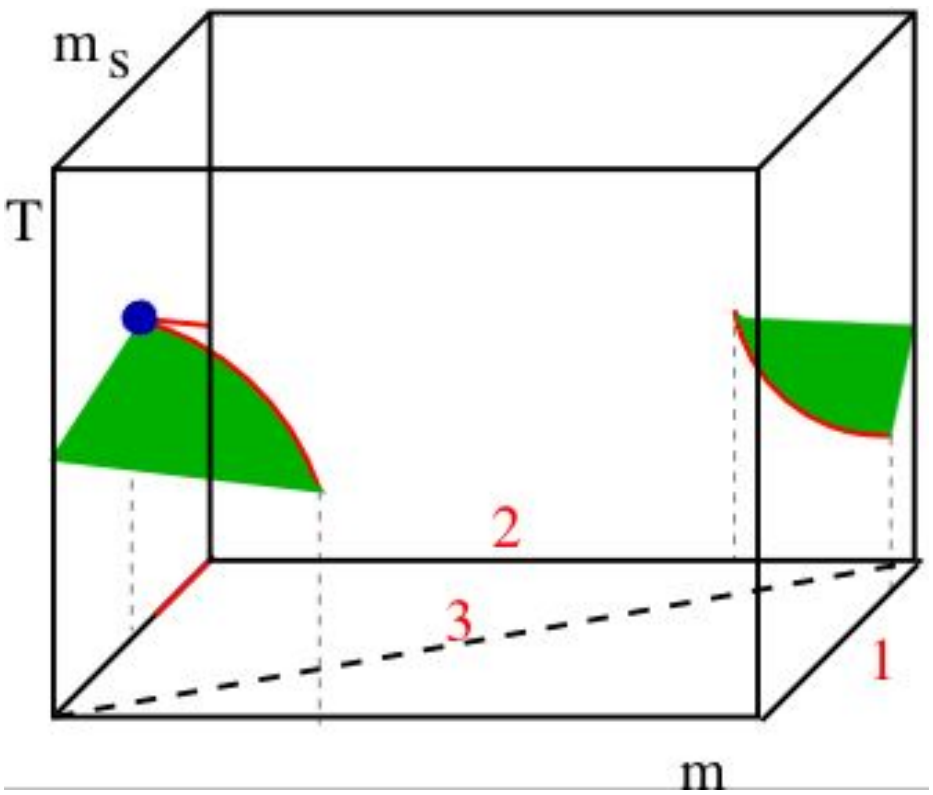
Flag diagrams make it easy



- For high-dimensional phase diagrams, project to lower dimension and make a **flag diagram**.
- A point in a flag diagram does not correspond to a thermodynamically stable phase, since it does not specify all thermodynamic state variables. Therefore it is not a phase diagram.
- A point in a flag diagram flags whether or not there is a phase transition "above" it.

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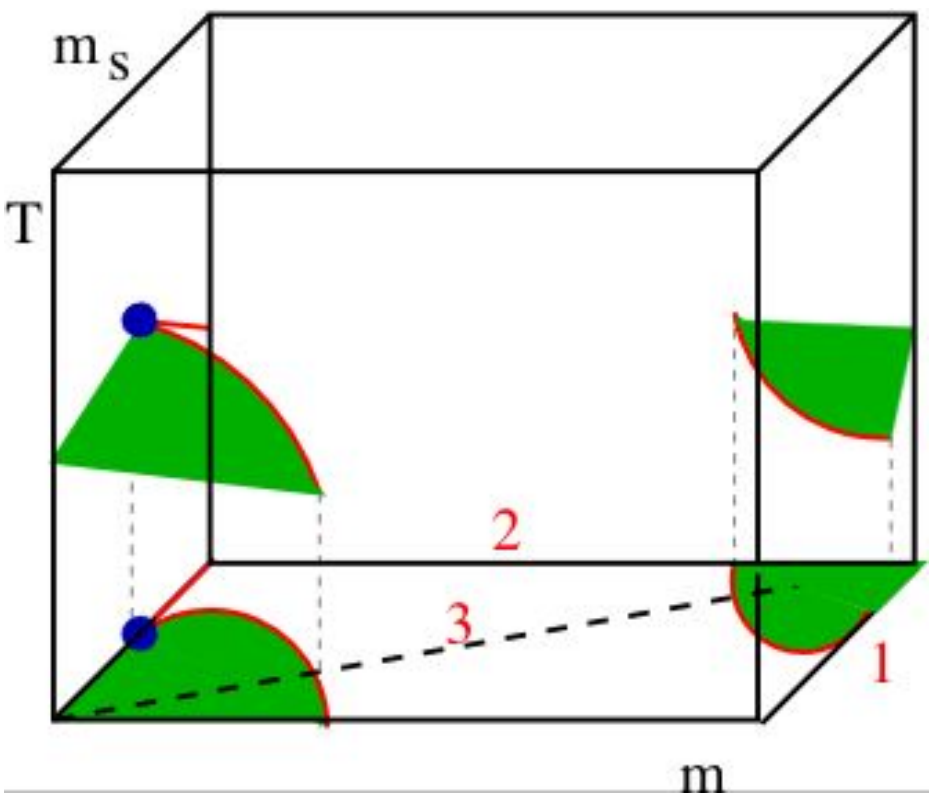

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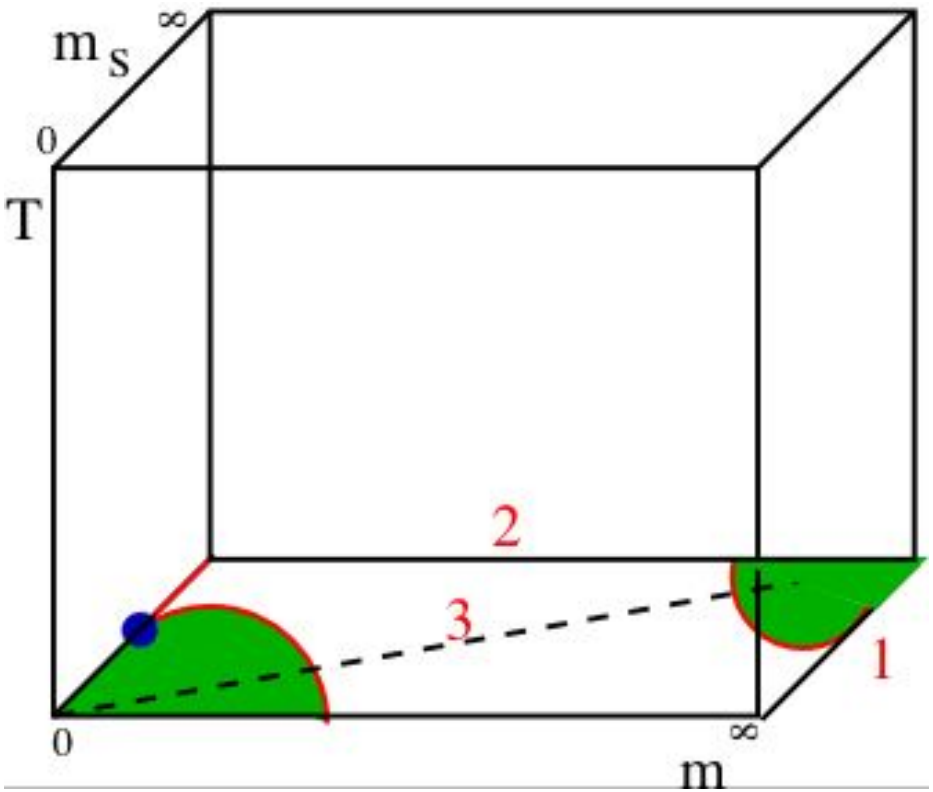

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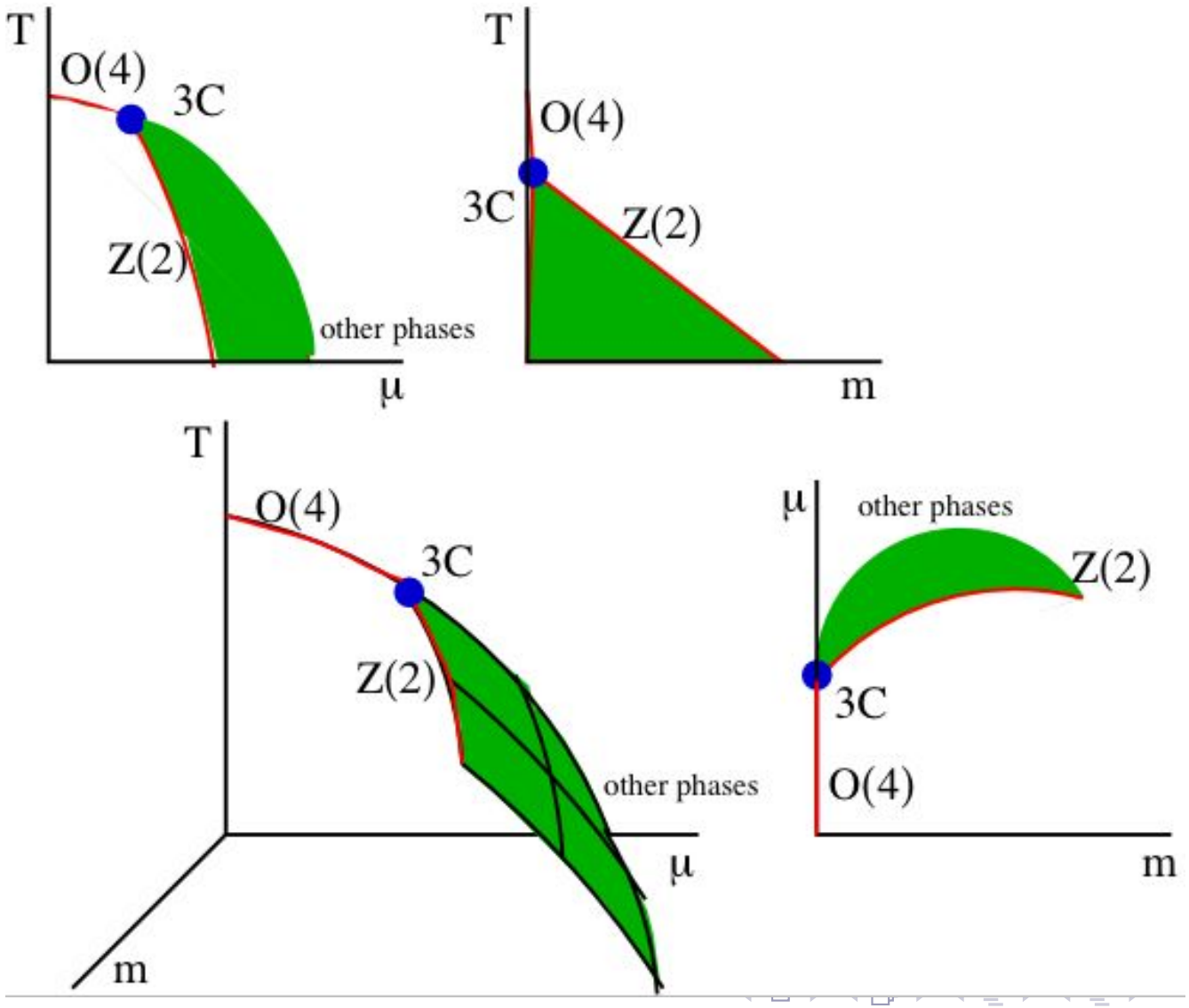
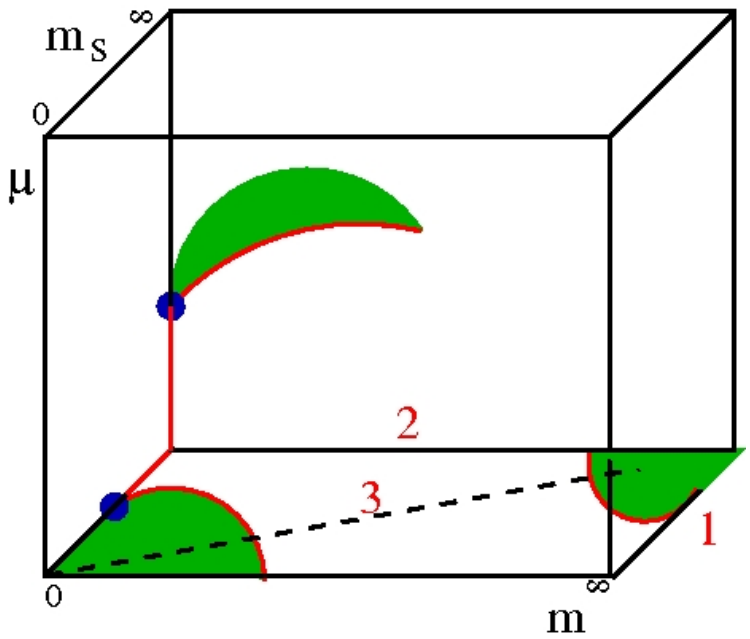
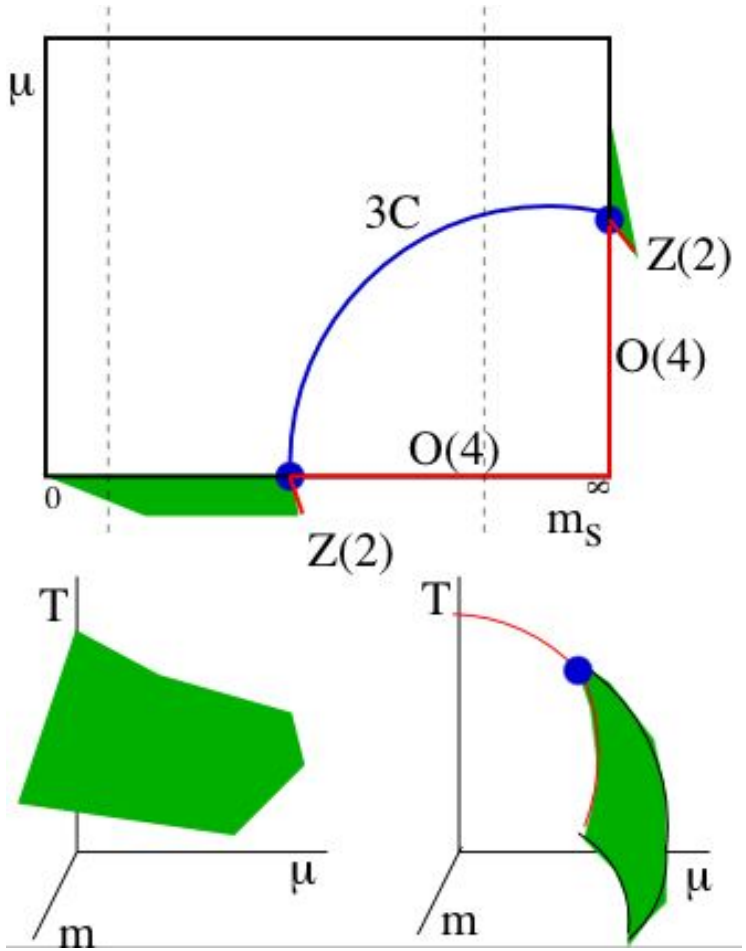


The flag diagram of QCD: some bits are known

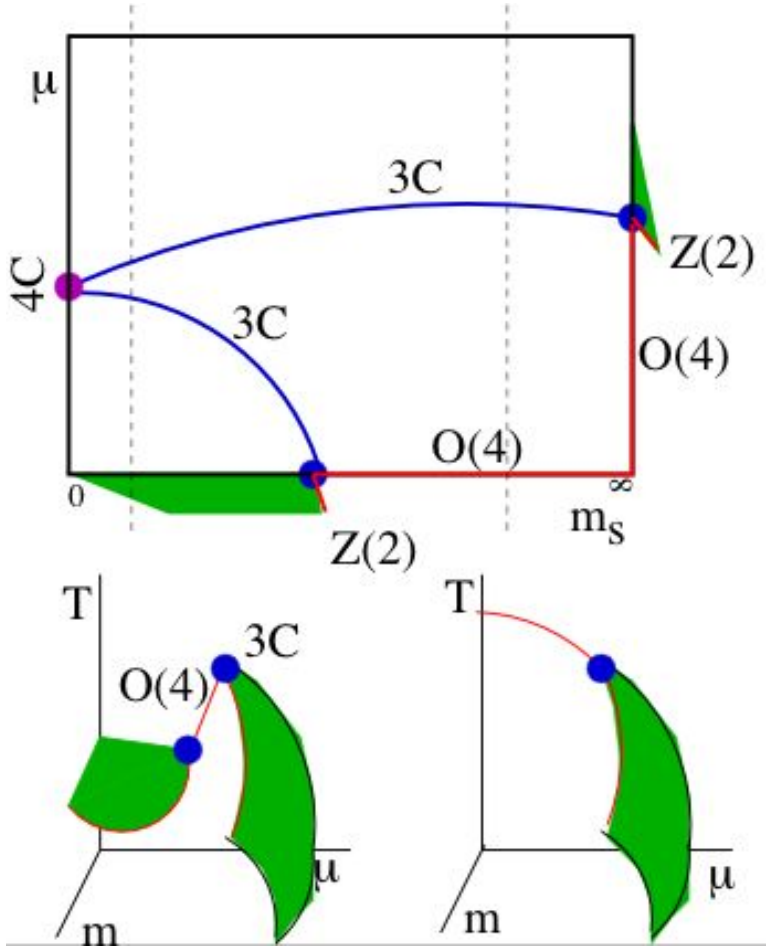


- Two planes in the flag diagram explored: $\mu_B = 0$, and $m_s = \infty$.
- Order parameter is 3-flavour chiral condensate ($N_f = 2 + 1$); 2-flavour condensate for $N_f = 2$.
- Light quark condensate remains an order parameter for $N_f = 2 + 1$. (Otherwise, strange quark condensate dissolves but chiral symmetry is still broken)
- Use this to stitch together the full phase diagram.

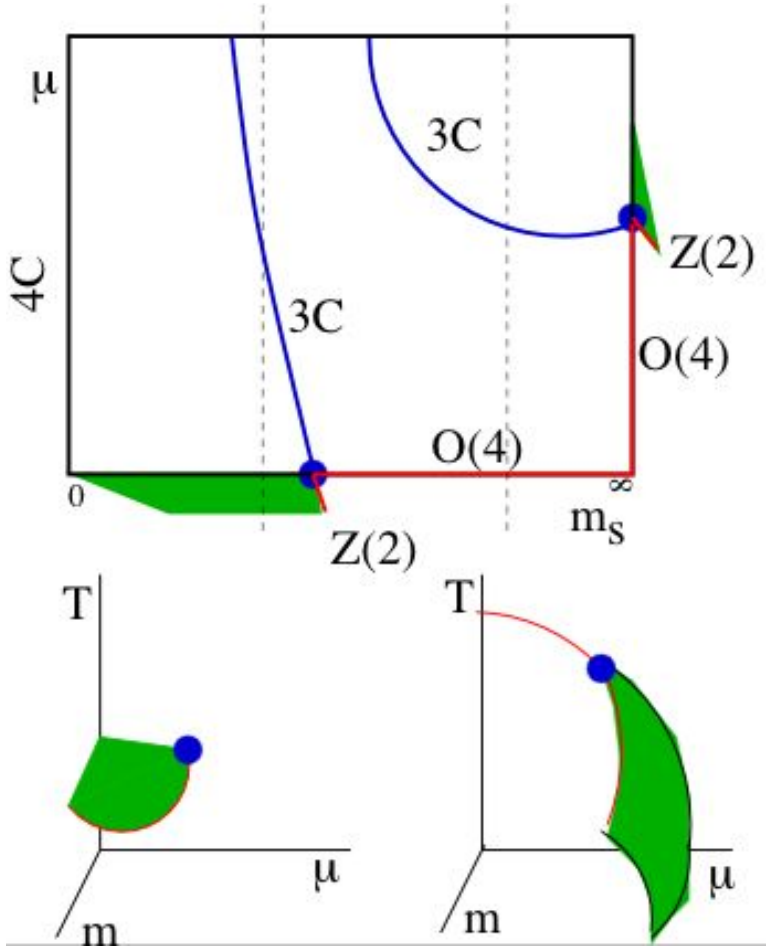
Possibility 1: the simplest



Possibility 2: ruled out



Possibility 3: needs search



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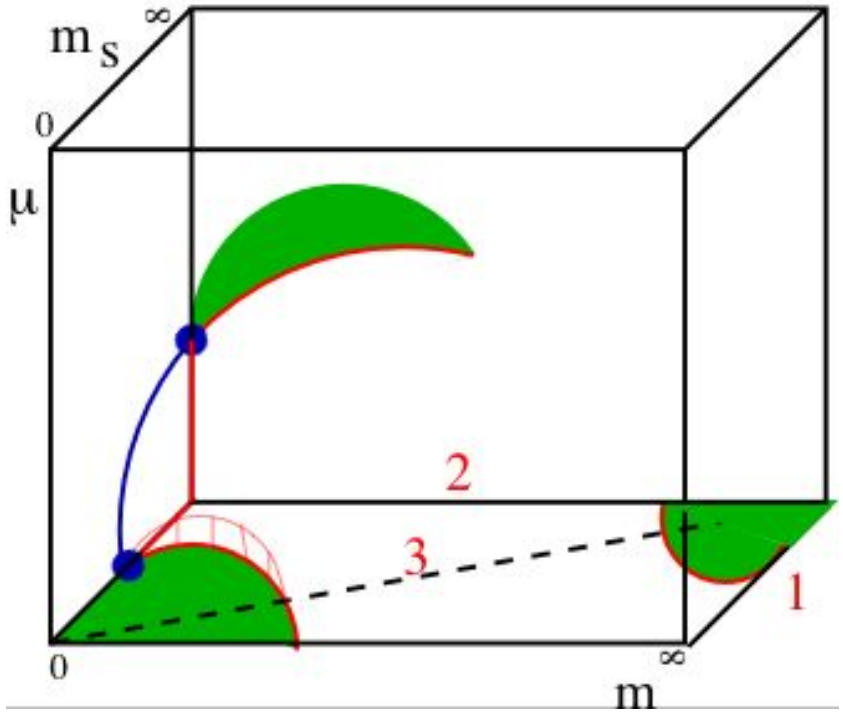

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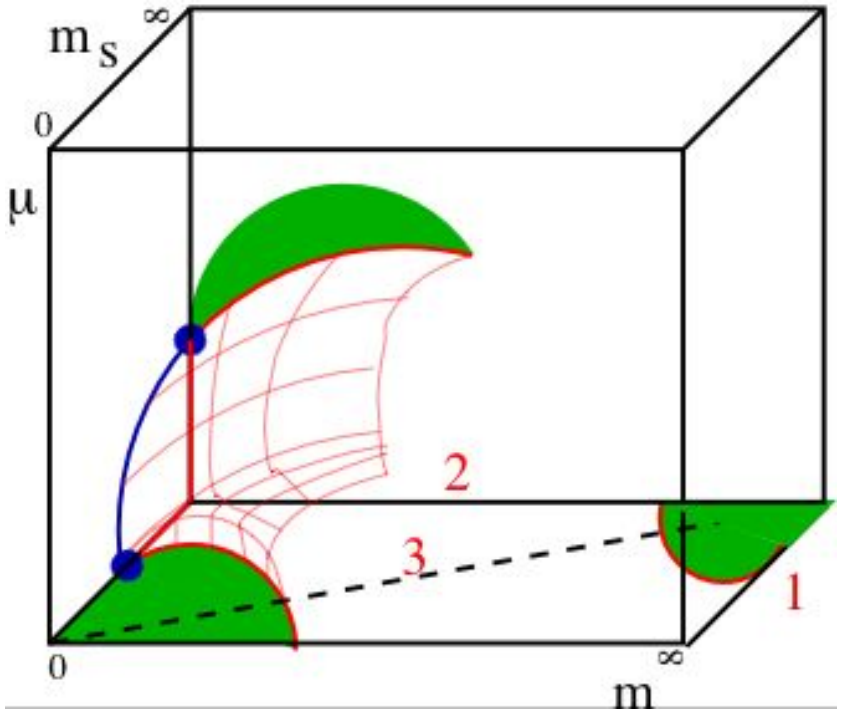

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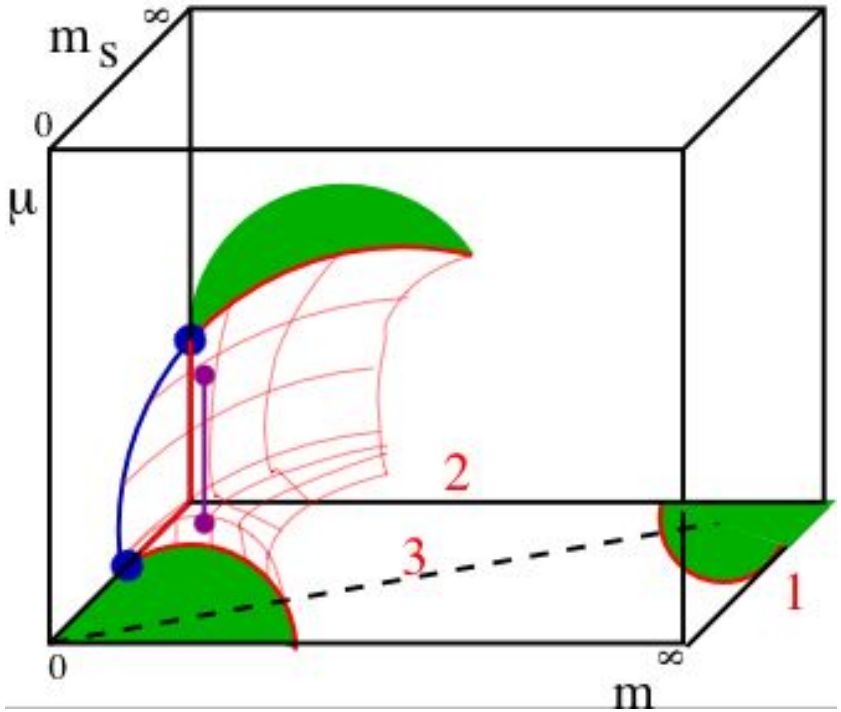

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$N_f = 2 + 1$

The phase diagram can resemble that of $N_f = 2$. Whether this is only qualitative (possibility 3 above) or quantitative as well (possibility 1 above) remains to be explored. An important element of the organization is the **tricritical strange quark mass**, m_s^{3C}. This is a hard computation since one needs $m = 0$.