First Order Scaling in N_f=2 QCD

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XQCD 2007

Based on: D'Elia, Di Giacomo, Pica, PRD72,114510(2005);
Cossu, D'Elia, Di Giacomo, Pica, arXiv:0706.4470 [hep-lat]

Outline

1. Order of the chiral transition from effective models
   - Effective model
   - Prediction for the order of the chiral transition
2. Previous Studies
3. Our Analysis of the $N_f = 2$ Chiral Transition
   - Strategy
   - Numerical results

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The idea

Suppose that the chiral transition is second order.

Find a simple model in the same universality class of N_f-flavor QCD.

1. must have the same symmetry breaking pattern (SBP).
2. must be constructed in terms of the relevant degrees of freedom.

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Construction of the model

The possible SBPs for the chiral transition ($m_q = 0$) are:

in the presence of the $U(1)_A$ anomaly:

$$U(1)_V \otimes SU(N_f)_L \otimes SU(N_f)_R \rightarrow U(1)_V \otimes SU(N_f)_V$$

if the anomaly disappears at the transition:

$$U(N_f) \otimes U(N_f) \rightarrow U(1)_V \otimes U(1)_A \otimes SU(N_f)_V$$

The relevant degrees of freedom are:

$$M_{ij}(x) = \langle \bar{\psi}_i^L(x) \psi_j^R(x) \rangle$$

The group $U(1)_A \otimes SU(N_f)_L \otimes SU(N_f)_R$ acts as:

$$M \rightarrow e^{i\alpha} V^\dagger M W$$

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If the axial anomaly is restored the effective lagrangean is:

$$\mathcal{L}_{eff} = \mathrm{Tr}(\partial_\mu M^\dagger)(\partial_\mu M) + r\mathrm{Tr}M^\dagger M + \frac{u_0}{4}\left(\mathrm{Tr}M^\dagger M\right)^2 + \frac{v_0}{4}\mathrm{Tr}\left(M^\dagger M\right)^2$$

If $U(1)_A$ is broken, additional terms are present:

$$\mathcal{L}_A = \mathcal{L}_{eff} + w_0\left(\det M + \det M^\dagger\right)$$

For $N_f = 2$ more terms are possible:

$$\mathcal{L}_2 = \frac{x_0}{4}\left(\mathrm{Tr}M^\dagger M\right)\left(\det M^\dagger + \det M\right) + \frac{y_0}{4}\left[(\det M^\dagger)^2 + (\det M)^2\right]$$

The effect of a mass term is modeled by the term:

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Expected order fot the chiral transition

At m_q=0:

	$SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V$	$U(N_f) \otimes U(N_f) \to U(1)_A \otimes SU(N_f)_V$
$N_f = 1$	F.O. / crossover	F.O. / $O(2)$
$N_f = 2$	F.O. / $O(4)$ / mean field	F.O. / $O(2) \otimes O(2)$
$N_f \geq 3$	F.O.	F.O.

At m_q>0 (the mass plays the role of a magnetic field) :

- if the transition were second order at m_q=0, an analytic crossover is expected.
- if it were first order, a first order transition line in the plane (T,m_q) is expected in a neighborhood of the chiral transition.

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(very) Incomplete summary of the existing literature

Lattice	m_L	Method	Conclusion	Ref.
$16^3 \times 4$	0.01,0.025	double peak	continuous?	Brown *et al.* '90
$(4-12)^3 \times 4$	0.0125,0.025	$(\chi_m)_{max}$, B_{min}	continuous?	Fukugita *et al.* '90
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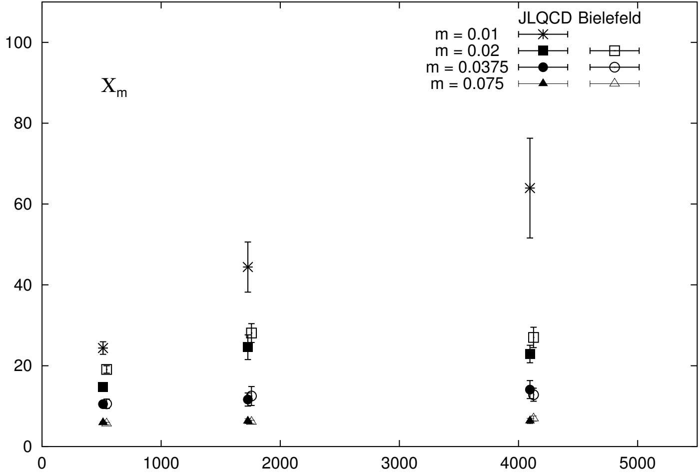
From: E. Laermann, Nucl.Phys.Proc.Suppl.63:114-125,1998



L_s^3

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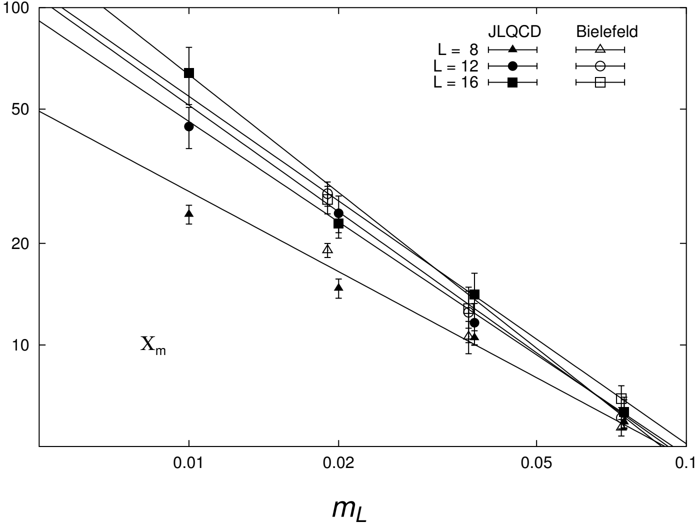

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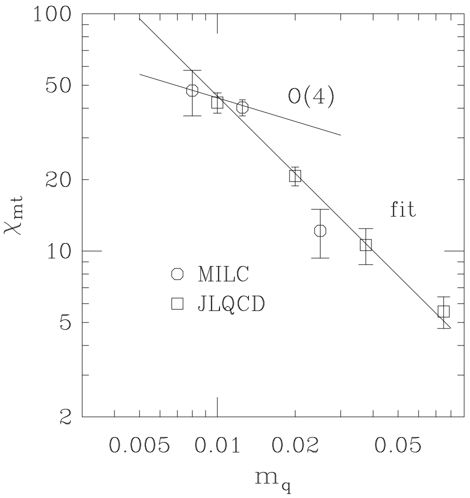
	O(2)	O(4)	MF	L=8	L=12	L=16
z_g	0.60	0.54	2/3	0.70(11)	0.74(6)	0.64(5)
				—	0.63(6)	—
z_m	0.79	0.79	2/3	0.70(4)	0.99(8)	1.03(9)
				0.84(5)	1.06(7)	0.93(8)
z_t	0.39	0.34	1/3	0.47(5)	0.81(9)	0.83(12)
				0.63(7)	0.94(12)	0.85(12)

Some results from [3] (MILC)

From: Bernard *et al.*, Phys.Rev.D61:054503,2000



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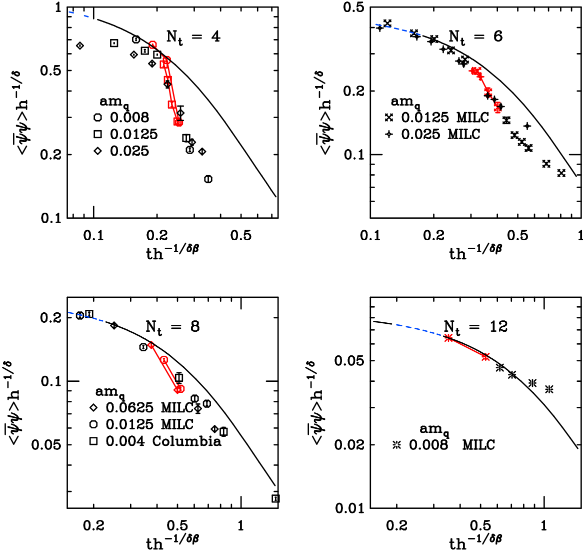
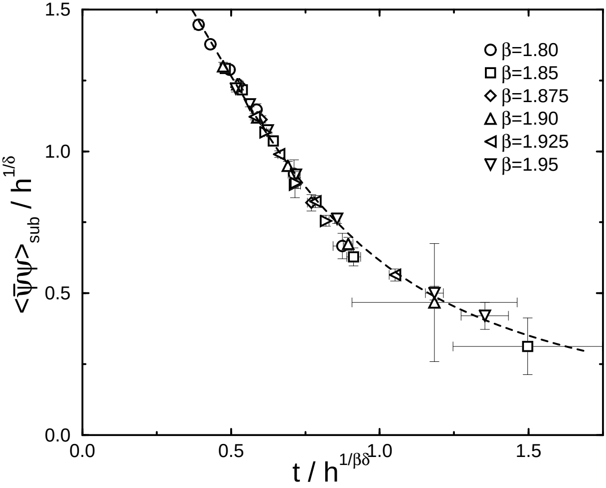


Some results from [4] (CP-PACS)

From: CP-PACS Coll. Phys.Rev.D63 (2001) 034502



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Critical Behavior

At the critical point the singular part of the free energy density $\mathcal{F}_s$ is an homogenous function of the relevant scaling variables. From the expression of $\mathcal{F}_s$ the scaling laws for thermodynamic quantities can be computed taking appropriate derivatives.

$$\mathcal{F}_s(\tau, m_L, L_s) = L_s^{-d}\mathcal{F}_s\left(\tau L_s^{y_t}, m_L L_s^{y_h}\right)$$

Critical Behavior

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At $m_L L_s^{y_h}$ fixed:

$$\mathcal{F}_s = L_s^{-d}\mathcal{F}_s\left(\tau L_s^{y_t}\right)$$

choosing y_h one can test different universality classes:
$y_h = 2.49$ for $O(4)$, $O(2)$;
$y_h = 3$ for 1st order.

Critical Behavior

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At $m_L L_s^{y_h}$ fixed:

$$C_V - C_0 = L_s^{\alpha/\nu} \phi_c(\tau L_s^{1/\nu})$$
$$\chi_m - \chi_0 = L_s^{\gamma/\nu} \phi_\chi(\tau L_s^{1/\nu})$$
$$\tau_c \propto L_s^{-1/\nu}$$

peaks $\propto$ power of L_s

widths $\propto L_s^{-1/\nu}$

T shift $\propto L_s^{-1/\nu}$

	y_t	y_h	ν	α	γ
$O(4)$	1.336(25)	2.487(3)	0.748(14)	-0.24(6)	1.479(94)
$O(2)$	1.496(20)	2.485(3)	0.668(9)	-0.005(7)	1.317(38)
MF	$3/2$	$9/4$	$2/3$	0	1
1^{st} *Order*	3	3	$1/3$	1	1

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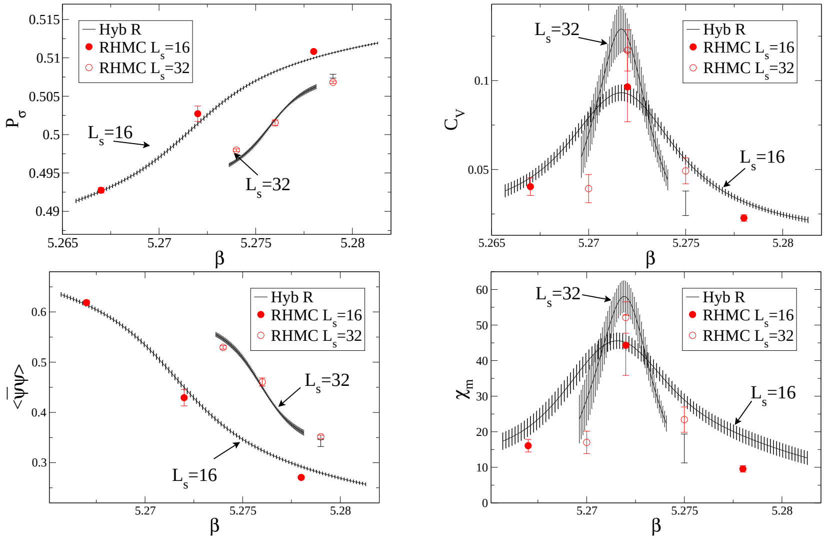
Monte Carlo Simulations - Parameters

- ⋆ 2 runs taking $m_L L_s^{y_h} = cost$ with $y_h = 2.49$ for $O(4)$-$O(2)$ and $y_h = 3$ for first order.
- ⋆ We used standard (non-improved) staggered fermions.
- ⋆ Lattice size: $L_s = 12, 16, 20, 24, 32 \quad L_t = 4$
- ⋆ bare quark masses: $m_L = 0.01335 - 0.153518$
- ⋆ Hybrid-R algorithm for the O(4) run; RHMC for the F.O. run
- ⋆ Trajectories: > 5000 for each β in the transition region

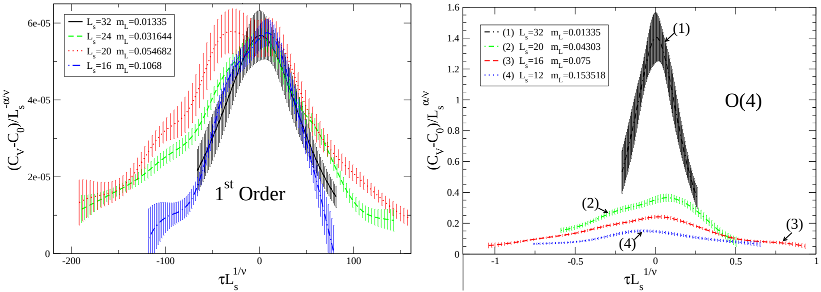
	O(4) run				First order run			
L_s	12	16	20	32	16	20	24	32
m_L	0.153518	0.075	0.04303	0.01335	0.1068	0.054682	0.031644	0.01335
# Traj.	22500	87700	14520	14500	90000	90000	90000	14500
$aL_s \cdot m_\pi^*$	11.9	11.0	10.0	8.9	12.0	11.2	10.8	8.9

* m_π taken from the parametrization by MILC Collaboration.

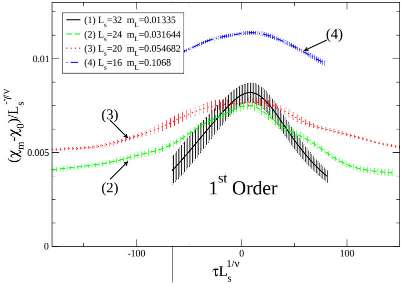
Sistematic errors of the Hybrid-R run

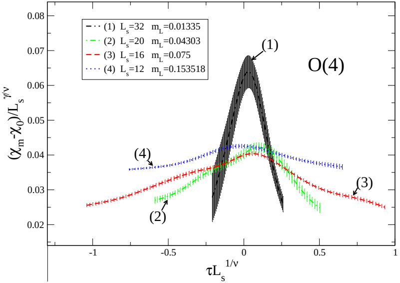


Scaling of C_V



Scaling of χ_m

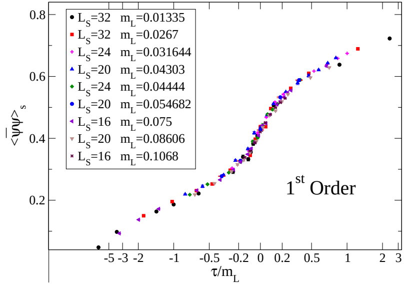


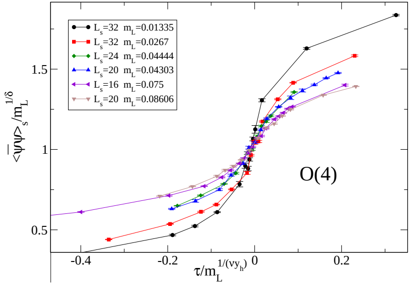


Magnetic Equation of State

Expected scaling law for the chiral condensate:

$$\langle \bar{\chi}\chi \rangle = m_L^{1/\delta} \phi_M(\tau / m_L^{1/\nu y_h})$$





Conclusions

- Staggered fermions at $L_t = 4$ give a clear indication for a first order chiral transition in $N_f = 2$ QCD, in agreement with all the literature where staggered fermions were used,

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- but in contrast with improved Wilson fermions at $L_t = 4$ seems to indicate an $O(4)$ transition instead (MF is also compatible).
- Other results also exist which indicate a second order tansition in a less direct way (e.g. Kogut and Sinclair '06).

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Conclusions

- Staggered fermions at $L_t = 4$ give a clear indication for a first order chiral transition in $N_f = 2$ QCD, in agreement with all the literature where staggered fermions were used, ...
- but in contrast with improved Wilson fermions at $L_t = 4$ seems to indicate an $O(4)$ transition instead (MF is also compatible).
- Other results also exist which indicate a second order tansition in a less direct way (e.g. Kogut and Sinclair '06).

We still lack a solid "proof" of the order of the chiral transition.