Finite temperature large N QCD

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QCD in extreme conditions

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Large N QCD in $d = 3$

- We will focus on large N QCD in $d = 3$ to discuss the deconfinement and related transitions.

- We will consider the 't Hooft limit:
  - $b = \frac{1}{g^2 N}$ fixed; $g \to 0$, $N \to \infty$.
  - Fermions will be ignored: Finite number of flavors of fermions in the fundamental representation are naturally quenched in the large N limit.

- We will work on a $L_t \times L^2$ lattice with $L_t \leq L$.

- We will use the tadpole improved coupling, $b_I = be(b)$ and define physical lengths of the lattice as $l = L/b_I$ and $l_t = L_t/b_I$.

Bulk transition

There is a bulk transition on the lattice at $b_B \approx 0.43$ that does not depend upon L or L_t.

- This is a lattice transition that does not have a continuum counterpart.
- The eigenvalue distribution of the plaquette operator develops a gap at b_B that grows for $b > b_B$.
- The continuum theory is always in the phase where the eigenvalue distribution of the plaquette operator has a gap.
- We will always consider $b > b_B$.

Polyakov loops

The action is given by $S = \frac{bN}{2}\sum_{n,i\neq j} Tr[U_{ij}(n) + U^{\dagger}_{ij}(n)]$

where the plaquette operator is $U_{ij}(n) = U_i(n)U_j(n+\hat{i})U^{\dagger}_i(n+\hat{j})U^{\dagger}_j(n)$

The Polyakov loop operator is $\mathcal{P}_{t,x,y}(n) = \prod_{m=1}^{L_{t,x,y}} U_i(n + m\hat{i})$.

$\mathcal{P}_t(n)$ transforms under $U_1(1, n_2, n_3) = e^{i\frac{2\pi k}{N}} U_1(1, n_2, n_3)$ but the action is invariant.

This is called the Z_N symmetry. Same holds for $\mathcal{P}_{x,y}(n)$.

$\langle Tr\mathcal{P}_{t,x,y}(n)\rangle$ is zero unless the Z_N symmetry is spontaneously broken in the corresponding direction.

The Z_N symmetry becomes $U(1)$ in the large N limit and there is no dependence on the size of the direction where it is not broken – Continuum reduction.

Spontaneous breaking of Z_N^3 symmetries

Use $P_{t,x,y} = \frac{1}{2L_tL^2}\sum_n 1 - \left|\frac{1}{N}Tr\mathcal{P}_{t,x,y}(n)\right|^2$

and $\bar{P}_{t,x,y} = \langle P_{t,x,y}\rangle$.

to study spontaneous symmetry breaking of Z_N^3.

- $\bar{P}_{t,x,y} = \frac{1}{2}$ – Uniform distribution of the eigenvalues of $\mathcal{P}_{t,x,y}(n)$ – Z_N is not broken.
- $0 \leq \bar{P}_{t,x,y} < \frac{1}{2}$ – Distribution of the eigenvalues of $\mathcal{P}_{t,x,y}(n)$ has a peak somewhere in the unit circle – Z_N is broken.

Continuum large N QCD is said to be in the Xc for $0 \leq X \leq 3$ if the $U(1)$ symmetries in the X directions are broken.

The 0c phase – confined phase

- All three $U(1)$ symmetries remain unbroken in this phase. Therefore, it is the confined phase.

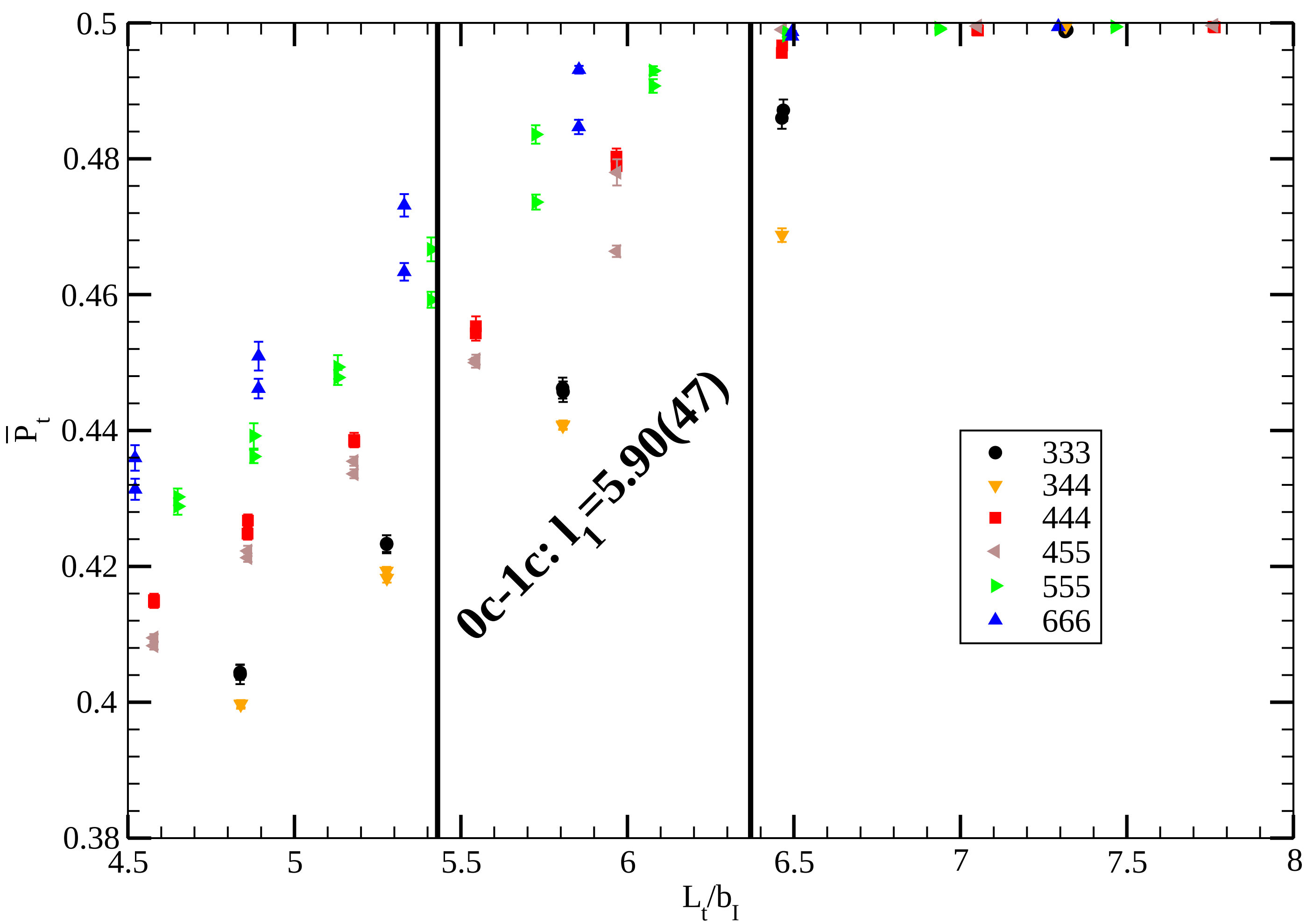
- The argument of Eguchi and Kawai hold and there is no dependence on L or L_t as long as the coupling, b, keeps it in the 0c phase.

- In particular, there is no dependence on L_t/L and this implies no dependence on the temperature in the confined phase.

Transition from the 0c to 1c phase

- Fix N, L and L_t with $L_t \leq L$. Find the coupling $b_1(L, L_t)$ such that the Z_N in the L_t direction starts to break but it remains unbroken in the other two directions.
  - If $L = L_t$, then call L_t as the direction where the Z_N is broken.
- $b_1(L, L_t)$ will have a limit as $N \rightarrow \infty$.
- $b_1(L, L_t)$ will be independent of L. This is independence in the length of the unbroken directions.
- $b_1(L_t)$ goes to infinity as L_t goes to infinity such that $l_1 = L_t / b_{1I}(L_t)$ will have a finite limit.
- l_1 is the critical size associated with the 0c to 1c transition.
  - $l_t > l_1$ is in the 0c phase and $l_t < l_1$ is in the 1c phase.
  - l_t is independent of $l \geq l_t$.
- $t_c = \frac{1}{l_1}$ is the deconfining temperature and $t_c = 0.86(7)\sqrt{\sigma}$.

The 0c to 1c transition



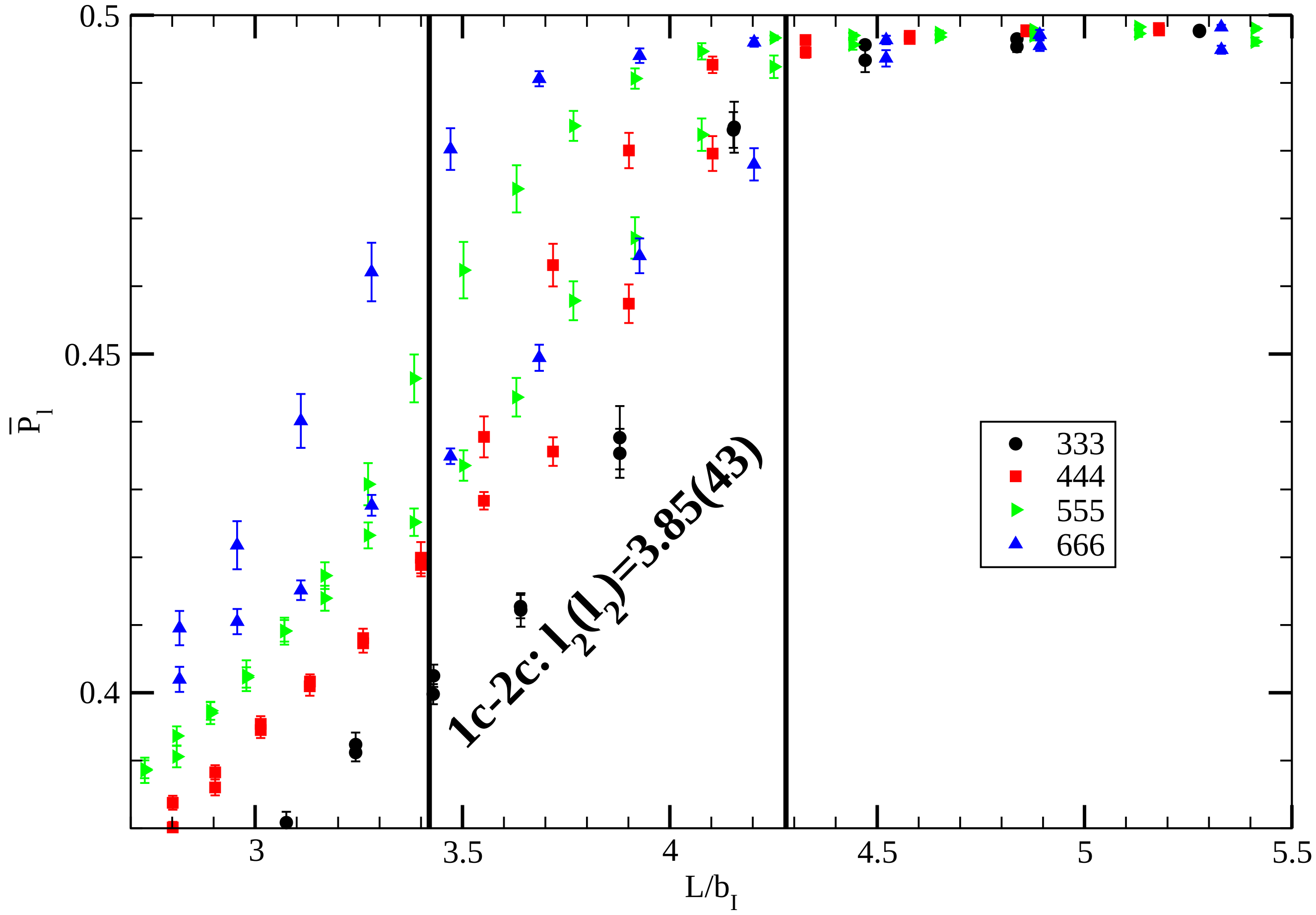
The 1c phase – deconfined phase

- Large N QCD on a $l_t \times l^2$ box with $l \geq l_t$ and $l_t < l_1$ is in the deconfined phase.
- There is no dependence on l but there is a dependence on l_t. Note that $\frac{t}{t_c} = \frac{l_1}{l_t} > 1$.
- What happens as the temperature increases to infinity? Is the theory always in the deconfined phase for all $l > l_t$ and $0 \leq l_t \leq l_1$?
- The answer is NO. The theory sometime goes into a $2c$ phase.

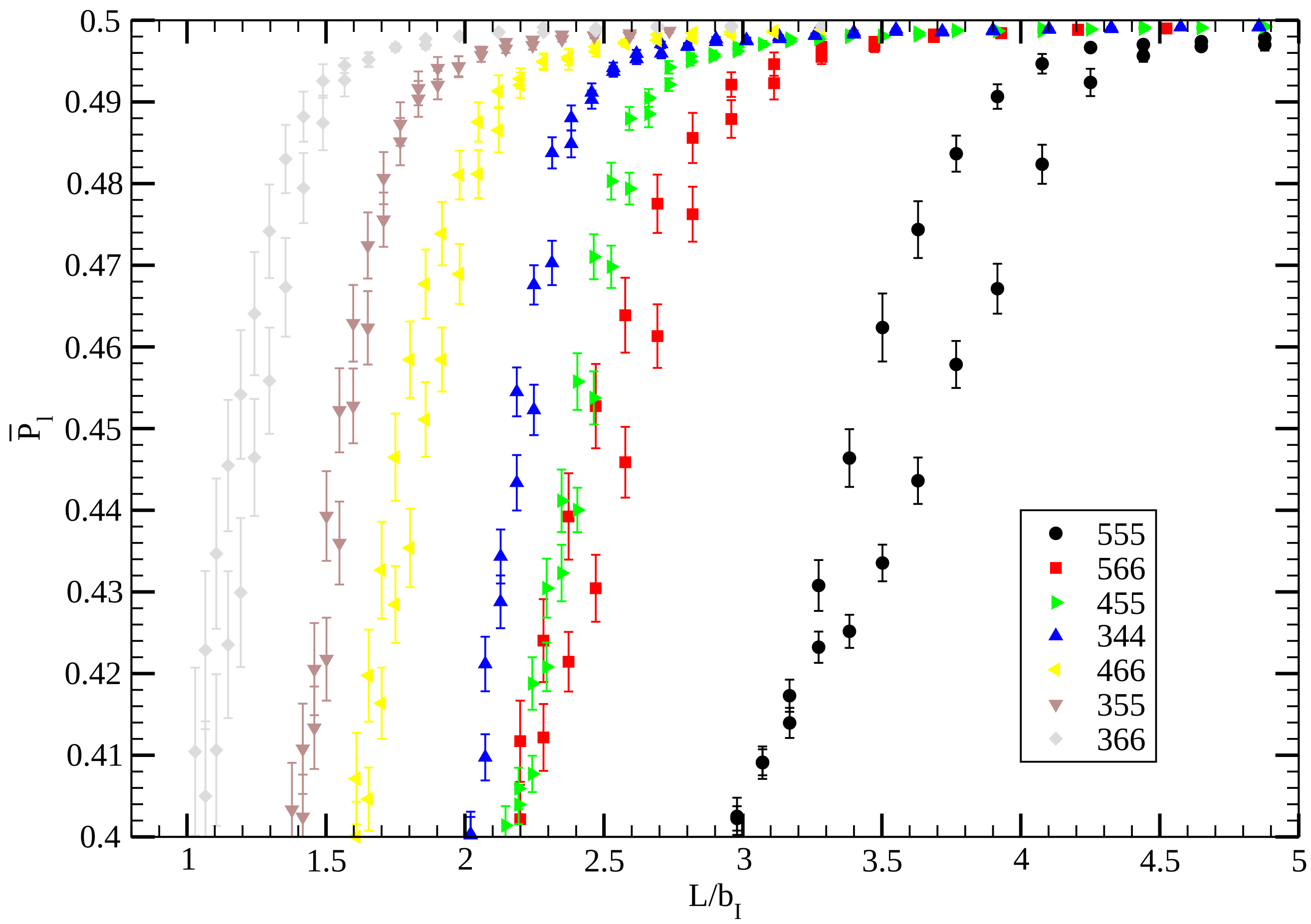
Transition from the 1c to 2c phase

- The phase transition from 1c phase to 2c phase occurs when
  - $l_t < l_1$
  - $l = l_2(l_t)$.
- This transition does not occur unless $l_t \leq 3.85(43)$. That is to say, we have $l_2(3.85) = 3.85$ and this is the transition on symmetric torus $l = l_t$.
- There is always a transition from 1c to 2c as long as $l_t \leq 3.85$ and the location of the transition is numerically estimated to be $l_2(l_t) = 0.56 + 1.08 l_t - 0.059 l_t^2$. In particular, $l_2(0) = 0.56 > 0$.

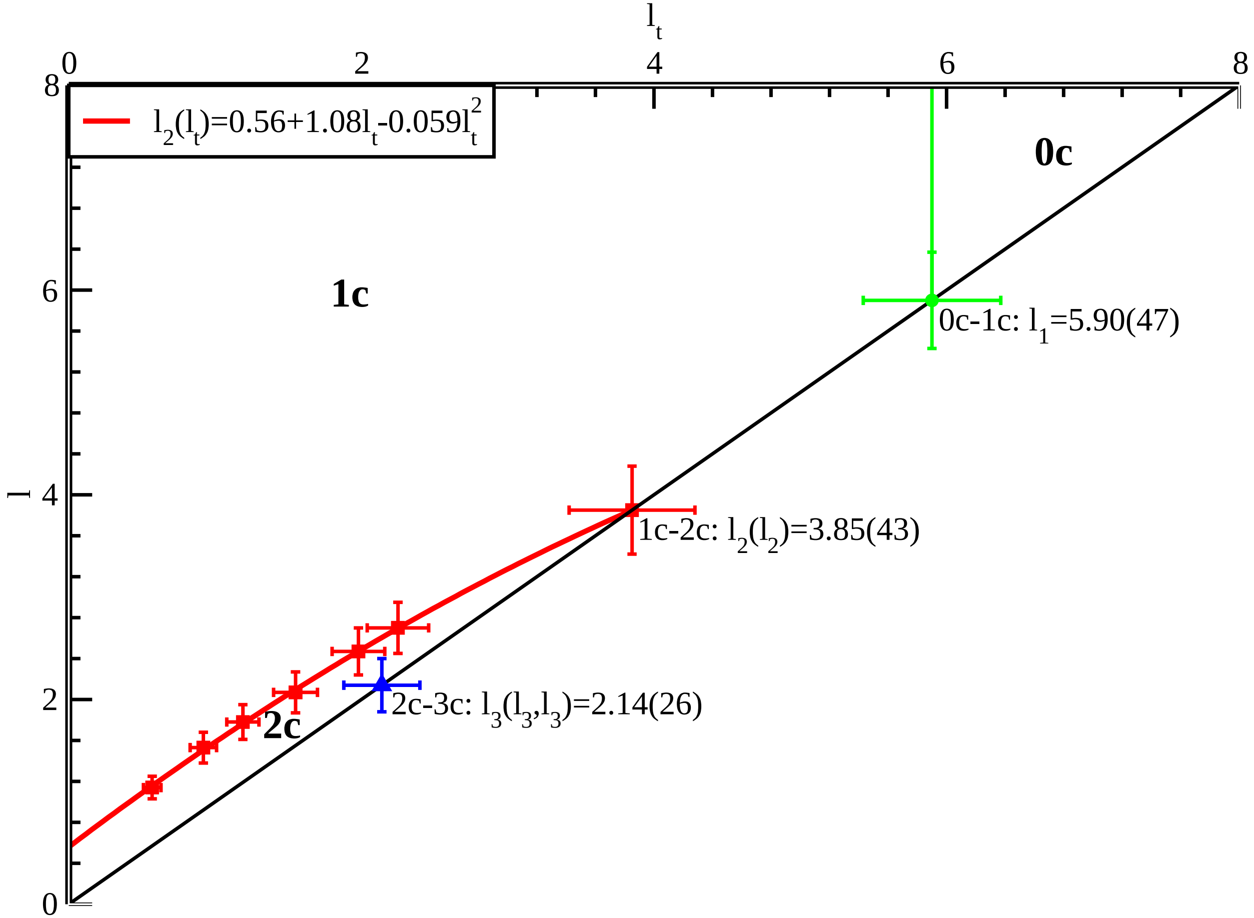
The 1c to 2c transition ($l = l_t$)



The 1c to 2c transition ($l > l_t$)



The phase diagram in the (l, l_t) plane



The 2c phase

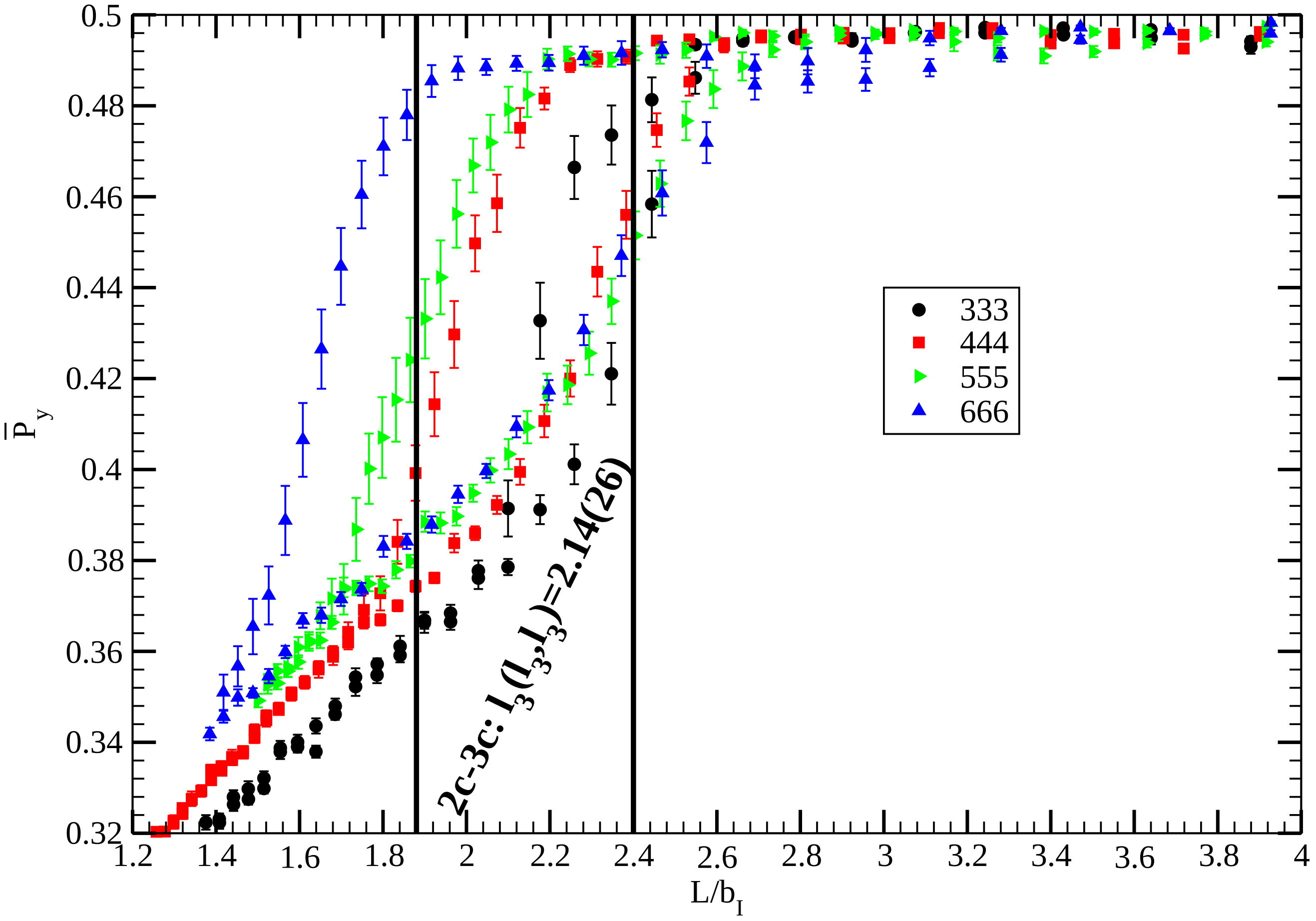
We have one direction where the $U(1)$ symmetry is not broken and there is no dependence on the size of this direction.

- We can interpret the unbroken direction as time and our results in the 2c phase are for an infinite extent in Euclidean time.
- The two broken directions are small in size and therefore we are describing large N QCD in a small two dimensional box of size $l \times l_t$.
- The theory goes into a high temperature phase when l for a fixed l_t is reduced further. This is the 3c phase.

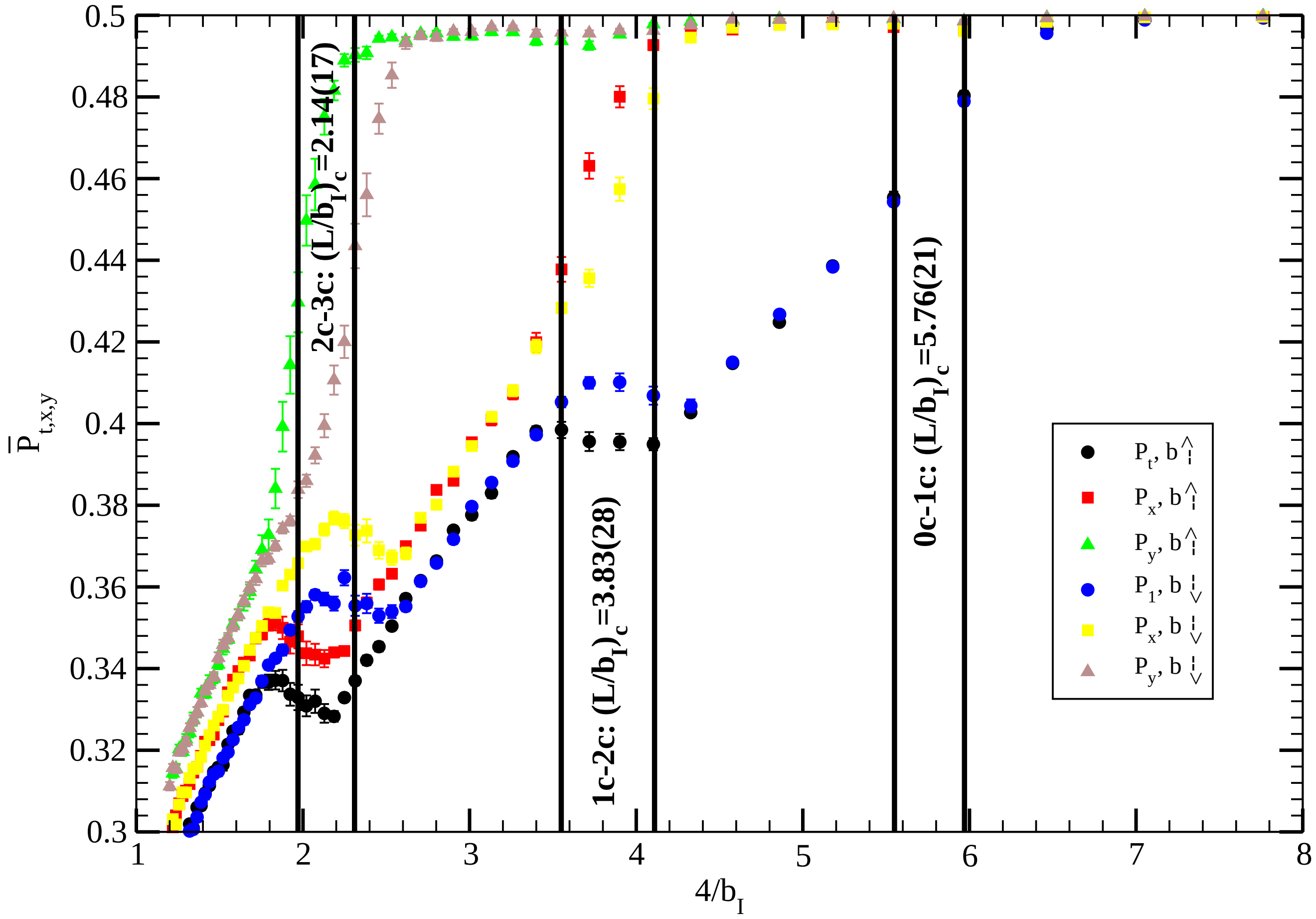
The transition from 2c phase to 3c phase

- The phase transition from 2c phase to 3c phase was numerically investigated only for the case of $l = l_t$.
- A complete investigation of this transition has to be done on a $l_t \times l_x \times l_y$ lattice with $l_t \leq l_x \leq l_y$.
- The transition will occur for some value of l_y which will depend on l_t and l_x. The critical size will therefore be $l_3(l_t, l_x)$.
- This transition will only occur if $l_t \leq l_x \leq 2.14(26)$.
- It is possible to investigate this transition using perturbative techniques for $l_x, l_t \to 0$ but one needs to account for the zero modes in the x and t directions.

The 2c to 3c transition with $l = l_t$



The cumulative picture on 4^4 lattice



Phase transitions on $L = L_t$ lattices

- There is good evidence that the rotational symmetry in the broken directions are preserved in the 2c and 3c phase.
- There is a good evidence for a hysteresis loop in the transition from 1c to 2c phase also from the 2c to the 3c phase.
- Why do we expect the 1c to 2c phase transition to be first order on a l^3 box?
  - Two loops are at $\frac{1}{2}$ and one loop has a value of $p < \frac{1}{2}$ in the 1c phase just before the transition.
  - One loop is at $\frac{1}{2}$ and two loops have a value of $q < \frac{1}{2}$ in the 2c phase just after the transition.
  - If $p = q$, the loop that just went through the transition will show a jump – First order transition.
  - Since all quantities will most likely show a jump across a first order transition, it is likely that $q \neq p$.
- Same logic holds for the 2c to 3c transition on a l^3 box.
- Numerical picture supports this argument.
- The two broken loops in the 2c phase do not have the same value if $l_t < l$.

Conclusions

- Large N gauge theories on a three dimensional continuum torus exhibit a rich phase structure.
- There are two more phases in addition to the confined and deconfined phase. These are QCD in a small two dimensional box at low and high temperatures.
- A similar cascade of phases also exist on a four dimensional torus.
- Chiral symmetry is spontaneously broken in the 0c phase and restored in the 1c phase in four dimensions.
  - Chiral condensate is independent of the temperature in the confined phase.
  - Therefore the chiral transition is first order.
- Massless pions exist in finite volume in the confined phase with massless quarks.
- Wilson loops undergo critical behavior as a function of their area in the confined phase.
  - This is the physical weak to strong coupling transition.
  - The universal behavior associated with this transition is the same in $d = 2, 3, 4$. This is called the Durhuus-Olesen transition.