QCD Matter above Tc

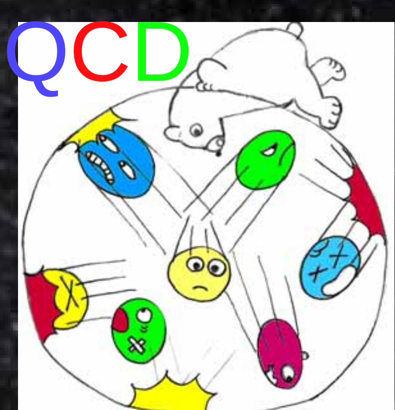
Atsushi NAKAMURA

RIISE, Hiroshima University

5th International Workshop on

Extreme QCD

Frascati, Aug.6-8, 2007



Collaboration with

- R. Gupta and S. Sakai
- M. Chernodub and V. Zakharov
- Y. Nakagawa, T. Saito, H. Toki and D. Zwanziger
- M.Hamada, M.Yahiro and H.Kono

Trial to construct

A Picture of QCD Matter above Tc

Contents

$$T \geq T_c$$

Temperature exceeds Tc

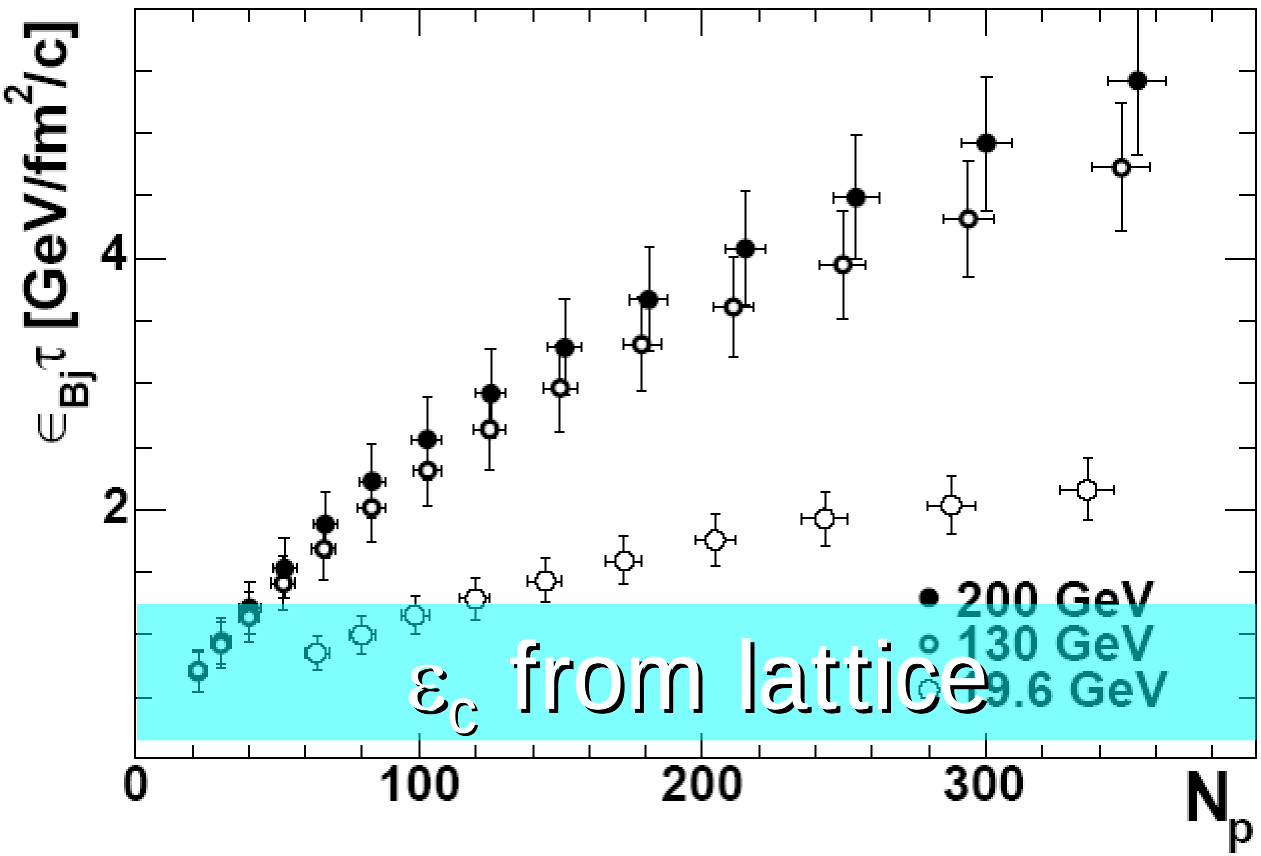
at RHIC, and probably at SPS

(and surely will exceed at LHC)

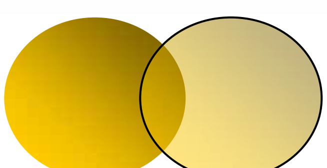
$$\varepsilon_{Bj}(\tau) = \frac{\langle m_T \rangle}{\tau \pi R^2} \frac{dN}{dy}$$

Bjorken('83)

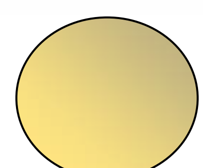
τ: proper time
y: rapidity
R: effective transverse radius
m_T: transverse mass



If we take
τ=1fm/c

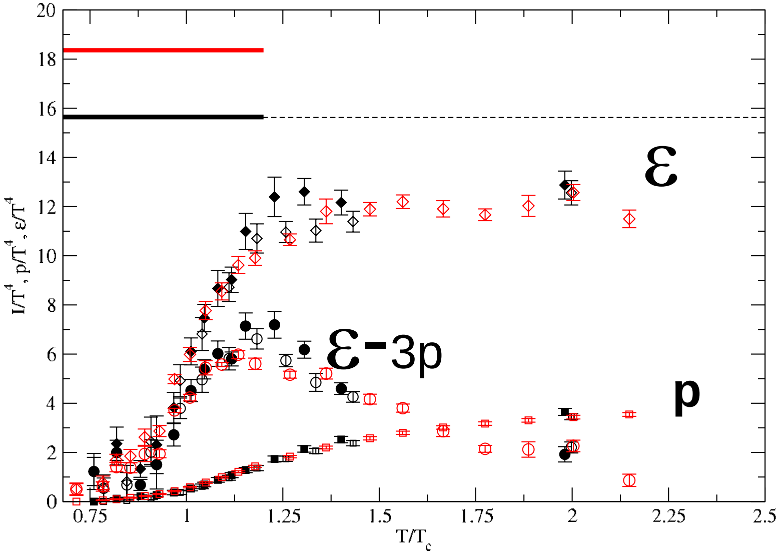


PHENIX('05)



$$T = 1/N_t a_t$$

$$a_t \to 0 \text{ (continuum limit)} \quad N_t \to \infty$$



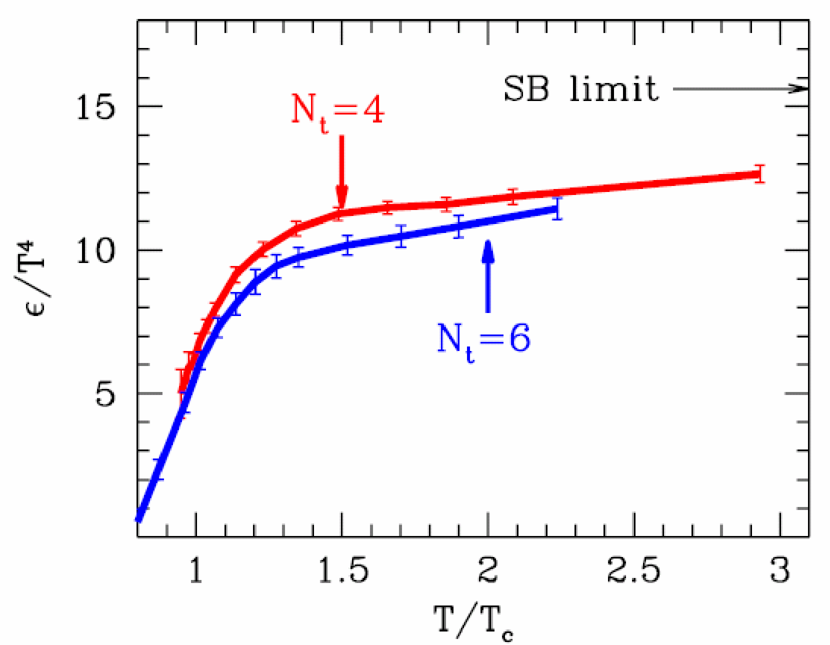
MILC Collaboration, Nf=2+1

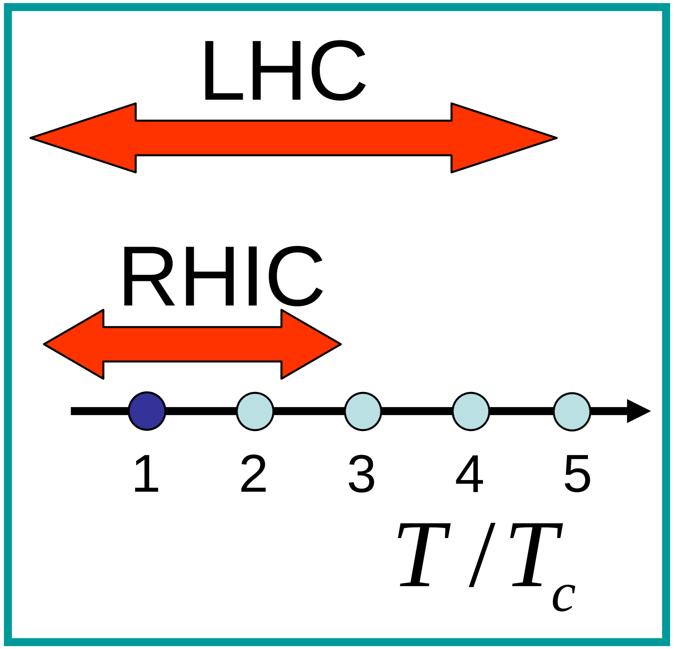
hep-lat/0509053

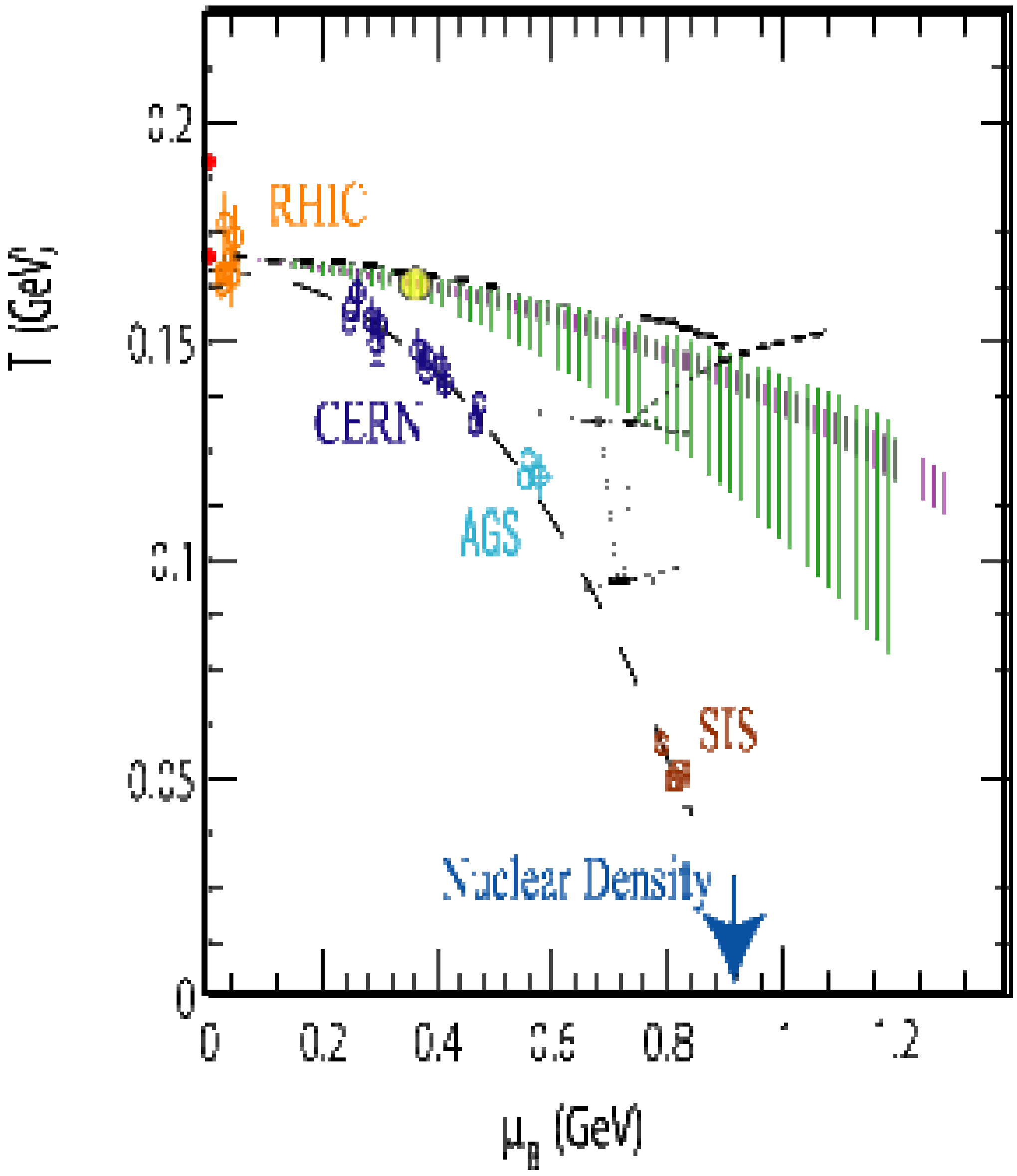
Red Nt=4 **Black Nt=6**

Y.Aoki et al.,

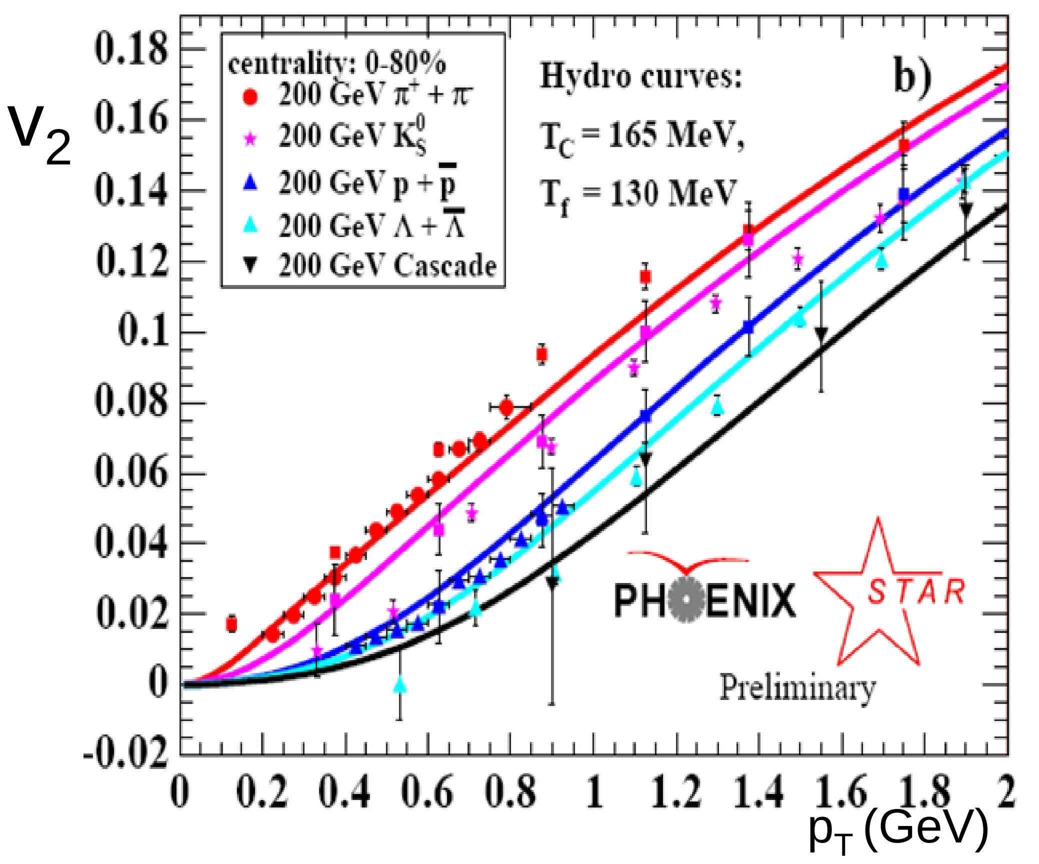
hep-lat/0510084




LHC
RHIC
1
2
3
4
5
T / T_c

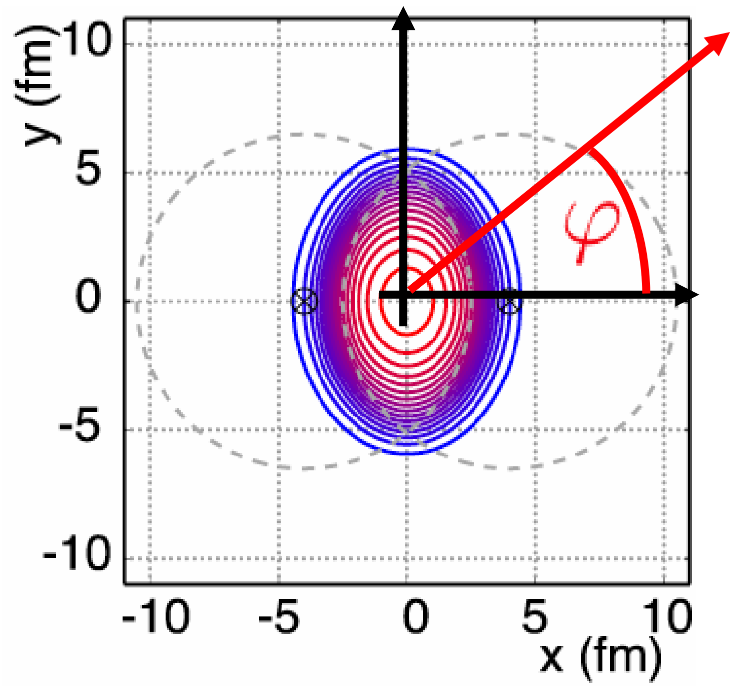

RHIC
CERN
AGS
SIS
Nuclear Density
T (GeV)
μ_B (GeV)
0
0.05
0.15
0.2
0.2
0.4
0.8

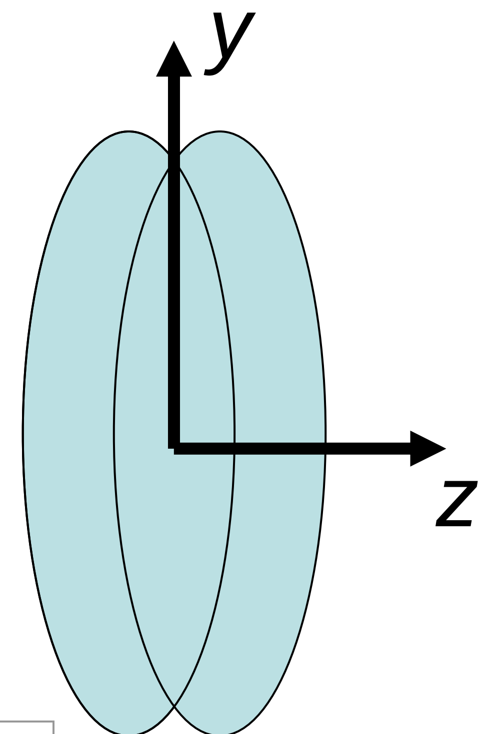
Like a Fluid?

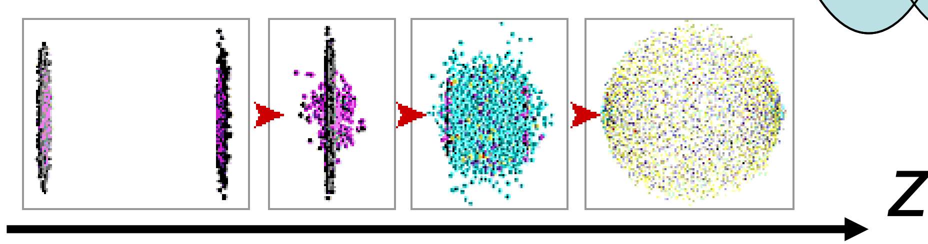


Lines:
Hydrodynamics calc.
with QGP type EoS.

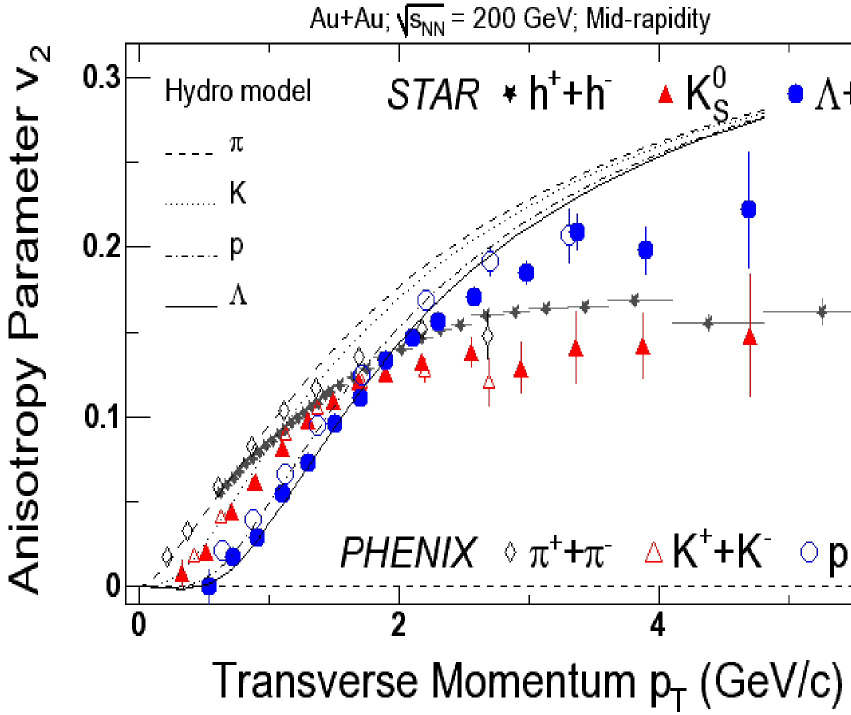
Collective Flow
(which does no exist if the matter is a gas)



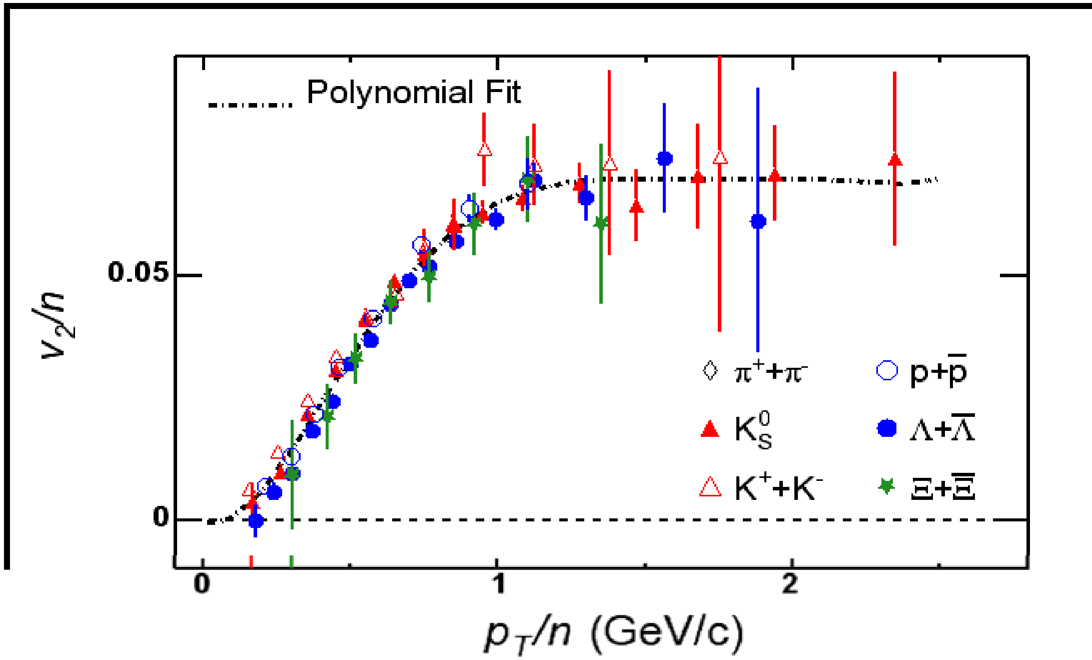




$$\frac{dN}{p_T dp_T dy d\varphi}(p_T, \varphi; b) = \frac{dN}{2\pi p_T dp_T dy}\left(1 + 2v_2(p_T; b)\cos(2\varphi) + ...\right)$$



Quarks flow !



n=2 for mesons

n=3 for baryons !

Matter interacts strongly with Partons.

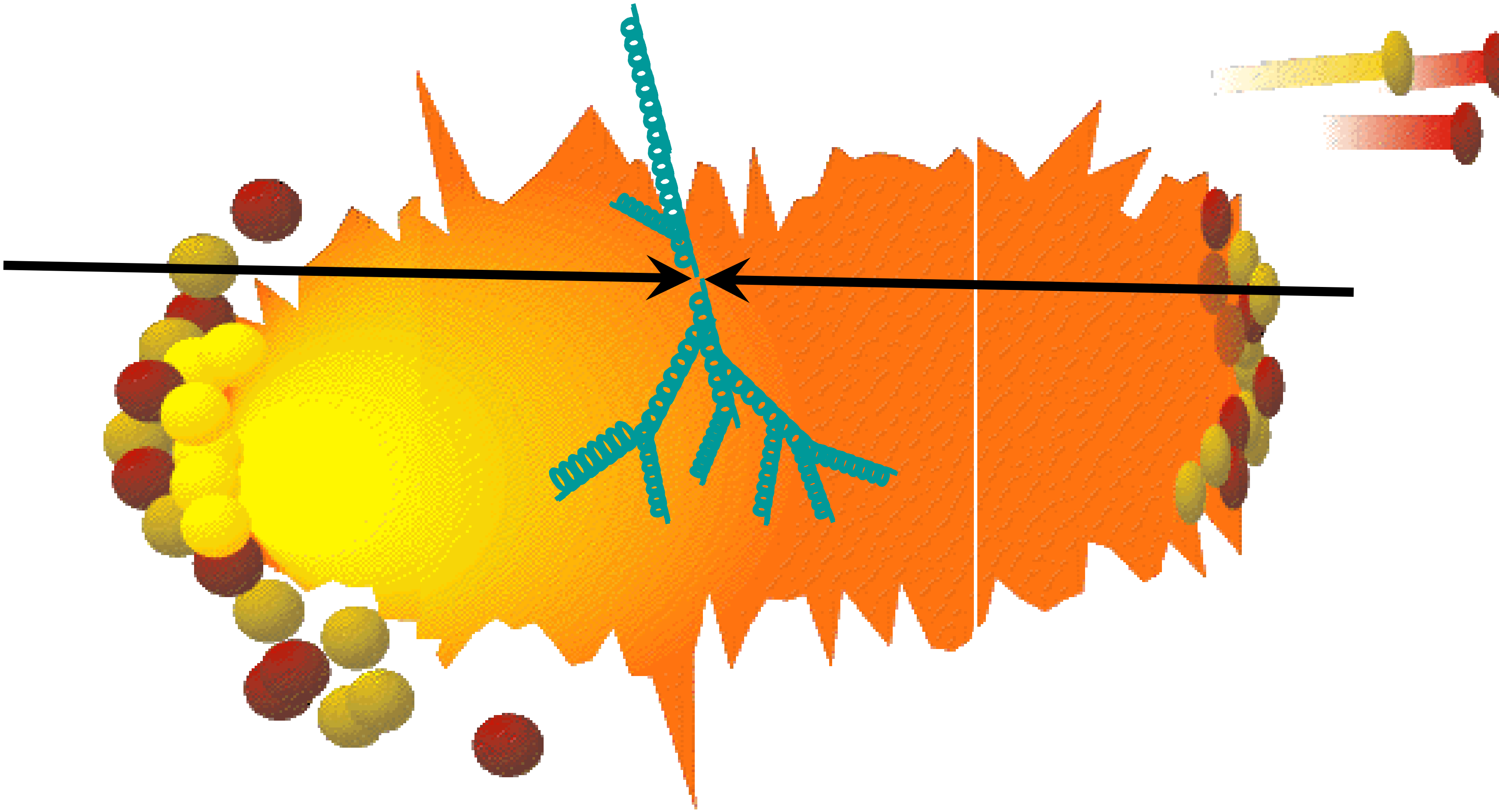
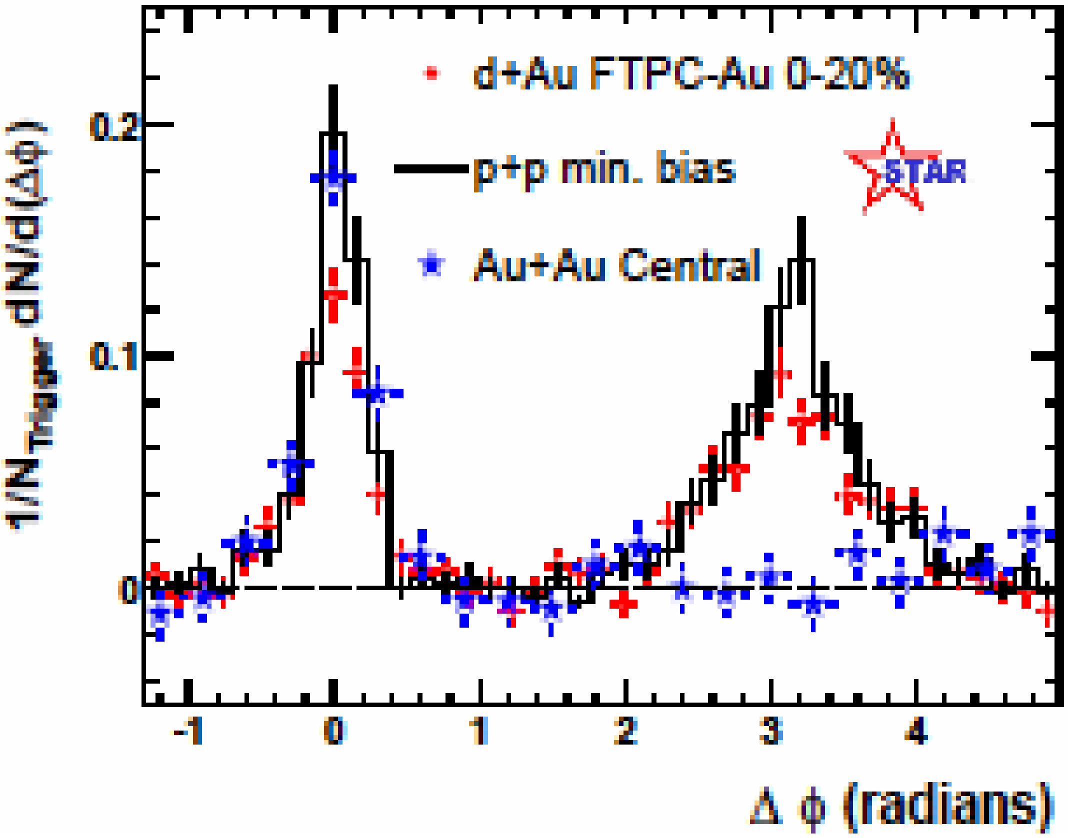
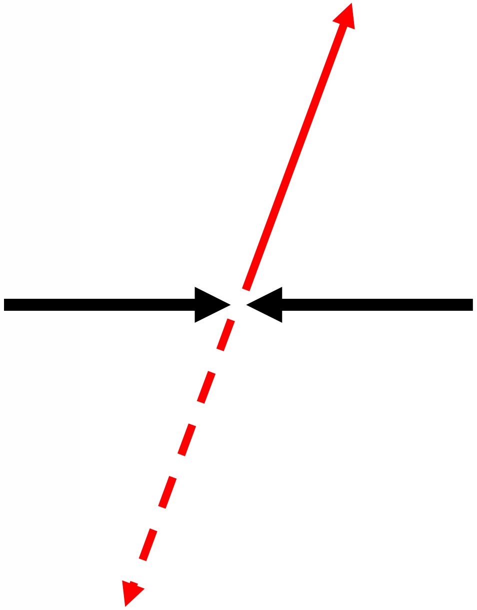


Figure stolen from Chujo's talk

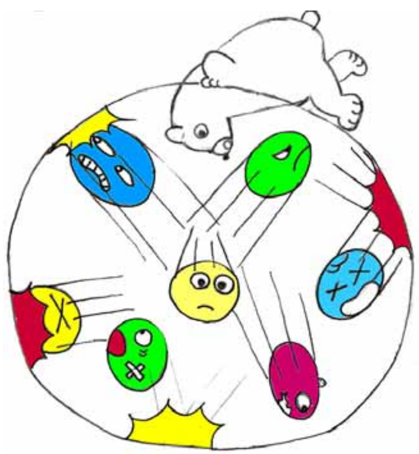
Jet Quenching



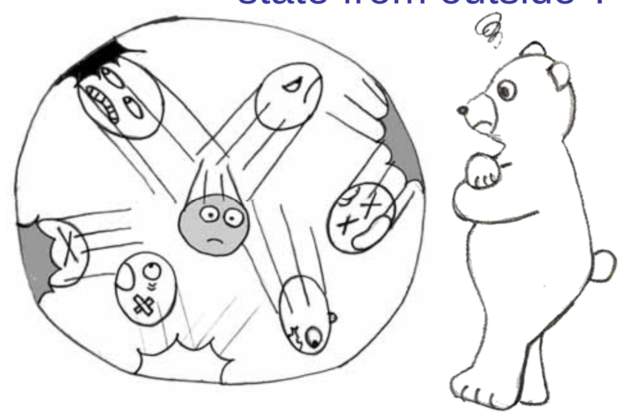


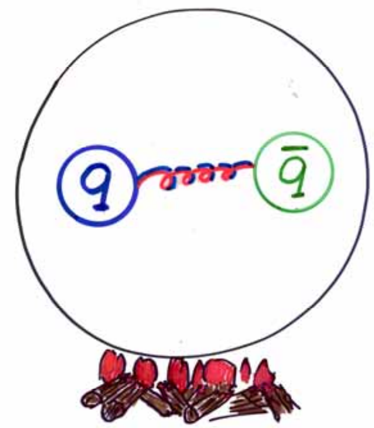
いない！

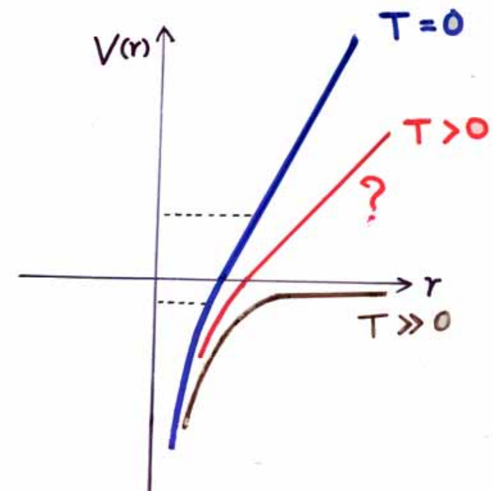
Confinement/Deconfinement – Simple Picture



I can see only a colorless state from outside ?







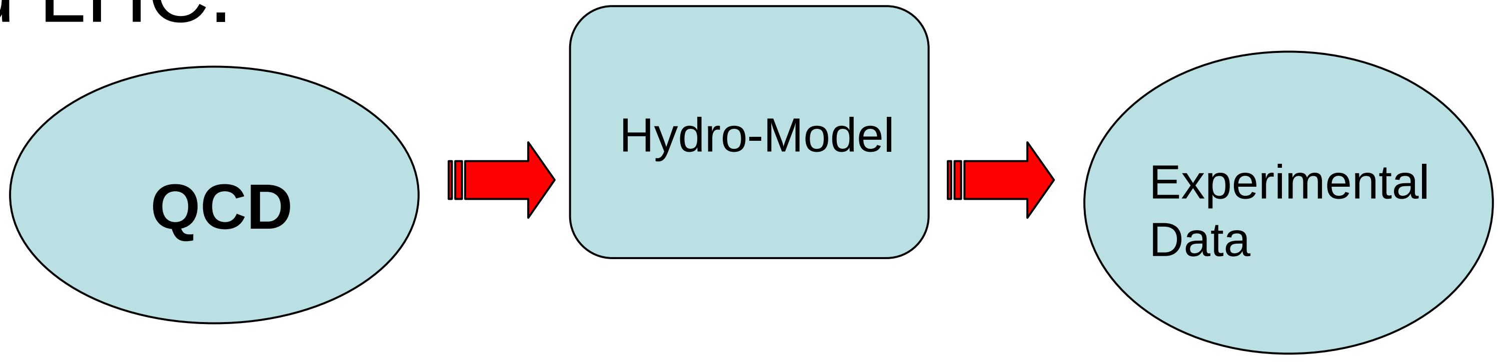
Confinement Potential is “screened” at finite temperature.

Deconfinement

Free Quark Gluon Gas

Transport Coefficients

- A Step towards Gluon Dynamical Behavior.
- They can be (in principle) calculated by a well established formula (Linear Response Theory).
- They are important to understand QGP which is realized in RHIC (and CERN-SPS) and LHC.



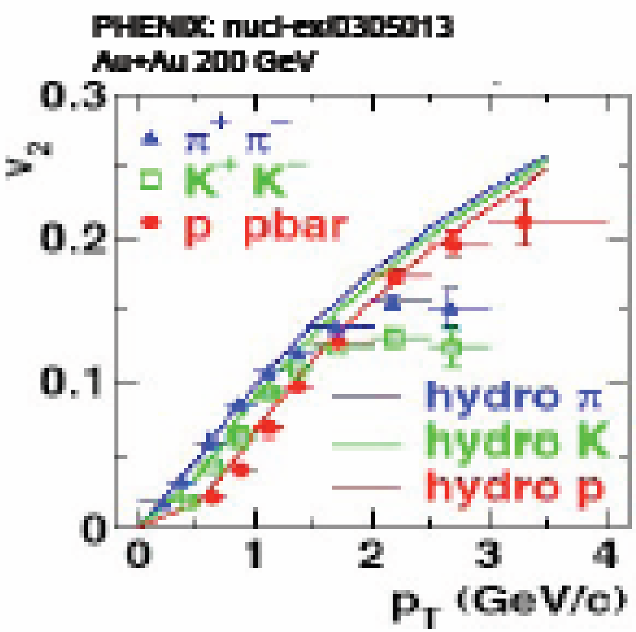
RHIC-data ⇨ Big Surprise !

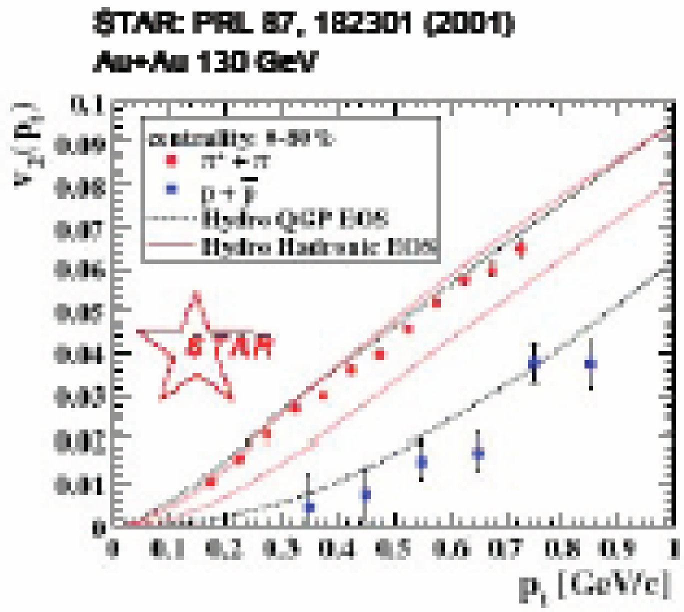
Hydro-dynamical
Model describes
RHIC data well !

At SPS, the Hydro describes well one-particle distributions,

HBT etc., but fails for the elliptic flow.

Hydro describes well v2



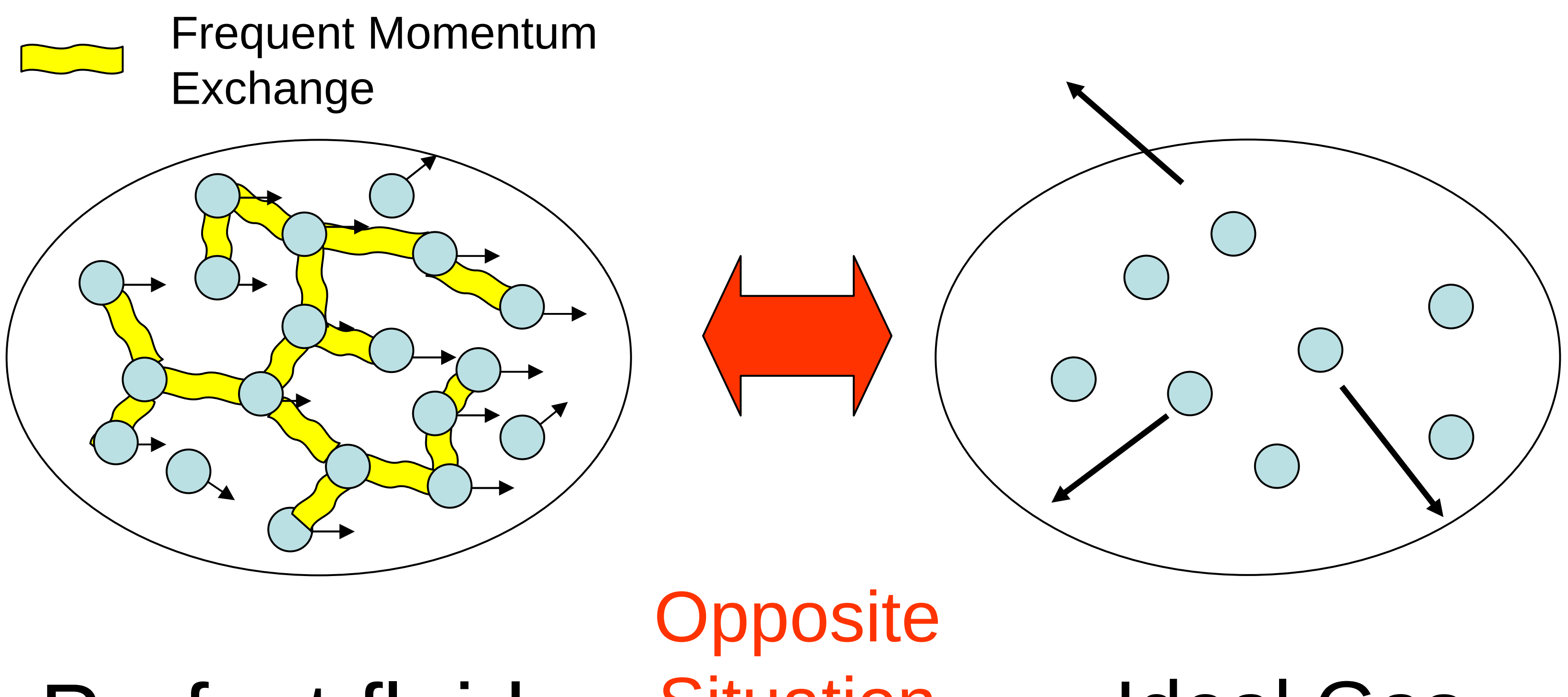


Hydrodynamical calculations are based on Ideal Fluid, i.e., zero shear viscosity.

Another Big Surprise !

- The Hydrodynamical model assumes zero viscosity,
  i.e., Perfect Fluid.
- Phenomenological Analyses suggest also small viscosity.

Liquid or Gas ?



Perfect fluid

Ideal Gas

Karsch and Wyld (1987)

Masuda, Nakamura and Sakai (Lattice 95)

Sakai, Nakamura, Saito(QM97,Lattice 98)
(Improved Action)

Aarts and Martinez-Resco (2002)

Sakai, Nakamura (2004) Anisotropic Lattice
caliburation for improved gauge actions

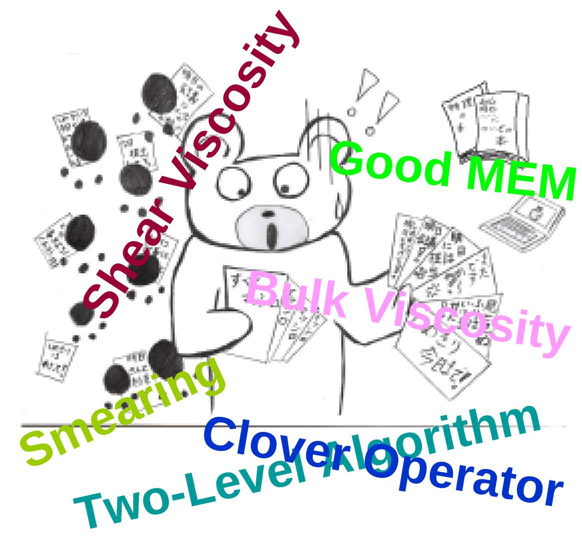
Nakamura and Sakai (2005)

Aarts, Allton, Foley, Hands, Kim (2007)

Meyer (2007) Luescher-Weiz 2-level

20th Century

21th Century



Linear Response Theory

- Zubarev
  “Non-Equilibrium Statistical Thermo-dynamics”
- Kubo, Toda and Saito
  “Statistical Mechanics”

$$\langle T_{\mu\nu} \rangle = \langle T_{\mu\nu} \rangle_{eq} +$$

$$+\int d^3x' \int_{-\infty}^{t} dt' e^{\varepsilon(t'-t)} (T_{\mu\nu}(x,t), T_{\rho\sigma}(x',t'))_{eq} \partial^{\rho}(\beta u^{\sigma})$$

where $(T_{\mu\nu}(x,t), T_{\rho\sigma}(x',t'))_{eq}$

$$\equiv \int_0^1 d\tau \left\langle T_{\mu\nu}(x,t) \left(e^{-A\tau} T_{\rho\sigma}(x',t') e^{A\tau} - \langle T_{\rho\sigma}(x',t') \rangle_{eq} \right) \right\rangle_{eq}$$

$$\langle T^{ij} \rangle = \eta (\partial^i u^j + \partial^j u^i)/2$$

$$\langle T^{0i} \rangle = -\chi (\beta^{-1}(x,t) \partial^i \beta + \partial_{\alpha} u^{\alpha})$$

$$\langle p \rangle - \langle p \rangle_{eq} = -\varsigma \partial_{\alpha} u^{\alpha}$$

$$p \equiv -\frac{1}{3} T^i{}_i$$

Energy Momentum Tensors

$$T_{\mu\nu} = 2Tr(F_{\mu\sigma}F_{\nu\sigma} - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma})$$

$$(T_{\mu\mu} = 0)$$

$$U_{\mu\nu}(x) = \exp(ia^2 gF_{\mu\nu}(x))$$

$$F_{\mu\nu} = \log U_{\mu\nu} / ia^2 g$$

or

$$F_{\mu\nu} = \left(U_{\mu\nu} - U_{\mu\nu}{}^{\dagger}\right) / 2ia^2 g$$

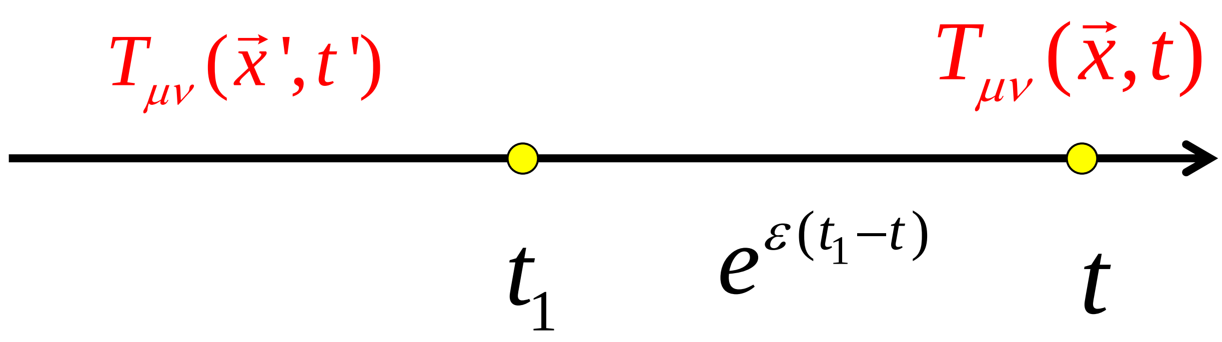
$$\eta = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{12}(\vec{x},t) T_{12}(\vec{x}',t') >_{ret}$$

$$\frac{4}{3}\eta + \varsigma = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{11}(\vec{x},t) T_{11}(\vec{x}',t') >$$

$$\chi = -\frac{1}{T} \int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{01}(\vec{x},t) T_{01}(\vec{x}',t') >_{ret}$$

η **: Shear Viscosity** ς **: Bulk Viscosity**

χ **: Heat Conductivity** ⇨ we do not consider in Quench simulations.



$$-\infty < t' < t_1 < t$$

Ansatz for the Spectral Functions

We measure Matsubara Green Function on Lattice (in coordinate space).

$$< T_{\mu\upsilon}(t,\vec{x}) T_{\mu\upsilon}(0) >= G_\beta(t,\vec{x}) = F.T.G_\beta(\omega_n,\vec{p})$$

$$G_\beta(\vec{p}, i\omega_n) = \int d\omega \frac{\rho(\vec{p},\omega)}{i\omega_n - \omega}$$

We assume (Karsch-Wyld)

$$\rho = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2} + \frac{\gamma}{(m+\omega)^2+\gamma^2}\right)$$

and determine three parameters,

A, m, γ.

We need large Nt !

Lattice and Statistics

Iwasaki Improved Action

$16^3 \times 8$

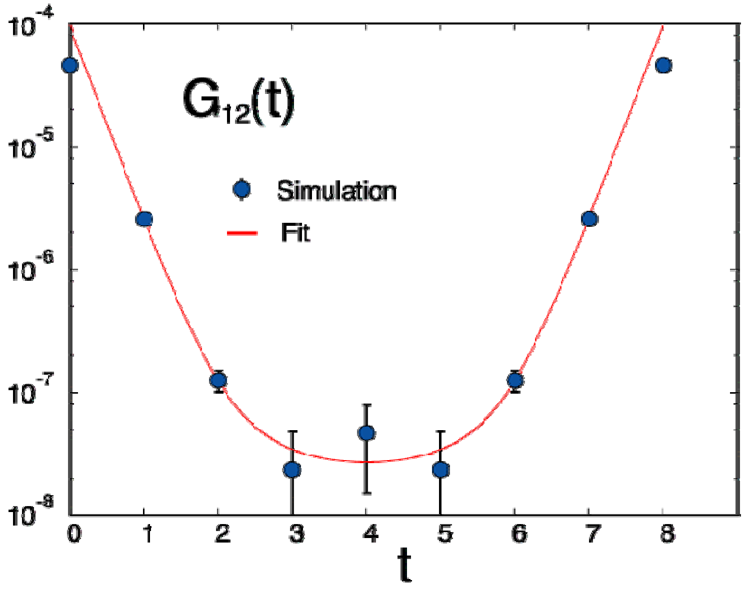
β=3.05 : 1.3M sweeps	β=3.05 : 3.0M sweeps
β=3.20 : 1.2M sweeps	β=3.20 : 2.5M sweeps
β=3.30 : 1.3M sweeps	β=3.30 : 2.0M sweeps

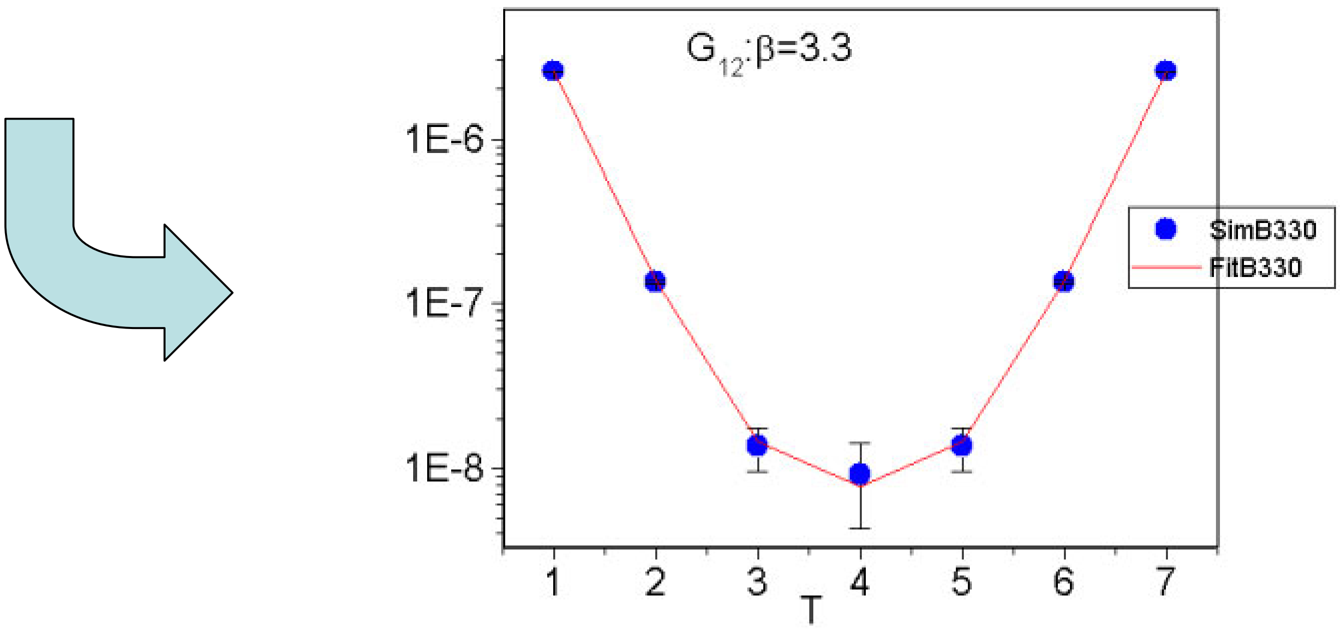
$24^3 \times 8$

β=3.05 : 0.6M sweeps	β=3.05 : 6.0M sweeps
β=3.30 : 0.8M sweeps	β=3.30 : 6.0M sweeps

Quench

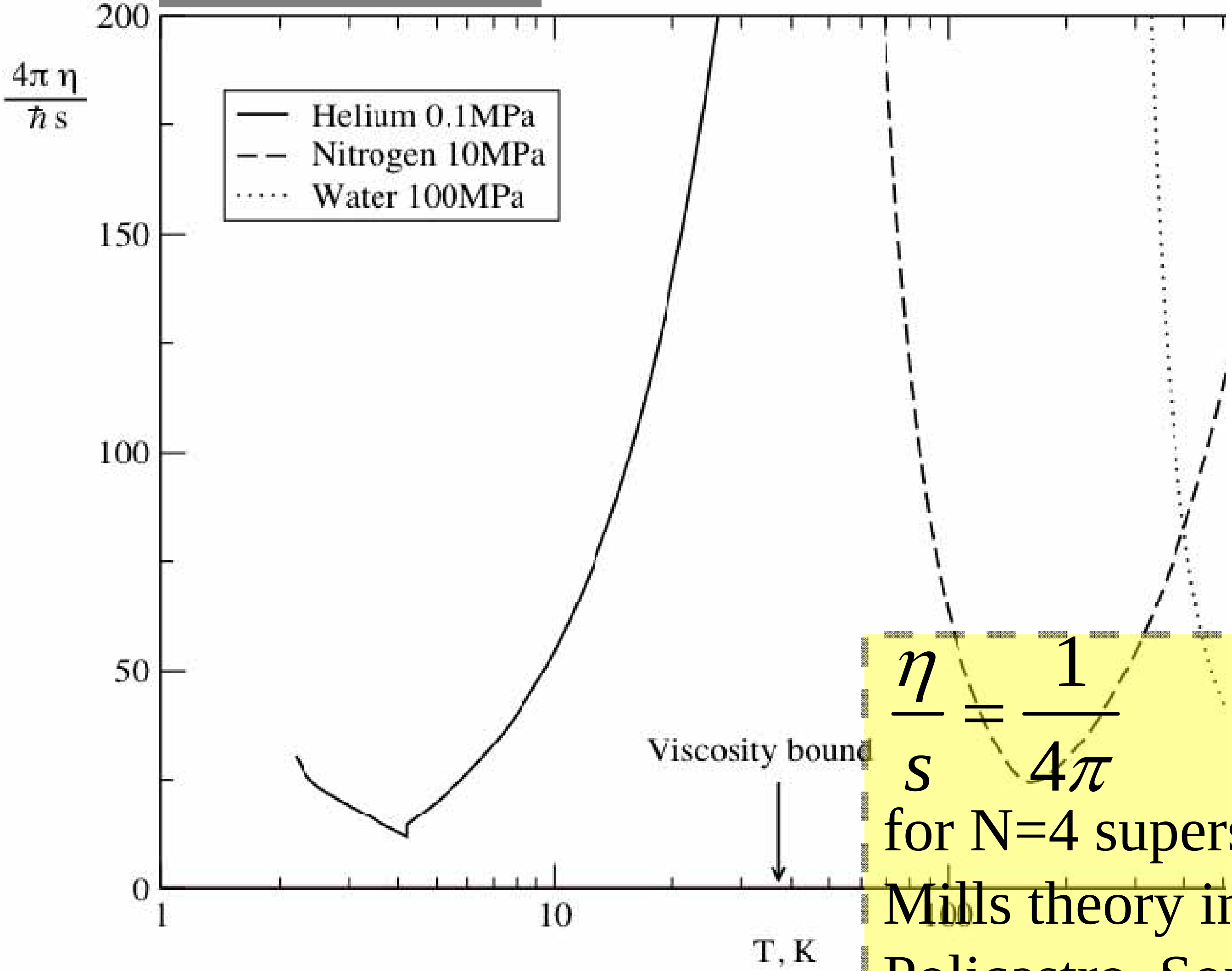
Nt=8





$$\frac{\eta}{s} \geq \frac{1}{4\pi} \; !$$

Kovtun, Son and Starinets, hep-th/0405231

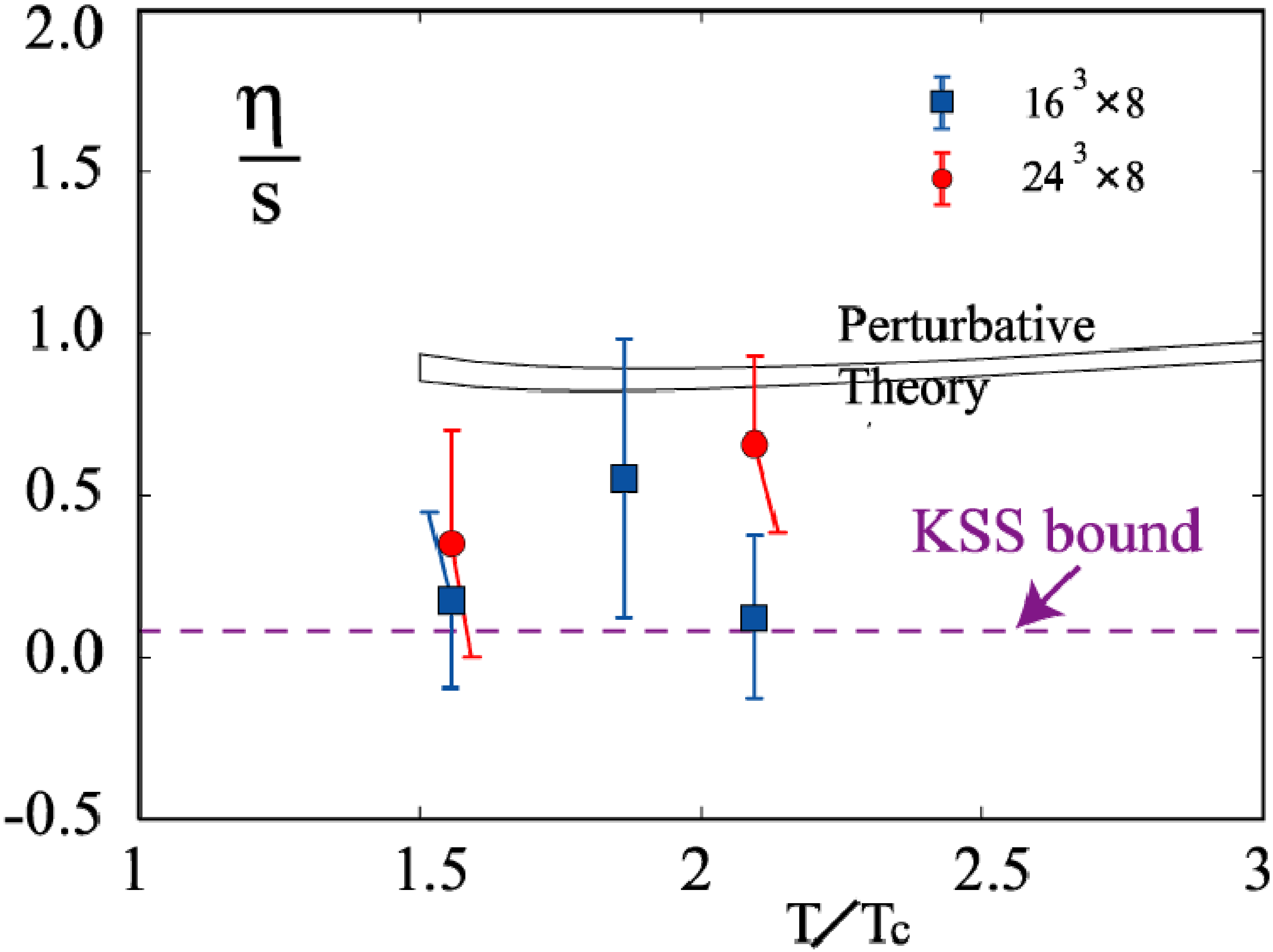


$$\frac{\eta}{s} = \frac{1}{4\pi}$$

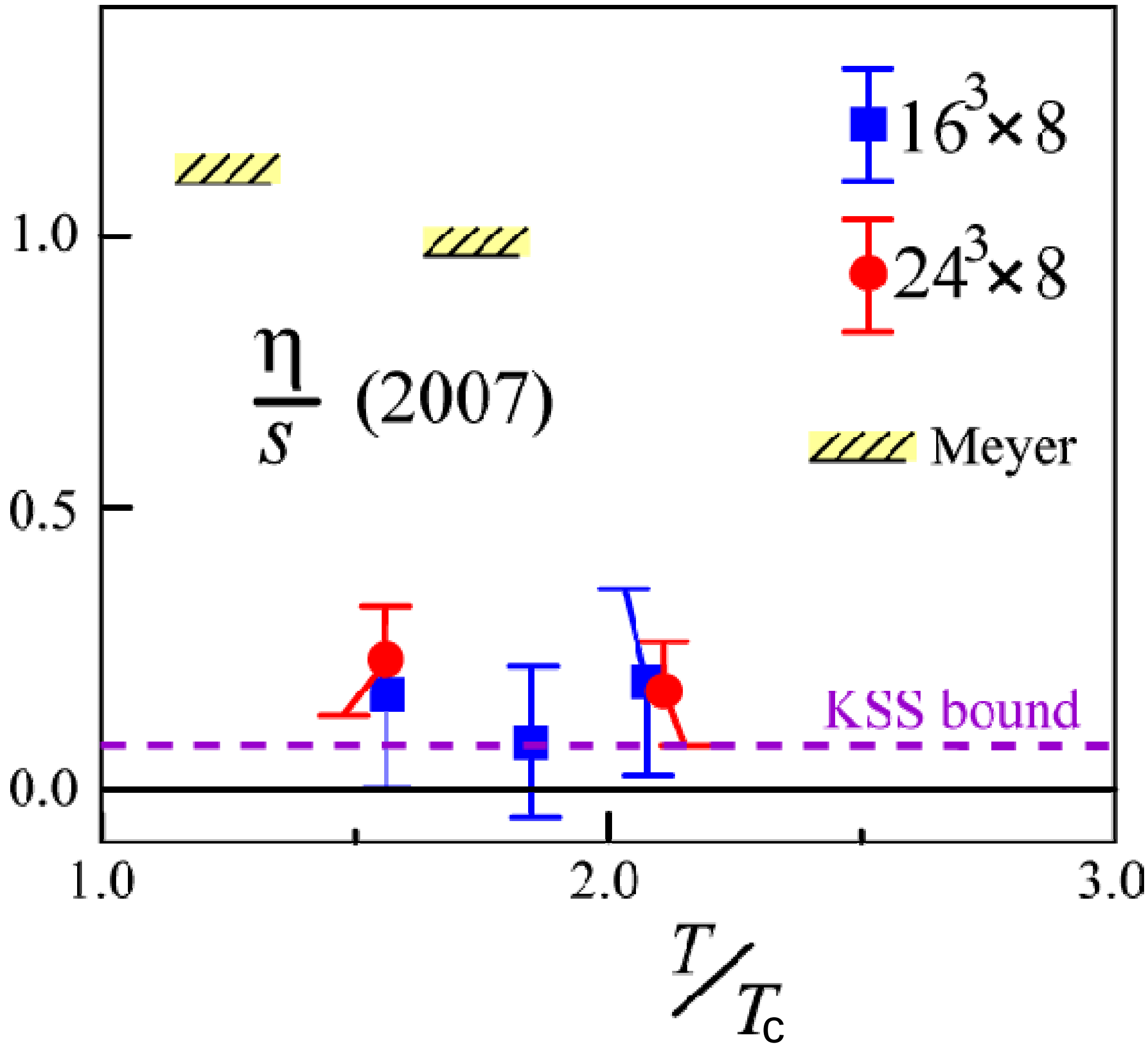
for N=4 supersymmetric Yang-Mills theory in the large N.

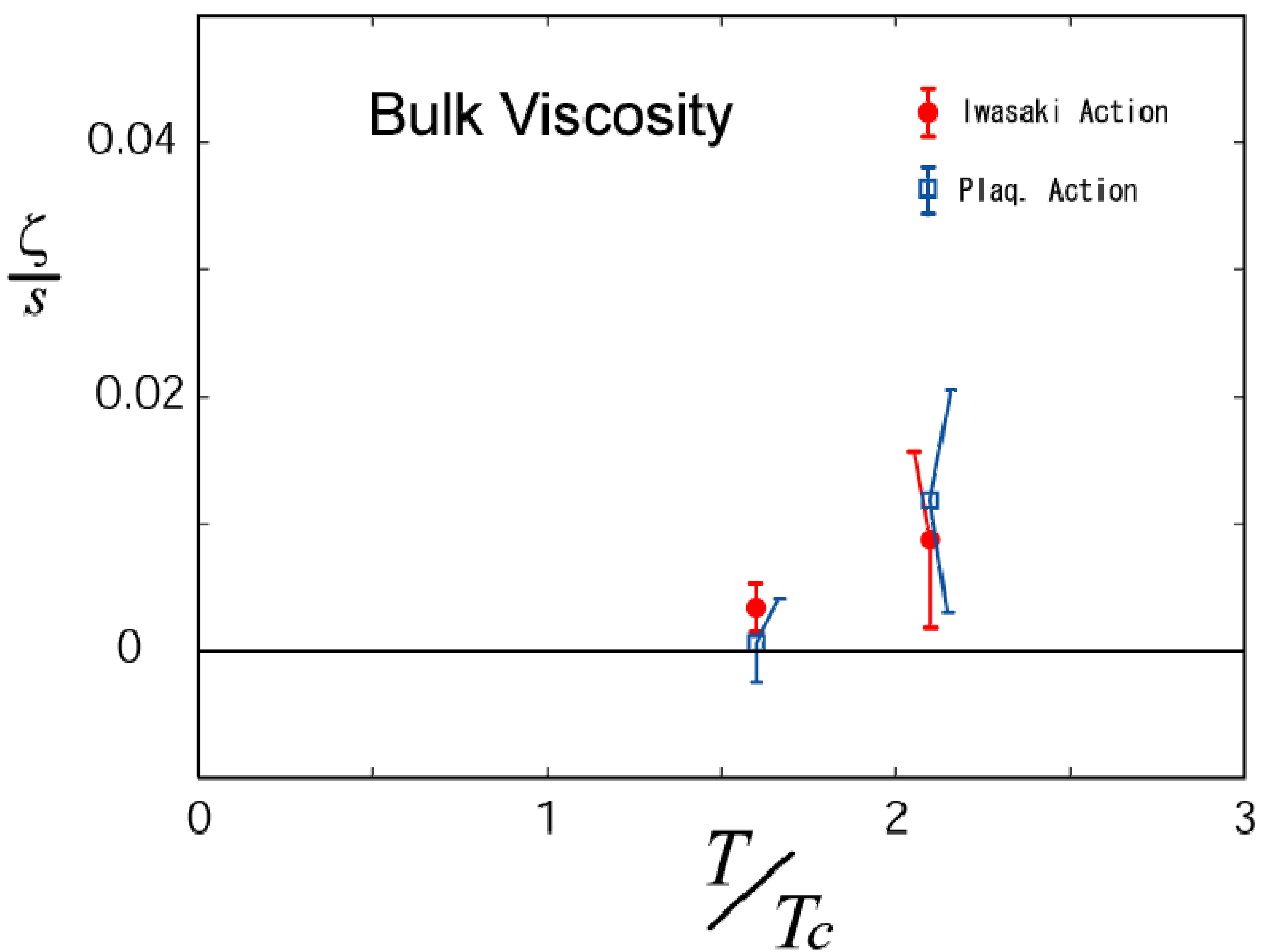
Policastro, Son and Starinets, Phys Rev. Lett. 87 (2001) 081601

Nakamura and Sakai, 2005



Viscosity by Lattice, 2007

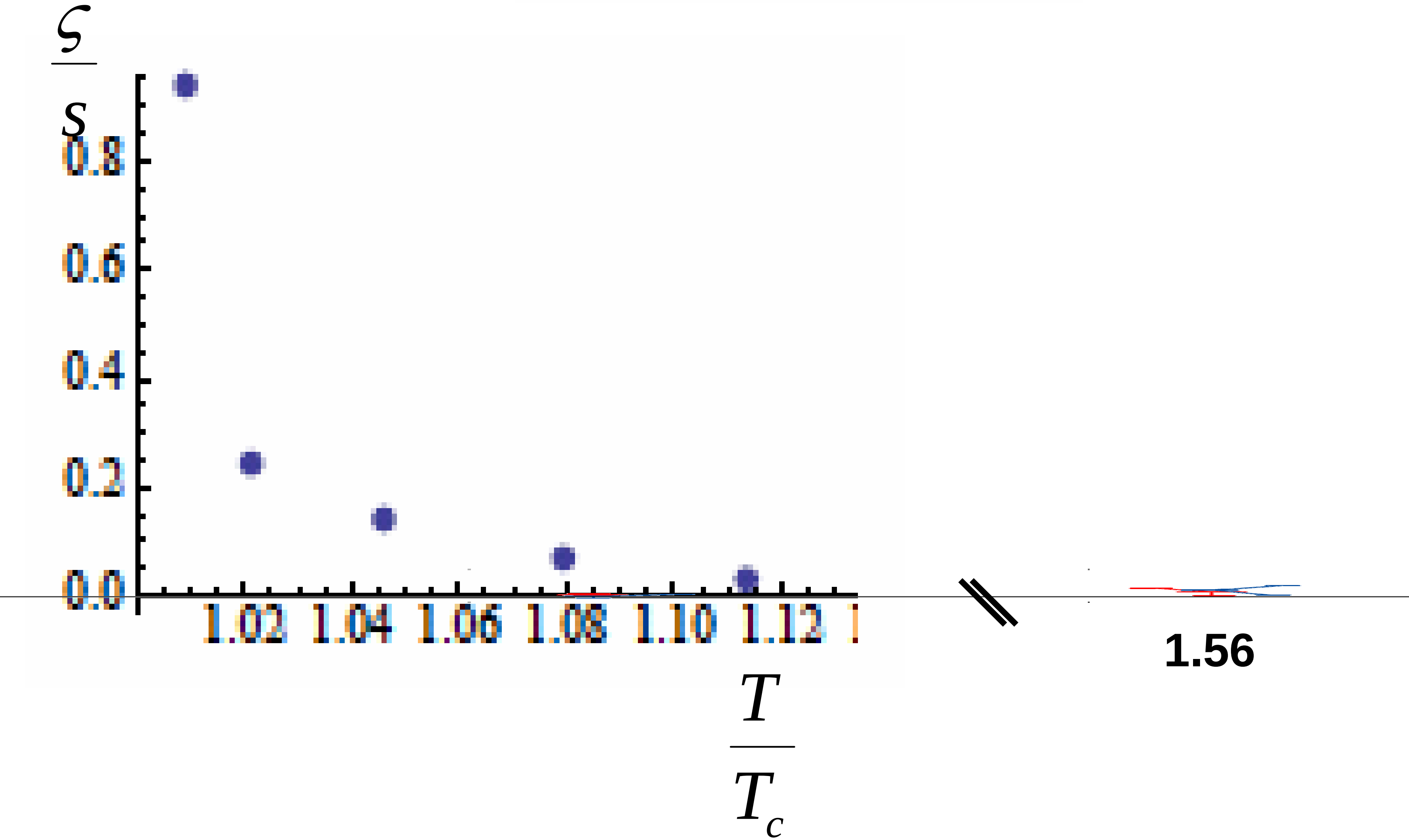




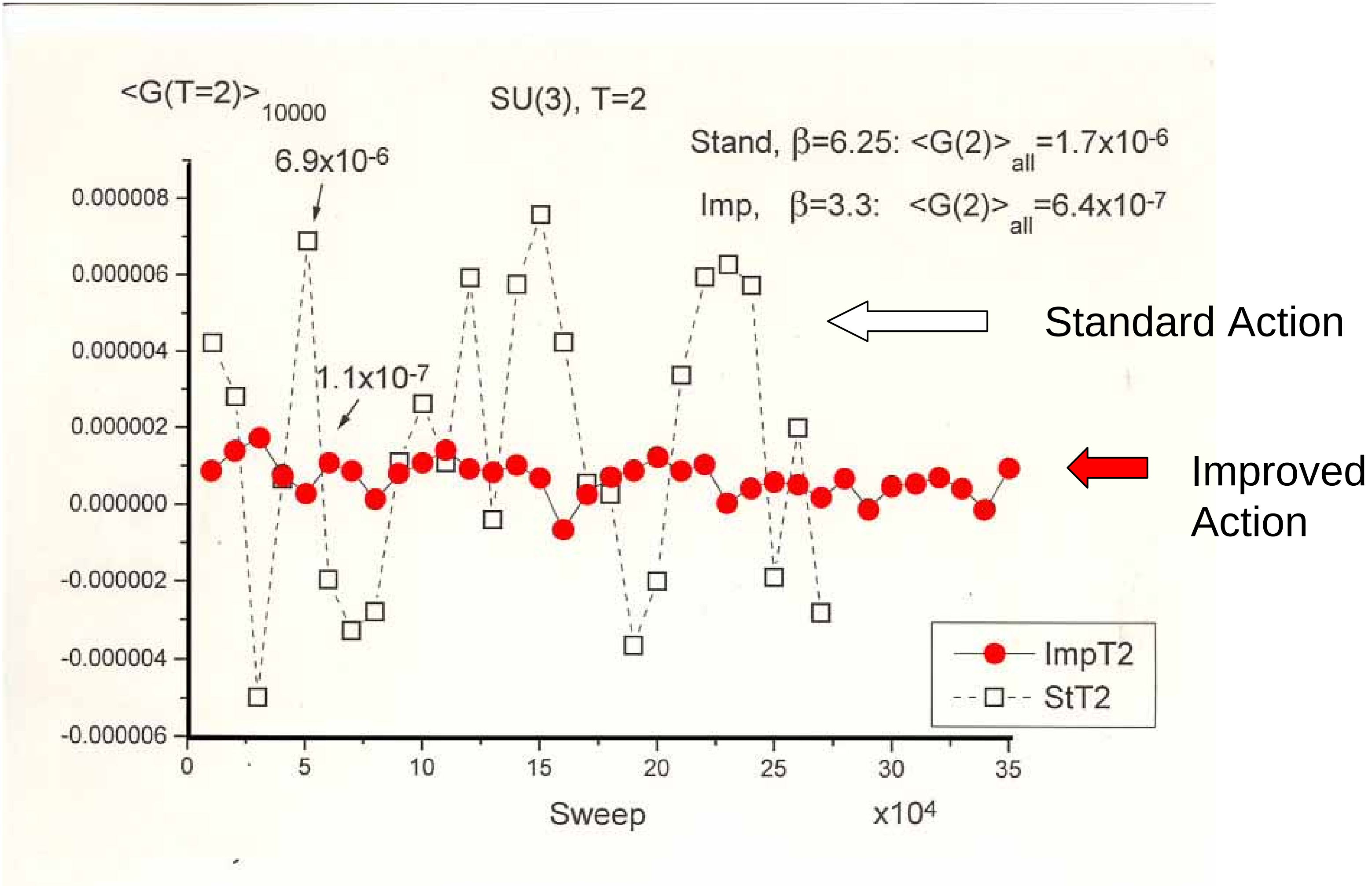
Bulk Viscosity
Iwasaki Action
Plaq. Action
0.04
0.02
0
$\frac{\zeta}{s}$
0
1
2
3
T/T_c

Kharzeeva and Tuchinb

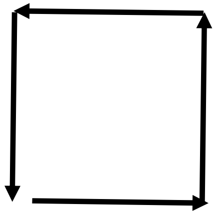
$$\zeta = \frac{1}{9\,\omega_0}\left\{T^5 \frac{\partial}{\partial T}\frac{(\mathcal{E}-3P)_{\rm LAT}}{T^4} + 16|\epsilon_v|\right\}$$

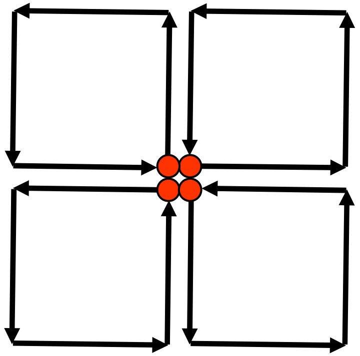


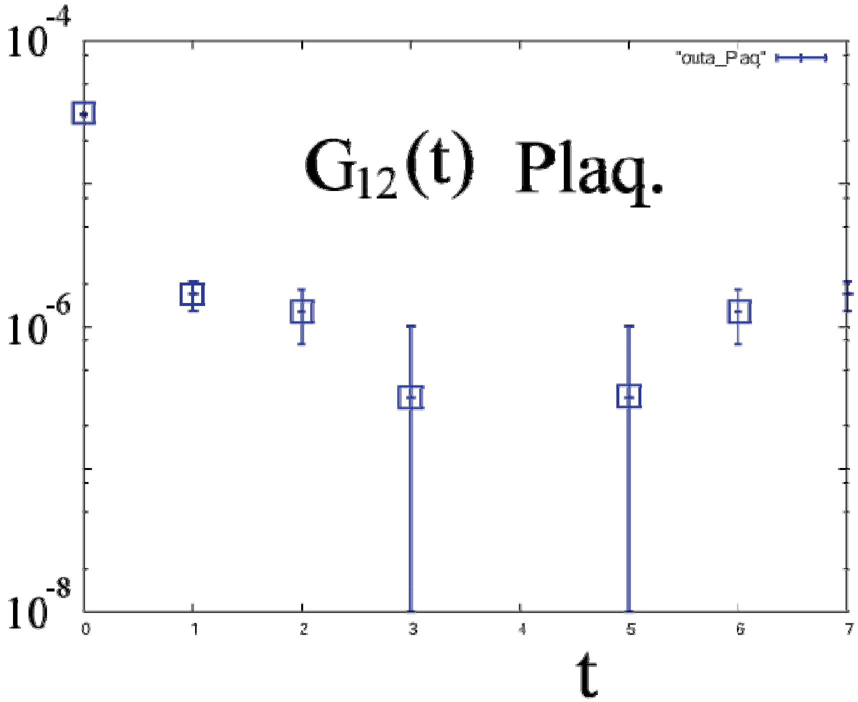
Fluctuations in MC sweeps

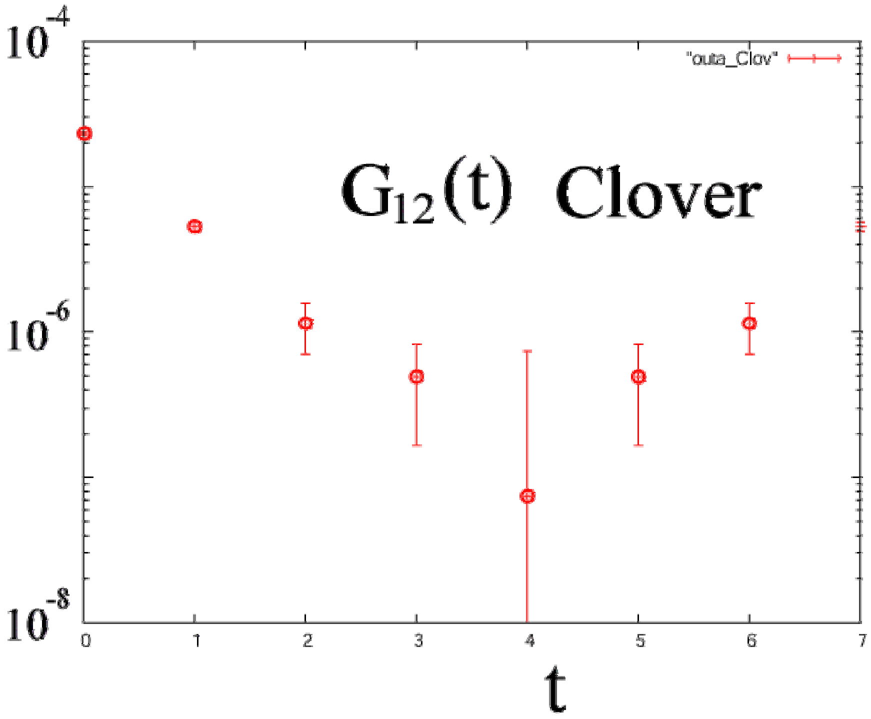


Improve Operator for Fμν

$F_{\mu\nu} =$ 

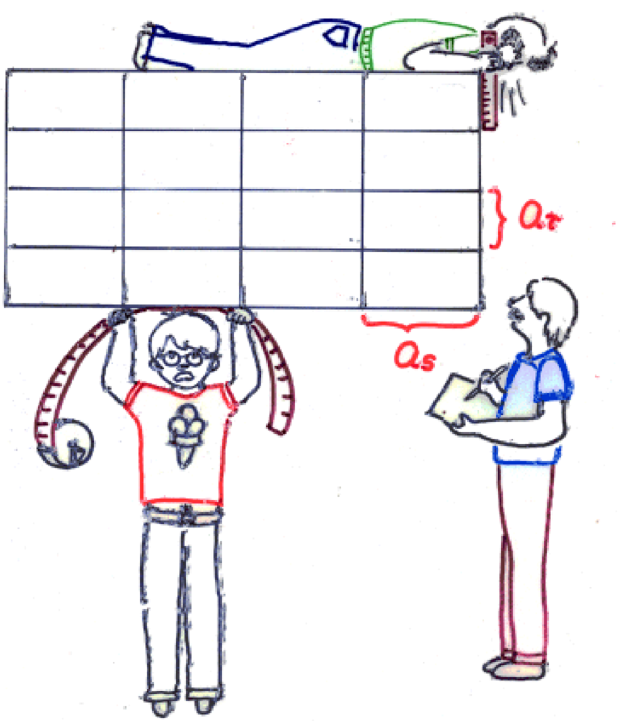
$F_{\mu\nu} =$ 



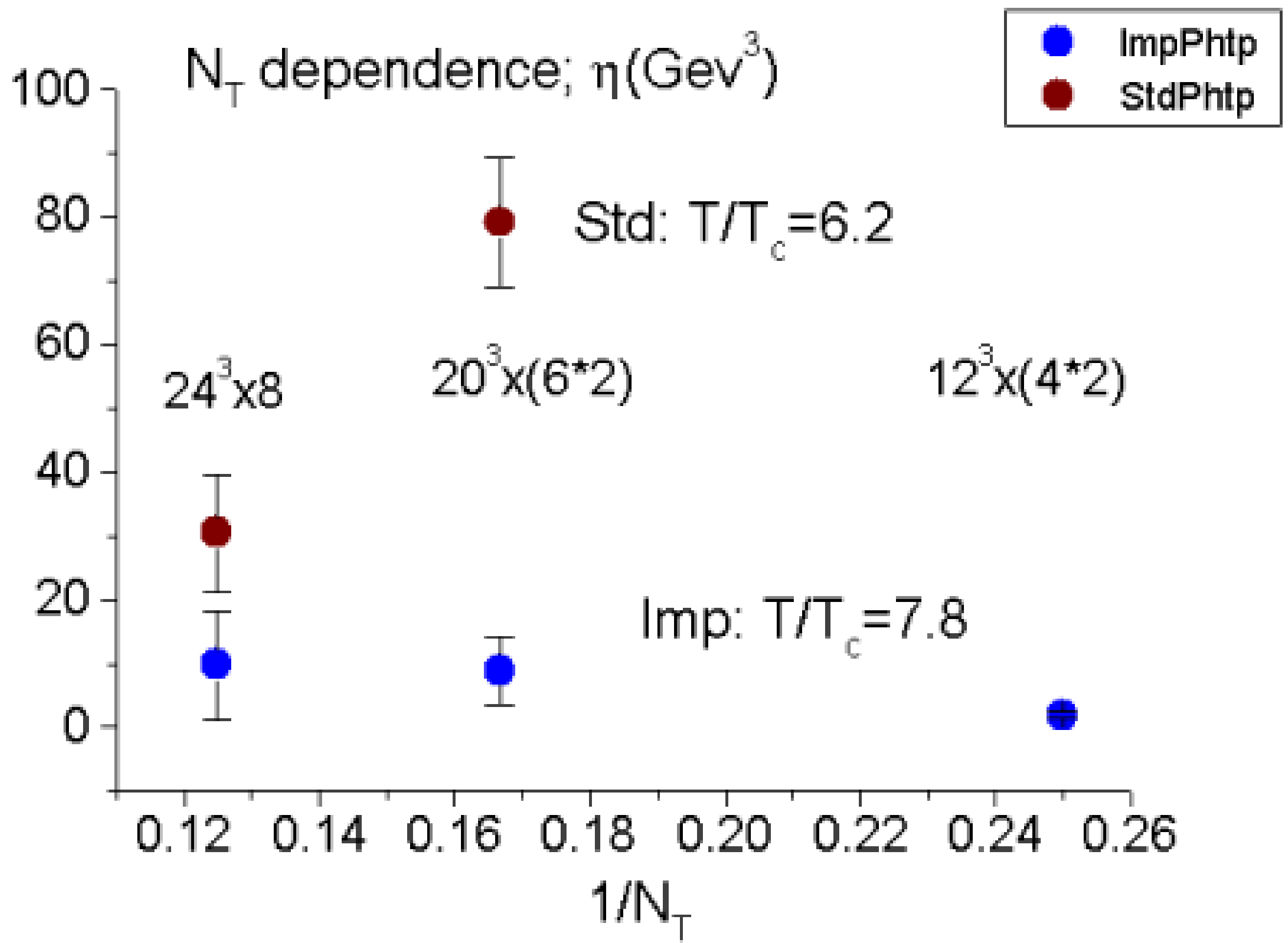


Anisotropic Lattice ?

- Anisotropic lattice has matured and will help us to get more data points to determine the spectral function.



η determined on different anisotropic lattices



Status of Transport Coefficients, 2007

- Old Method (We employ an Ansaz for the spectral function)
  - Numerically, no discrepancy is observed agains the Ansaz, so far.
  - High statistic reduces (2 ~ 6 M. 10 times more than before) error bars, and $\eta \sim 0.1$
  - Bulk viscosity has small positive values.
- Many techniques are tested (improved Kernel of MEM, improved operators, 2-level-Algorithm by Luescher-Weiz etc).
  - All seem to work
  - Then let’s go

In order to understand Deconfinement, we need to understand the Confinement Mechanism

- Gribov-Zwanziger Picture
- Monopole Picture

$$H = \frac{1}{2}\int d^3x\Big((E_i^{tr}(x))^2 + B_i^2(x)\Big) + \frac{1}{2}\int d^3x d^3y\big(\rho(\vec{x})V(\vec{x},\vec{y})\rho(\vec{y})\big)$$

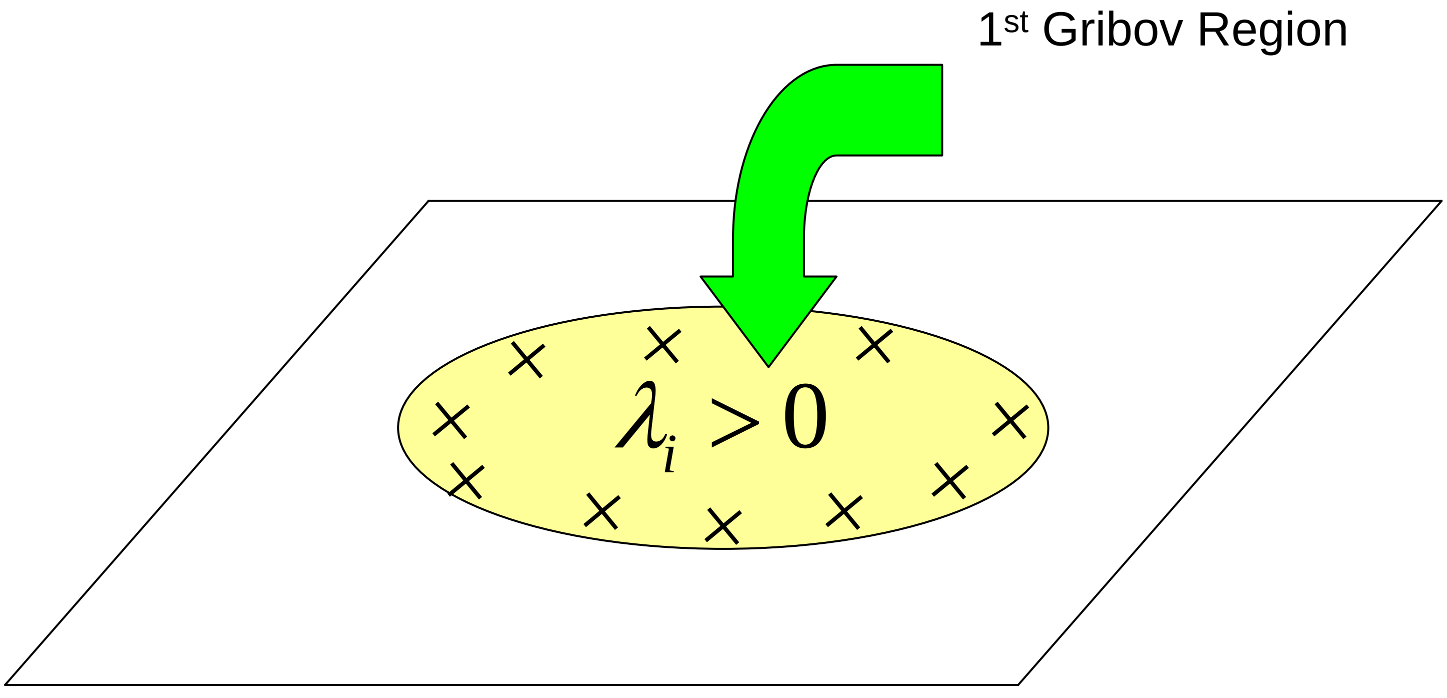
$$M = -\vec{D}\vec{\partial} = -\left(\vec{\partial}^2 + g\vec{A}\times\vec{\partial}\right)$$ Faddeev-Popov Operator

$$V(\vec{x},\vec{y}) = \int d^3z\left[\frac{1}{M(\vec{x},\vec{z})}(-\vec{\partial}_z^2)\frac{1}{M(\vec{z},\vec{y})}\right]$$

If $M = -\partial^2$, i.e. **Abelian Case**

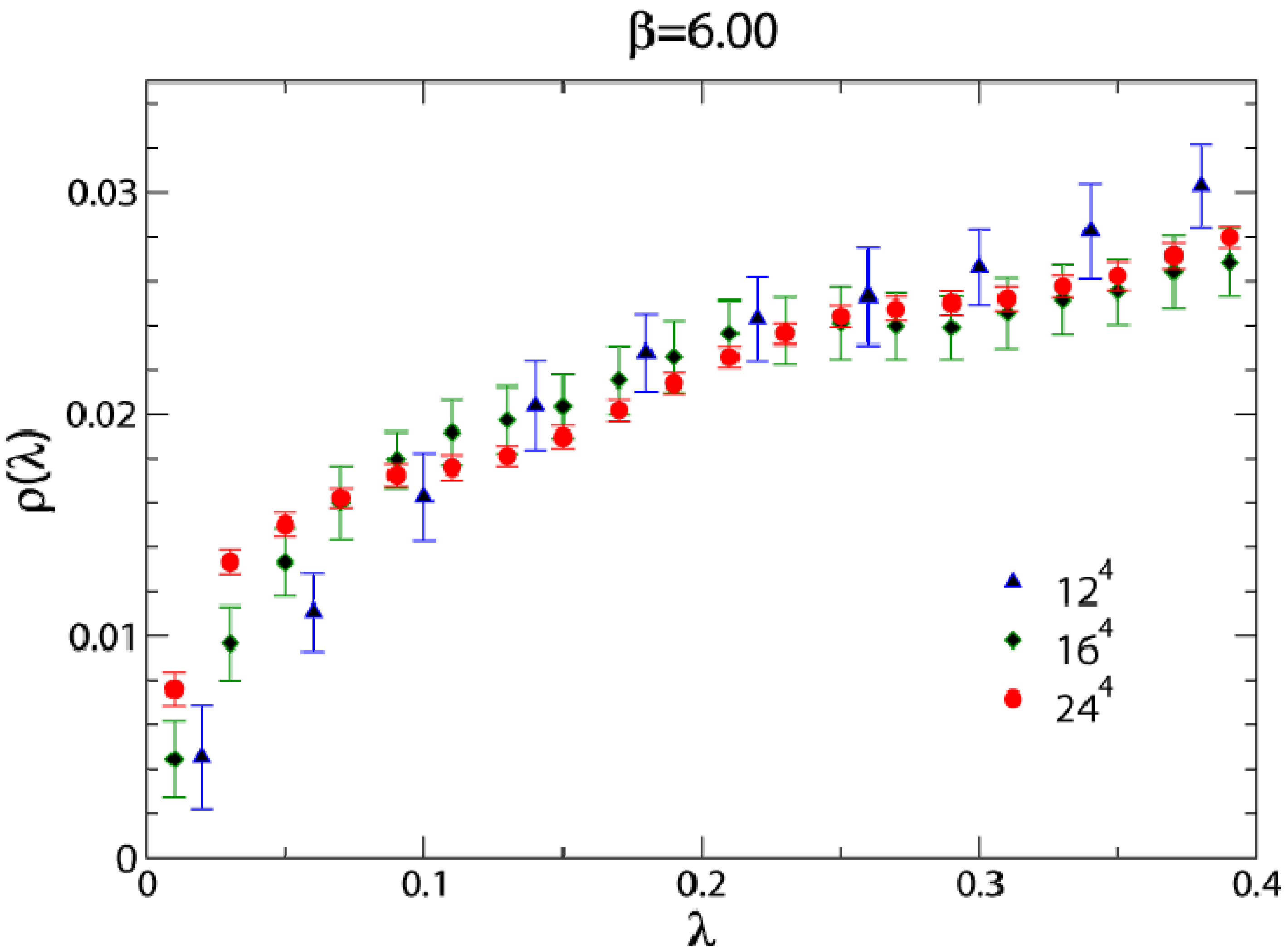
$$V \simeq \frac{1}{\left|\vec{x}-\vec{y}\right|}$$ i.e, **Coulomb Potential**

Gribov picture



λ_i: Eigen values of Faddeev -Popov

Eigen Values locate near the boundary, and they make the confinement Potential.



β=6.00
ρ(λ)
λ
0.03
0.02
0.01
0
0.1
0.2
0.3
0.4
12^4
16^4
24^4

Time-time gluon propagator in Coulomb gauge

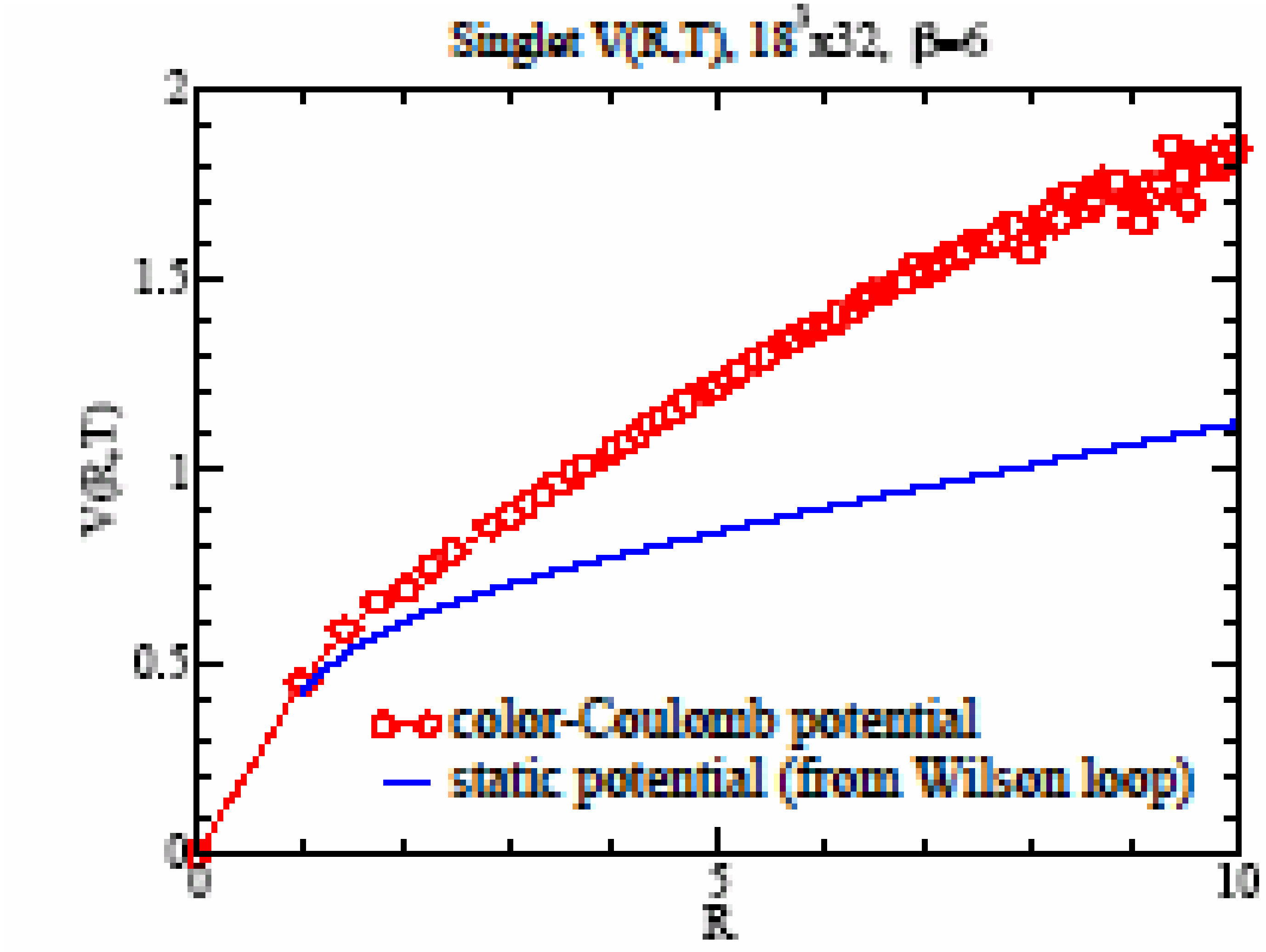
$$D_{00}(x,t) = < A_0(x,t) A_0(0,0) >$$

$$= V_{coul}(r)\delta(t) + P(x,t), \; r = |x|$$

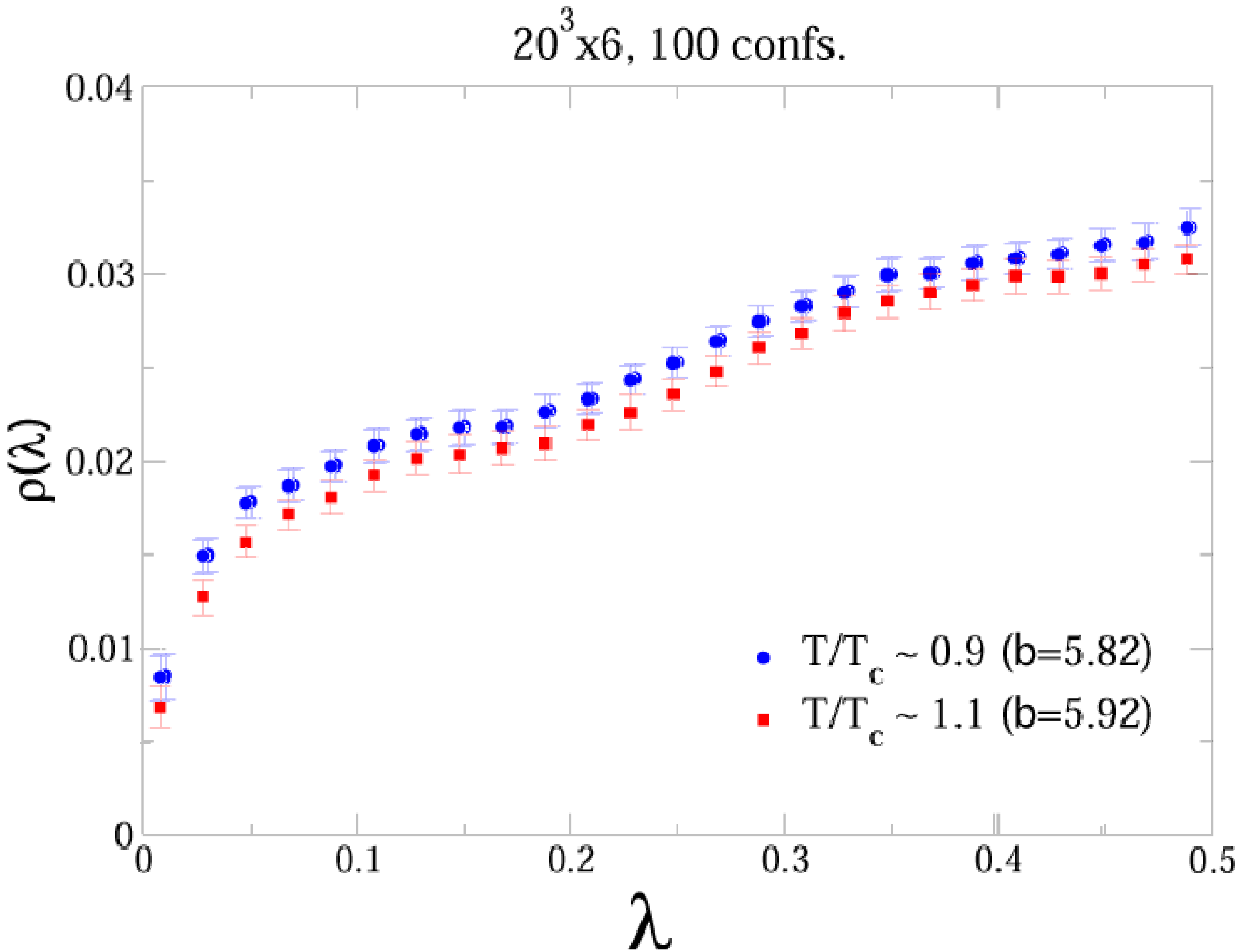
Color-Coulomb instantaneous part (antiscreening); *this term may produce the color confinement*.

Non-instantaneous vacuum polarization part (screening) ; *this term causes a screening effect (quark-pair creation).*

D_{00} is invariant under the renormalization.



Singlet V(R,T), 18³x32, β=6
V(R,T)
R
0
0.5
1
1.5
2
0
5
10
color-Coulomb potential
static potential (from Wilson loop)



20^{3}x6, 100 confs.
0.04
0.03
0.02
0.01
0
0
0.1
0.2
0.3
0.4
0.5
ρ(λ)
λ
T/T$_c$ ~ 0.9 (b=5.82)
T/T$_c$ ~ 1.1 (b=5.92)

- Eigen-Value Distribution of Faddeev-Popov Operator does not change drastically below and above Tc.
- Consequently, the Instantanous Potential remains Linear-Rising behavior even above Tc.
- The Polarization Part *P* makes the Potential screened.

$$D_{00}(x,t) = < A_0(x,t) A_0(0,0) >$$

$$= V_{coul}(r)\delta(t) + P(x,t), \quad r = |x|$$

Always Linear Rising

Quark Propagators above Tc

Quarks become free from the Confinement, but we suspect they are not quasi-free.

Then how they behave ?

Quark Propagators

$$G(p_4, p_i) = \frac{1}{iA(p_4^2, p_i^2)\gamma_4 p_4 + iB(p_4^2, p_i^2)\gamma_i p_i + C(p_4^2, p_i^2)}$$

$$= \frac{1}{A(p_4^2, p_i^2)} \frac{1}{i\gamma_4 p_4 + iB'(p_4^2, p_i^2)\gamma_i p_i + C'(p_4^2, p_i^2)}$$

$$= \frac{1}{A(p_4^2, p_i^2)} \frac{-i\gamma_4 p_4 - iB'(p_4^2, p_i^2)\gamma_i p_i + C'(p_4^2, p_i^2)}{p_4^2 + B'^2(p_4^2, p_i^2) p_i^2 + C'^2(p_4^2, p_i^2)}$$

$$B'(p_4^2, p_i^2) = \frac{B(p_4^2, p_i^2)}{A(p_4^2, p_i^2)} \qquad C'(p_4^2, p_i^2) = \frac{C(p_4^2, p_i^2)}{A(p_4^2, p_i^2)}$$

C. D. Roberts and S. M. Schmidt, nucl-th/0005064

Quark Propagators

Wave Function: $$Z(q_4, q_i) = \frac{A_{free}(q_4^2, q_i^2)}{A(q_4^2, q_i^2)}$$

J-I. Skullerud and A. G. Williams Phys. Rev. D63, 054508

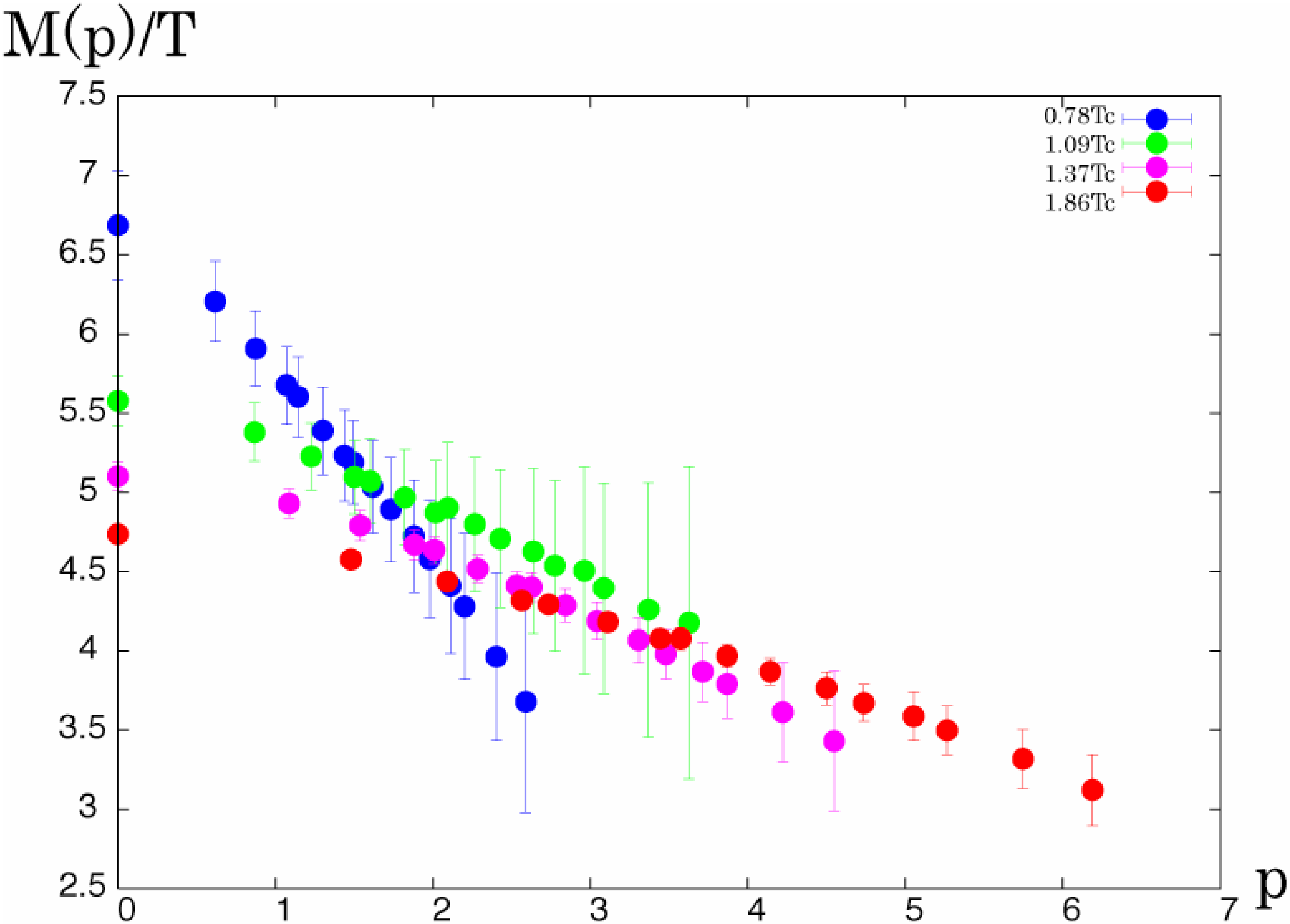
C. D. Roberts and A. G. Williams, hep-ph/9403224

P. O. Bowman et. al., *Lattice Hadron Physics*, (Springer)

Mass Function: $$M(q_4, q_i) = C'(q_4^2, q_i^2) - C'_{free}(q_4^2, q_i^2)$$

P. O. Bowman et. al., *Lattice Hadron Physics*, (Springer)

Momentum Dependence (Wilson fermion)



Pole Mass

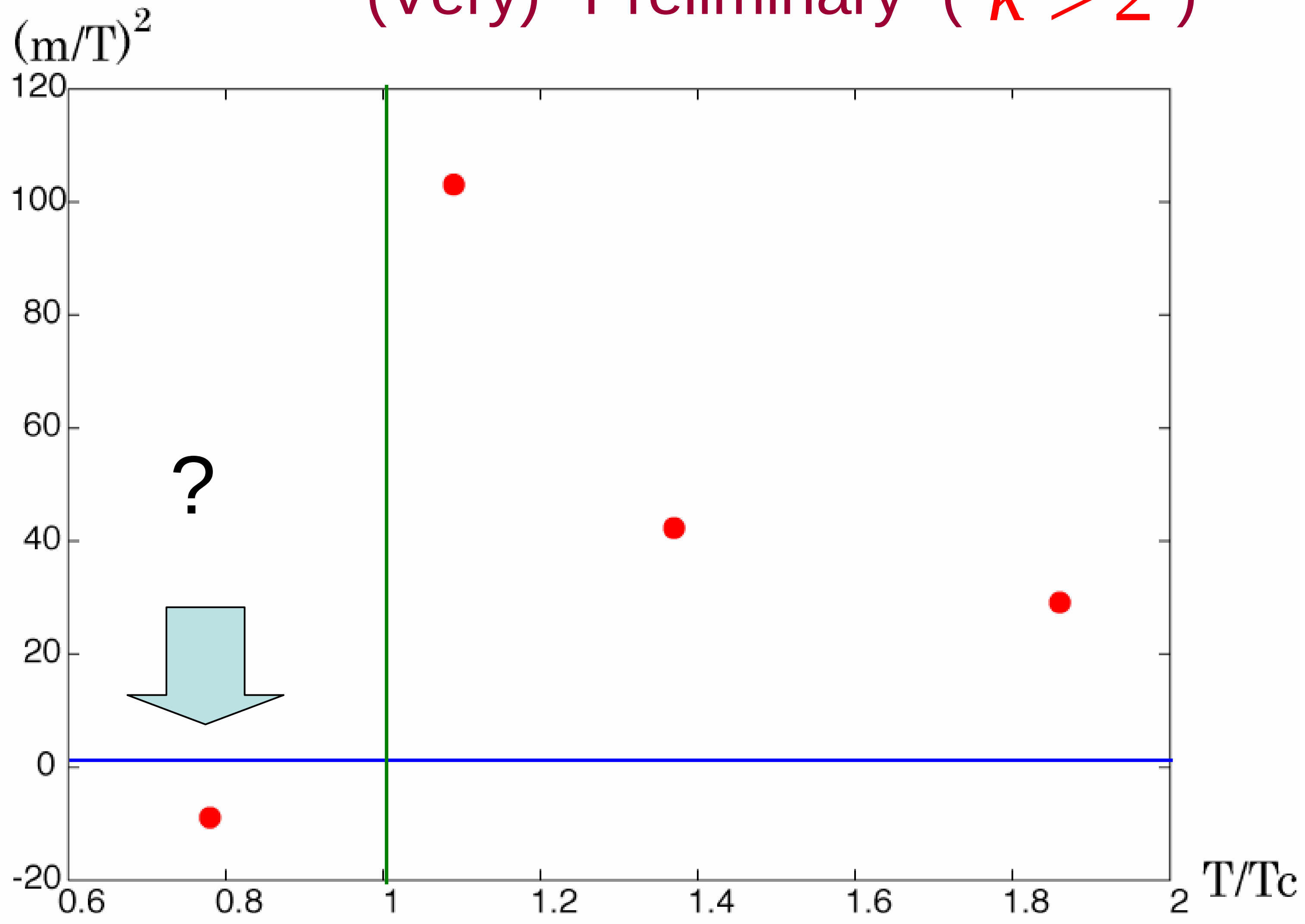
$$p_4^2 + C'^2(p_4^2, p_i^2) = p_4^2 + \left(C'(0) + \left.\frac{\partial C'(p_4^2)}{\partial p_4^2}\right|_{p_4^2=0} p_4^2 \right)^2$$

$$\simeq \left(1 + 2C'(0) \left.\frac{\partial C'(p_4^2)}{\partial p_4^2}\right|_{p_4^2=0} \right) \left(p_4^2 + \frac{C'^2(0)}{1 + 2C'(0) \left.\frac{\partial C'(p_4^2)}{\partial p_4^2}\right|_{p_4^2=0}} \right)$$

$$M_{pole}^2 = \frac{C'^2(0)}{1 + 2C'(0) \left.\frac{\partial C'(p_4^2)}{\partial p_4^2}\right|_{p_4^2=0}}$$

Pole Mass in Quark Propagators (Quench)

(Very)^k Preliminary ($k > 2$)

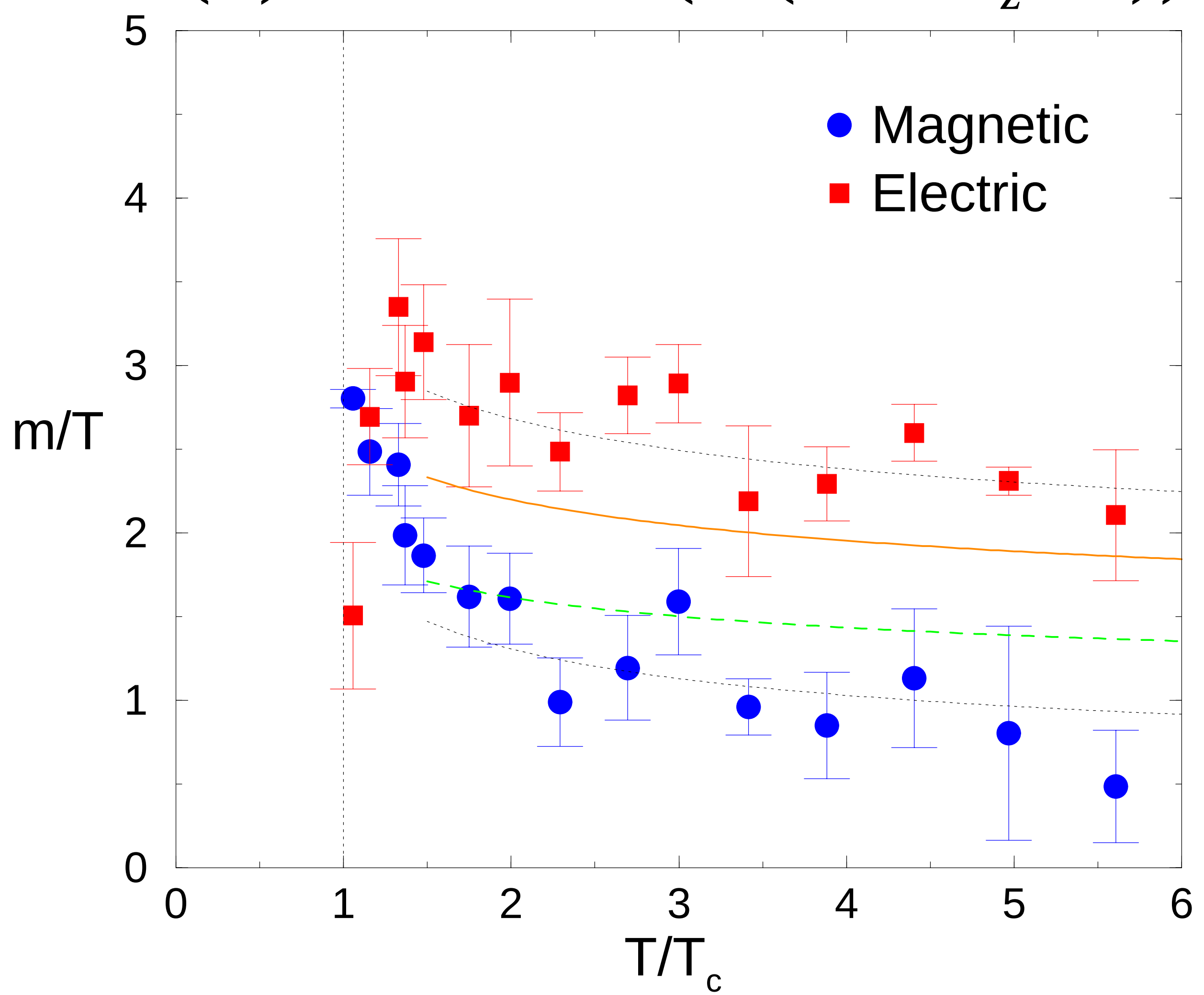


Magnetic Degrees of Freedom ?

- M. Chernodub and V. Zakharov
  - hep-lat/0611228
- J. Liao and E. Shuryak
  - hep-ph/0611131

Fitting to extrapolate mass

$$G(z) = C \cdot \cosh(m(z - N_z/2)) \quad \text{at } z > 1/T$$

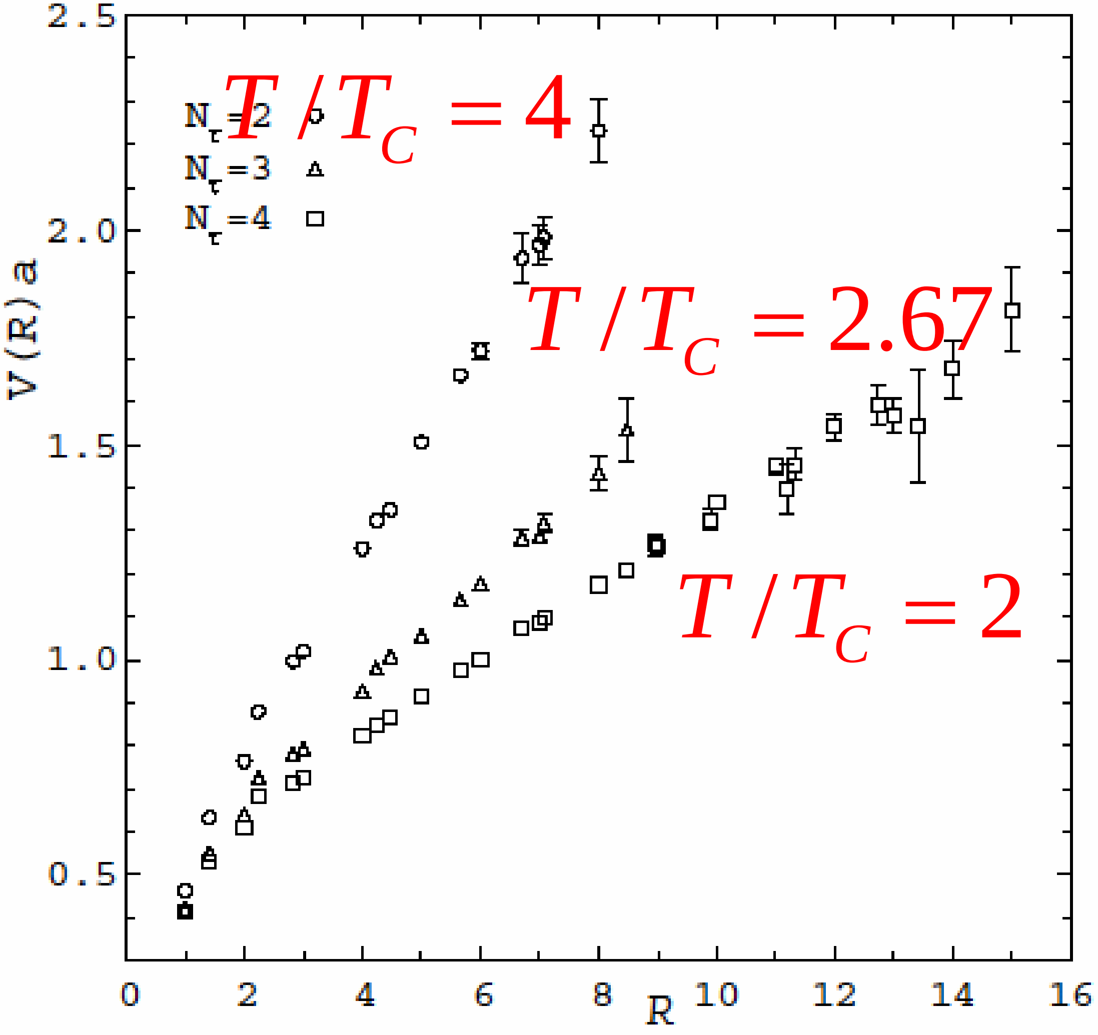


Nakamura, Sakai and Saito,

Phys. Rev. D69, (2004) 014506.

hep-lat/0311024

Spatial Wilson Loops

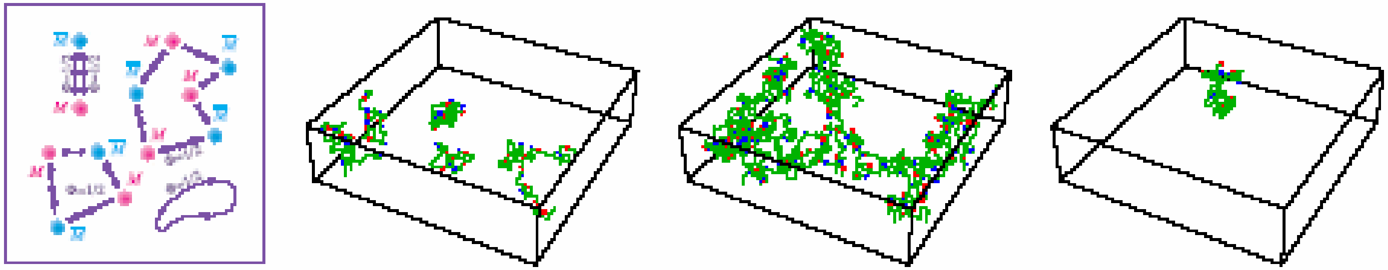


Karsch, Laermann, and Luetgemeier,

Phys.Lett. B346 (1995) 94

It is well known that Spatial Wilson Loops give a Confinement Potential. even above Tc.

- Confinement is due to monopole condensation
- **Center vortex mechanism**
  - Del Debbio, Faber, Greensite, Olejnik, ’97
- a realization of spaghetti (Copenhagen) vacuum
- Center strings are classified with respect to the center
  Z_N of the $SU(N)$ gauge group
- Confinement is due to vortex percolation



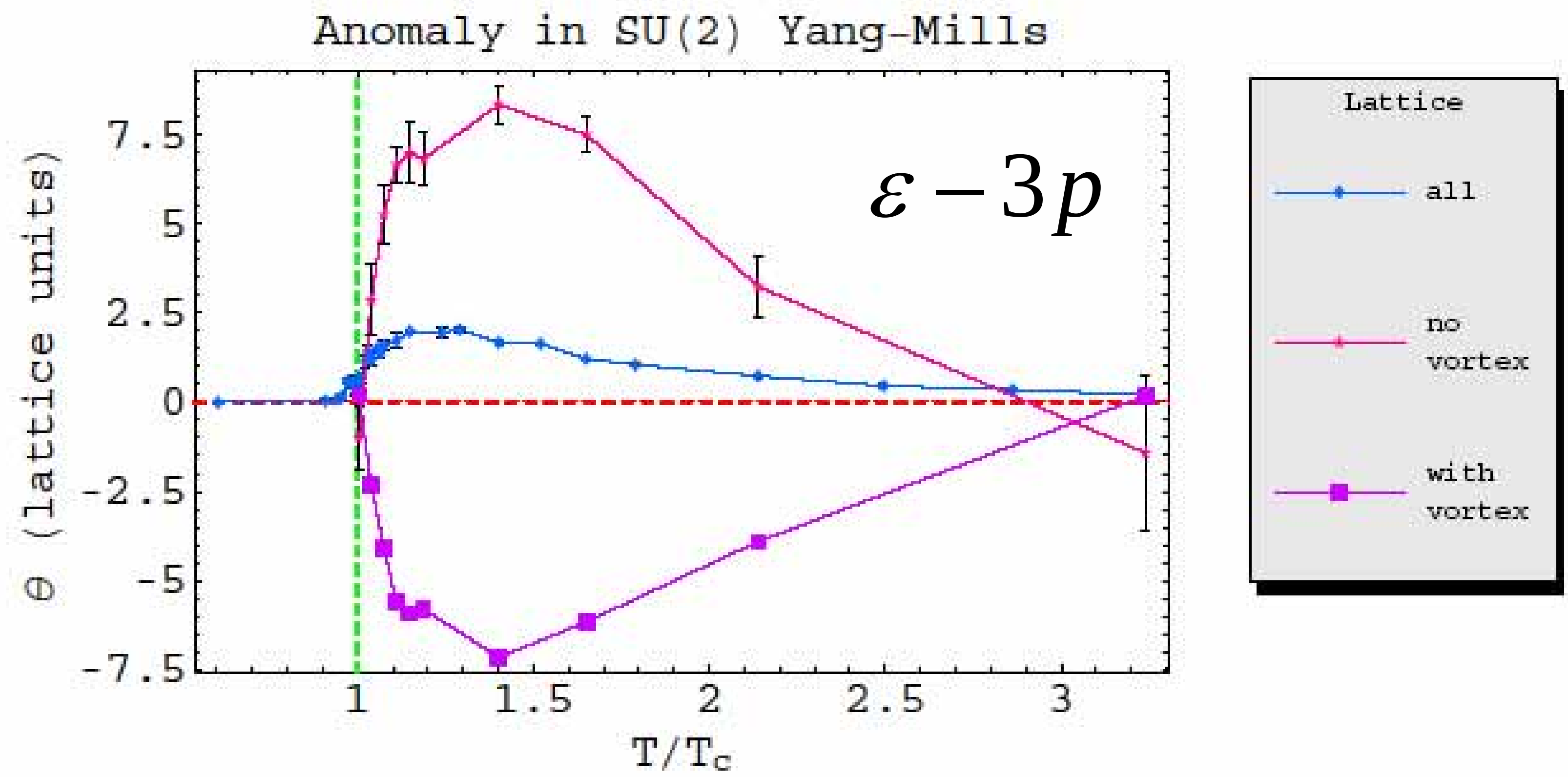
[results of numerical simulations are taken from Feldmann, Ilgenfritz, Schiller & M.Ch. '05]

- Observation of monopoles in the vortex chains:
- monopole is a defect, at which the flux of the vortex alternates.

$SU(2) \quad 12^3 \times 4$

T_C

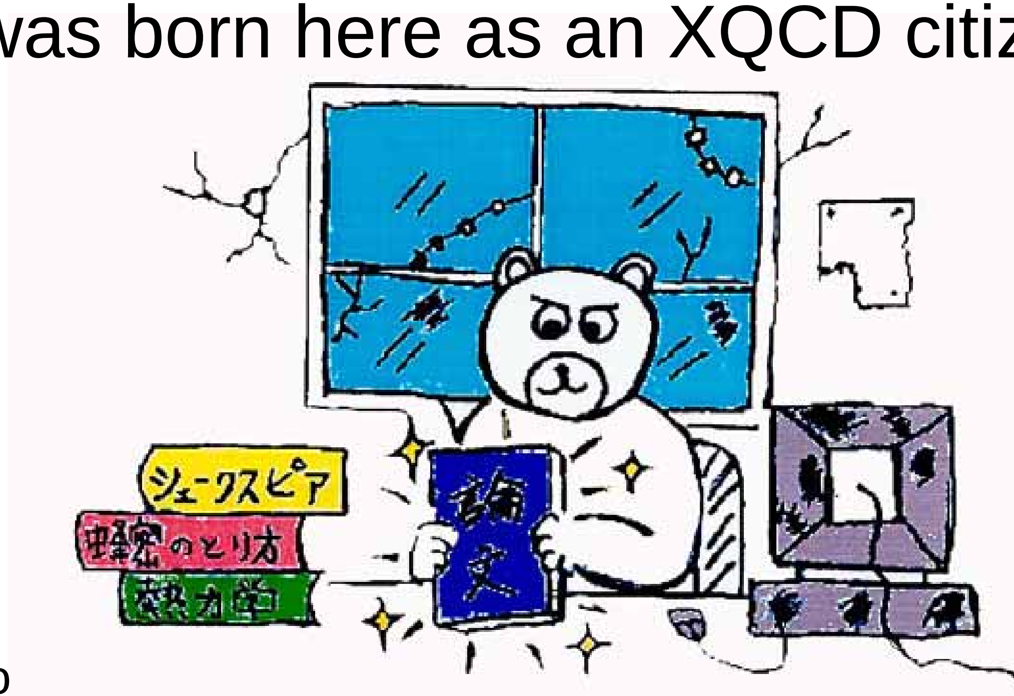
(Very)^k Preliminary ($k > 2$)



Sono ritornato, e molto contento.
Grazie !!

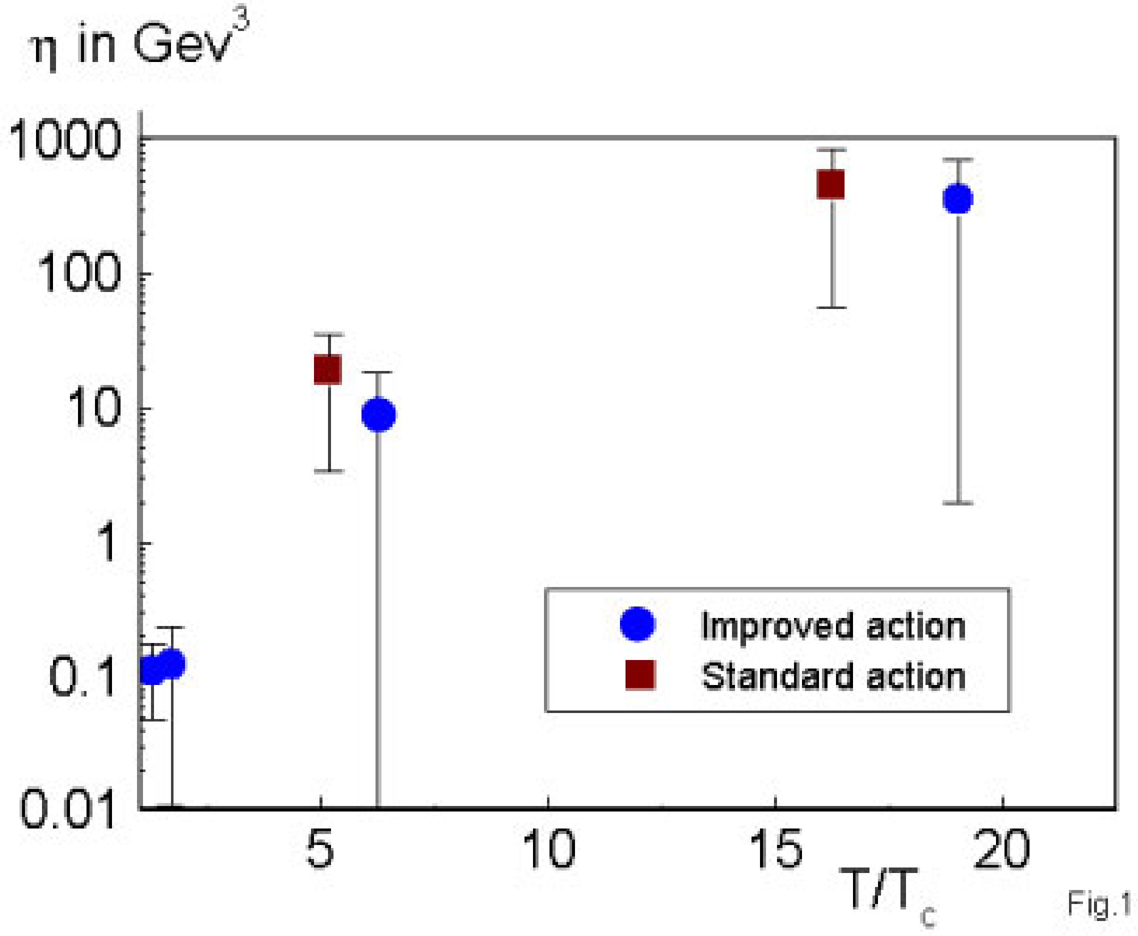
I was a Pos-Doc from 1981-1983 at Frascati.
I started Lattice QCD Study here and wrote the first paper about SU(2) Color Lattice at Finite Density (Phys.Letters 149B, 1984) ,
i.e., I was born here as an XQCD citizen.

Vino Vianco



Backup Slides

Very high Temperature



Entropy Density

$$F = fV$$

$$f = -p$$

$$U - TS = -T \log Z = F$$

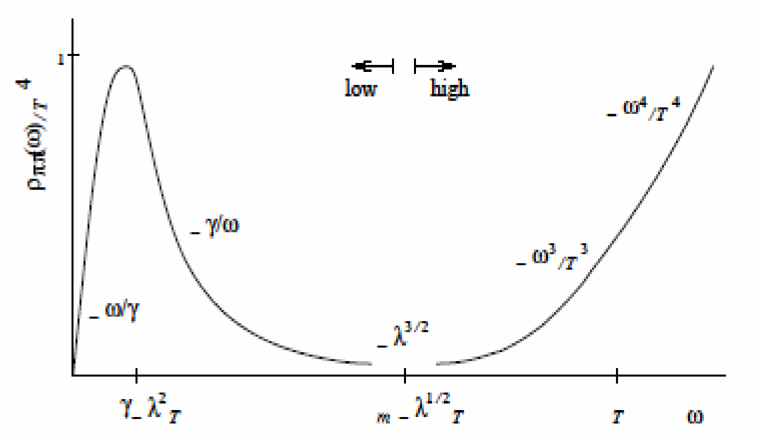
$$\Rightarrow \quad s = \frac{S}{V} = \frac{\varepsilon + p}{T}$$

$$\left. \frac{p}{T^4} \right|_{\beta_0}^{\beta} = \int_{\beta_0}^{\beta} d\beta' \frac{d}{d\beta'} \frac{p}{T^4}$$

We reconstruct *p* from Raw-Data by CP-PACS
(Okamoto et al., Phys.Rev.D (1999) 094510)

Spectral Function by Aarts and Resco

$$\rho(\omega) = \rho^{low}(\omega) + \rho^{high}(\omega)$$



$$\frac{\rho^{low}(\omega)}{T^4} = x \frac{b_1 + b_2 x^2 + \cdots}{1 + c_1 x^2 + c_2 x^4 + \cdots}$$

$$x \equiv \frac{\omega}{T}$$

$$\rho^{high}(\omega) = \theta(\omega - 2m_{th}) \frac{(N_c{}^2 - 1)(\omega^2 - 4m_{th}{}^2)^{5/2}}{80\pi^2\omega} [n(\omega) + 0.5]$$

Fitting with three parameters, $b_1 \;\; c_1 \;\; m$

⇒ $c_1 < 0$?

Effect of High-Frequency part

$$\rho = \rho^{BW} + \rho^{high}$$

$$\frac{\rho^{low}(\omega)}{T^4} = x\frac{b_1 + b_2 x^2 + \cdots}{1 + c_1 x^2 + c_2 x^4 + \cdots} \qquad x \equiv \frac{\omega}{T}$$

$$\rho^{BW} = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2 + \gamma^2} + \frac{\gamma}{(m+\omega)^2 + \gamma^2}\right)$$

β=3.3

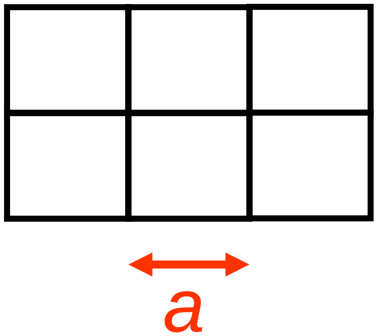
ηa^3	m_{th}
0.00225(201)	∞
0.00223(191)	5.0
0.00194(194)	3.0
0.00126(204)	2.0

$m_{th} = 1.8$

ρ^{high} contribution is larger than ρ^{BW} at t=1.

Why they are so noisy ?

- RG improved action helps lot.
  - Noise from Lattice Artifact ? (Finite *a* correction ?)
  - Once we checked that there is not so much difference between

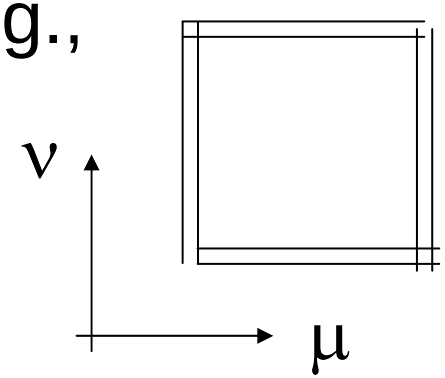


$$F_{\mu\nu} = \left(U_{\mu\nu} - U_{\mu\nu}{}^{\dagger}\right)/2i \text{ and } F_{\mu\nu} = \log U_{\mu\nu}/i$$

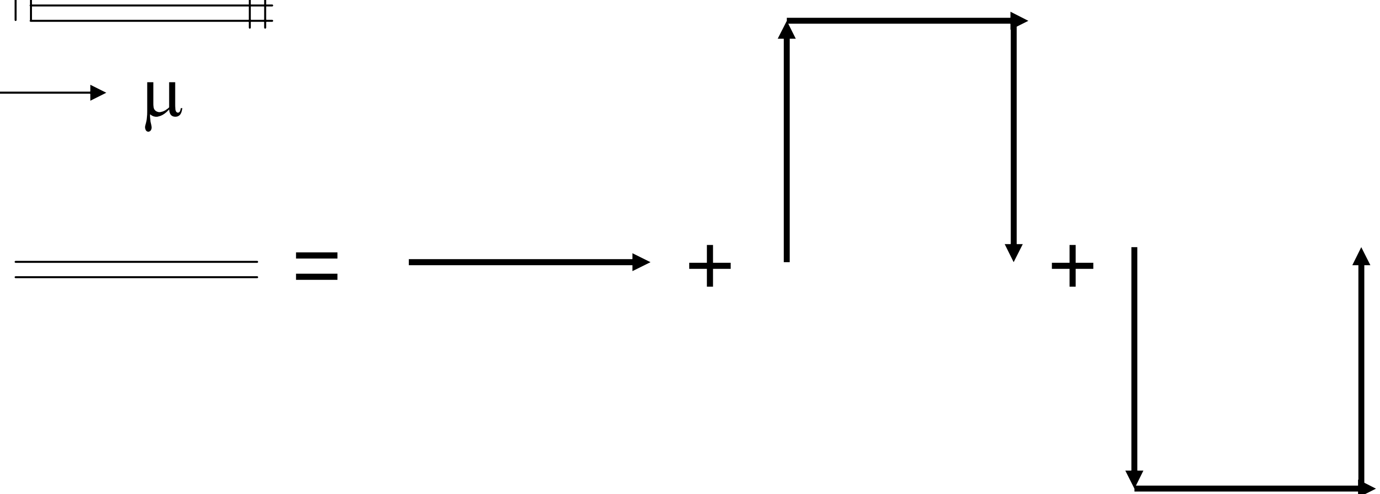
for SU(2). But we should check it again.

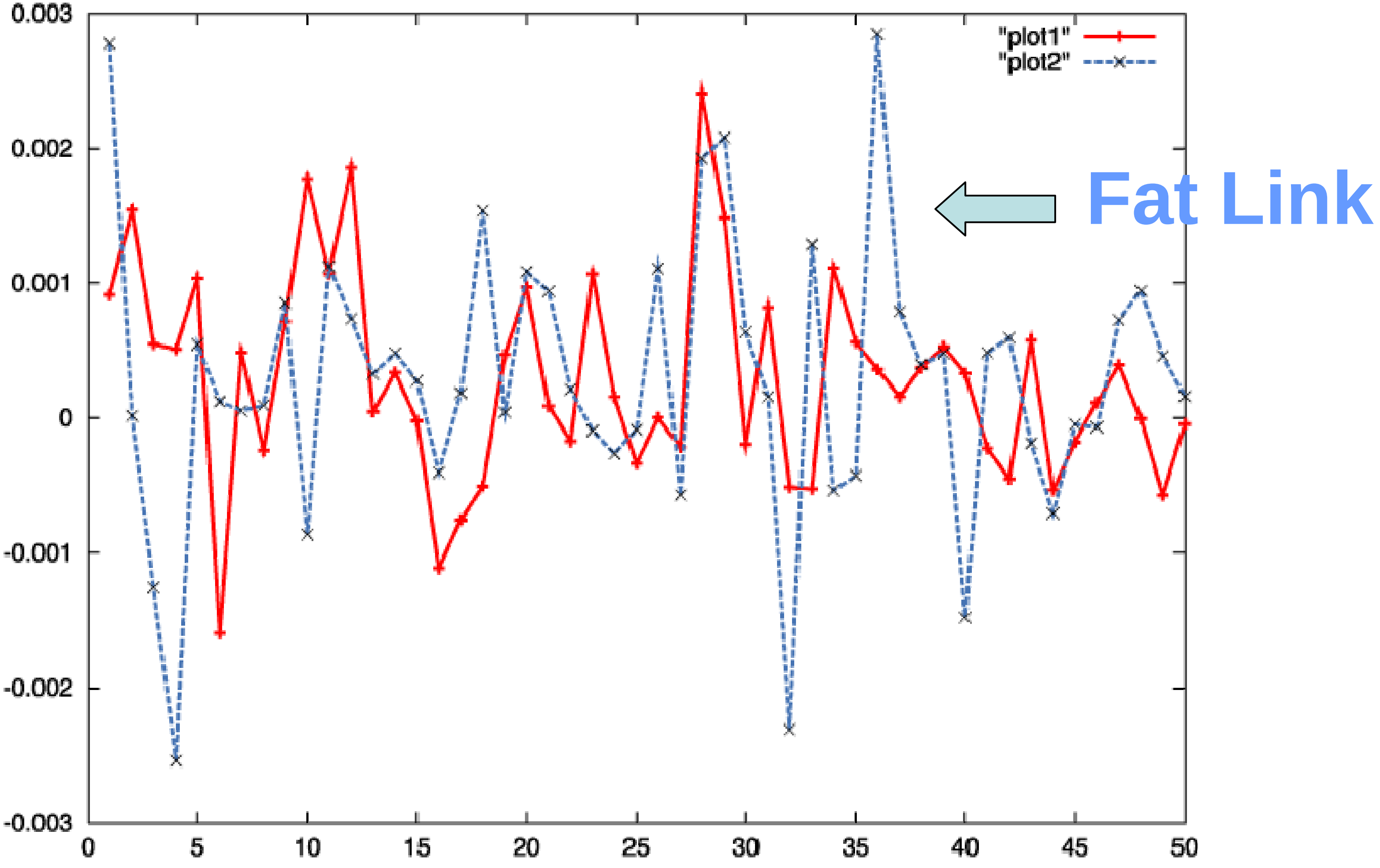
The situation reminds us Glue-Ball Case. (I thank Ph.deForcrand for discussions on this point.)

- Glue-Ball Correlators = $\left\langle \Box(\tau)\,\Box(0) \right\rangle$
- Large (extended) Operators work better, e.g.,



where

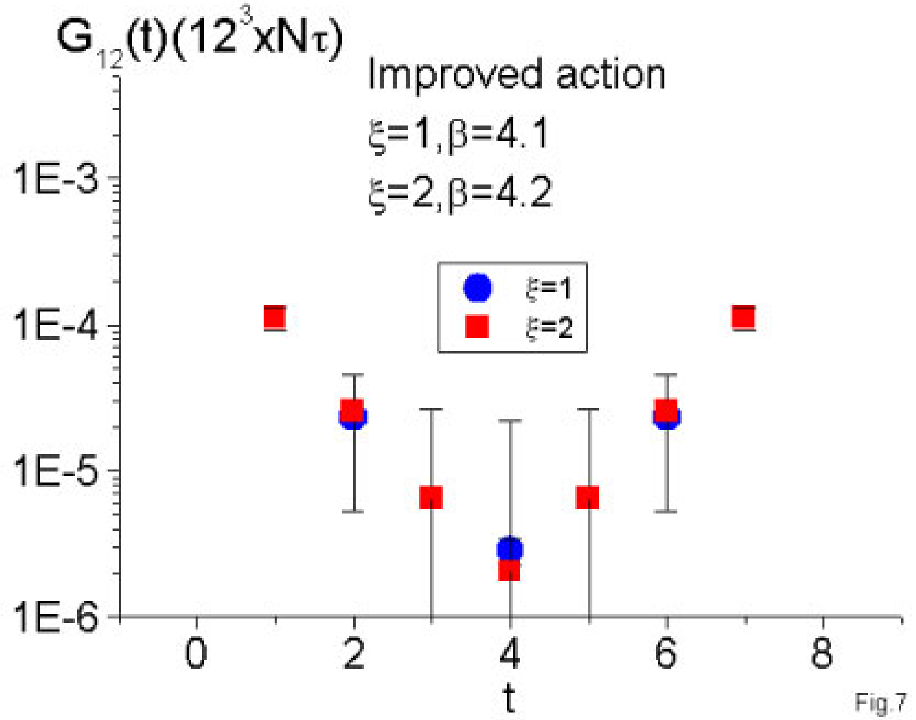


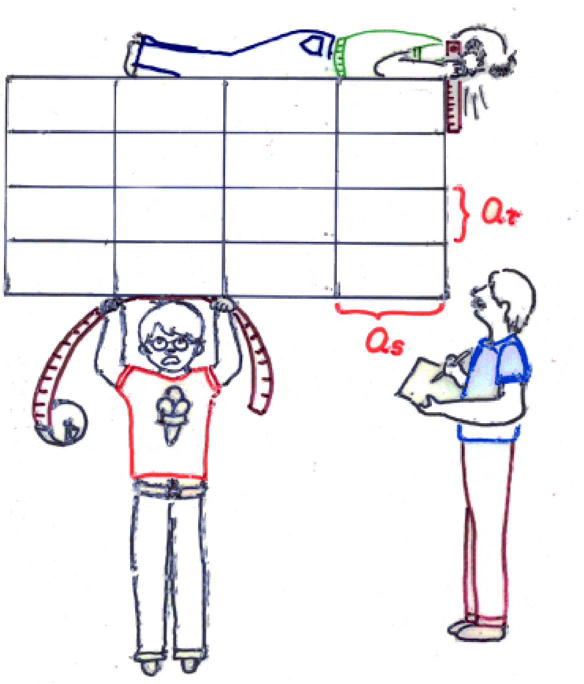


- Mmmm… not works …

Anisotropic Lattice ?

- Anisotropic lattice has matured and will help us to get more data points to determine the spectral function.





フーリエ変換

格子QCD計算で得られたクォークプロパゲータ

$$G(n',n) = G(n'-n) = \frac{1}{N}\sum_i \left\langle \bar{\psi}(n')\psi(n) \right\rangle_i = \frac{1}{N}\sum_i \left\langle W^{-1}(n'-n) \right\rangle_i$$

を運動量空間へフーリエ変換する

$$G(p_4, p_i) = \sum_{n_\mu=0}^{N_\mu} \exp(i(p \cdot n))G(n)$$

$$p_4 = \frac{2\pi}{N_4} \times (n_4 + \frac{1}{2})$$ ：反周期的境界条件

$$p_i = \frac{2\pi}{N_i} \times n_i$$ ：周期的境界条件

Quark Propagators

$$G(p_4, p_i) = \frac{1}{iA(p_4^2, p_i^2)\gamma_4 p_4 + iB(p_4^2, p_i^2)\gamma_i p_i + C(p_4^2, p_i^2)}$$

$$= \frac{1}{A(p_4^2, p_i^2)} \frac{1}{i\gamma_4 p_4 + iB'(p_4^2, p_i^2)\gamma_i p_i + C'(p_4^2, p_i^2)}$$

$$= \frac{1}{A(p_4^2, p_i^2)} \frac{-i\gamma_4 p_4 - iB'(p_4^2, p_i^2)\gamma_i p_i + C'(p_4^2, p_i^2)}{p_4^2 + B'^2(p_4^2, p_i^2)p_i^2 + C'^2(p_4^2, p_i^2)}$$

$$B'(p_4^2, p_i^2) = \frac{B(p_4^2, p_i^2)}{A(p_4^2, p_i^2)} \qquad C'(p_4^2, p_i^2) = \frac{C(p_4^2, p_i^2)}{A(p_4^2, p_i^2)}$$

$$p_4 \to p_4 + i\mu$$

C. D. Roberts and S. M. Schmidt, nucl-th/0005064

クォークプロパゲータから得られる量

波動関数： $Z(q_4,q_i) = \dfrac{A_{free}(q_4^2,q_i^2)}{A(q_4^2,q_i^2)}$

J-I. Skullerud and A. G. Williams Phys. Rev. D63, 054508

クォーク閉じ込めのシナリオと関わる重要な量

クォークが閉じ込められている ⟺ 4元運動量がゼロの点で, ゼロになる

C. D. Roberts and A. G. Williams, hep-ph/9403224

ゼロ温度における格子計算でゼにならないことが確認された

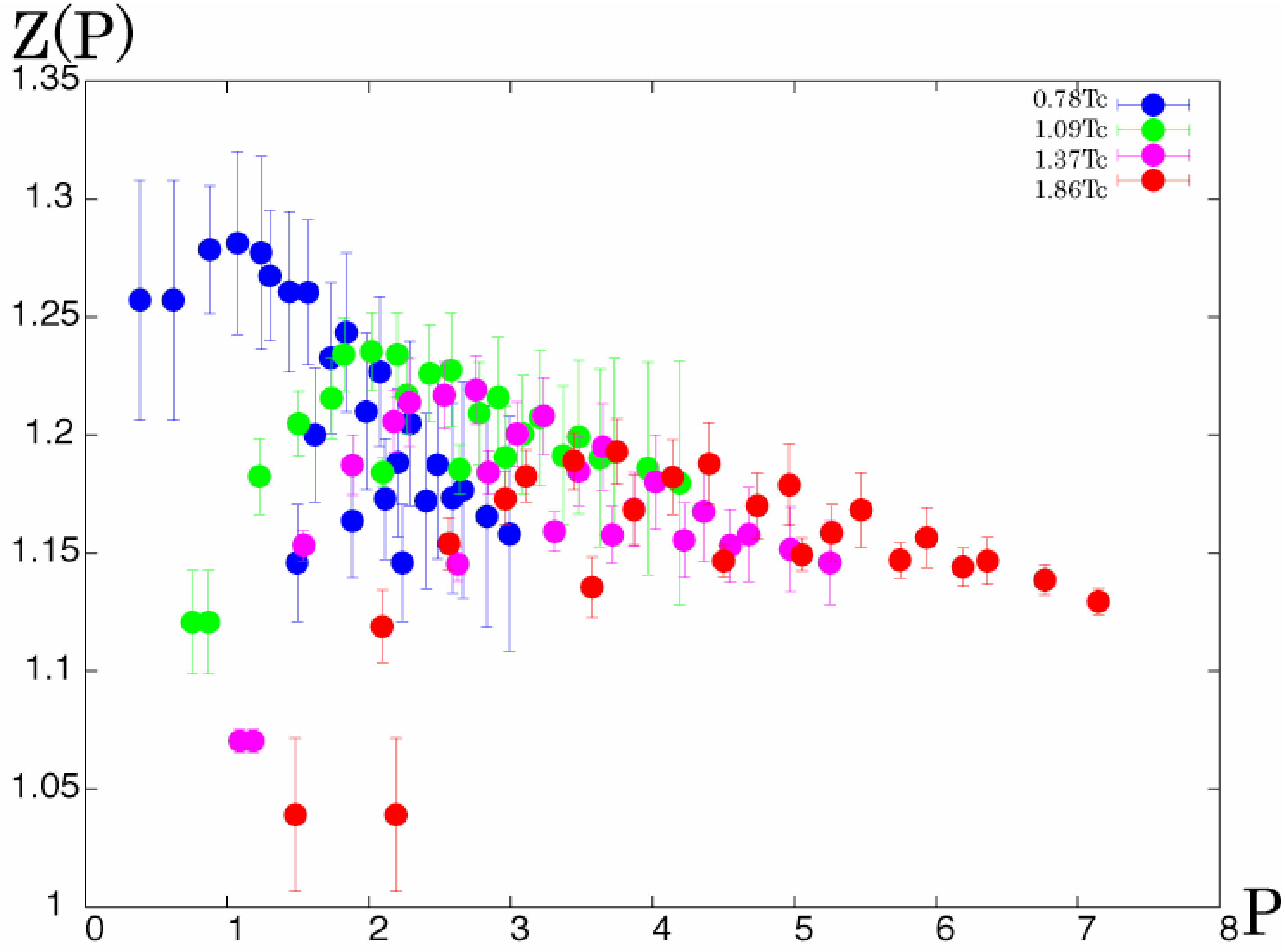
P. O. Bowman et. al., *Lattice Hadron Physics*, (Springer)

質量関数： $M(q_4,q_i) = C'(q_4^2,q_i^2) - C'_{free}(q_4^2,q_i^2)$

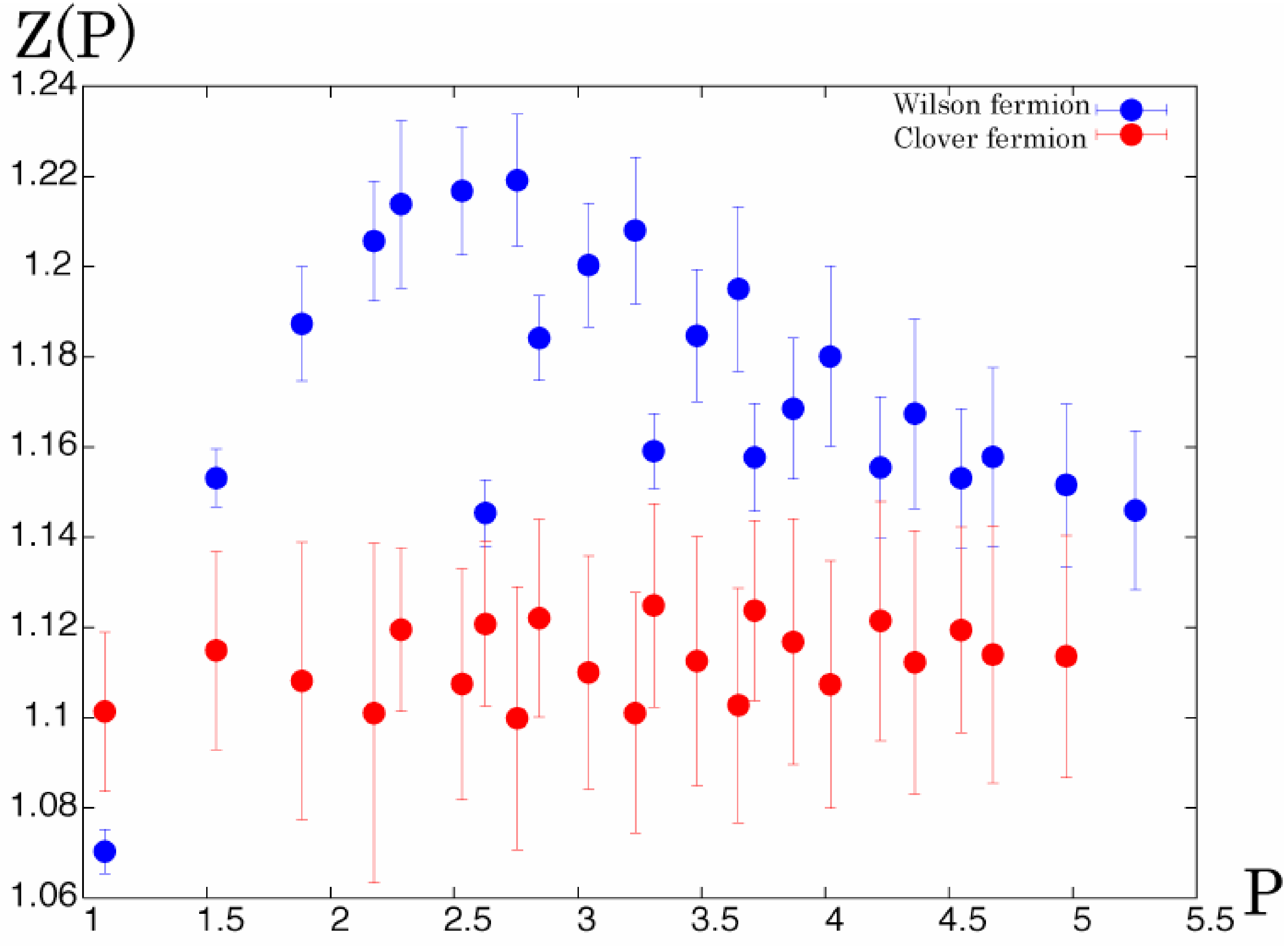
カイラル対称性の自発的破れの効果があらわれる量

P. O. Bowman et. al., *Lattice Hadron Physics*, (Springer)

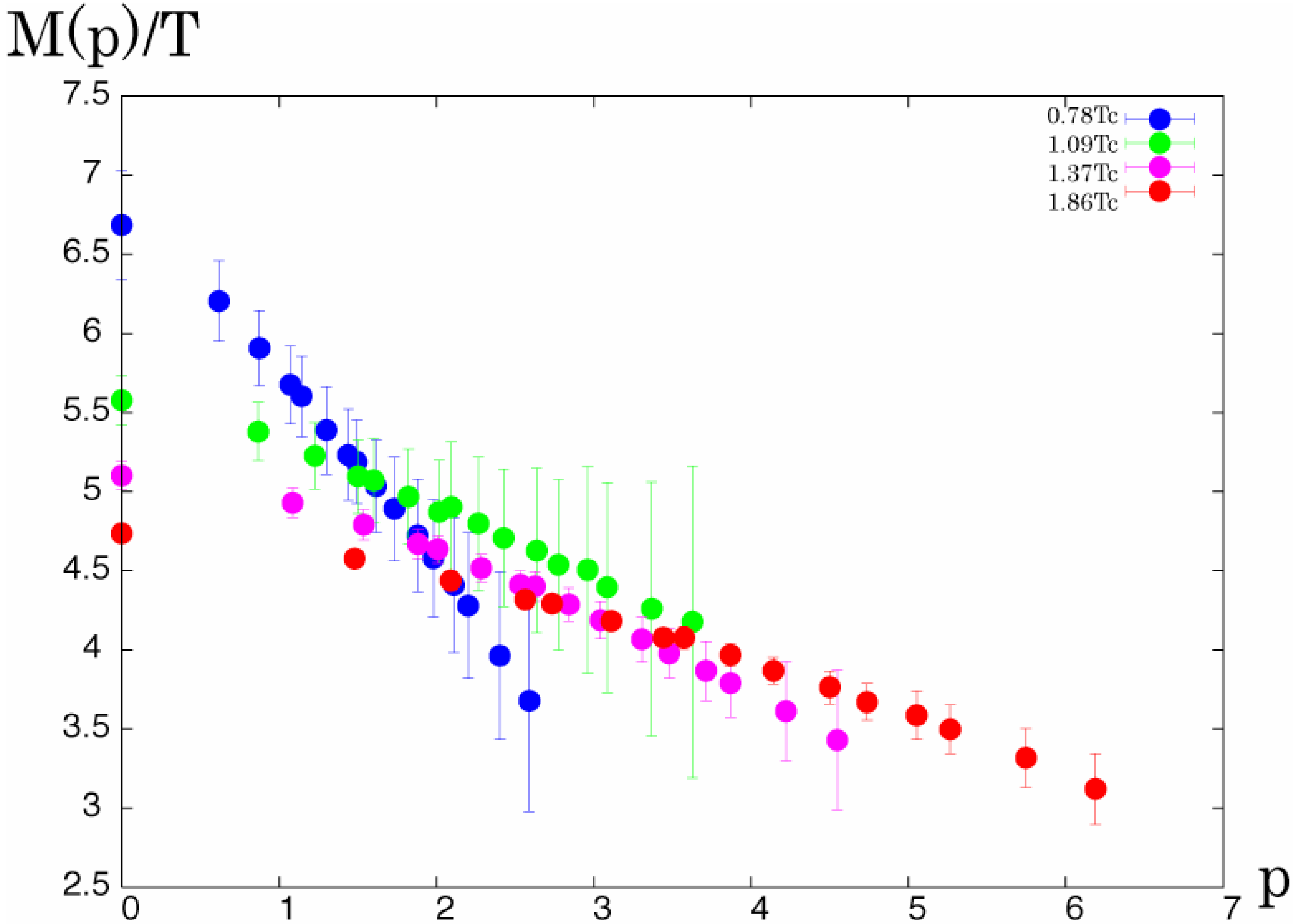
Momentum Dependence (Wilson fermion)



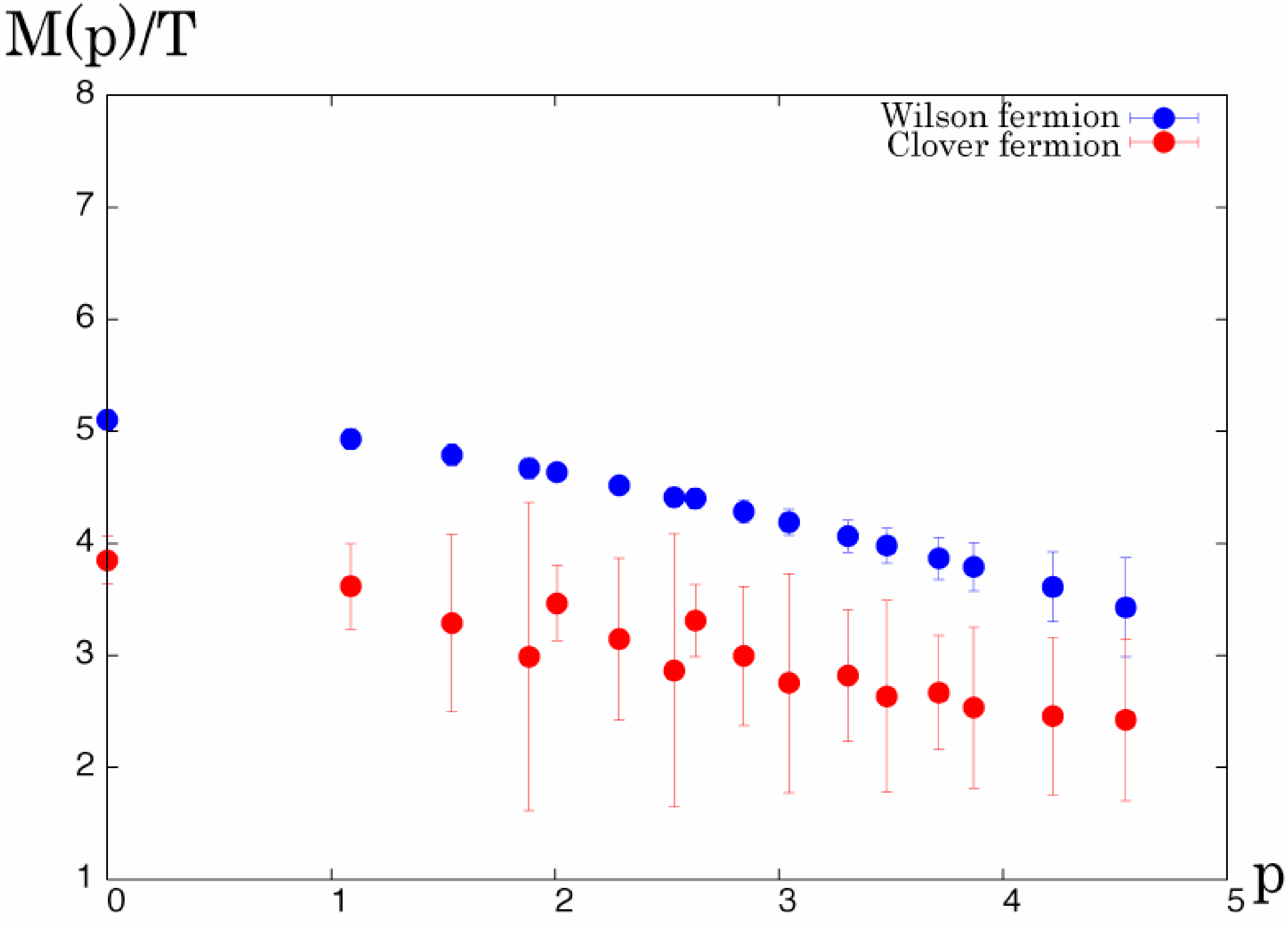
Momentum Dependence (Wilson fermion vs. Clover fermion)



Momentum Dependence (Wilson fermion)



質量関数の3元運動量依存性(Wilson fermion vs. Clover fermion)



ポール質量1

ポール質量は，実時間形式のプロパゲータの分母がゼロになる点で与えられる．

$$p_0^2 - M(p_i^2) = 0$$

虚時間形式のプロパゲータの分母を考える　$(p_0^2 \to -p_4^2)$

$$p_4^2 + B'^2(p_4^2, p_i^2)p_i^2 + C'^2(p_4^2, p_i^2)$$

空間の運動量がゼロの点のポールが熱質量 ⇒ $p_i = 0$

$C'(p_4^2, 0)$ を p_4^2 で展開する．

$$C'(p_4^2) = C'(0) + \left.\frac{\partial C'(p_4^2)}{\partial p_4^2}\right|_{p_4^2=0} p_4^2$$

Pole Mass

$$p_4^2 + C'^2(p_4^2, p_i^2) = p_4^2 + \left(C'(0) + \left.\frac{\partial C'(p_4^2)}{\partial p_4^2}\right|_{p_4^2=0} p_4^2 \right)^2$$

$$\simeq \left(1 + 2C'(0) \left.\frac{\partial C'(p_4^2)}{\partial p_4^2}\right|_{p_4^2=0} \right) \left(p_4^2 + \frac{C'^2(0)}{1 + 2C'(0) \left.\frac{\partial C'(p_4^2)}{\partial p_4^2}\right|_{p_4^2=0}} \right)$$

$$M_{pole}^2 = \frac{C'^2(0)}{1 + 2C'(0) \left.\frac{\partial C'(p_4^2)}{\partial p_4^2}\right|_{p_4^2=0}}$$