Universality and scaling at the chiral transition in two-flavor QCD at finite temperature

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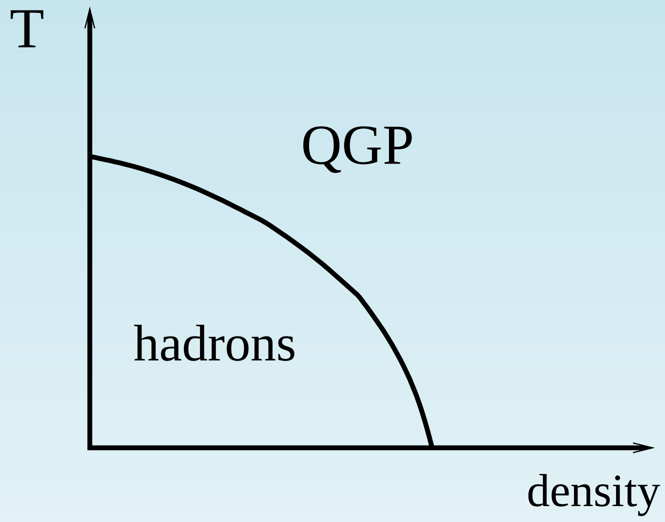
Abstract

It is clear that the order of the deconfining phase transition in QCD at nonzero temperature and density (with realistic quark masses) is of great experimental interest. The importance of the transition at zero baryon density with only two (degenerate) quark flavors is perhaps less evident. As a matter of fact, the order of the transition in this case is still an open problem and corresponds to the last question mark in the zero-density phase diagram of QCD. We argue that establishing the nature of the transition in this case is also a crucial test for numerical simulations of lattice QCD, allowing precise estimates of the possible systematic errors related to the choice of fermion-simulation algorithm and of discretized formulation for fermions.

QCD Phase Transition



- deconfinement and restoration of chiral symmetry at high temperatures or densities; what are the properties of the high-temperature phase (QGP)? what is the nature of the transition?

- a first-order transition for $\mu = 0, T \neq 0$ would mean a real transition in the general case, perhaps corresponding to cosmological relics.

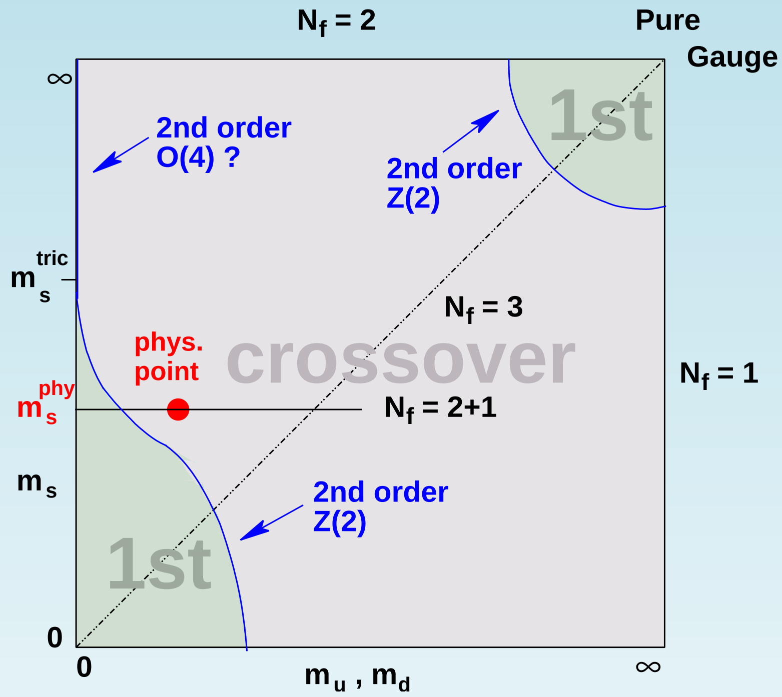
- a second-order transition in the $\mu = 0$ two-flavor case (two light quarks) would mean no transition, or a crossover in the general case, with the possibility of a critical endpoint in the phase diagram.

Study of the QCD Phase Transition

- non-perturbative study through numerical simulations of lattice QCD $\Rightarrow$ finite temperature and finite density
- for two flavors: prediction of critical behavior in the universality class of the $O(4)$ spin model has not been verified (see F. Wilczek, NPB Proc. Suppl. 2003)
- predicted scaling is observed for Wilson fermions but not for staggered fermions
- indications of first-order transition by M. D'Elia et al., PRD 2005 and arXiv:0707.1987
- if a second-order transition takes place, then systematic effects in the simulation may be large around the transition
- in this case, the prediction of universality presents a stringent test for full-QCD simulations

Lattice QCD at Finite Temperature

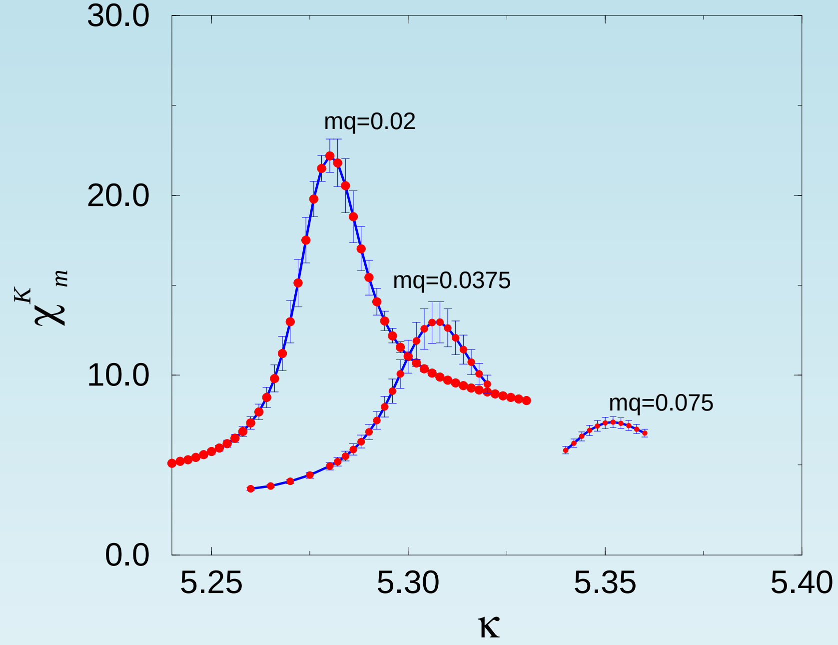
$$\mathcal{S} = -\beta \sum_{\square} \frac{1}{N_c} \text{Re Tr}\, U_{\square} + \sum_{x,y} \overline{\psi}_x K_{x,y} \psi_y$$



In the path integral: inverse temperature ⇔ temporal extension

- pure gauge (quenched): order parameter given by the Polyakov loop $< L >$, breaking of the symmetry $Z(N_c)$
  - $SU(3)$ first order (3-state Potts model)
  - $SU(2)$ second order ($3d$ Ising)
- with dynamical fermions: chiral condensate $< \overline{\psi}\,\psi >$, sensitive to the breaking of the chiral symmetry.
- Effective σ model (Ginzburg-Landau theory respecting the chiral symmetry). For two flavors: critical behavior in the universality class of the $O(4)$ spin model (if the transition is second order).

QCD with 2 staggered quark flavors



- order parameter: Chiral condensate $<\overline{\psi}\,\psi>$
- magnetic field: quark mass m_q
- reduced temperature: $\tau \sim 6/g^2 - 6/g_c^2(0)$ (from $T = (L_t\,a)^{-1}$)
- peak of the chiral susceptibility: $t_p \sim {m_q}^{1/\beta\delta}$

Bielefeld (1994), JLQCD (1998), MILC (2000) $\longrightarrow$ peaks scale with the predicted exponents

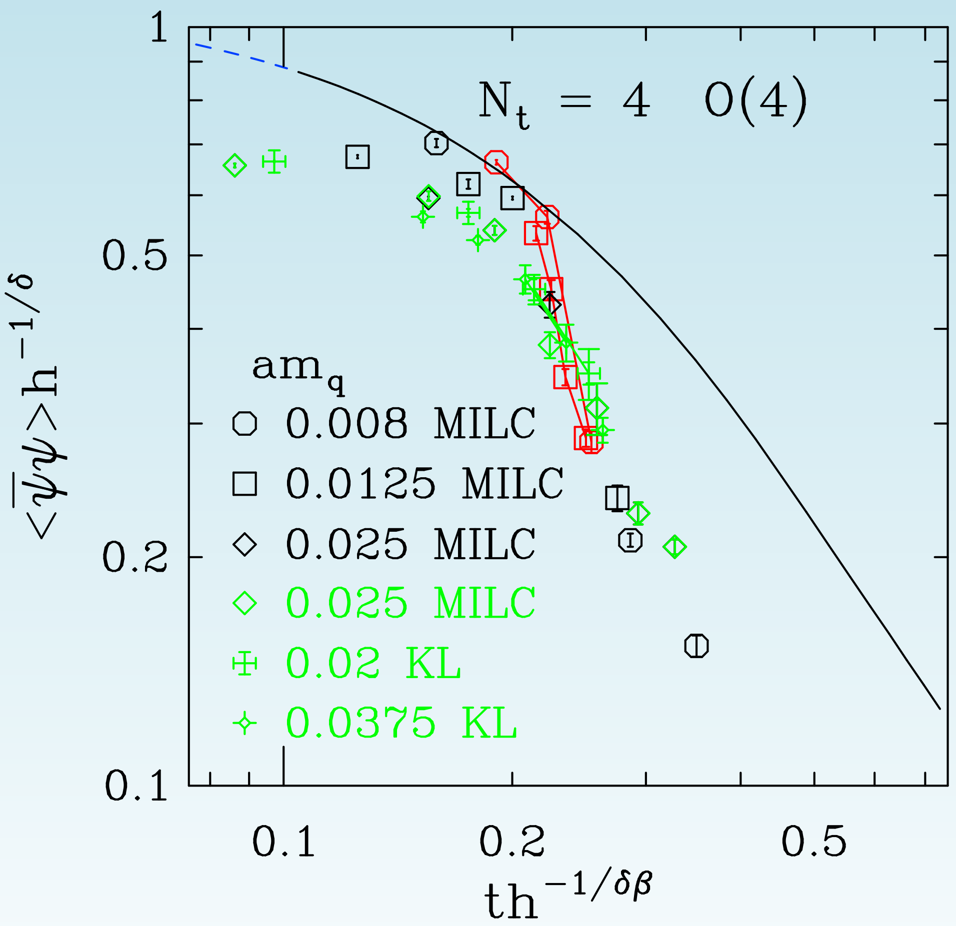
$3d\;O(4)$ model: precise numerical determination of the critical properties.

M is described by the universal function f_M for all systems in the universality class

$$M/h^{1/\delta} = f_M(t/h^{1/\beta\delta})$$

Engels, T. M., Nucl. Phys. B '00; Engels et al., Phys. Lett. B '01; Cucchieri, T.M., JPA 2005.

Scaling Function for 2 Staggered Quarks



Bernard et al., Phys. Rev. D '00

No Scaling! Possible Problems...

- not chiral enough (large quark masses)
- the transition is not second-order
- effects due to the staggered formulation for quarks
- finite-size effects
- systematic effects (due to the use of the R algorithm)

in any case, may perform a better comparison by an unambiguous normalization of the data [assuming $O(4)$ behavior and using the observed scaling of susceptibility peaks]

Universality

- **Critical Exponents**: for Magnetization M (order parameter) and Susceptibility χ

$$M_{h=0} \sim |t|^{\beta}, \qquad \chi_{h=0} \sim |t|^{-\gamma}, \qquad M_{t=0} \sim h^{1/\delta}$$

where $t = (T - T_c)/T_0$, $h = H/H_0$. (Exponents are known for the $3d$ $O(4)$-model)

- **Scaling Function**: given T_0 and H_0, M is described by the universal function f_M for all systems in the universality class

$$M/h^{1/\delta} = f_M(t/h^{1/\beta\delta})$$

Normalized f_M calculated for $3d$ $O(4)$-model

Engels, T. M., Nucl. Phys. B572, 289 (2000)

Pseudo-Critical Line ($H \neq 0$)

Defined by the (finite) peaks of χ, corresponding to the divergence for $H = 0$, $T = T_c$.

Scaling Function for χ

$$\chi \;=\; \partial M/\partial H \;=\; (1/H_0)\, h^{1/\delta - 1}\, g(t/h^{1/\beta})$$

with $g(z)$ universal (related to f_M)

Peak for each fixed h: $\quad t_p \;=\; z_p\, h^{1/\beta\delta}$

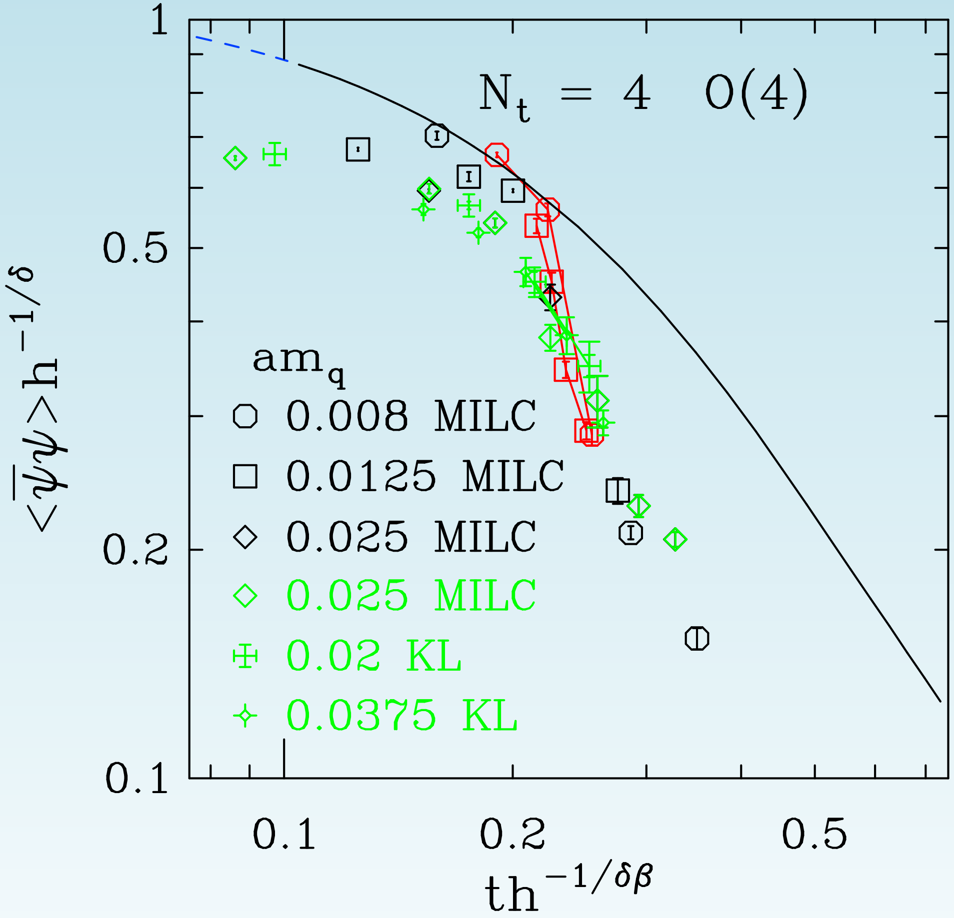
$$M_p \;=\; h^{1/\delta}\, f_M(z_p), \qquad H_0\, \chi_p \;=\; h^{1/\delta - 1}\, g(z_p)$$

Thus the pseudo-critical line is given by the universal constants z_p, $f_M(z_p)$, $g(z_p)$.

For the $3d\; O(4)$-model $z_p = 1.33(5)$

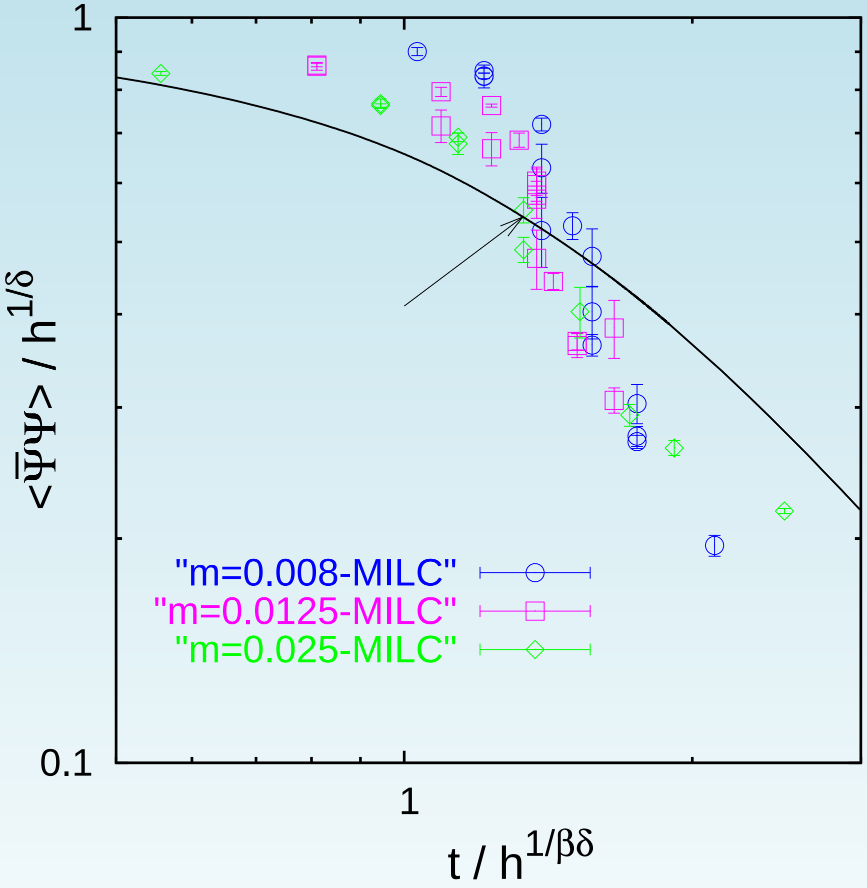
Engels et al., Phys. Lett. B514, 299 (2001)

Scaling Function



Bernard et al., Phys. Rev. D '00

After Normalization

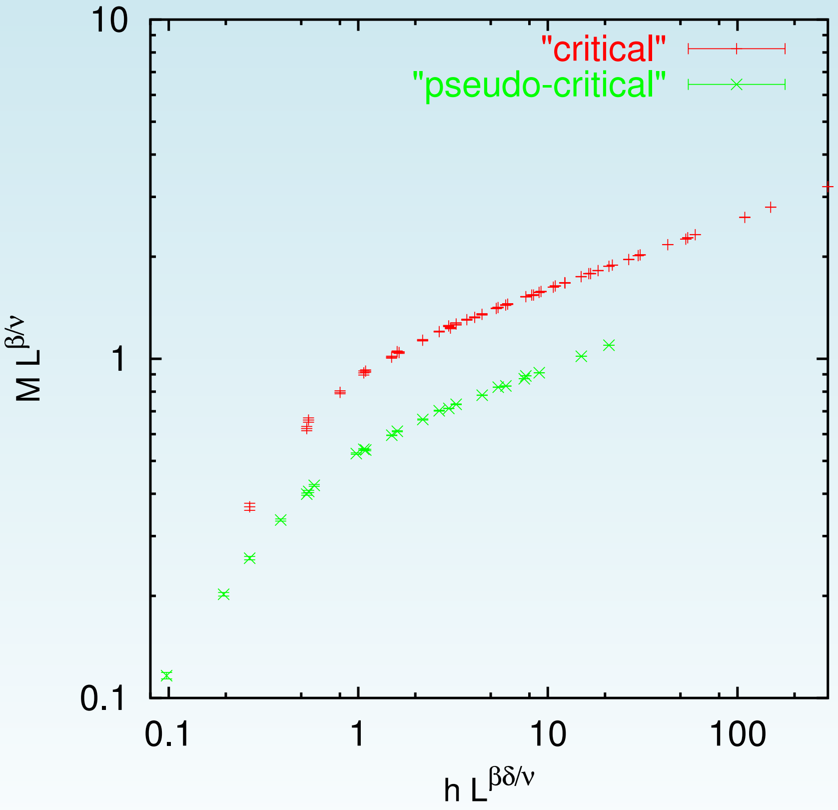


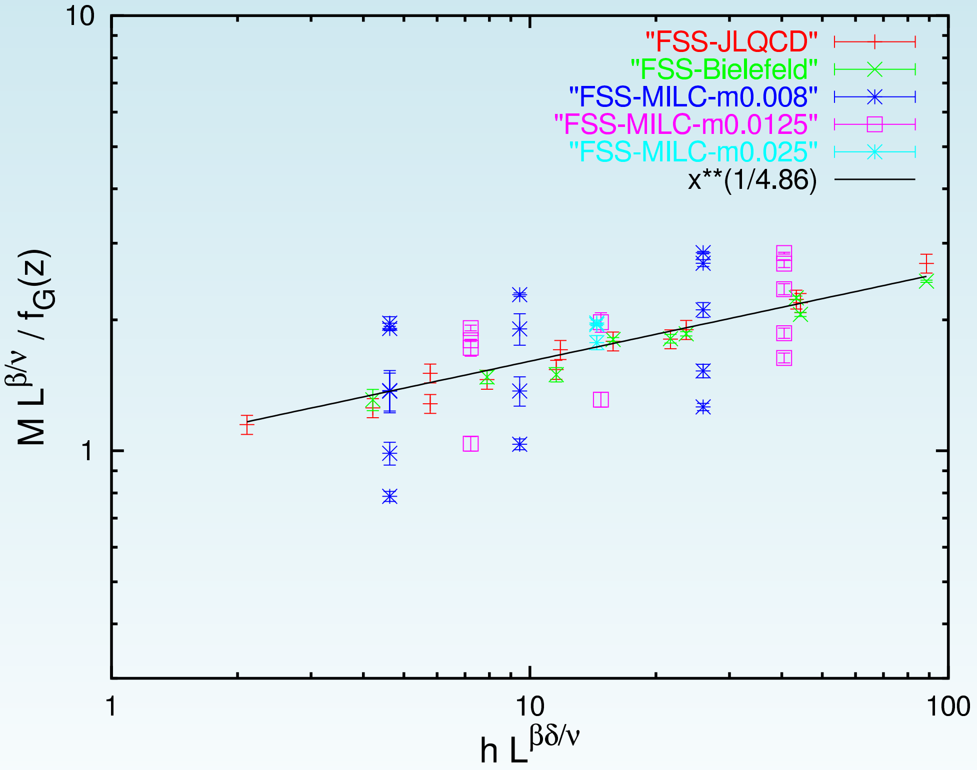
T.M., Nucl. Phys. A '02

Finite-Size Scaling

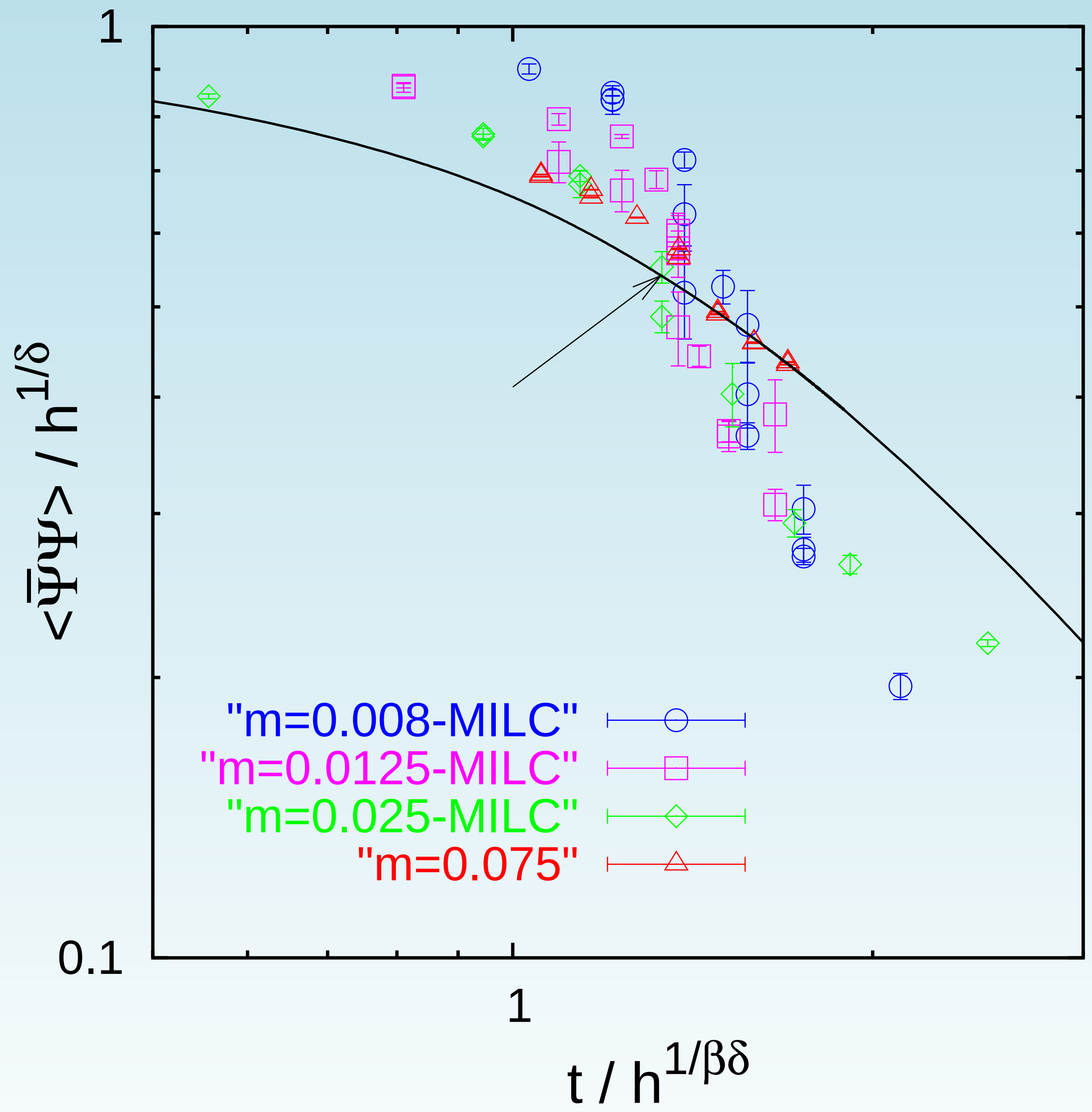
$M = L^{-\beta/\nu}\, Q_M(h\, L^{\beta\delta/\nu})$ for fixed $z = t/h^{1/\beta\delta}$ (e.g. $z = 0$, $z = z_p$)

Note: $M = h^{1/\delta}\, f_M(z)$ for $L \to \infty$. Thus $Q_M(u) \to f_M(z)\, u^{1/\delta}$ for large u

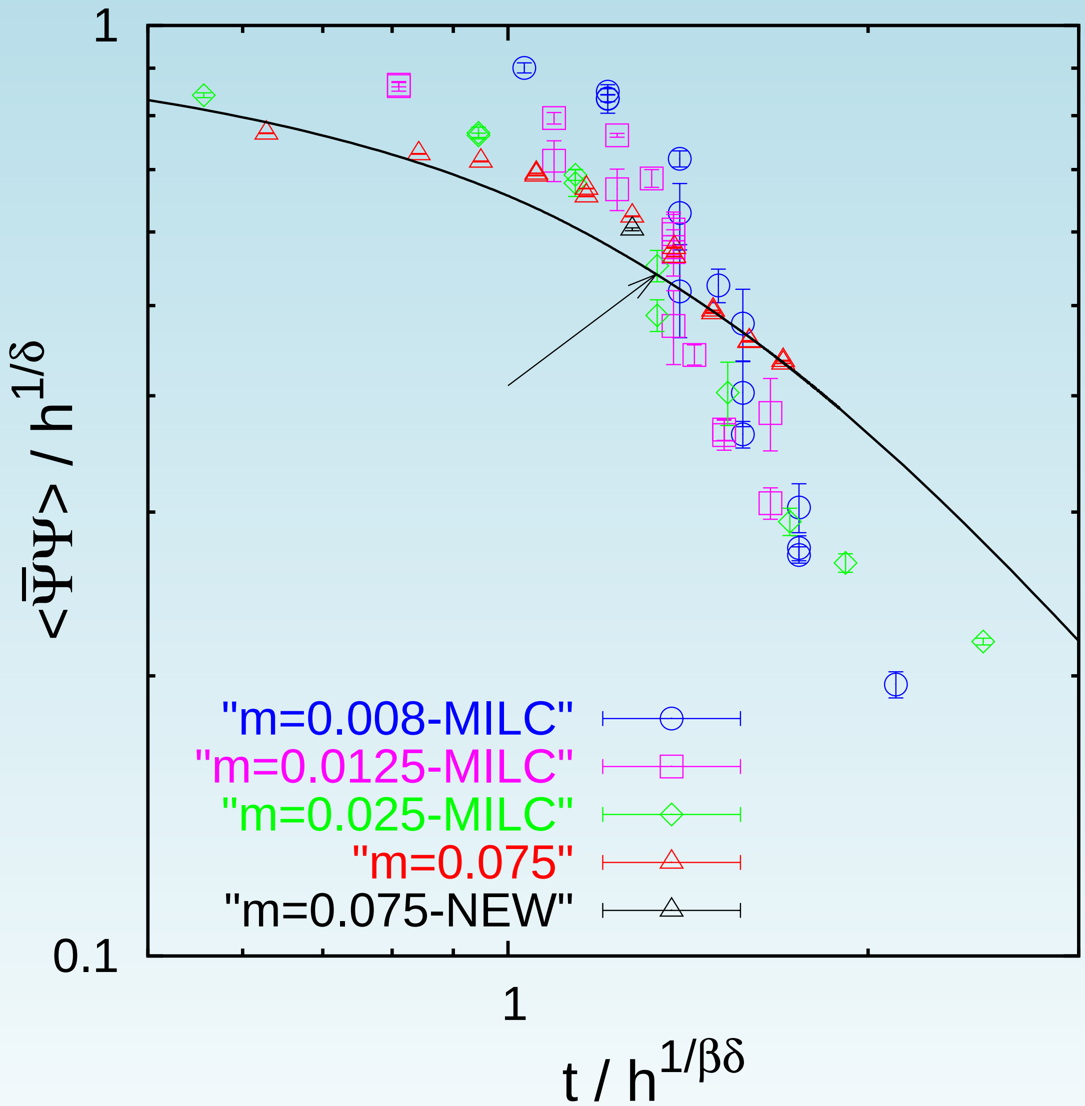




Including New Data at Larger Masses



T.M., Cucchieri, AIP Conf. Proc. (2004)



T.M., AIP Conf. Proc. (2005)

In Favor of a First Order Transition

D'Elia, Di Giacomo, Pica (PRD 2005)

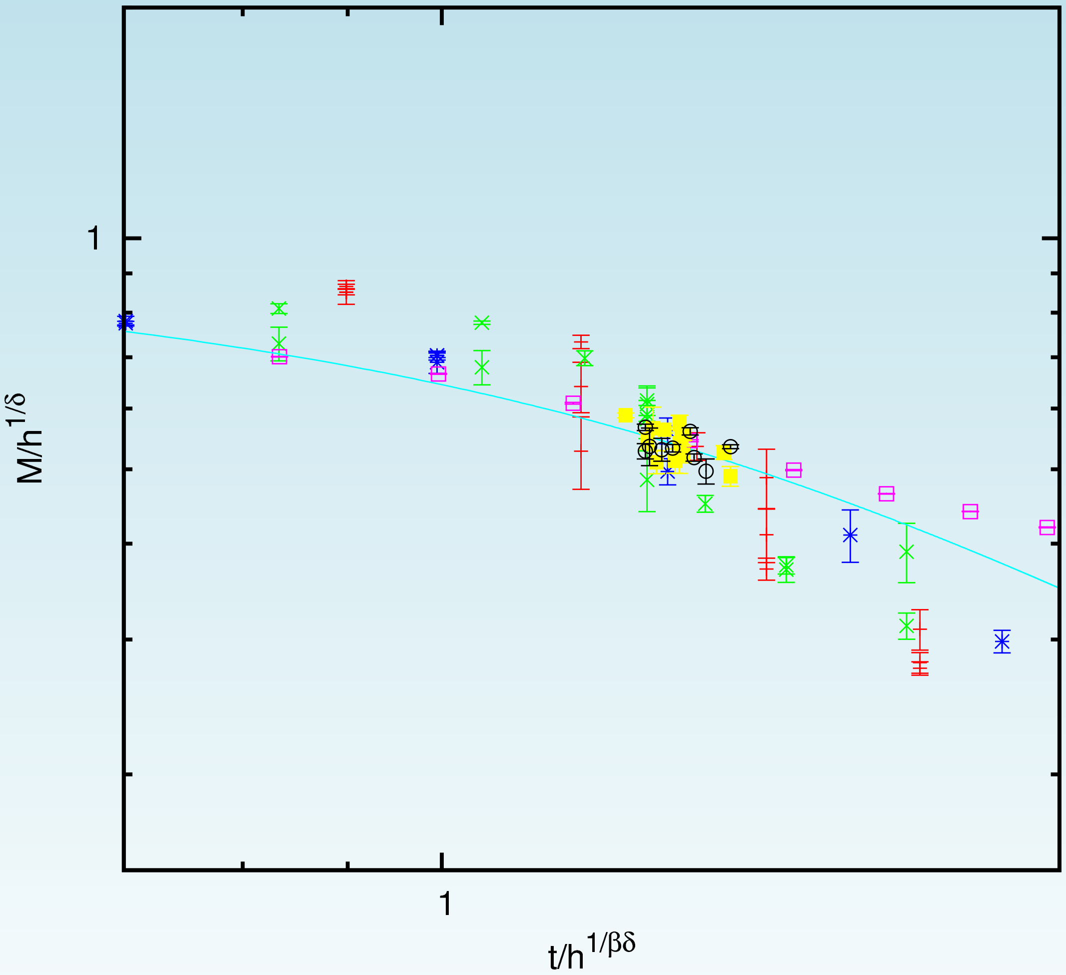
- better definition for temperature in terms of lattice spacing:

$$\tau \sim 6/g^2 - 6/g_c^2(0) + k_m\, m_q$$

  for full QCD $k_m \neq 0$

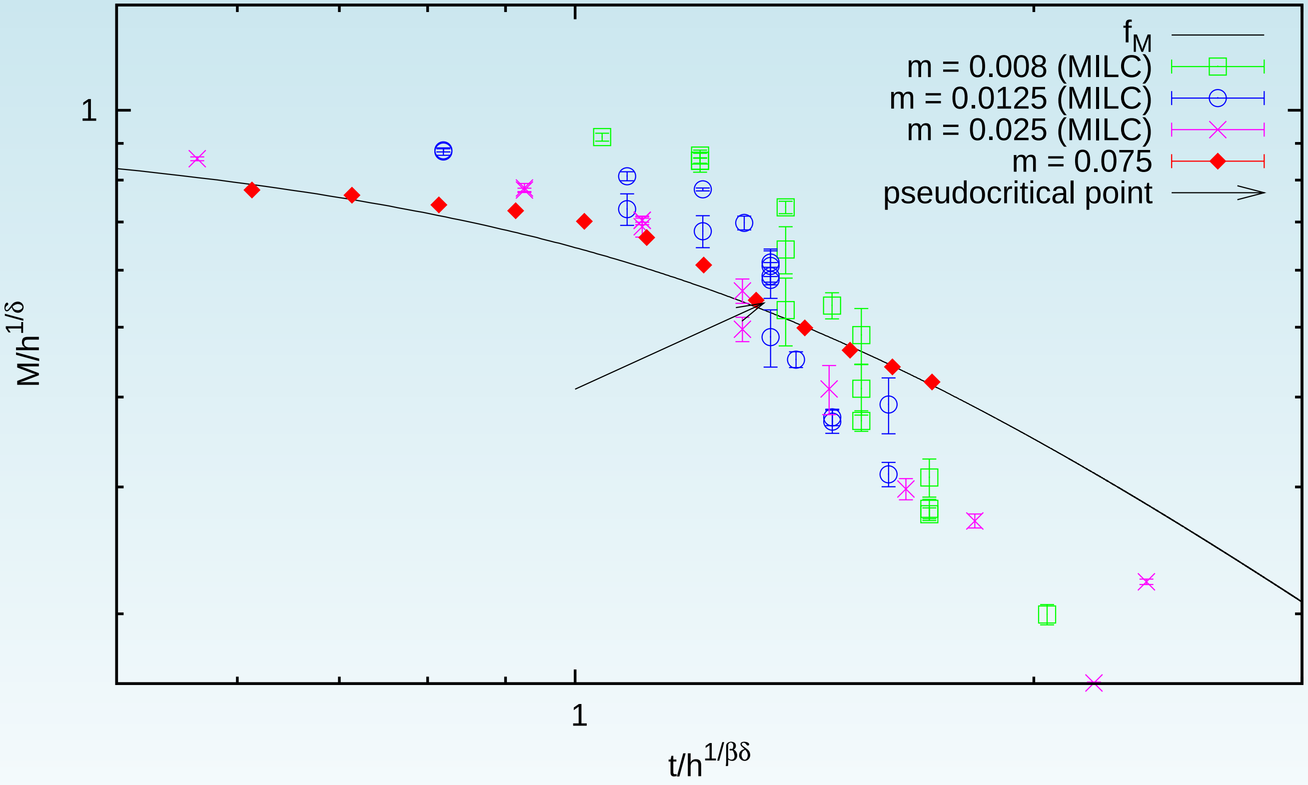
- data are produced taking finite-size scaling into account
- largest lattices to date (recently: improved fermions, RHMC)
- no evidence for discontinuities
- seems to exclude $O(4)$ scaling and roughly consistent with 1st order scaling

∞-Volume Scaling for New Definition of τ



In Favor of a Second-Order Transition

Plot of scaling function including new data at the largest mass (and smaller δt), combined with older MILC data



T.M., IJMP E, to appear

Conclusions

- By an unambiguous normalization of QCD data for 2 staggered fermions the predicted $O(4)$ scaling seems to be achieved for larger quark masses
- The universal scaling range around the pseudo-critical point z_p seems to be rather large, with data at smaller masses describing a wiggle around z_p
- Simulation points for $m = 0.075$ show good scaling, even to the right of z_p
- Using a significantly smaller integration step in the R-algorithm the deviations for $m = 0.075$ decrease visibly
- Results suggest that the (large) deviations at small masses might be due to systematic errors in the simulations
- Agreement with Wilson case (second-order transition)
- 2-flavor case is especially suitable for testing better algorithms (RHMC)