

Frithjof Karsch, BNL

Introduction:

deconfinement and chiral symmetry restoration

- The transition temperature in (2+1)-flavor QCD
- Bulk thermodynamics
- Conclusions

Deconfinement and χ -symmetry

- The chiral phase transition (i.e. at $m_q = 0$) is deconfining
 - true in QCD, i.e. SU(3) + fermions in the fundamental representation
 - SU(3) + fermions in the adjoint representation: $T_{deconf} < T_{\chi}$
- The transition in QCD with physical quark masses is a crossover

In which sense is the transition

deconfining and chiral symmetry restoring?

- deconfinement: heavy hadrons \Rightarrow light quarks and gluons; liberation of many new light degrees of freedom \Rightarrow rapid change in ϵ/T^4 , s/T^3 ,
- chiral symmetry restoration: vanishing mass splittings, no new degrees of freedom

⇒ minor effect on bulk thermodynamics, but rapid change of chiral condensate

Deconfinement and χ -symmetry

- The chiral phase transition (i.e. at $m_q = 0$) is deconfining
 - true in QCD, i.e. SU(3) + fermions in the fundamental representation
 - SU(3) + fermions in the adjoint representation: $T_{deconf} < T_{\chi}$
- The transition in QCD with physical quark masses is a crossover

In which sense is the transition

deconfining and chiral symmetry restoring?

- deconfinement: heavy hadrons \Rightarrow light quarks and gluons; liberation of many new light degrees of freedom \Rightarrow rapid change in ϵ/T^4 , s/T^3 ,
- chiral symmetry restoration: vanishing mass splittings, no new degrees of freedom

⇒ minor effect on bulk thermodynamics, but rapid change of chiral condensate

Summary of recent results on T_c



deconfinement

chiral

chiral+deconfinement

DECONFINEMENT and χ - SYMMETRY RESTORATION

Susceptibilities: χ_l , χ_L



200,000/10 trajectories enter Ferrenberg-Swendsen sample

DECONFINEMENT and χ - SYMMETRY RESTORATION

Susceptibilities: χ_l , χ_L



200,000/10 trajectories enter Ferrenberg-Swendsen sample

Chiral susceptibility, $N_{ au}=4,~6$





- weak volume dependence
- peak location consistent with that of Polyakov loop susceptibility and maximum of quartic fluctuation of quark number density

Ambiguities in locating the crossover point

0.025 $\beta_L - \beta_l$ $\beta_L - \beta_l$ $\beta_L - \beta_l$ 0.020 $\beta_l - \beta_s$ $\beta_l - \beta_s$ $\beta_l - \beta_s$ 0.015 8^3 x4 $16^{3}x4$ $16^{3}x6$ 0.010 differences of pseudo-critical couplings 0.005 locating peaks in 0.000 light (β_l), strange (β_s) and Polyakov loop (β_L) -0.005 susceptibilities m_//m_s m_l/m_s m_l/m_s -0.010 0 0.5 0 0.5 0 0.5 2.5% ($N_{\tau} = 4$) or 4% ($N_{\tau} = 6$) error band \Leftrightarrow 5 or 8 MeV

differences in the location of pseudo-critical couplings are taken into account as systematic error

Deconfinement and χ -symmetry

hotQCD, preliminary results for $32^3 \times 8$, $m_q = 0.1 m_s$ consistent with RBC-Bielefeld: $16^3 \times 4$ and $24^3 \times 6$, $m_q = 0.1 m_s$



$T_c r_0$: continuum and quark mass extrapolation

extrapolation to chiral and continuum limit



Transition temperature: $N_{ au}=4,6$

- RBC-Bielefeld (p4fat3 (p4)) vs. MILC (asqtad (Naik)) and Wuppertal (stout (stand. staggered))
- **asqtad** results for $N_{\tau} = 4$ and 6 agree with p4 results within statistical errors; (C.Bernard et al., PR D71, 034504 (2005))
- results obtained with stout action for $N_{\tau} = 4$ and 6 are about 15% lower; β_c from $N_{\tau} = 8$, 10 covers (151 - 176) MeV; (Y. Aoki et al., hep-lat/0609068)



asqtad data for $T_c r_1$ rescaled with $r_0/r_1 = 1.4795$

Transition temperature: $N_{ au}=4,6$

- RBC-Bielefeld (p4fat3 (p4)) vs. MILC (asqtad (Naik)) and Wuppertal (stout (stand. staggered))
- **asqtad** results for $N_{\tau} = 4$ and 6 agree with p4 results within statistical errors; (C.Bernard et al., PR D71, 034504 (2005))
- results obtained with stout action for $N_{\tau} = 4$ and 6 are about 15% lower; β_c from $N_{\tau} = 8$, 10 covers (151 - 176) MeV; (Y. Aoki et al., hep-lat/0609068)



Transition temperatures

RBC-Bielefeld (p4fat3 (p4)) vs. MILC (asqtad (Naik)) and Wuppertal (stout (stand. staggered)) vs. DIK-collaboration (clover improved Wilson)



extrapolations to phys. point

- RBC-Bielefeld (p4fat3 (p4)) vs. Wuppertal (stout (stand. staggered))
- **stout** results for different observables no longer consistent with each other for $N_{ au} = 8, 10$



Y. Aoki et al, Phys. Lett. B643 (2006) 46

uses f_k to set the scale

extrapolations to phys. point

- RBC-Bielefeld (p4fat3 (p4)) vs. Wuppertal (stout (stand. staggered))
- **stout** results for different observables no longer consistent with each other for $N_{\tau} = 8, 10$
- If the second second second second second structure $N_{ au}=4,6$ differ by 15% but show similar cut-off dependence



Quark number susceptibility... ...and its susceptibility

- rapid change in quark/baryon/strangeness number susceptibility reflects change in mass of the carrier of these quantum numbers DECONFINEMENT
- quark number susceptibility feels nearby singular point just like the energy density

scaling field:
$$t = \left| \frac{T - T_c}{T_c} \right| + A \left(\frac{\mu_q}{T_c} \right)^2$$
, $\mu_{crit} = 0$
singular part: $f_s(T, \mu_q) = b^{-1} f_s(tb^{1/(2-\alpha)}) \sim t^{2-\alpha}$

Y. Hatta, T. Ikeda, PRD67 (2003) 014028

$$egin{aligned} c_2 &\equiv \chi_q \sim rac{\partial^2 \ln \mathcal{Z}}{\partial \mu_q^2} \sim t^{1-lpha} &, \quad c_4 \sim rac{\partial^4 \ln \mathcal{Z}}{\partial \mu_q^4} \sim t^{-lpha} & (\mu=0) \ & \epsilon \sim rac{\partial \ln \mathcal{Z}}{\partial T} \sim t^{1-lpha} &, \quad C_V \sim rac{\partial^2 \ln \mathcal{Z}}{\partial T^2} \sim t^{-lpha} & (\mu=0) \end{aligned}$$

 $\Rightarrow 2^{nd}$ derivative w.r.t μ_q "looks like energy density" $\Rightarrow 4^{th}$ derivative w.r.t μ_q "looks like specific heat"

p4: RBC-Bielefeld, preliminary

Light and Strange Susceptibilities

$$\left(\frac{p}{T^4}\right)_{\mu} - \left(\frac{p}{T^4}\right)_0 = \frac{1}{2} \frac{\chi_l}{T^2} \left(\frac{\mu_l}{T}\right)^2 + \frac{1}{2} \frac{\chi_s}{T^2} \left(\frac{\mu_s}{T}\right)^2 + \frac{\chi_{ls}}{T^2} \frac{\mu_l}{T} \frac{\mu_s}{T} + \mathcal{O}(\mu^4)$$

$$\begin{array}{c} 1.2 \\ 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1.0 \\ \hline & & & & & & & & \\ 1.0 \\ & & & & & & & & \\ 1.2 \\ & & & & & & & \\ 1.2 \\ & & & & & & \\ 1.2 \\ & & & & & & \\ 1.2 \\ & & & & & \\ 1.2 \\ & & & & & \\ 1.2 \\ & & & & & \\ 1.2 \\ & & & & & \\ 1.2 \\ & & & & \\ 1.2 \\ & & & & \\ 1.2 \\ & & & & \\ 1.2 \\ & & & & \\ 1.2 \\ & & & & \\ 1.2 \\ &$$

F. Karsch, xQCD, August 2007 - p. 13/28

p4: RBC-Bielefeld, preliminary

Light and Strange Susceptibilities

$$\left(\frac{p}{T^4}\right)_{\mu} - \left(\frac{p}{T^4}\right)_0 = \frac{1}{2} \frac{\chi_l}{T^2} \left(\frac{\mu_l}{T}\right)^2 + \frac{1}{2} \frac{\chi_s}{T^2} \left(\frac{\mu_s}{T}\right)^2 + \frac{\chi_{ls}}{T^2} \frac{\mu_l}{T} \frac{\mu_s}{T} + \mathcal{O}(\mu^4)$$

$$\begin{array}{c} 1.2 \\ 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1.0 \\ \hline & & & & & & & & \\ 1.0 \\ & & & & & & & & \\ 1.2 \\ & & & & & & & \\ 1.2 \\ & & & & & & \\ 1.2 \\ & & & & & & \\ 1.2 \\ & & & & & \\ 1.2 \\ & & & & & \\ 1.2 \\ & & & & & \\ 1.2 \\ & & & & & \\ 1.2 \\ & & & & \\ 1.2 \\ & & & & \\ 1.2 \\ & & & & \\ 1.2 \\ & & & & \\ 1.2 \\ & & & & \\ 1.2 \\ &$$

F. Karsch, xQCD, August 2007 - p. 13/28

Light and Strange Susceptibilities



F. Karsch, xQCD, August 2007 - p. 13/28

Light and Strange Susceptibilities

$$\left(rac{p}{T^4}
ight)_\mu - \left(rac{p}{T^4}
ight)_0 = rac{1}{2}rac{\chi_l}{T^2}\left(rac{\mu_l}{T}
ight)^2 + rac{1}{2}rac{\chi_s}{T^2}\left(rac{\mu_s}{T}
ight)^2 + rac{\chi_{ls}}{T^2}rac{\mu_l}{T}rac{\mu_s}{T} + \mathcal{O}(\mu^4)$$



Light and Strange Susceptibilities

In hotQCD, preliminary results for $32^3 imes 8$, $m_q = 0.1 m_s$

good agreement between p4 and asqtad persists on finer lattices; small shifts in the transition temperature (inflection points)



Light and Strange Susceptibilities

1-link, stout, physical quark masses;

T-scale from f_K , but $f_K r_0$ consistent with asquad value for r_0 in the continuum limit

p4fat3, $m_q = 0.1 m_s$, i.e. $m_\pi \simeq 220$ MeV; T-scale from r_0 using the asqtad r_0 deduced from 'gold plated observables'



strange quark number susceptibility:

- $T
 ightarrow\infty$, ideal gas limit: χ_l/T^2 , $\chi_s/T^2
 ightarrow 1$
- similar cut-off dependence as pressure \Rightarrow strong cut-off dependence in simulations with not $\mathcal{O}(a^2)$ improved actions

1-link, stout Y. Aoki et al., PLB643, 46 (2006)

p4fat3, $\mathcal{O}(a^2)$ improved RBC-Bielefeld, preliminary $N_{\tau} = 8$: hotQCD, preliminary

Light and Strange Susceptibilities

1-link, stout, physical quark masses;

T-scale from f_K , but $f_K r_0$ consistent with asquad value for r_0 in the continuum limit

p4fat3, $m_q = 0.1 m_s$, i.e. $m_\pi \simeq 220$ MeV; T-scale from r_0 using the asqtad r_0 deduced from 'gold plated observables' expect still a shift of T-scale $\sim 5~{\rm MeV}$ for physical quark masses



F. Karsch, xQCD, August 2007 - p. 16/28

Polyakov loop expectation value $\langle L \rangle = \exp(-F_q(T)/T)$; needs renormalization of divergent quark self-energies:

. **N**T

$$L_{ren}(T) = Z(\beta)^{N_{\tau}} \langle L \rangle(T)$$



used T = 0 potential to determine $Z(\beta)$ for each T > 0 parameter set

Polyakov loop expectation value $\langle L \rangle = \exp(-F_q(T)/T)$; needs renormalization of divergent quark self-energies:

$$L_{ren}(T) = Z(\beta)^{N_{\tau}} \langle L \rangle(T)$$



- used T = 0 potential to determine $Z(\beta)$ for each T > 0 parameter set
- good scaling behavior of the renormalized Polyakov loop

no significant cut-off dependence; confirms SU(3) experience

Polyakov loop expectation value $\langle L \rangle = \exp(-F_q(T)/T)$; needs renormalization of divergent quark self-energies:

$$L_{ren}(T) = Z(\beta)^{N_{\tau}} \langle L \rangle(T)$$



Polyakov loop expectation value $\langle L \rangle = \exp(-F_q(T)/T)$; needs renormalization of divergent quark self-energies:

$$L_{ren}(T) = Z(\beta)^{N_{\tau}} \langle L \rangle(T)$$

expect still a shift of T-scale ~ 5 MeV for physical quark masses



p4 versus standard

EoS with $\mathcal{O}(a^2)$ improved SF

Goal: QCD thermodynamics with realistic quark masses in (2+1)-f QCD and controlled extrapolation to the continuum limit;

 $\Rightarrow T_c$, EoS,.. for $\mu_q \geq 0$

RBC-Bielefeld collaboration

- use an improved staggered fermion action that removes O(a²) errors in bulk thermodynamic quantities and reduces flavor symmetry breaking inherent to the staggered formulation
 RBC-Bielefeld choice: p4-action + 3-link smearing (p4fat3)
 MILC: Naik-action + (3,5,7)-link smearing (asqtad)
 C. Bernard et al., PRD75, 094505 (2007)
- use RHMC algorithm to remove 'step-size errors'
- perform detailed T = 0 study of vacuum subtractions and scale setting for ALL T > 0 parameter sets

Calculating the EoS on lines of constant physics (LCP)

The pressure

$$\begin{split} \frac{p}{T^4} \Big|_{\beta_0}^{\beta} &= N_{\tau}^4 \int_{\beta_0}^{\beta} d\beta' \left[\frac{1}{N_{\sigma}^3 N_t} (\langle S_g \rangle_0 - \langle S_g \rangle_T) \right. \\ &\left. - \left(2(\langle \bar{\psi}\psi \rangle_{l0} - \langle \bar{\psi}\psi \rangle_{lT}) + \frac{\hat{m}_s}{\hat{m}_l} (\langle \bar{\psi}\psi \rangle_{s0} - \langle \bar{\psi}\psi \rangle_{sT}) \right) \left(\frac{\partial \hat{m}_l}{\partial \beta'} \right)_{\hat{m}_s/\hat{m}_l} \right. \\ &\left. - \hat{m}_l \left(\langle \bar{\psi}\psi \rangle_{s0} - \langle \bar{\psi}\psi \rangle_{sT} \right) \left(\frac{\partial \hat{m}_s/\hat{m}_l}{\partial \beta'} \right)_{\hat{m}_l} \right] \end{split}$$

The interaction measure for $N_f = 2 + 1 \iff$ Trace Anomaly

$$egin{array}{lll} rac{\epsilon-3p}{T^4} &=& Trac{\mathrm{d}}{\mathrm{d}T}\left(rac{p}{T^4}
ight) = \left(arac{\mathrm{d}eta}{\mathrm{d}a}
ight)_{LCP}rac{\partial p/T^4}{\partialeta} \ &=& \left(rac{\epsilon-3p}{T^4}
ight)_{gluon} + \left(rac{\epsilon-3p}{T^4}
ight)_{fermion} + \left(rac{\epsilon-3p}{T^4}
ight)_{\hat{m}_s/\hat{m}_l} \end{array}$$

T = 0 scale setting using the heavy quark potential

use r_0 or string tension to set the scale for $T_c = 1/N_{ au} a(eta_c)$

$$V(r) = -rac{lpha}{r} + \sigma r \ , \ r^2 rac{{
m d} V(r)}{{
m d} r}|_{r=r_0} = 1.65$$



no significant cut-off dependence when cut-off varies by a factor 5

i.e. from the transition region on $N_{\tau} = 4$ lattices ($a \simeq 0.25$ fm) to that on $N_{\tau} = 20$ lattices ($a \simeq 0.05$ fm) !!

T = 0 scale setting using the heavy quark potential



use r_0 to set the scale for $T_c = 1/N_{\tau}a(\beta_c)$: good control over r_0/a $\Rightarrow Tr_0 \equiv (r_0/a)/N_{\tau}$

 r_0/r_1 and $r_0\sqrt{\sigma}$ vary by less than 2% for $a{\lesssim}0.2$ fm

no hint for large $\mathcal{O}(a^2)$ corrections

T = 0 scale setting using the heavy quark potential



use r_0 to set the scale for $T_c = 1/N_{\tau}a(\beta_c)$: good control over r_0/a $\Rightarrow Tr_0 \equiv (r_0/a)/N_{\tau}$

we use $r_0 = 0.469(7)$ fm determined from quarkonium spectroscopy A. Gray et al, Phys. Rev. D72 (2005) 094507

 r_0/r_1 and $r_0\sqrt{\sigma}$ vary by less than 2% for $a{\lesssim}0.2$ fm

no hint for large $\mathcal{O}(a^2)$ corrections

$(\epsilon-3p)/T^4$ on LCP

- overall good agreement between $N_{ au} = 4$ and 6
- $N_{\tau} = 4$ data in peak region and below are sensitive to non-universal features of the β -functions



- 16^34 and 24^36 lattices
- T = 0 subtraction for each T-value
- statistical errors within symbol size

Note:

T-scale from T = 0 potential is an absolute scale, *i.e.* not dependent on T_c determination

RBC-Bielefeld, preliminary

$(\epsilon-3p)/T^4$ on LCP

- overall good agreement between $N_{ au} = 4$ and 6
- $N_{\tau} = 4$ data in peak region and below are sensitive to non-universal features of the β -functions
- $N_{\tau} = 6$ data in good agreement with asqtad simulations;
 C. Bernard et al. (MILC), PRD75, 094505 (2007)



- 16³4 and 24³6 lattices
- T = 0 subtraction for each T-value
- statistical errors within symbol size

Note:

T-scale from T = 0 potential is an absolute scale, *i.e.* not dependent on T_c determination and totaly independent for p4-action and asquad calculations

RBC-Bielefeld, preliminary

Pressure, Energy and Entropy

 p/T^4 from integration over $(\epsilon - 3p)/T^5$; systematic error arises from starting the integration at $T_0 = 100$ MeV with $p(T_0) = 0$;

use hadron resonance gas to estimate systematic error: $[p(T_0)/T_0^4]_{HRG} \simeq 0.265$



Pressure, Energy and Entropy

 p/T^4 from integration over $(\epsilon - 3p)/T^5$; systematic error arises from starting the integration at $T_0 = 100$ MeV with $p(T_0) = 0$;

use hadron resonance gas to estimate systematic error: $[p(T_0)/T_0^4]_{HRG} \simeq 0.265$



F. Karsch, xQCD, August 2007 - p. 23/28

Deconfinement and χ -symmetry and bulk thermodynamics

most prominent features of bulk thermodynamics are related to deconfinement



Deconfinement and χ -symmetry and bulk thermodynamics

most prominent features of bulk thermodynamics are related to deconfinement

 0.4
 0.5
 0.6
 0.7
 0.8
 0.9
 1



Lattice EoS: energy density \Leftrightarrow temperature \Rightarrow conditions for heavy $q\bar{q}$ bound states

 $\begin{array}{l} \mathsf{LGT:}\ T_c\simeq 190\ \mathsf{MeV}\\ T=T_c:\ \epsilon_c/T_c^4\simeq 6\ \Rightarrow\ \epsilon_c\simeq 1\ \mathsf{GeV/fm^3}\\ T\ \geq\ 1.5T_c:\ \epsilon/T^4\simeq (13-14)\\ T=1.5T_c:\ \epsilon\simeq 11\ \mathsf{GeV/fm^3}\\ T=2.0T_c:\ \epsilon\simeq 35\ \mathsf{GeV/fm^3}\\ \Downarrow\end{array}$

observable consequences: J/ψ suppression

RHIC

 $R_{Au} \simeq 7$ fm; $au_0 \simeq 1 ~{
m fm}$ $\langle E_T \rangle \simeq 1 \ {
m GeV}$ $dN/dy \simeq 1000$ \downarrow $\epsilon_{Bj}\simeq 7~{
m GeV/fm^3}$ maybe: $au_0 \simeq 0.5$ fm \downarrow $\epsilon_{Bj} \simeq 14~{
m GeV/fm^3}$

Lattice EoS: energy density \Leftrightarrow temperature

LGT: $T_c \simeq 190 \text{ MeV} - 170 \text{ MeV} - 150 \text{ MeV}$

 $T = T_c$: $\epsilon_c/T_c^4 \simeq 6 \implies \epsilon_c \simeq 1 \text{ GeV/fm}^3 - 0.64 \text{ GeV/fm}^3 - 0.39 \text{ GeV/fm}^3$

let's assume: $\epsilon_c(T_\chi) \simeq 0.5 \epsilon_c(T_{dec})$

$$\Rightarrow~\epsilon_c/T_\chi^4\simeq 0.2~{
m GeV/fm^3}$$

Conclusions

$\mathcal{O}(a^2)$ improved actions drastically reduce cut-off effects

- p4 and asqtad actions lead to consistent thermodynamics on lattices of temporal extent $N_{\tau} = 6$, although the handling of flavor symmetry breaking (fat-links) and $\mathcal{O}(a^2g^2)$ corrections as well as cut-off effects in the free limit are quite different
- deconfinement and chiral symmetry restoration happen at roughly the same temperature that also characterizes the crossover region seen in bulk thermodynamics
- T_c needs to be confirmed on larger lattices

Extreme QCD 2007

This was an **EXTREME**ly interesting meeting

in an EXTREMEly pleasant sourounding

I guess we all are EXTREMEly thankful to Maria for

arranging this years xQCD meeting