# REAL TIME DYNAMICS IN THE Z(N) INTERFACE AT HIGH TEMPERATURE

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# Condensed Matter Physics of QCD

## Strongly Coupled Plasma

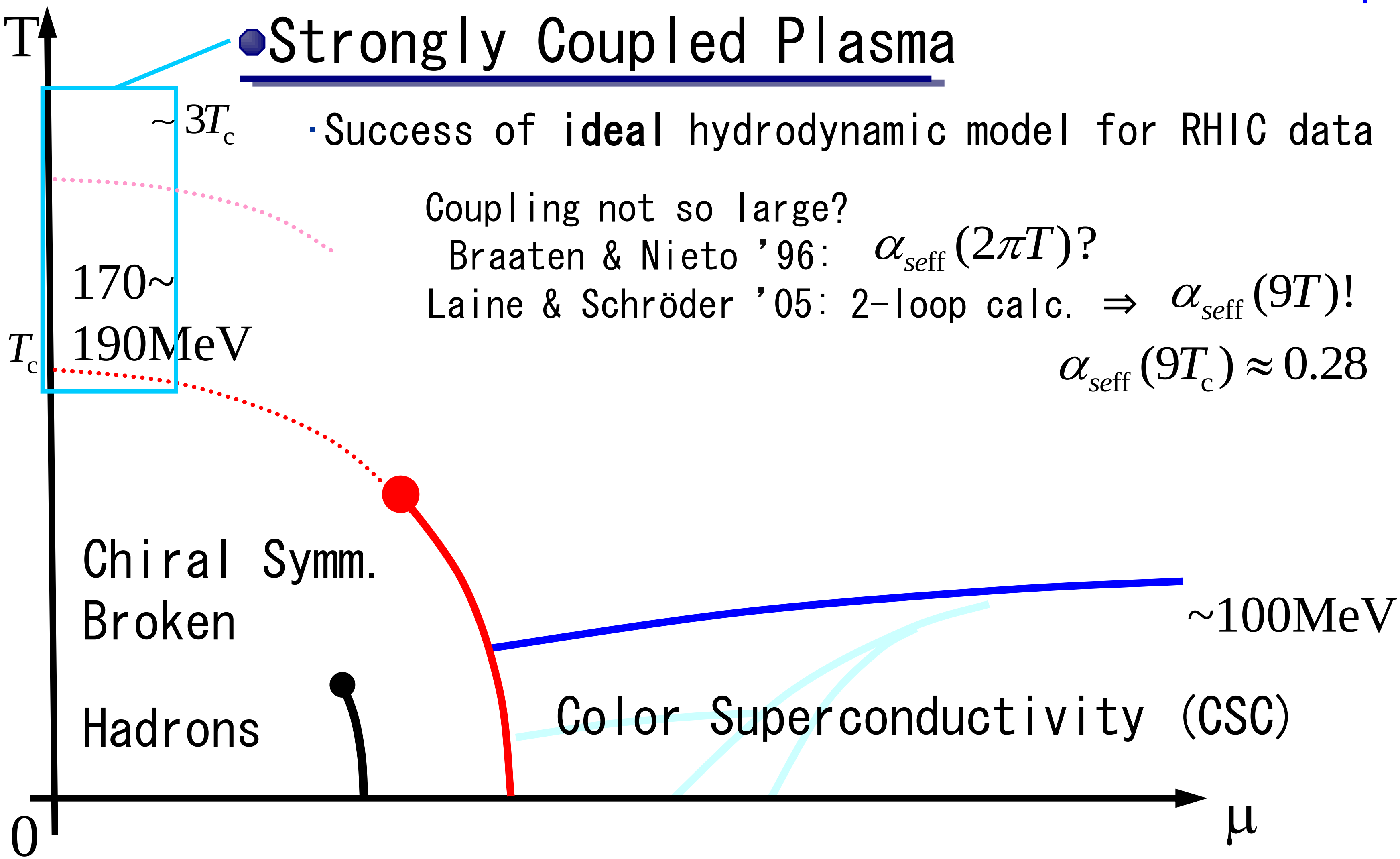
- Success of **ideal** hydrodynamic model for RHIC data

Coupling not so large?

Braaten & Nieto '96: $\alpha_{seff}(2\pi T)$?

Laine & Schröder '05: 2-loop calc. $\Rightarrow$ $\alpha_{seff}(9T)$!

$\alpha_{seff}(9T_c) \approx 0.28$



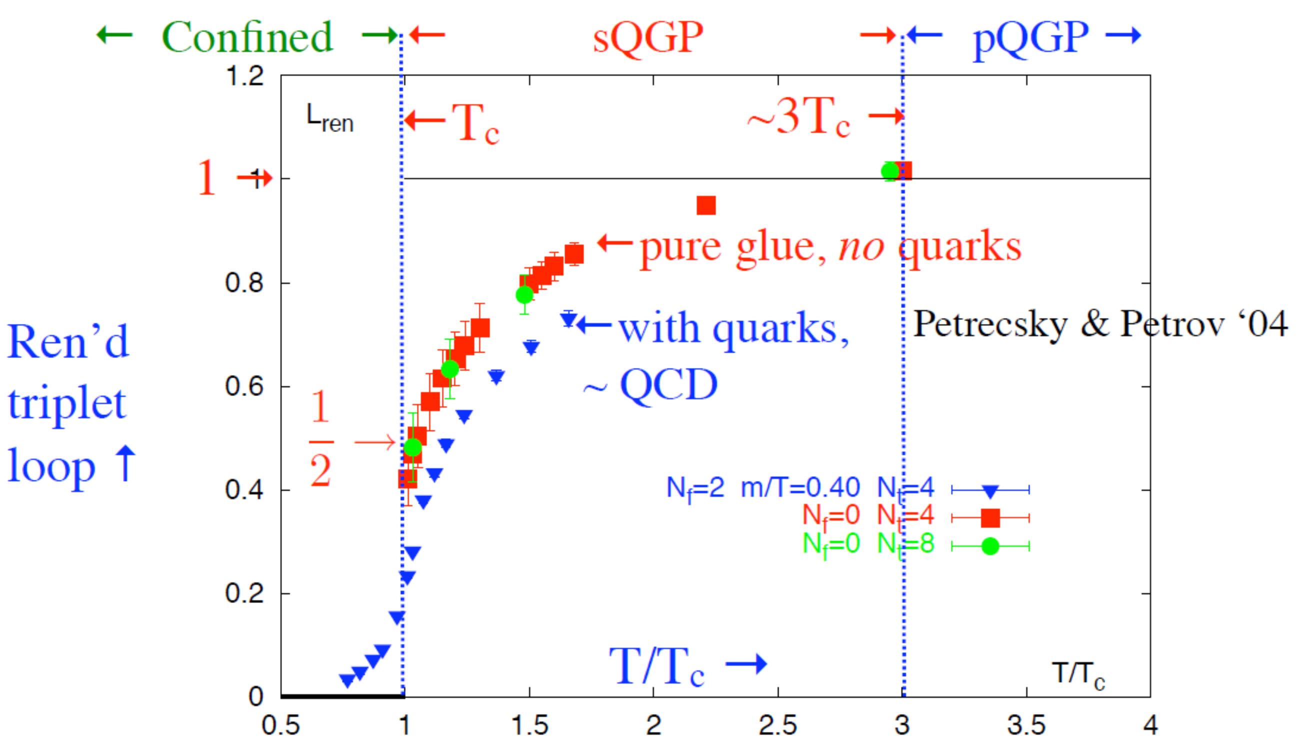
# Partial deconfinement

$T > 3T_c \quad \left\langle \frac{1}{N_c} \mathrm{tr} L \right\rangle \simeq 1$ perturbative QGP, "pQGP". Eff. thy.: small $A_0$

$T_c < T < 3T_c \quad \left\langle \frac{1}{N_c} \mathrm{tr} L \right\rangle < 1$ partial deconfinement, "sQGP" Eff. thy.: large $A_0$

$$L = \frac{1}{N_c} \exp[ig \int_0^\beta d\tau A_0]$$

For sQGP, need effective theory for *large* $A_0$



# Static and real time properties

**Static**

- Pressure
- Entropy
- Susceptibilities

⋮

**Real time**

- Transport coefficient
  - Share viscosity
  - Bulk viscosity
  - Heat conductivity
- Plasma oscillation
- Particle production rate
  - Dilepton emission from QGP

Many observables are obtained by not only static but also real time properties based on linear response theory.

High temperature

$T$

Static

Real time

Polyakov loop

Perturbative calculation

Hard thermal loop resummation

$$\left\langle \frac{1}{N_c} \mathrm{tr} L \right\rangle \simeq 1$$

$\sim 3T_c$

Real time effective theory?

3D effective theory

We want to construct an effective theory with large $A_0$.

$$\left\langle \frac{1}{N_c} \mathrm{tr} L \right\rangle < 1$$

$T_c$

Lattice simulation

$$\left\langle \frac{1}{N_c} \mathrm{tr} L \right\rangle \simeq 0$$

# Effective Lagrangian

Hard Thermal loop resummation: Small A0

$$L_{\text{eff}} = -m_{\text{T}}{}^2 \int \frac{d\Omega}{4\pi} F^a_{\rho\mu} \frac{K^\mu K^\nu}{(K\cdot D)^2} F^a_{\nu\rho} + \cdots$$

$D_\mu$ Covariant derivative $K^\mu = (1, \hat{\mathrm{k}})$ $m_T = \frac{g^2 T^2}{6} N_c$

Effective Lagrangian with large A0

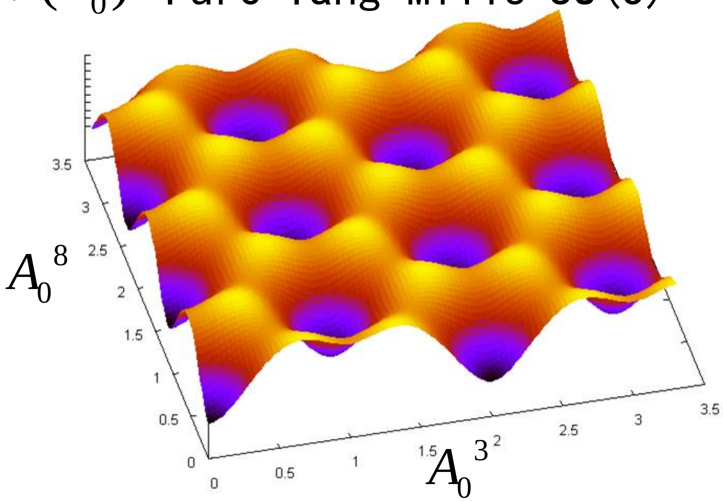
$$L_{\text{eff}} = L_{A_0^{\text{cl}}}(A_0^{\text{cl}}) + F^a_{\rho\mu} \Delta^{\mu\nu\rho\lambda}_{ab}(A_0^{\text{cl}}, D_\alpha) F^b_{\nu\lambda} + \cdots$$

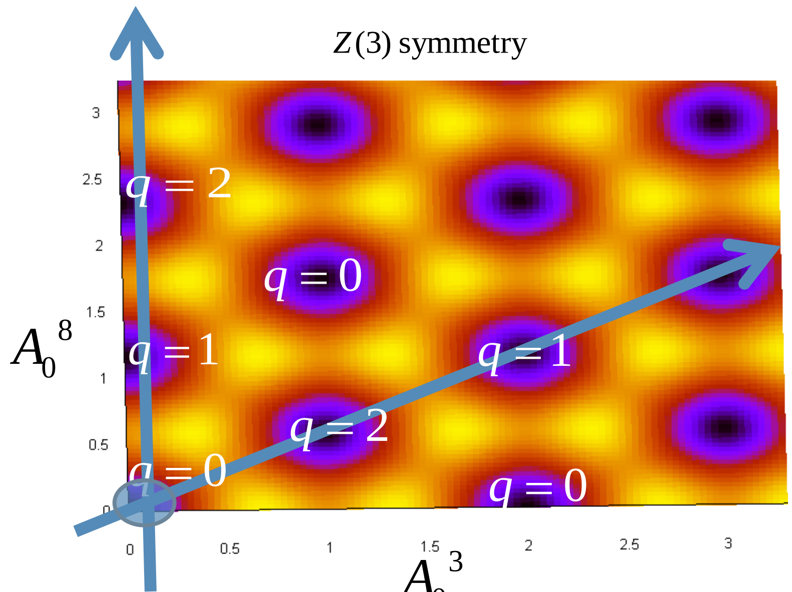
# Degenerate vacua at high T

D. Gross, et.al.('81), N. Weiss ('81)

One-loop effective potential in the background $A_0$ field.

$V(A_0)$ Pure Yang-Mills SU(3)




$A_0{}^3$ $A_0{}^8$ diagonal components of gluons

$\langle L \rangle = \exp(2\pi i q/3), \quad q = 0,1,2$

Large gauge transform

spatially constant, time dependent

$\Omega = \exp(2\pi i t_8 T\tau / N_c)$

Vacuum is changed from q to q+1

$gA_\mu \to \Omega^\dagger g A_\mu \Omega + i\Omega^\dagger \partial_\mu \Omega$

$L \to \Omega^\dagger(\beta) L \Omega(0) = e^{2\pi i l N_c} L \qquad q \to q+1$

# Z(N) interfaces

Korthals-Altes et al ('93, '99, '01, '02, '04)

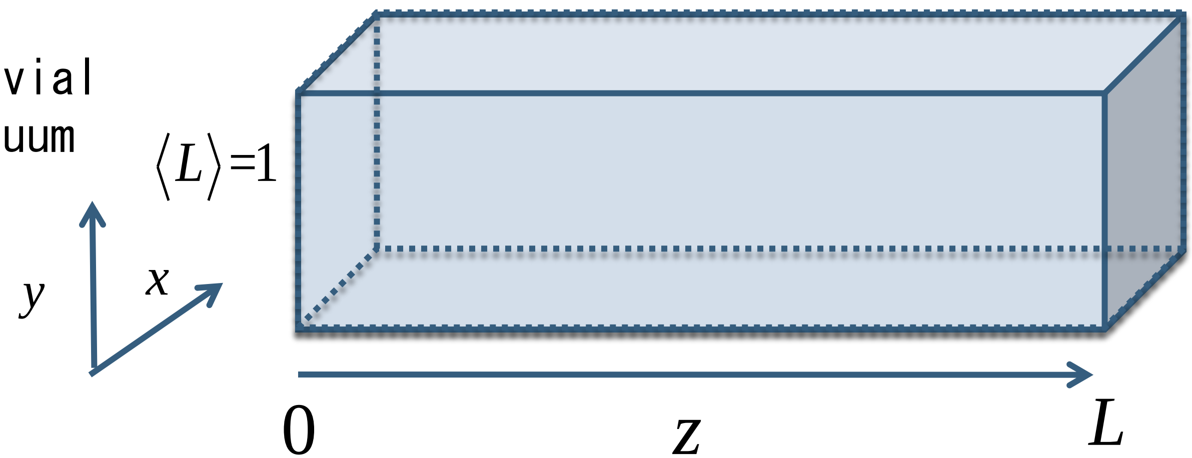
One way to probe large $A_0$: $Z(N)$ interface related to gauge transformation

Polyakov line: $L = \exp[i\int_0^\beta d\tau g A_0]$ $\qquad A_0 \sim \frac{T}{g}$ Large

Z(N) vacua are spatially separated.

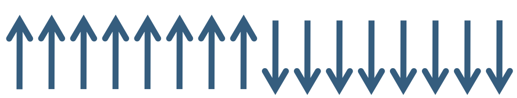
Trivial vacuum

$\langle L \rangle = 1$




$\langle L \rangle = e^{2\pi i / N_c} 1$

Analogy: spin system Z(2)



$\langle \sigma \rangle = 1$ $\qquad \langle \sigma \rangle = -1$

# Z(N) interfaces

Korthals-Altes et al ('93, '99, '01, '02, '04)

Classical + one-loop potential

$$L_{\text{eff}} = \frac{1}{2}(E^i)^2 + V_{\text{1-loop}}(A_0) \sim \# \left( \frac{1}{T^2 g^2}\left(\frac{dq}{dz}\right)^2 + q^2(1-q)^2 \right)$$

where $A_0 = \frac{2\pi T}{g N_c} q(z) t_N$

One dimension soliton problem

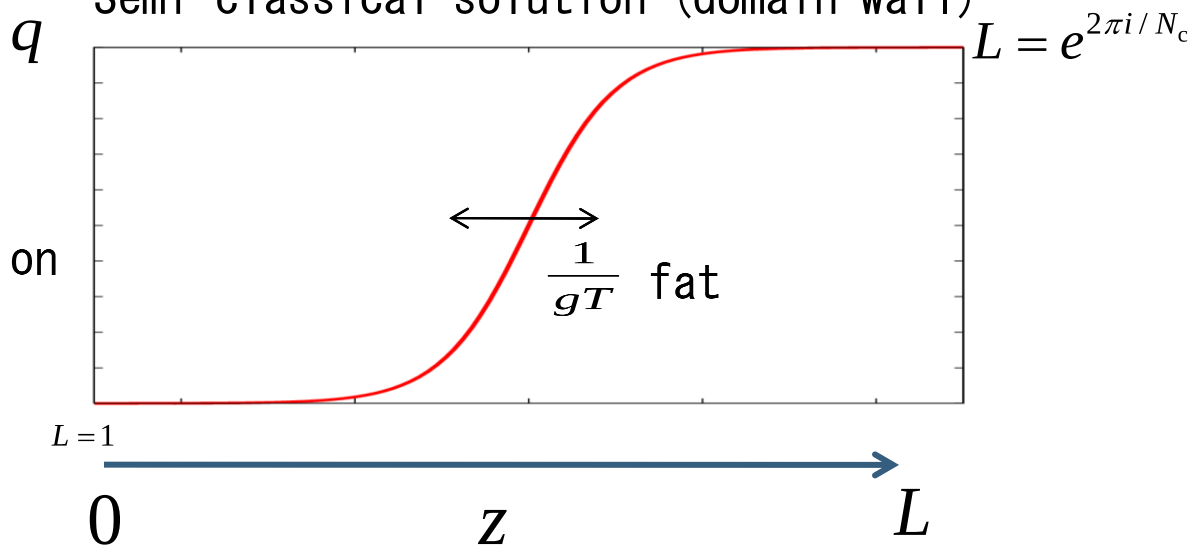
$$S \sim \frac{1}{g} \quad (\text{Instanton} \sim \frac{1}{g^2})$$

Interface is fat

width $\sim \frac{1}{gT} \gg \frac{1}{T}$,

so can use derivative expansion

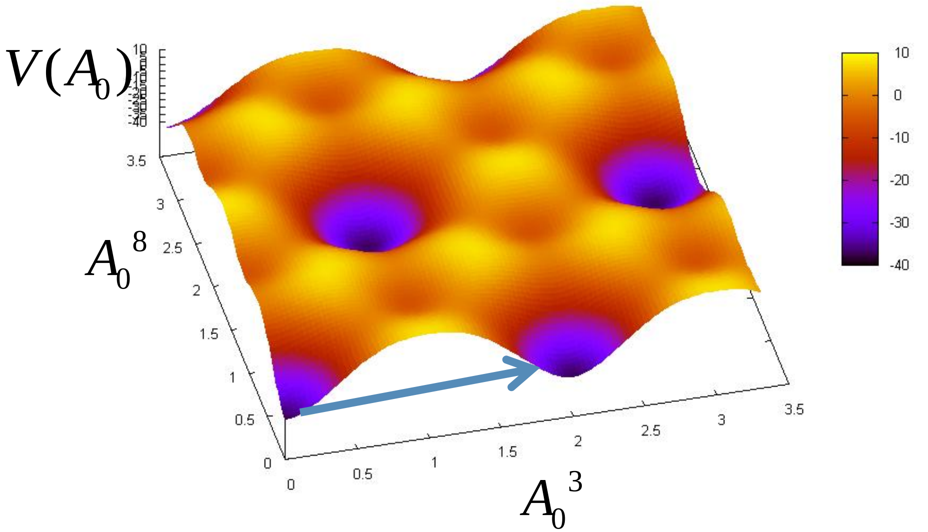
Semi-classical solution (domain wall)



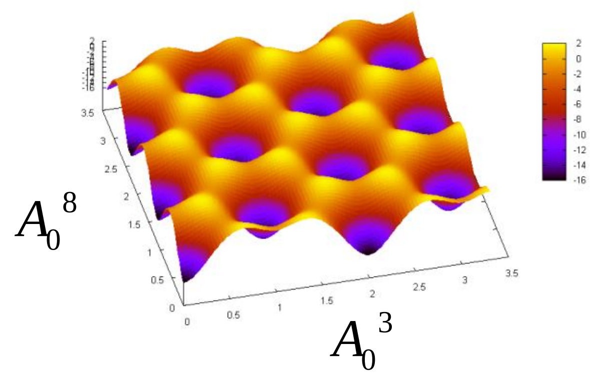
# With quarks

No Z(3) symmetry.  Still have "$U(1)$" interface: $\langle L \rangle$ : 1 → 1

with quarks ($N_f = 2$)




$V(A_0)$  Pure Yang-Mills




Use "$U(1)$" interfaces to probe large $A_0$

$$L = 1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} e^{2\pi i} & 0 & 0 \\ 0 & e^{-2\pi i} & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1$$

Step 1

Decompose $A_0$ field to large classical part and quantum fluctuations.

Calculate effective potential in the imaginary time formalism with a background field.

Step 2

Calculate a correlation function in the imaginary time formalism with the background field.

Step 3

Analytical continuation from imaginary time to real time with the background. Infrared resummations

# Step 1:Imaginary time

Decompose gauge field to $A_0 = A_0^{\text{cl}} + A_0^{\text{qu}}$

Background gauge field is large $A_0^{\text{cl}} / T \sim \dfrac{1}{g}$

Slowly changing $\partial A_0^{\text{cl}} / T^2 \sim 1$

Derivative expansion can be done.

**Background field**

$$A_0^{\text{cl}}(x) = \frac{1}{g}\begin{pmatrix} Q_1(x) & 0 & 0 & 0 \\ 0 & Q_2(x) & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & Q_N(x) \end{pmatrix}$$

$Q_1 + Q_2 + \cdots + Q_3 = 0$

$Q_i \sim T$ hard

$$L(x) = \exp(ig\int d\tau A_0^{\text{cl}}(x)) = \exp(i\int d\tau Q(x))$$

# Step 1 : Imaginary time

We choose the basis of Lie algebra as eigenstates of the background field.

Quarks and gluons carry color "charge".

$$iD_0\psi^i = (k_0 + Q_i)\psi^i \quad k_0 = 2\pi(n + \frac{1}{2})T$$

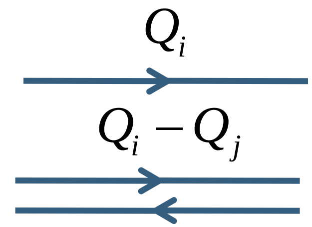
$$iD_0 A_\mu^{ij} = (k_0 + Q_i - Q_j)A_\mu^{ij} \quad k_0 = 2\pi n T$$

hard

Double line notation

fundamental

adjoint




$$\mathrm{SU(2)} \quad t^+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, t^- = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, t^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$Q_j$ Corresponds to imaginary chemical potential

Propagator (leading order)

$$\frac{1}{(\omega_n + Q_j)^2 + k^2 + m^2}$$

Bose distribution

$$n_Q = \frac{1}{\exp(\omega - iQ)/T - 1}$$

# From imaginary time to real time

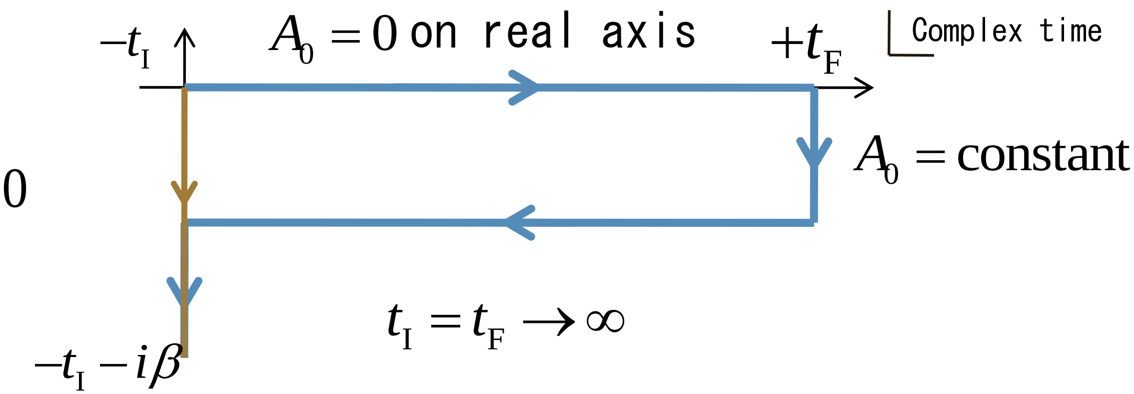
**Without background**

$$i\omega_n \to p_0 \qquad \omega_n = 2\pi nT$$ Matsubara frequency

Assume $A_0(t)$ does not depend on $x$

Furuuchi '05

Consider complex time path




Free case

$$(-(\partial_0 - iQ_i)_C{}^2 + \partial_i{}^2 - m^2)\phi_i(x) = 0$$

solution

$$\phi(x) = \exp(i\int_C^t dt'(Q_i(t') - k_0) + \mathbf{k}\cdot\mathbf{x})$$

$$L = \exp(i\int_C^{-i\beta} dt A_0(t))$$

We require that L is independent from a choice of complex time path.

A reasonable gauge choice is
$$\begin{cases} A_0 = 0 & \text{on real axis.} \\ A_0 = \text{constant} & \text{on imaginary axis.} \end{cases}$$

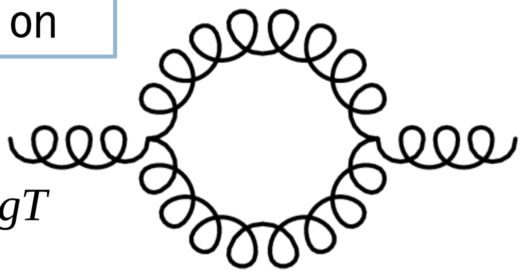
$$\Rightarrow \quad \boxed{i\omega_n + iQ_i \to p_0}$$

# Step 2 & 3: Hard Thermal Loop approximation

Internal momentum
$K, Q_i \sim T$

**Two-point function**

$\Pi_{\mu\nu} =$



$+$ Tad pole, ghost diagrams

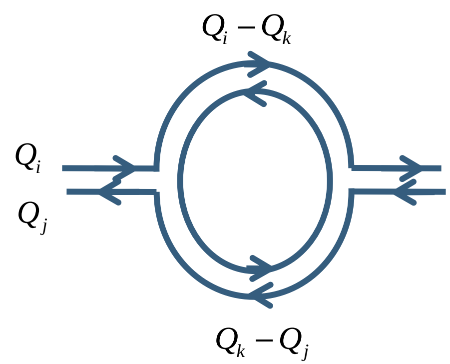
Soft momentum $P \sim gT$

**Hard Thermal Loop approximation**

External momentum $P \sim gT$ soft

Loop momentum $K, Q \sim T$ hard

⇨ Pick up $T^2$ order $\Pi_{\mu\nu} \sim g^2T^2$




Diagonal gluon (neutral charge) OK

Off diagonal gluon has another part $\sim i\dfrac{g^2T^3}{P} \sim gT^2 \gg g^2T^2$ Need infrared resummations

# Two point function

$$\Pi^{ij,kl}_{\mu\nu}(P) = -2m_T^{2ij,kl}(Q)\left(-\delta^{\mu 0}\delta^{\nu 0} + \int \frac{d\Omega}{4\pi}\frac{P^0 K^\mu K^\nu}{K\cdot P}\right) + if^{ij,kl}{}_{mn} J^{mn0}(Q)\int \frac{d\Omega}{4\pi}\frac{K^\mu K^\nu}{K\cdot P}$$

$$L_{\mathrm{eff2}} = \frac{1}{2} A^{ij}_\mu \Pi^{ij,kl}_{\mu\nu} A^{kl}_\nu$$

thermal mass

$$m_T{}^2(Q) = \frac{g^2T^2}{6} f^{ij,kl,mn} f^{i'j'}{}_{kl,mn}(1-6q^{kl}(1-q^{kl})) \qquad q^{ij} = \frac{Q^{ij}}{2\pi T} \qquad Q^{ij} = g\left(\left(A_0^{\mathrm{cl}}\right)^i - \left(A_0^{\mathrm{cl}}\right)^j\right)$$

$$J^{ij0}(Q) = \frac{\pi T^3}{3} g f^{ij,kl}{}_{kl} q^{kl}(1-q^{kl})(1-2q^{kl})$$

Debye mass

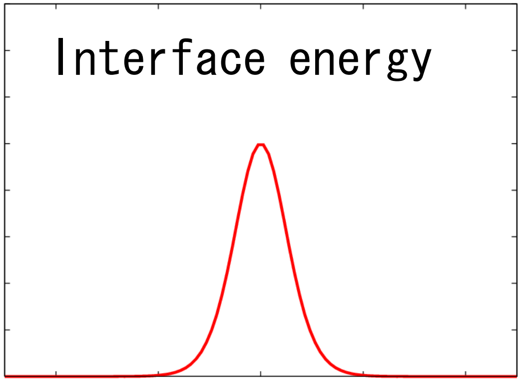
$$m_D{}^2(Q) = -\Pi_{00}(p_0 = 0, p \to 0) = 2m_T{}^2$$ For diagonal gluon

Thermal mass square become negative in some region.

$$m_T{}^2 = \frac{g^2T^2}{6} f^{ij,kl,mn} f^{i'j'}{}_{kl,mn}(1-6q^{kl}(1-q^{kl})) < 0$$

Tunneling effect

$S$




Is this a real wall?

# Is the interface physical observable?

The interface lives on the imaginary time.

The interface is not real domain wall in Minkowski space-time.

V.M. Belyaev et. al. ('92) , A Smilga ('93)

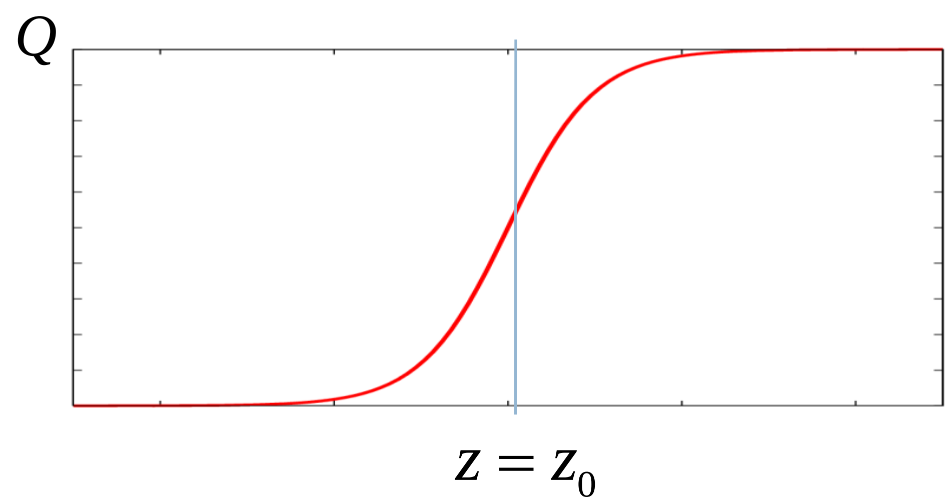
**Anomalous charge density**

$\langle J^{ij0} \rangle =$ pure imaginary because the background has a Euclidean electric field.

Gauss' law $D \cdot E^{ij} = iJ^{ij0}$

Sum over corrective coordinates and multi solitons.

$\Rightarrow \langle J^{ij0} \rangle = 0$



# Summary

- We have calculated two-point function to construct an effective Lagrangian in the real time with a large $A_0$.

  Thermal math is modified. $m_T^{\,2}(Q) = \frac{g^2 T^2}{6} f^{ij,kl,mn} f^{i'j'}{}_{kl,mn} (1 - 6q^{kl}(1 - q^{kl}))$

  another term proportional to charge density

  $$J^{ij0}(Q) = \frac{\pi T^3}{3} g f^{ij,kl}{}_{kl} q^{kl} (1 - q^{kl})(1 - 2q^{kl})$$

- We have explored interfaces background, large $A_0$ but semi-classical calculation can be done.
- For HTL resummation, we need to calculate three and four point functions.