Two Color Quark Matter from Lattice Simulations

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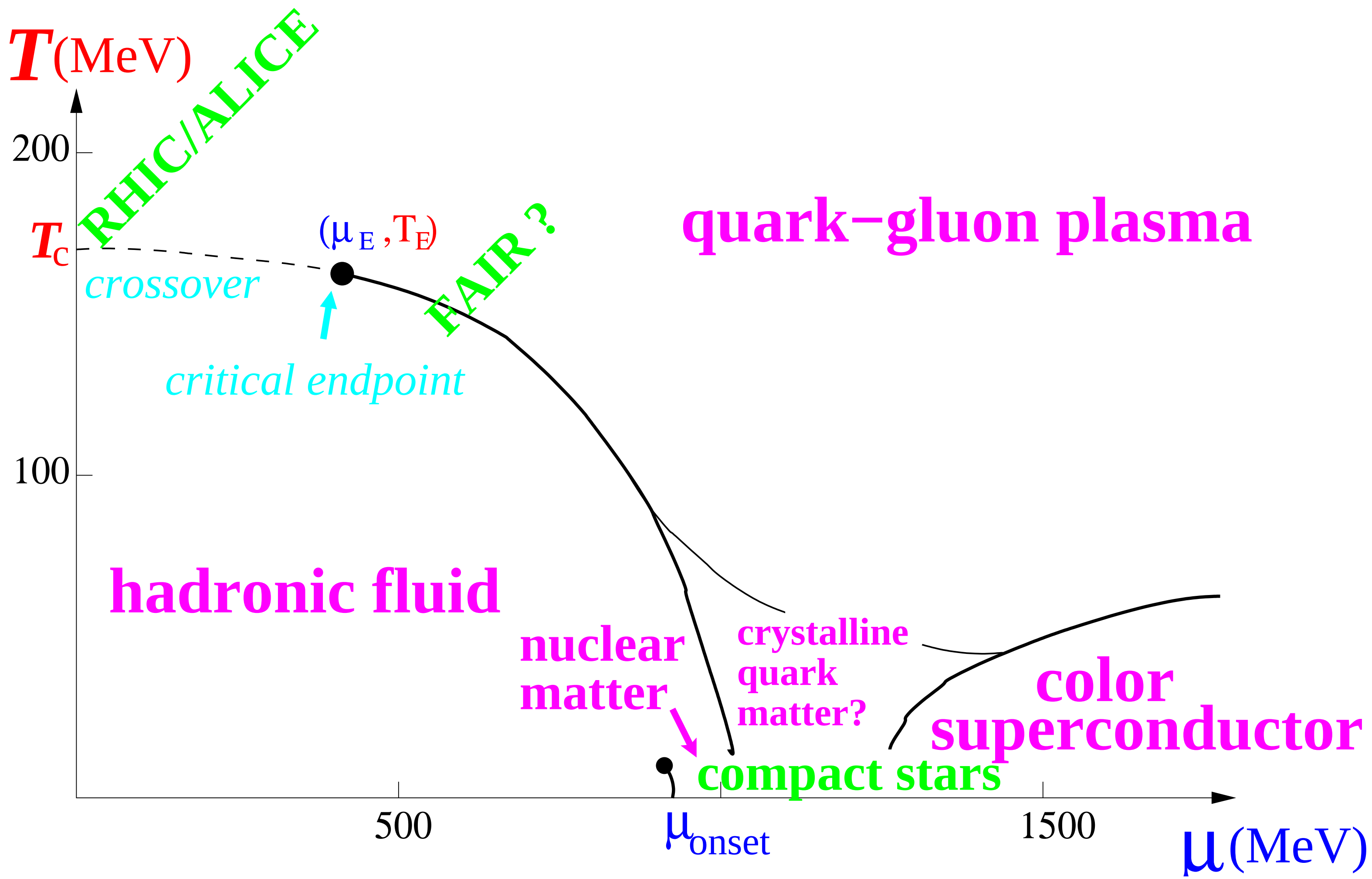
- Overview of QC_2D
- Thermodynamic observables and deconfinement
- Hadron spectrum
- Gludynamics

Collaborators: Phil Kenny, Seyong Kim, Peter Sitch, Jon-Ivar Skullerud

XQCD Frascati, 6th August 2007

The QCD Phase Diagram



QC$_2$D – the large N_c^{-1} limit

QCD with gauge group SU(2) and non-zero quark chemical potential μ has a real functional measure; it is one of the few dense matter systems with long-range interactions amenable to study with standard LGT methods.

Since q and $\bar{q}$ live in equivalent reps. of the color group, chiral multiplets contain both $q\bar{q}$ mesons and qq baryons. For $m_\pi \ll m_\rho$ the behaviour as μ is varied can be studied using chiral perturbation theory (χPT)

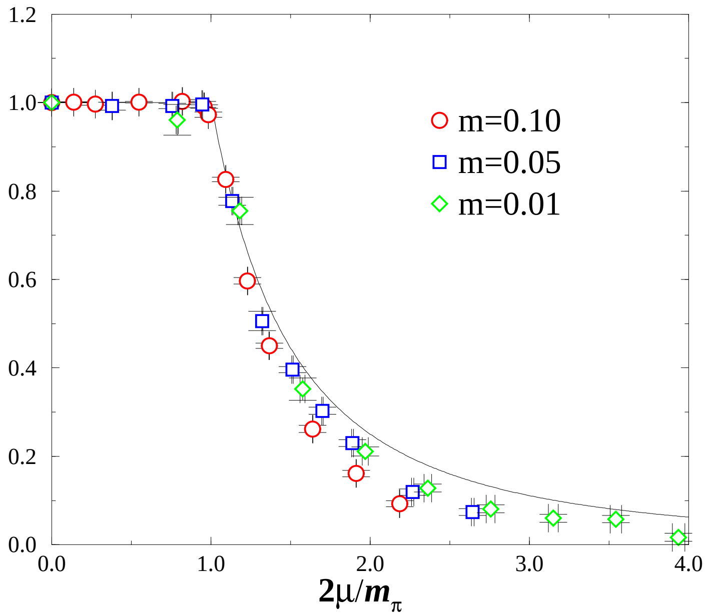
Key result: for $\mu \geq \mu_o = \frac{1}{2} m_\pi$ a baryon charge density develops, $n_q > 0$, along with a gauge invariant superfluid condensate $\langle qq \rangle \neq 0$. For $\mu \gtrsim \mu_o$, the system is a dilute Bose Einstein Condensate (BEC) consisting of weakly interacting scalar qq baryons.

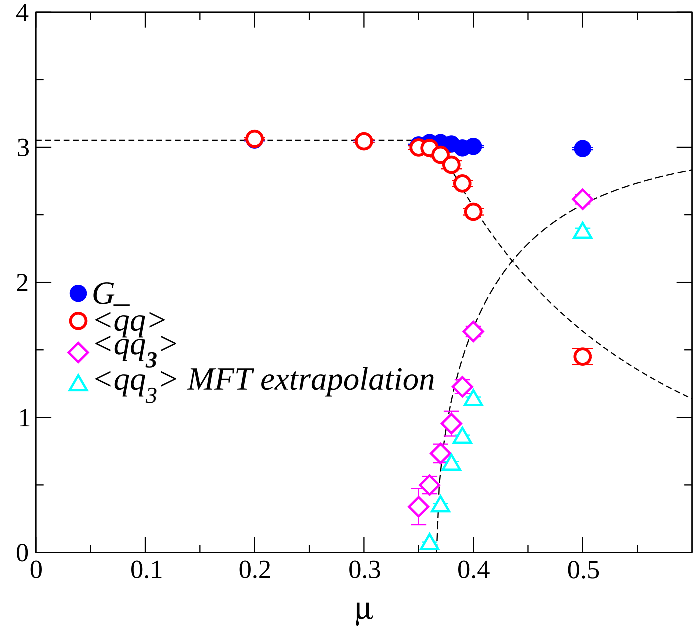
Quantitatively, for $\mu \gtrsim \mu_o$ χPT predicts

$$\frac{\langle\bar\psi\psi\rangle}{\langle\bar\psi\psi\rangle_0} = \left(\frac{\mu_o}{\mu}\right)^2; \quad n_q = 8N_f f_\pi^2 \mu \left(1 - \frac{\mu_o^4}{\mu^4}\right); \quad \frac{\langle qq\rangle}{\langle\bar\psi\psi\rangle_0} = \sqrt{1 - \left(\frac{\mu_o}{\mu}\right)^4}$$

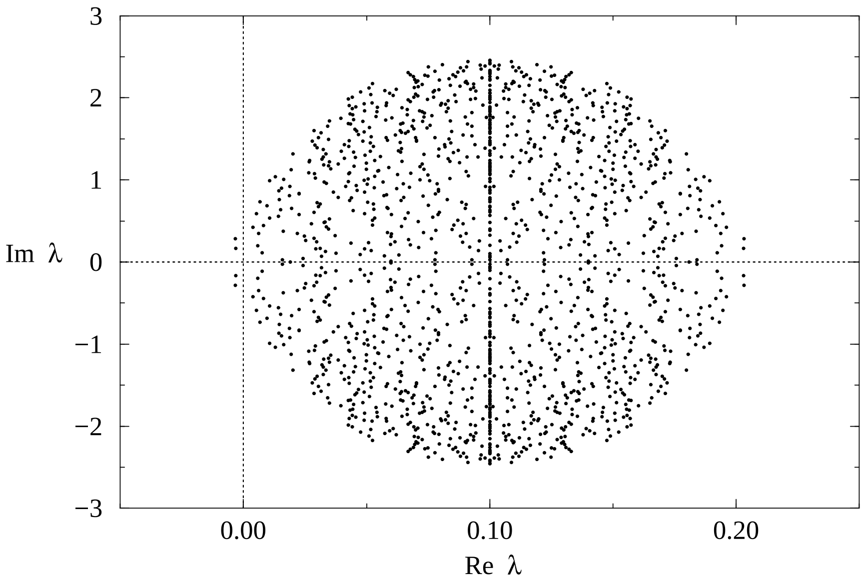
[Kogut, Stephanov, Toublan, Verbaarschot & Zhitnitsky, Nucl.Phys.B582(2000)477]

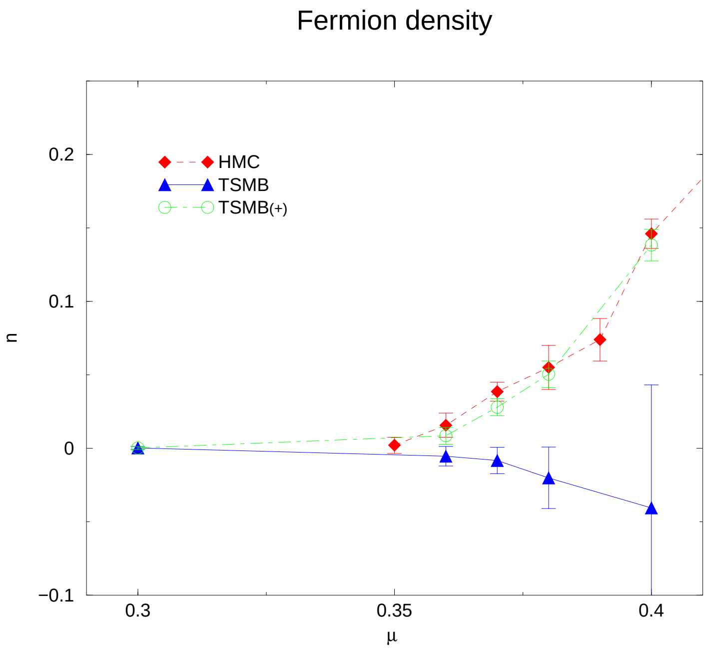
confirmed by QC_2D simulations with staggered fermions





[SJH, I. Montvay, S.E. Morrison, M. Oevers, L. Scorzato J.I. Skullerud, Eur.Phys.J.C17(2000)285, *ibid* C22(2001)451]





If you insist on a Sign Problem, try simulations of QC_2D with $N = 1$ adjoint staggered quarks.

The fake onset of a superfluid phase at $\mu = m_\pi/2$, whose condensate is forbidden by the Pauli Exclusion Principle, disappears once configurations with $\det M < 0$ are included with the correct weight.

SJH,Montvay,Scorzato,Skullerud, EurPJ C22 (2001) 451

Thermodynamics at $T = 0$ from χPT

quark number density $n_{\chi PT} = 8N_f f_\pi^2 \mu \left(1 - \frac{\mu_o^4}{\mu^4}\right)$ [KSTVZ]

pressure $p_{\chi PT} = -\frac{\Omega}{V} = \int_{\mu_o}^{\mu} n_q d\mu = 4N_f f_\pi^2 \left(\mu^2 + \frac{\mu_o^4}{\mu^2} - 2\mu_o^2\right)$

energy density $\varepsilon_{\chi PT} = -p + \mu n_q = 4N_f f_\pi^2 \left(\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 2\mu_o^2\right)$

interaction measure

$$\delta_{\chi PT} = T_{\mu\mu} = \varepsilon - 3p = 8N_f f_\pi^2 \left(-\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 4\mu_o^2\right)$$

NB $\delta_{\chi PT} < 0$ for $\mu > \sqrt{3}\mu_o$

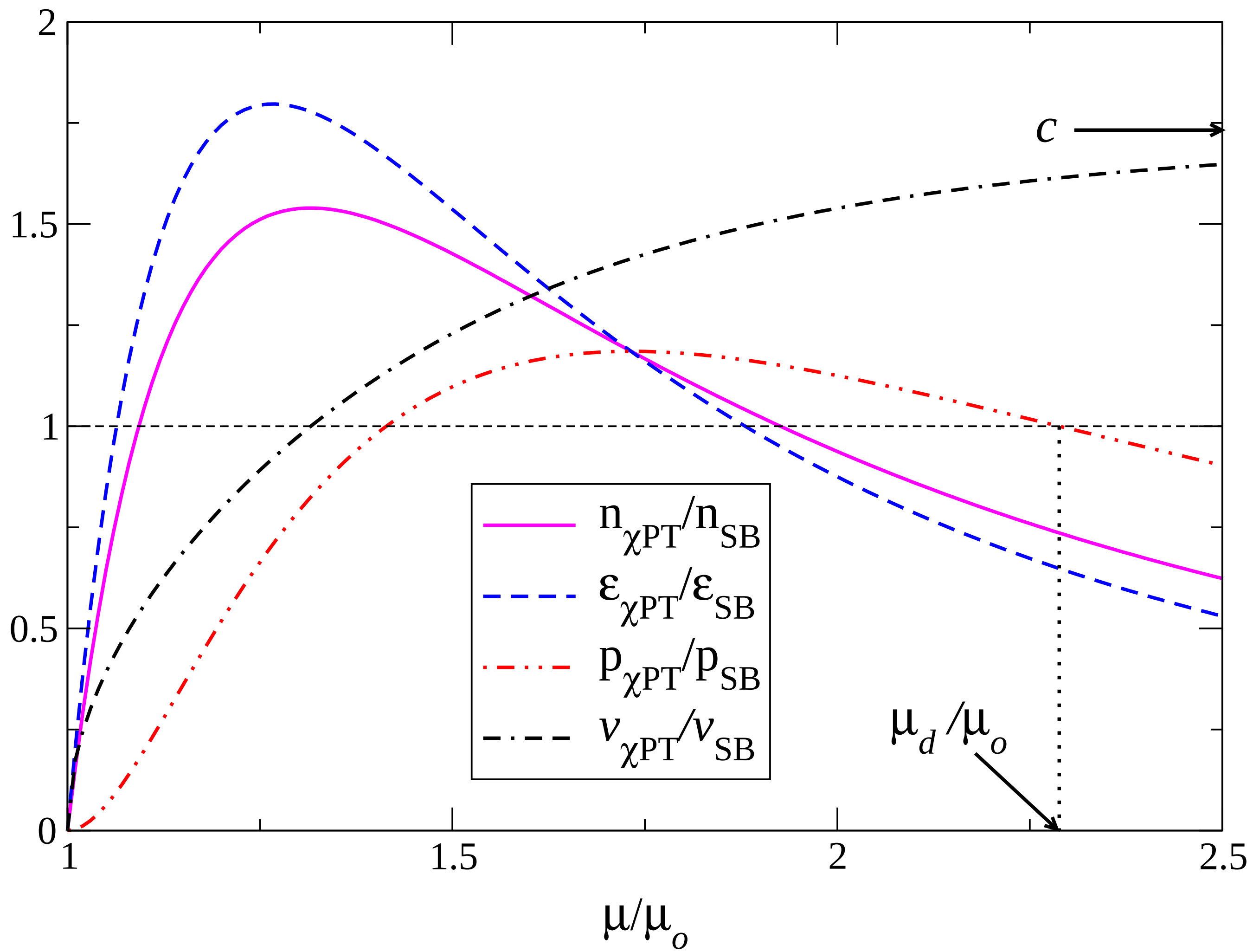
speed of sound $v_{\chi PT} = \sqrt{\frac{\partial p}{\partial \varepsilon}} = \left(\frac{1-\frac{\mu_o^4}{\mu^4}}{1+3\frac{\mu_o^4}{\mu^4}}\right)^{\frac{1}{2}}$

This is to be contrasted with another paradigm for cold dense matter, namely a degenerate system of weakly interacting (deconfined) quarks populating a Fermi sphere up to some maximum momentum $k_F \approx E_F = \mu$

$$\Rightarrow \quad n_{SB} = \frac{N_f N_c}{3\pi^2}\mu^3; \quad \varepsilon_{SB} = 3p_{SB} = \frac{N_f N_c}{4\pi^2}\mu^4;$$
$$\delta_{SB} = 0; \quad v_{SB} = \frac{1}{\sqrt{3}}$$

Superfluidity arises from condensation of diquark Cooper pairs from within a layer of thickness Δ centred on the Fermi surface:

$$\Rightarrow \quad \langle qq \rangle \propto \Delta\mu^2$$



By equating free energies, we naively predict a first order deconfining transition from BEC to quark matter;
eg. for $f_\pi^2 = N_c/6\pi^2$, $\qquad \mu_d \approx 2.3\mu_o$.

Simulation Details ($N_f = 2$ Wilson flavors)

Initial runs used a $8^3 \times 16$ lattice with parameters $\beta = 1.7$, $\kappa = 0.1780$ (Wilson gauge action)

$\Rightarrow$ $a = 0.220$ fm, $m_\pi a = 0.79(1)$, $m_\pi / m_\rho = 0.779(4)$

Now have preliminary data from a matched $12^3 \times 24$ lattice with $\beta = 1.9$, $\kappa = 0.1680$

$\Rightarrow$ $a = 0.15$ fm, $m_\pi a = 0.68(1)$, $m_\pi / m_\rho = 0.80(1)$

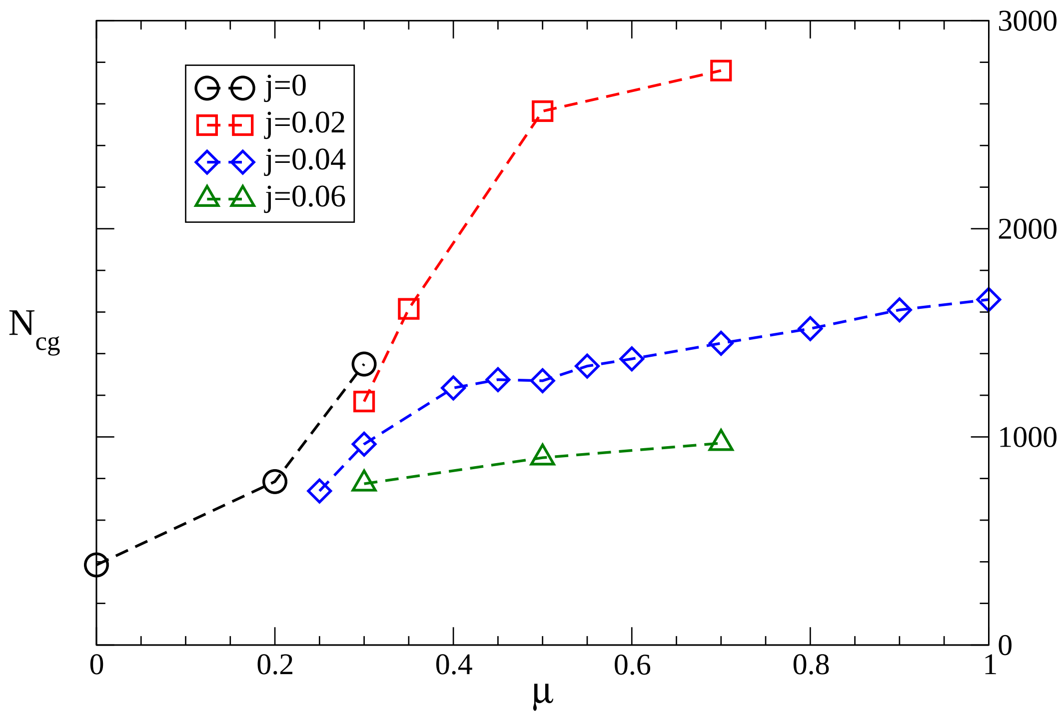
$\Rightarrow$ $T \approx 60$MeV in both cases

To counter IR fluctuations and to maintain ergodicity, we introduce a diquark source $j\kappa(-\bar{\psi}_1 C\gamma_5\tau_2\bar{\psi}_2^{tr} + \psi_2^{tr} C\gamma_5\tau_2\psi_1)$

So far have accumulated roughly 300 trajectories of mean length 0.5 on $8^3 \times 16$ and 100 trajectories on $12^3 \times 24$

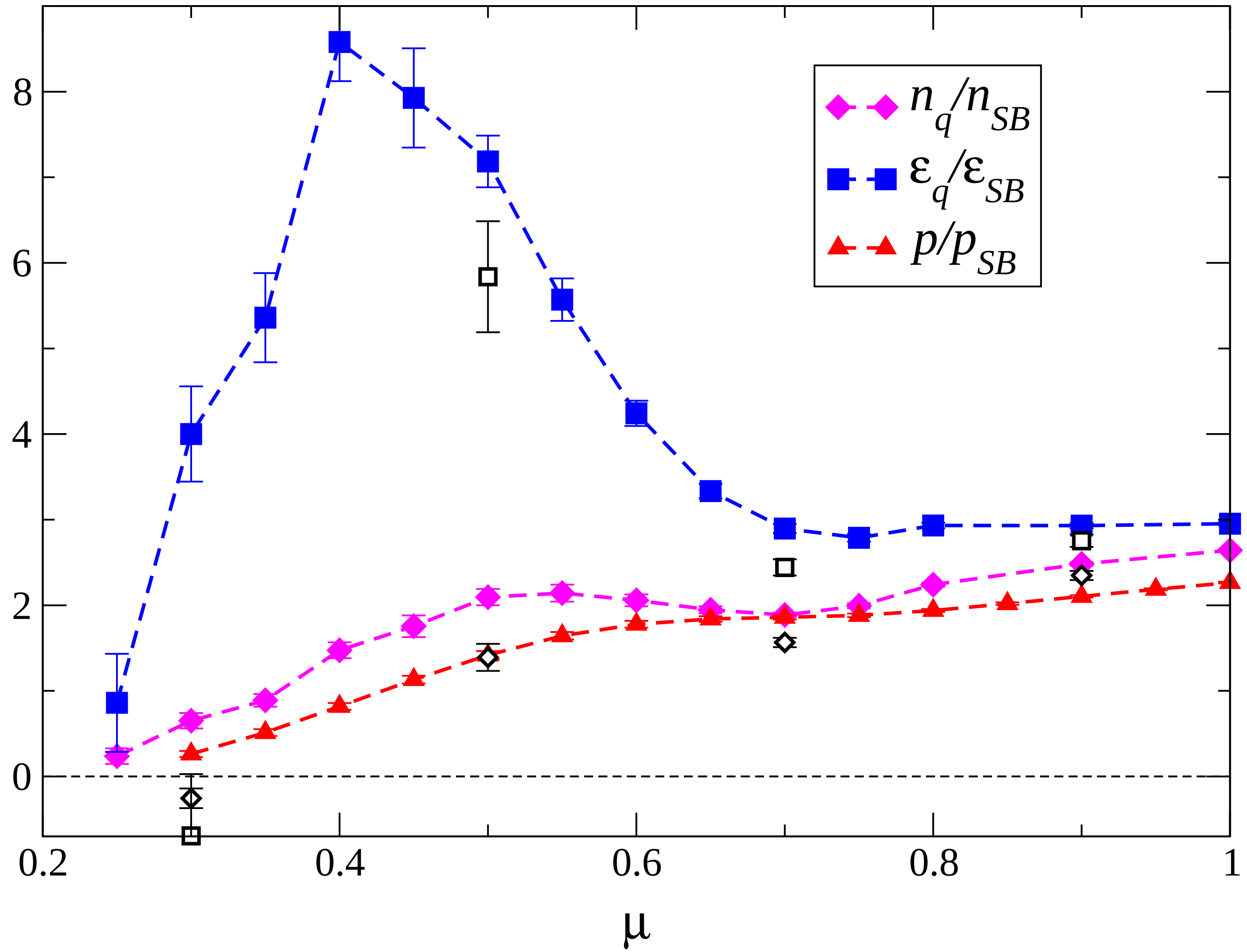
SJH, Kim, Skullerud, Eur.Phys. J. C48 (2006)193

Computer effort



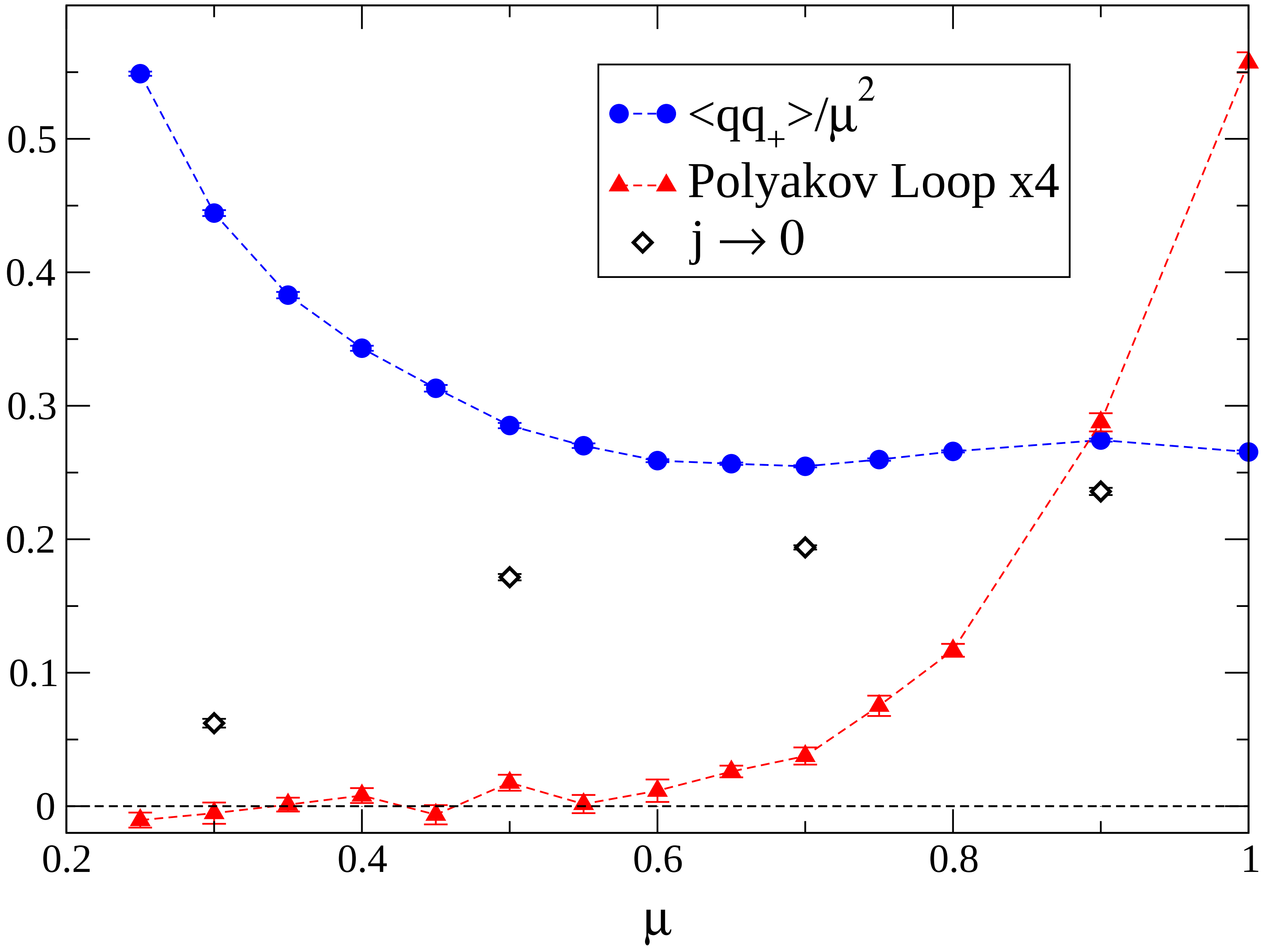
The number of `congrad` iterations required for convergence during HMC guidance rises as μ increases $\Leftrightarrow$ accumulation of small eigenvalues of M.

Equation of State ($8^3 \times 16$)



Open symbols denote $j \to 0$ extrapolation

Evidence for Deconfinement at $\mu a \simeq 0.65$



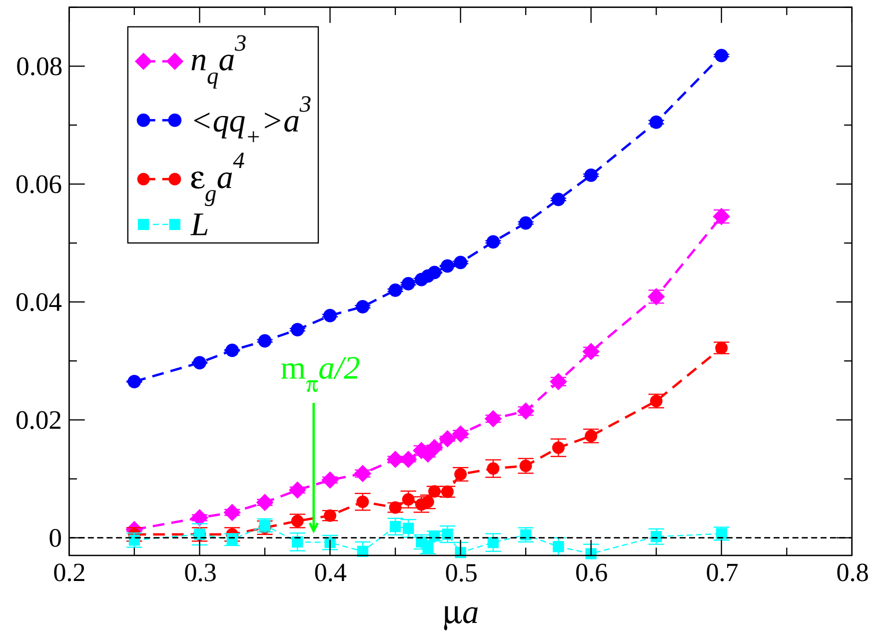
We conclude there is a transition from confined bosonic "nuclear matter" to deconfined fermionic "quark matter" at $\mu_d \approx 0.65$. Both phases are superfluid, but for $\mu > \mu_d$ the scaling is that expected of a degenerate system.

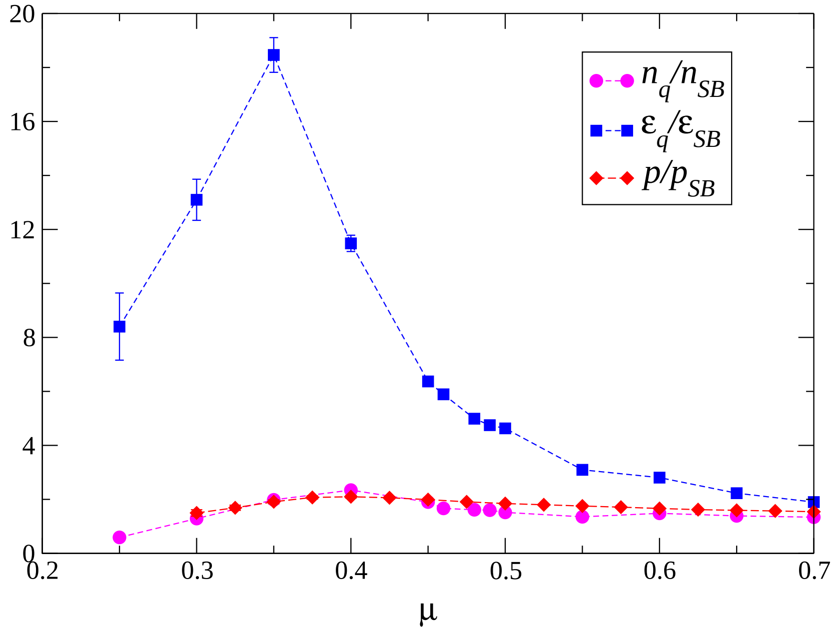
In condensed matter parlance we are observing a BEC/BCS crossover.

Similar conclusions reached using staggered fermions with $N_f = 8$ by studying topological charge susceptibility $\chi(\mu)$.

[Alles, d'Elia & Lombardo NPB752(2006)124]

Towards the continuum limit. . .





$12^3 \times 24$ results at $\beta = 1.9$ $\kappa = 0.168$ $ja = 0.04$

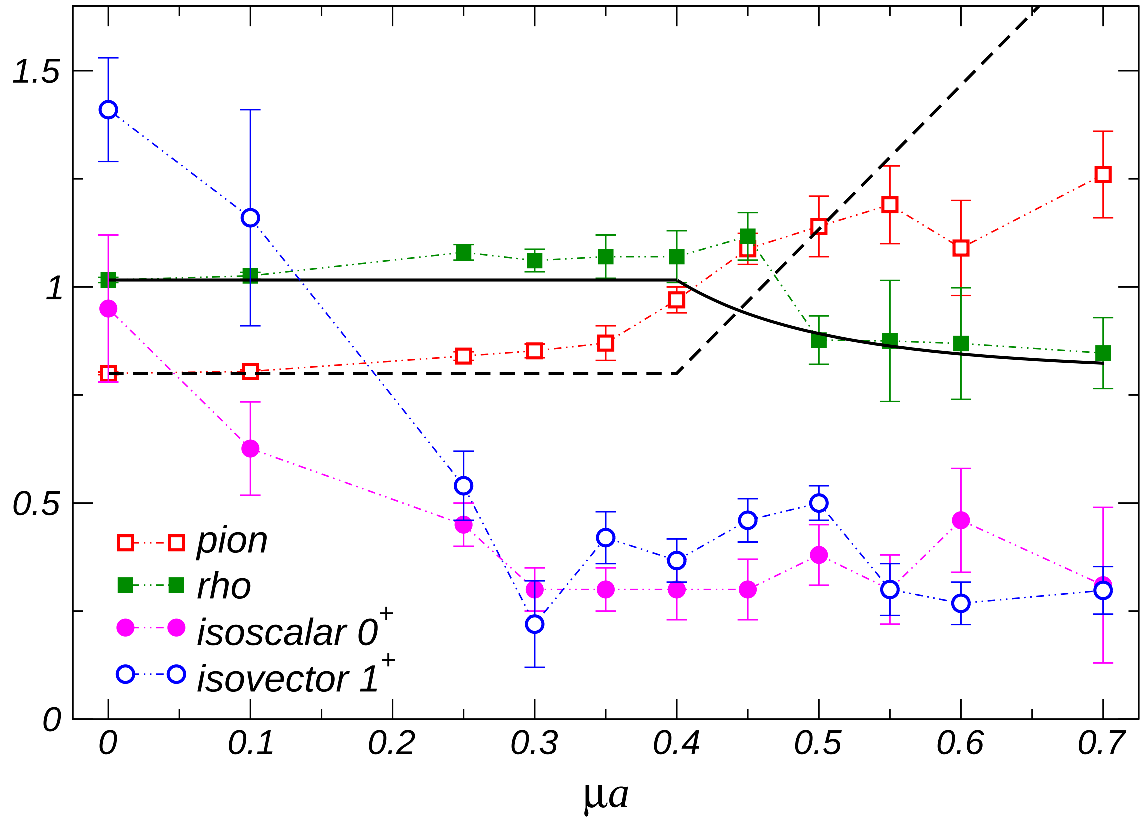
Identify onset transition at $\mu a \approx 0.32$ and (very tentatively) a deconfining transition at $\mu a \approx 0.5$

i.e. with $\mu_q \approx 670$ MeV, $n_q \approx 5\,\text{fm}^{-3}$, $\Delta\varepsilon_g \lesssim 2\,\text{GeVfm}^{-3}$

On $N_\tau = 24$ the unrenormalised Polyakov loop L has very poor signal:noise

Meson Spectrum on $8^3 \times 16$

SJH, Sitch, Skullerud

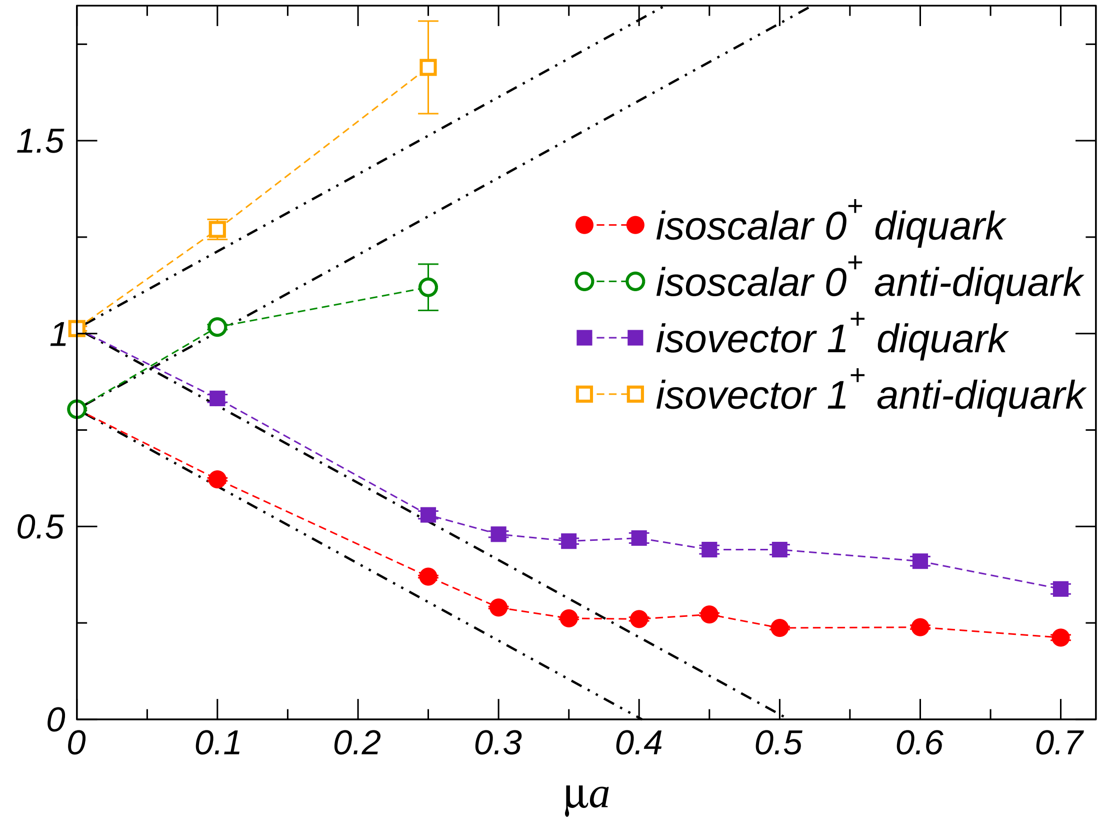


Meson spectrum roughly constant up to onset. Then $m_\pi \approx 2\mu$ in accordance with χPT, while m_ρ decreases once $n_q > 0$, in accordance with effective spin-1 action

[Lenaghan, Sannino & Splittorff PRD65:054002(2002)]

Cf. Hiroshima group [Muroya, Nakamura & Nonaka PLB551(2003)305]

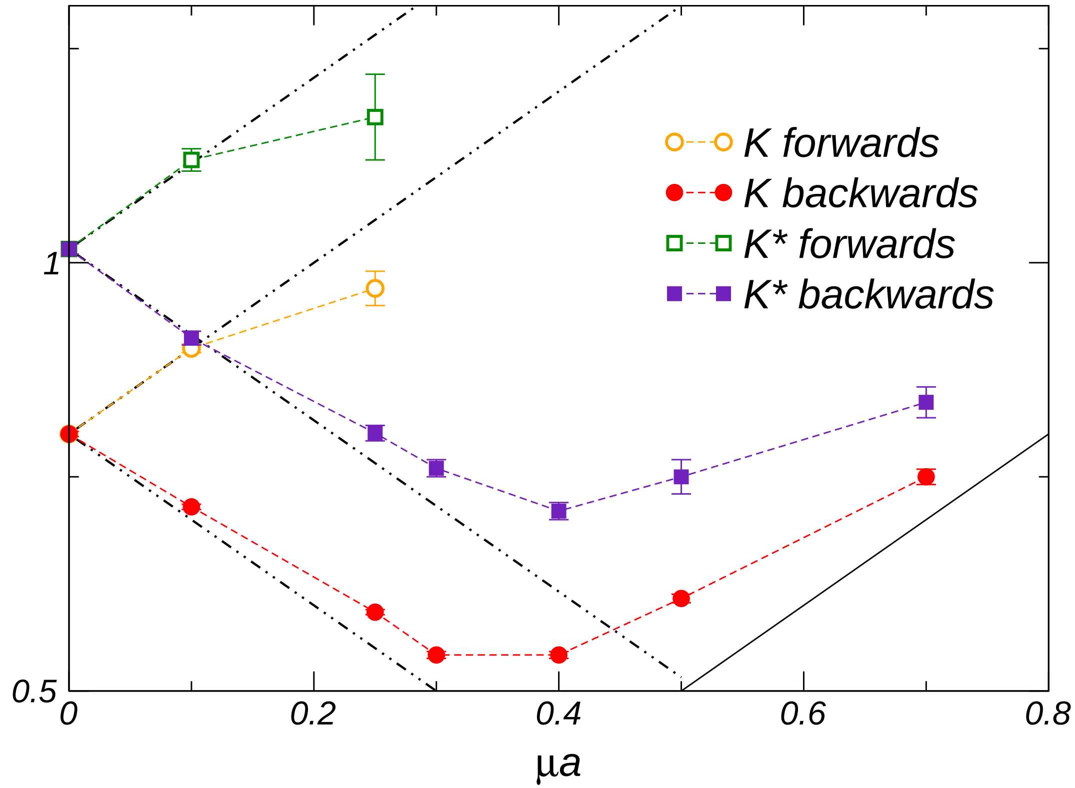
Diquark Spectrum on $8^3 \times 16$



Diquark spectrum modelled by $m_{\pi,\rho} \pm 2\mu$ up to onset, while post-onset:

- Splitting of "Higgs/Goldstone" degeneracy in $I = 0\ 0^+$ channel
- Meson/Baryon degeneracy in $I = 0\ 0^+$ and $I = 1\ 1^+$ channels

Kaon Spectrum on $8^3 \times 16$

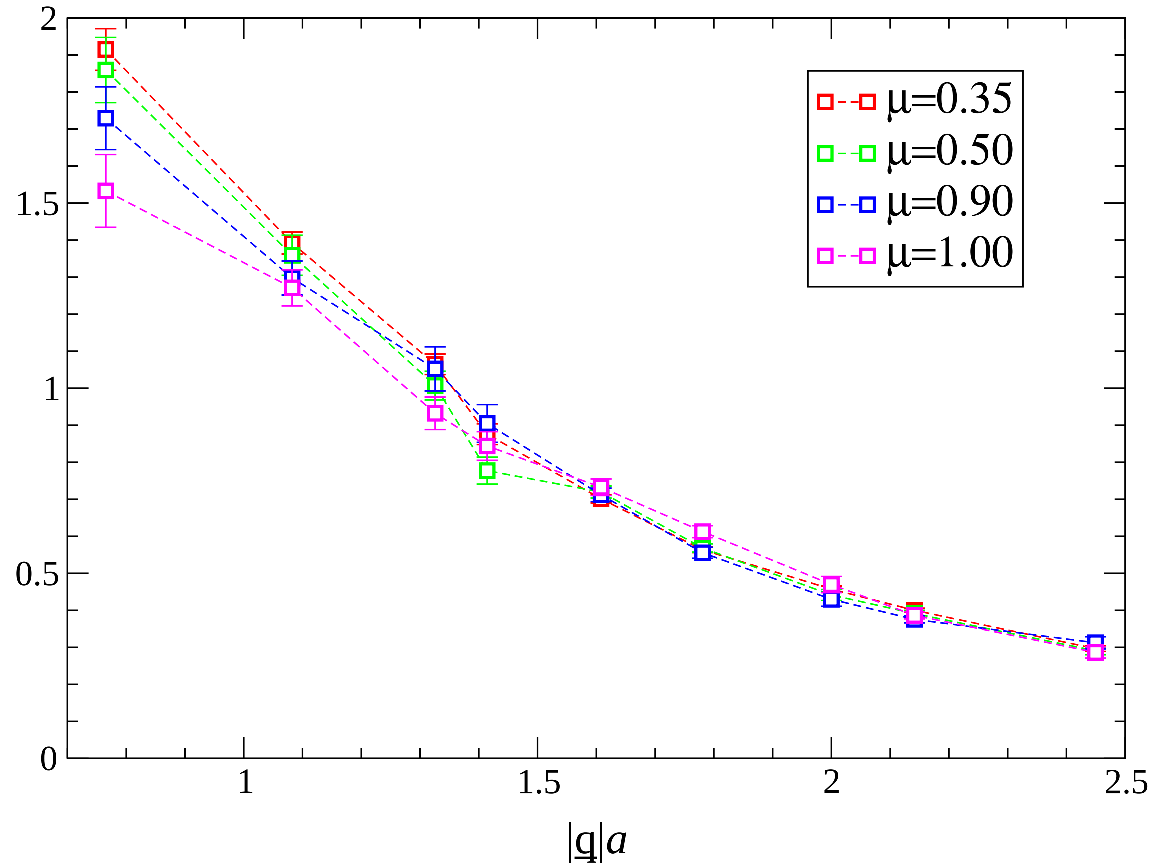


"Kaons" have one quark propagating with $\mu_s \equiv 0$.

Kaon spectrum modelled by $m_{\pi,\rho} \pm \mu$ up to onset, while post-onset $m_{K^-} \gtrsim \mu$, as if it were a weakly-bound state of an s-quark and a u-hole.

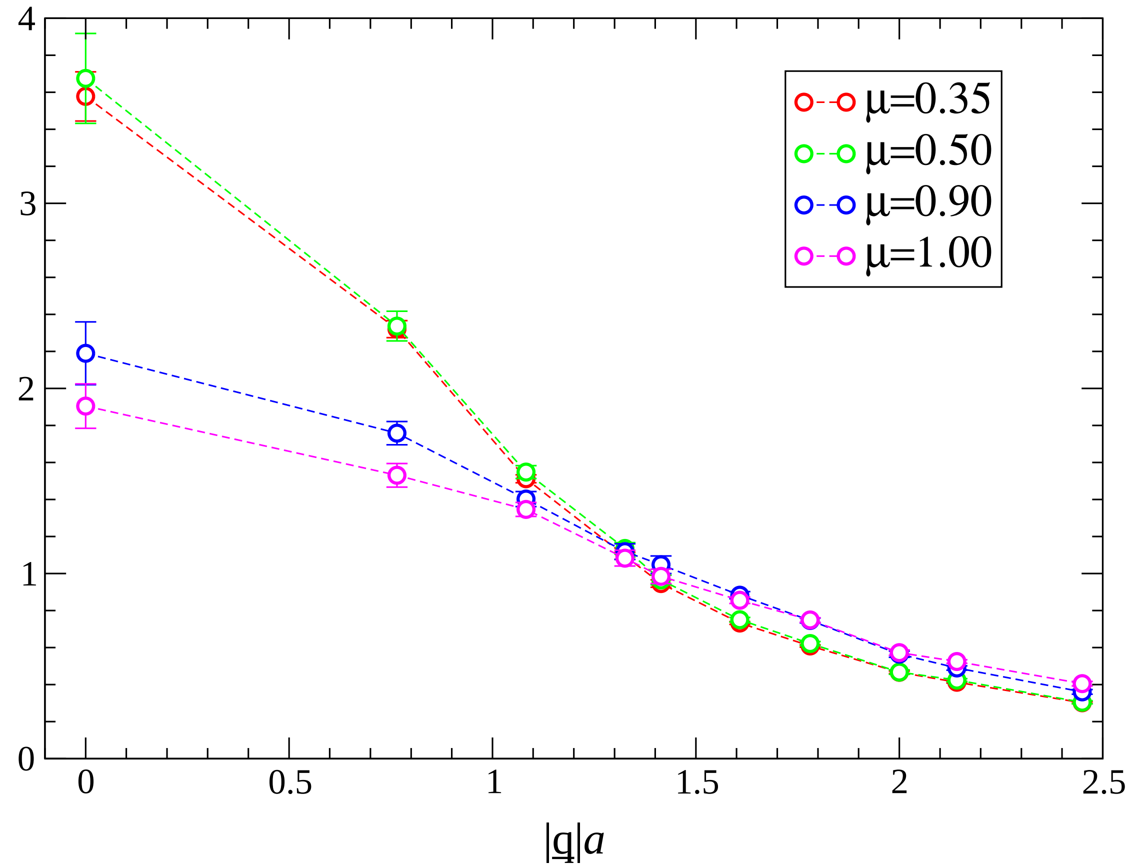
Suggestive of bound kaonic states in nuclear medium?

Electric gluon propagator (Landau gauge)



Plot $D^E(q_0, \vec{q})$ for fixed q_0 as a function of $|\vec{q}|$. The electric gluon in the static limit $q_0 = 0$ shows some evidence of Debye screening as $|\vec{q}| \to 0$ for $\mu \gtrsim 0.9$.

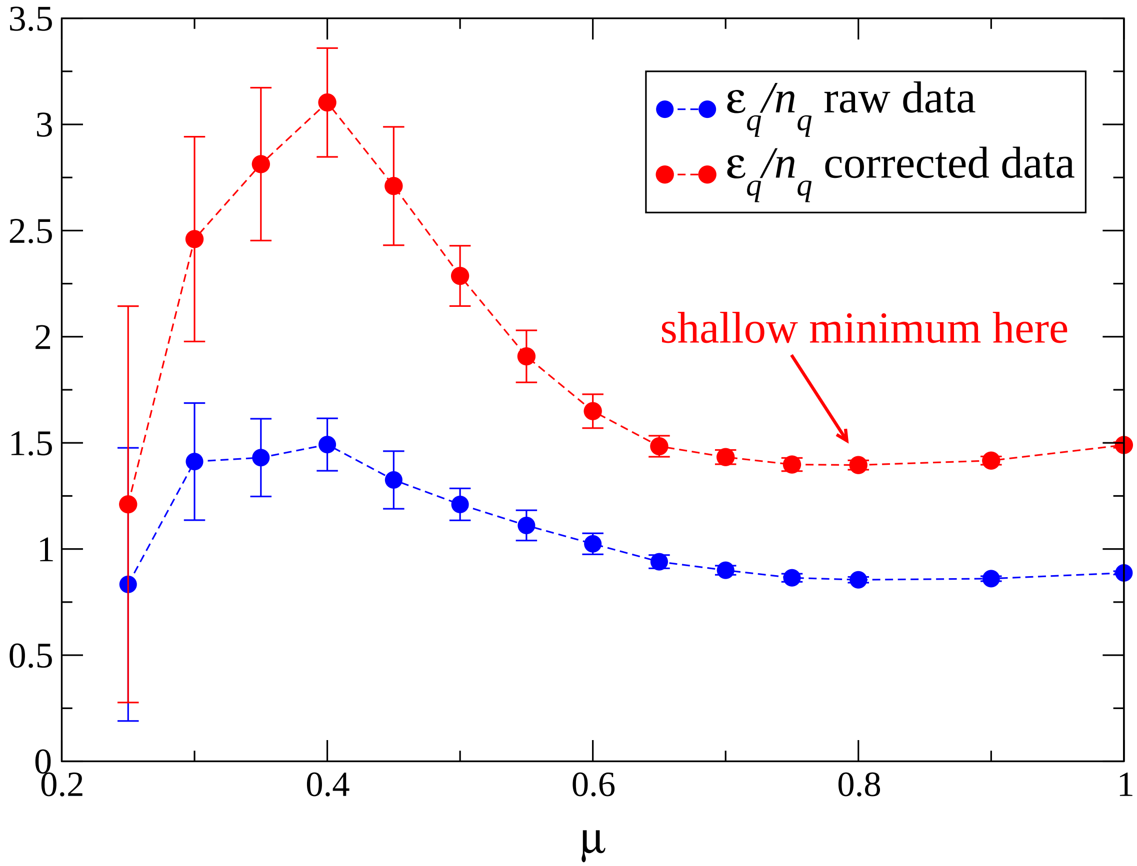
Magnetic gluon



The effect of $\mu \neq 0$ is much more dramatic in the magnetic sector.

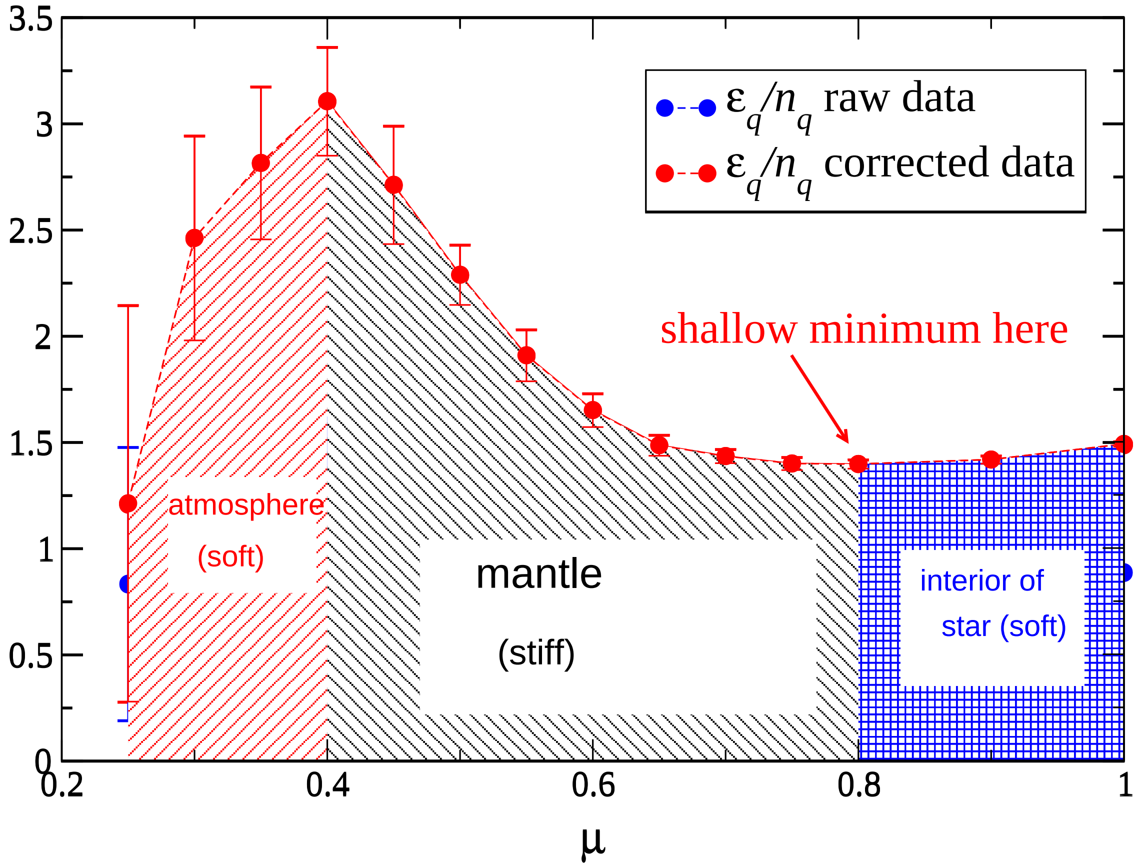
This is significant because in perturbation theory magnetic gluons are not screened in the static limit.

A Star is Born?



Remarkably, ε_q/n_q exhibits a robust minimum for $\mu \gtrsim \mu_d$, implying that macroscopic objects such as Two Color Stars are largely made of quark matter…

A Star is Born?



Remarkably, ε_q/n_q exhibits a robust minimum for $\mu \gtrsim \mu_d$, implying that macroscopic objects such as Two Color Stars are largely made of quark matter. . .

Summary & Outlook

- Thermodynamic results support a BEC at intermediate μ, and deconfined BCS superfluid at large μ
- A non-vanishing gluon energy density may be a more reliable indicator of deconfinement than the Polyakov line as $T \to 0$
- In-medium decrease of 1^- mass, meson/baryon degeneracy, and kaonic-nuclear bound states
- Non-perturbative screening of gluon propagator in magnetic sector
- Bulk quark matter may be more stable energetically than predicted by χPT
- Future analysis to include: nature of deconfinement, topological excitations. . .

EXPLORING QCD: DECONFINEMENT, EXTREME ENVIRONMENTS AND HOLOGRAPHY

20 August to 24 August 2007

Isaac Newton Institute for Mathematical Sciences, Cambridge, UK

Organisers: Nick Evans (*Southampton*), Simon Hands (*Swansea*) and Mike Teper (*Oxford*)

in association with the Newton Institute programme **Strong Fields, Integrability and Strings** (23 July to 21 December 2007)

Draft Programme | **Participants** | **Application** | **Accommodation and Cost**

Theme of Workshop:

QCD is the accepted theory of the strong interaction, but fundamental questions remain unanswered, eg. the dynamics behind the confinement of color and generation of a mass gap; the behaviour of the spectrum as either temperature is raised, or the number of colors or supersymmetries is varied; the ground state of matter at high baryon density.

Interest in these questions is as topical now as at any time in the last 25 years, driven by the heavy-ion collision experimental programmes at RHIC and LHC; the advent of Teraflop-scale computer resources enabling systematic and quantitative approach to QCD beyond perturbation theory; and dramatic theoretical progress in non-perturbative gauge theory exploiting a conjectured duality between gauge theory and gravity, which promises to fulfil a longstanding dream of finding a theoretical description of the QCD string.

The workshop's aim is to initiate and sustain a dialogue between different communities of researchers, with the aim both of reviewing and communicating progress, and of suggesting new and fruitful directions for collaborative exploration.

Invited Speakers:

- Ofer Aharony (Weizmann Institute)
- Johanna Erdmenger (MPP Munich)
- Philippe de Forcrand (ETH Zurich)
- Clifford Johnson (Southern California) *

- Frithjof Karsch (Brookhaven National Laboratory)
- Mikko Laine (Bielefeld)
- Aneesh Manohar (San Diego) *
- Rob Myers (Perimeter Institute) *
- Horatiu Nastase (Tokyo Institute of Technology)
- Peter Petreczky (Brookhaven National Laboratory)
- Alex Pomarol (UAB Barcelona)
- Krishna Rajagopal (MIT)
- Francesco Sannino (NBI Copenhagen and SDU Odense)
- Thomas Schaefer (North Carolina)
- Edward Shuryak (SUNY Stony Brook)
- Andrei Starinets (Perimeter Institute)
- Misha Stephanov (Illinois)
- Guy de Teramond (Costa Rica)
- Jac Verbaarschot (SUNY Stony Brook)
- Laurence Yaffe (Washington).

* to be confirmed

Poster Session/Contributed Talks:

If you wish to be considered to present a poster or contributed talk please indicate your request on the application form.

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