Chiral Phase Boundary of QCD with Many Flavors

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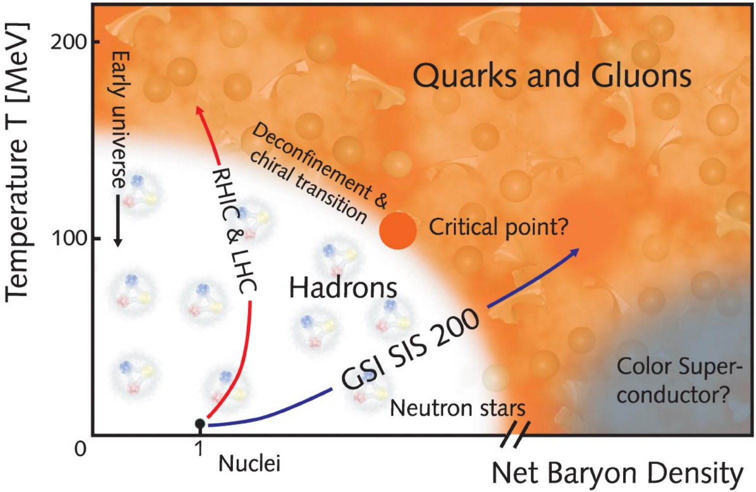
QCD Phase Diagram



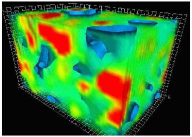
[FAIR@WWW.GSI.DE]

“Learning by D o ing”

“Learning by Deforming”

QCD

Lattice QCD



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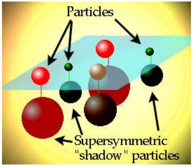
Large N_c

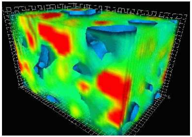
SUSY QCD



“Learning by Deforming”

QCD

Lattice QCD



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Large N_c

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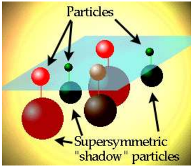


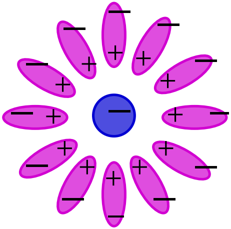
Large N_f

“many-flavor QCD”

Many-Flavor QCD

Many-flavor QCD

▷ charge screening:

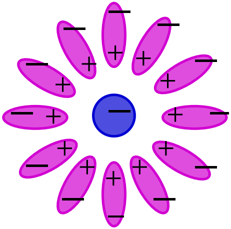


▷ β function

$$\beta = -2\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)\frac{g^4}{16\pi^2} - 2\left(\frac{34N_c{}^3 + 3N_f - 13N_c{}^2N_f}{3N_c}\right)\frac{g^6}{(16\pi^2)^2} + \dots$$

Many-flavor QCD

▷ charge screening:



▷ β function

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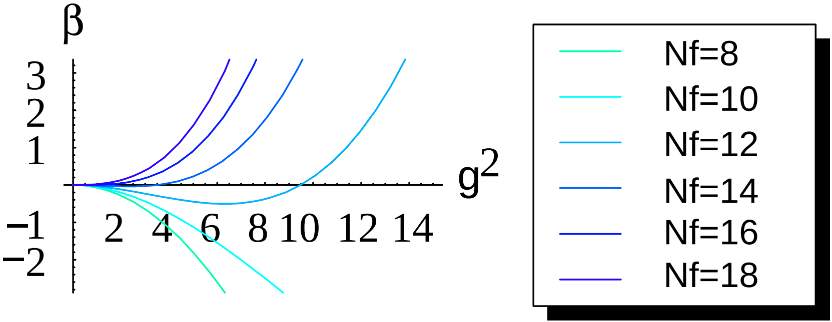
$$\text{for } N_f > \frac{34N_c^3}{13N_c^2 - 3} \overset{SU(3)}{\simeq} 8.05$$

▷ β function

$$\beta = -2\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)\frac{g^4}{16\pi^2} - 2\left(\frac{34N_c{}^3 + 3N_f - 13N_c{}^2N_f}{3N_c}\right)\frac{g^6}{(16\pi^2)^2} + \dots$$

▷ e.g., SU(3): IR fixed point α_* (BANKS&ZAKS'82)



Many-flavor QCD

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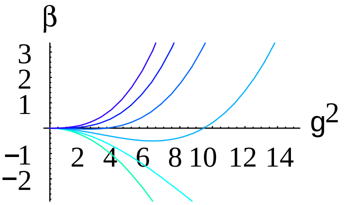
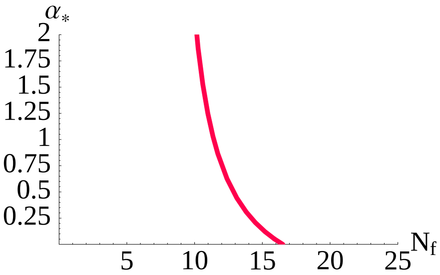
▷ N_f dependence of α_*





Many-flavor QCD

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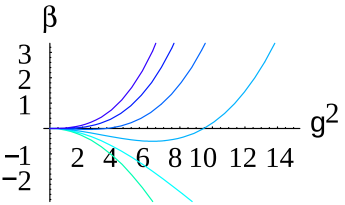
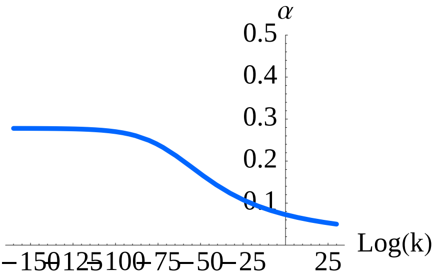
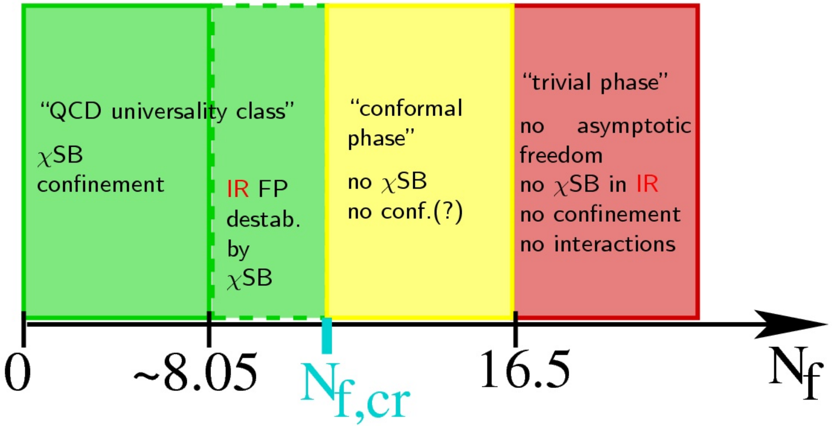
▷ e.g. $N_f = 14$ $\Longrightarrow$ IR fixed point



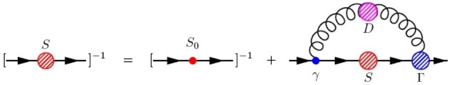


Many-flavor QCD



Many-flavor QCD

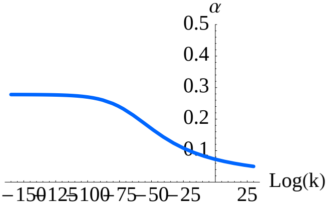
▷ gap equation



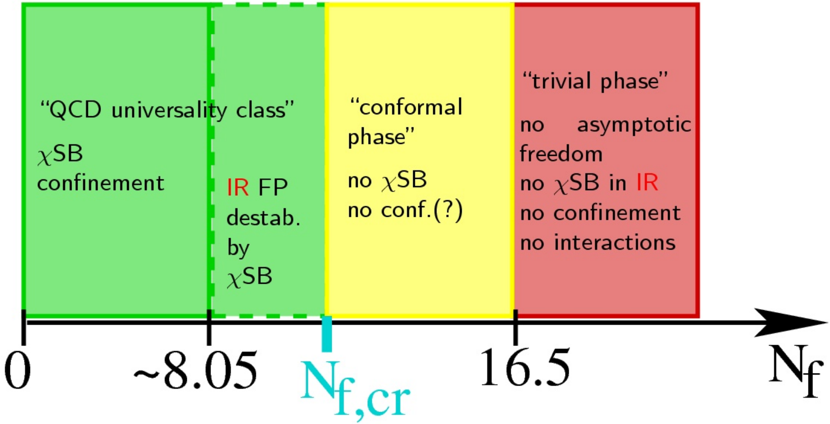
▷ rainbow/ladder approximation: $\Gamma \to g$

$$\Longrightarrow \qquad \chi\text{SB} \quad \text{for} \quad \alpha > \alpha_{\text{cr}}$$

▷ α_* vs. α_{cr}

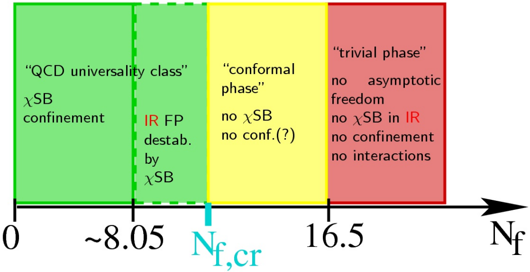


Many-flavor QCD



$$N_{\mathrm{f,cr}} = ?$$

Many-flavor QCD



$$N_{f,cr} = \begin{cases} 5 & \text{(Harada\&Yamawaki'00)} \\ 6 & \text{(Iwasaki et al.'03)} \\ \gtrsim 6 & \text{(Velkovsky\&Shuryak'97, Appelquist\&Selipsky'97)} \\ \gtrsim 10 & \text{(Sannino\&Schechter'99)} \\ \simeq 12 & \text{(Miransky\&Yamawaki'96, Appelquist et al.'96)} \end{cases}$$

LGT: (Kogut&Sinclair'88; Brown et al.'92; Iwasaki et al.'96; Damgaard et al.'97)

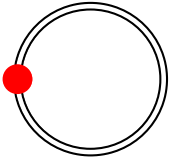
adjoint matter: (Dietrich&Sannino'06)

Functional RG

Functional RG Flow Equation

$$\partial_t \Gamma_k \equiv k\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)} + R_k}\,\partial_t R_k$$

$=$

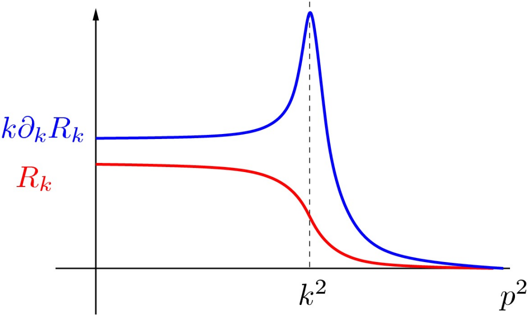


(Wilson'71; Wegner&Houghton'73; Polchinski'84; Wetterich'93)

Functional RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k$$

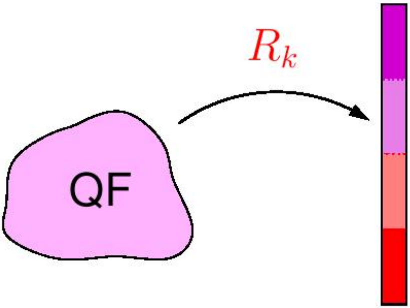
▷ regulator



Functional RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)} + R_k}\,\partial_t R_k$$

▷ quantum fluctuations:



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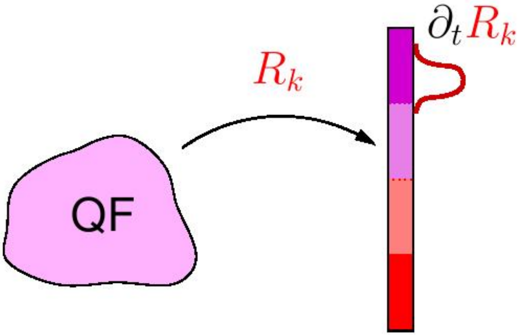

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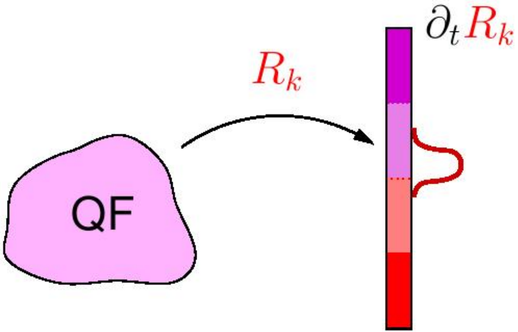

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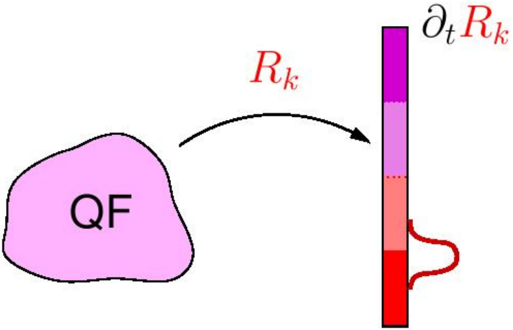
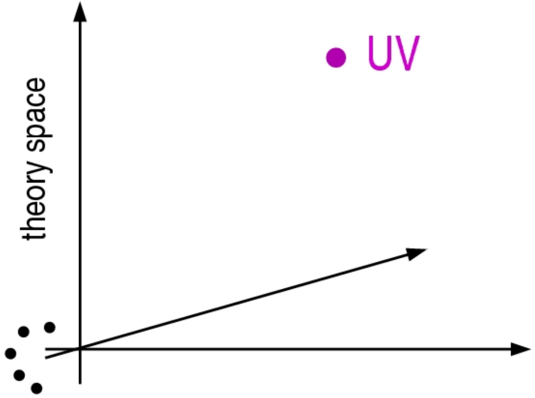


Functional RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k$$

▷ RG trajectory: $\Gamma_{k=\Lambda} = S_{\mathrm{bare}} = \int \frac{1}{4} F^z_{\mu\nu} F^z_{\mu\nu} + \bar{\psi} (\mathrm{i}\partial\!\!\!/ + gA\!\!\!/) \psi$



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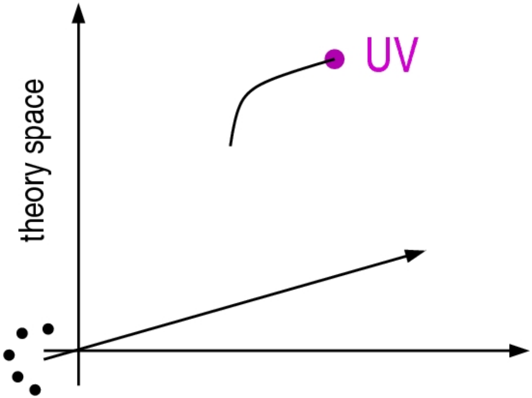

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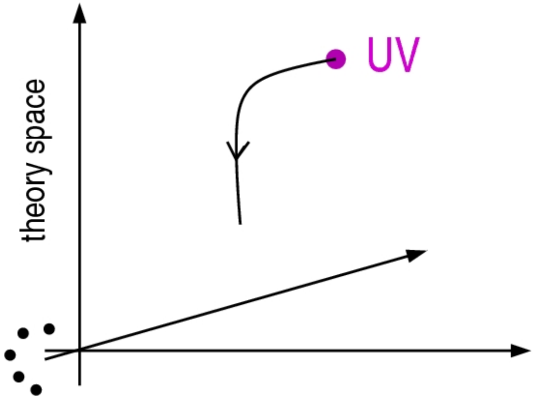
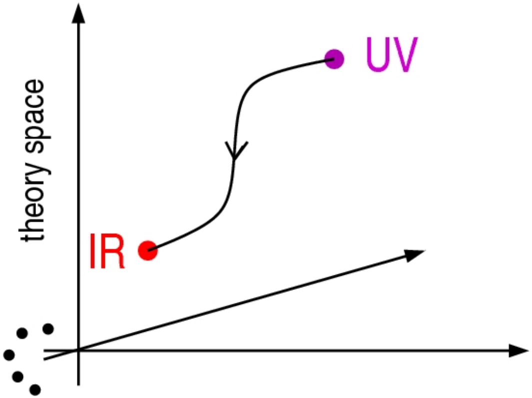


Functional RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k$$

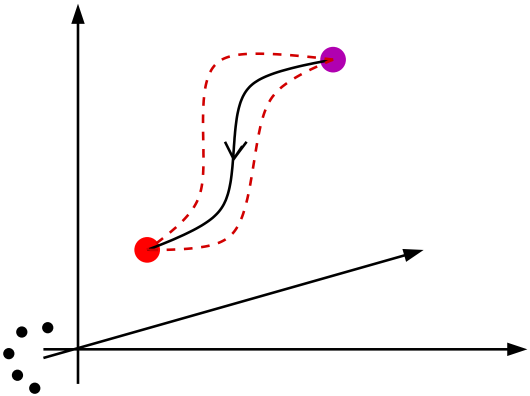
▷ RG trajectory: $\Gamma_{k\to 0} = \Gamma$



Functional RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k$$

- RG trajectory: R_k scheme independence



Functional RG Flow Equation

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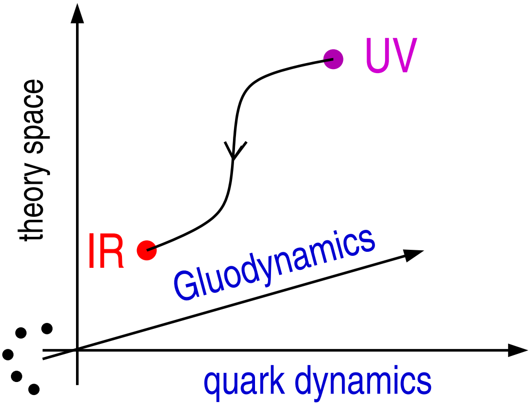
▷ RG trajectory: truncation



Functional RG Flow Equation

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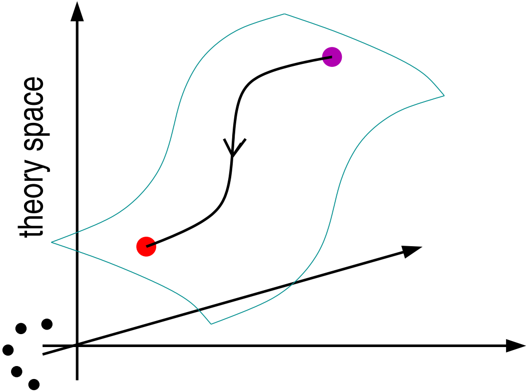
- RG trajectory:



Functional RG Flow Equation

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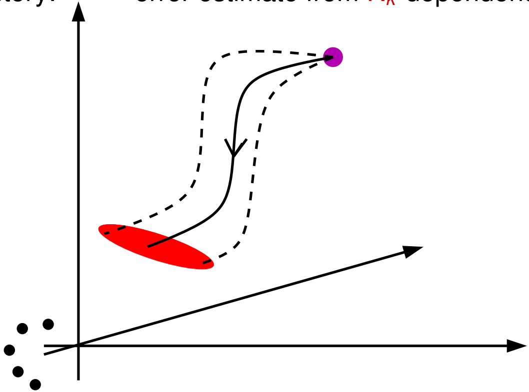
▷ RG trajectory: error estimate from R_k dependence



RG Flow towards the Chiral Transition

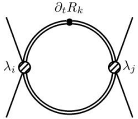
RG Flow of the Chiral Sector

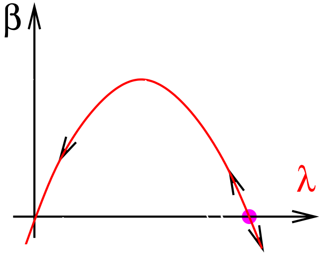
- effective action:

$$\Gamma_k = \int \frac{1}{2} \frac{\lambda_\sigma}{k^2} \left[(\bar{\psi}^a \psi^b)^2 - (\bar{\psi}^a \gamma_5 \psi^b)^2 \right]$$

- RG flow

$$\partial_t \lambda_\sigma = 2\lambda_\sigma - \frac{N_c}{4\pi^2} \lambda_\sigma{}^2$$





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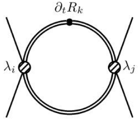
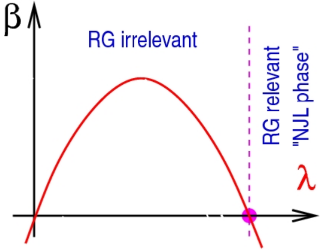




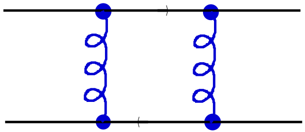
RG Flow of the Chiral Sector

▷ effective action:

$$\Gamma_k = \int \frac{1}{4} F^z_{\mu\nu} F^z_{\mu\nu} + \dots + \bar\psi \, (\mathrm{i}\partial\!\!\!/ + \bar{g} A\!\!\!/) \, \psi + \frac{1}{2} \frac{\lambda_\sigma}{k^2} \left[(\bar\psi^a \psi^b)^2 - (\bar\psi^a \gamma_5 \psi^b)^2 \right]$$

▷ RG flow

$$\partial_t \lambda_\sigma = 2\lambda_\sigma - \frac{N_c}{4\pi^2} {\lambda_\sigma}^2 - \frac{3}{8\pi^2} \frac{{N_c}^2 - 1}{N_c} g^2 \lambda_\sigma - \frac{9}{256\pi^2} \frac{3{N_c}^2 - 8}{N_c} g^4$$



RG Flow of the Chiral Sector

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▷ RG flow

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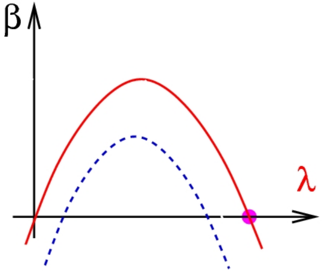
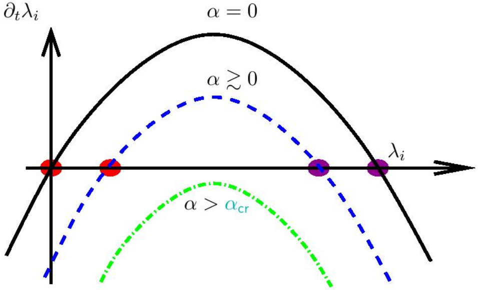


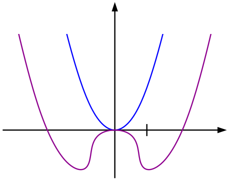
Chiral Criticality

▷ critical gauge coupling α_{cr}:



$\Longrightarrow$ bosonization $\rightarrow \chi$SB:

if $\alpha > \alpha_{\text{cr}}$: $\quad \lambda \sim \frac{1}{m_\phi^2} \rightarrow \infty$



RG Flow of the Chiral Sector

▷ effective action: SU(N_c), $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$

$$
\begin{aligned}
\Gamma_k \;=\; \int & \frac{Z_F}{4} F^z_{\mu\nu} F^z_{\mu\nu} + \ldots + \bar\psi\,(\mathrm{i} Z_\psi \partial\!\!\!/ + Z_1 \bar g A\!\!\!/)\,\psi \\
& + \frac{1}{2}\frac{\lambda_\sigma}{k^2}\,(\text{S-P}) + \frac{1}{2}\frac{\lambda_{VA}}{k^2}\,\left[2(\text{V-A})^{\text{adj.}} + (1/N_c)(\text{V-A})\right] \\
& + \frac{1}{2}\frac{\lambda_+}{k^2}\,(\text{V+A}) + \frac{1}{2}\frac{\lambda_-}{k^2}\,(\text{V-A})
\end{aligned}
$$

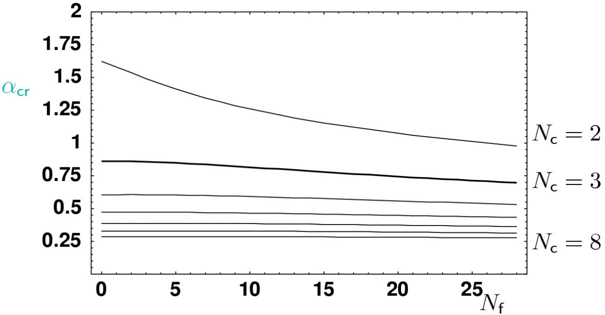
▷ RG flow, e.g.,

$$
\begin{aligned}
\partial_t \lambda_\sigma \;=\;& 2\lambda_\sigma - \frac{1}{4\pi^2} l_1^{(F)}[R_k]\left\{2N_c\,{\lambda_\sigma}^2 - 2\lambda_-\lambda_\sigma - 2N_f\lambda_\sigma\lambda_{VA} - 6\lambda_+\lambda_\sigma\right\} \\
& - \frac{1}{8\pi^2}\, l_{1,1}^{(FB)}[R_k]\left[3\frac{{N_c}^2-1}{N_c}\, g^2\lambda_\sigma - 6g^2\lambda_+\right] \\
& - \frac{3}{128\pi^2}\, l_{1,2}^{(FB)}[R_k]\,\frac{3{N_c}^2-8}{N_c}\, g^4
\end{aligned}
$$

(HG, JAECKEL, WETTERICH'04)

χSB Critical Coupling

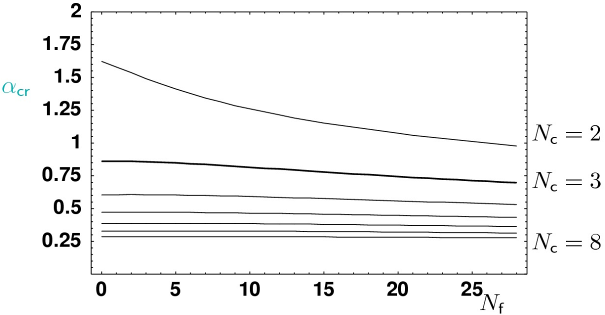
(HG,JAECKEL'05)



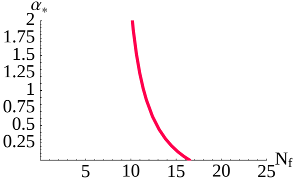
e.g., for $N_c = 3 = N_f$: $\alpha_{cr} \simeq 0.85$

χSB Critical Coupling

(HG,JAECKEL'05)



▷ compare with
Banks-Zaks IR fixed point:



χSB Critical Coupling

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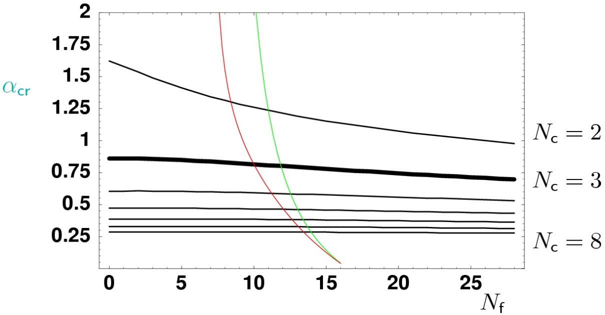


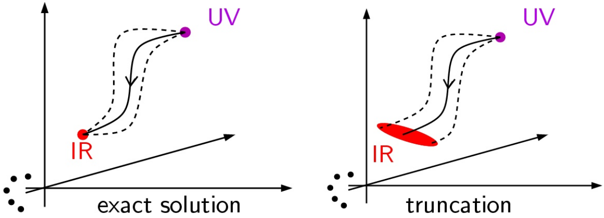
—— 2-loop SU(3) β function in $\overline{\rm MS}$ scheme

—— 4-loop SU(3) β function in $\overline{\rm MS}$ scheme $\Longrightarrow$ $N_{\rm f,cr} \simeq 10.0$

(RITBERGEN ET AL.'97)

Error Estimate

▷ regulator dependence



▷ fermion sector: "optimized" regulator vs. "sharp cutoff" (LITIM'01)

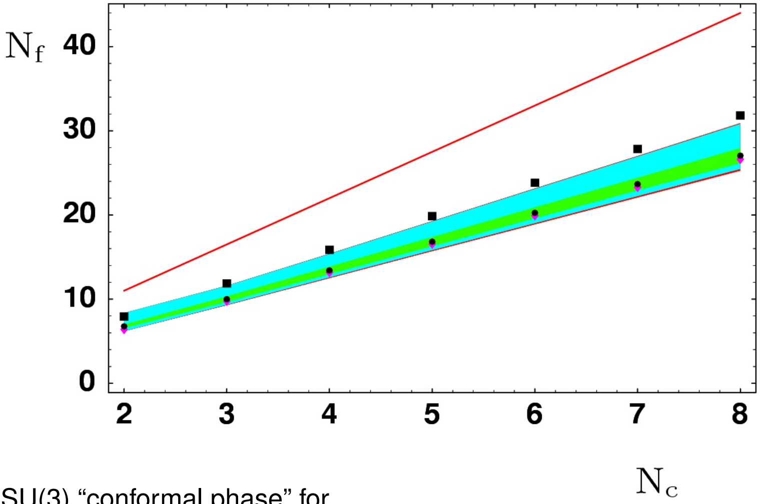
$$l_1^{(F),4} = \frac{1}{2},\ l_{1,1}^{(FB),4} = 1,\ l_{1,2}^{(FB),4} = \frac{3}{2} \qquad \text{vs.} \qquad l_1^{(F),4} = l_{1,1}^{(FB),4} = l_{1,2}^{(FB),4} = 1$$

▷ anomalous dimensions, momentum dependencies, higher-order operators $\sim \psi^8$, etc. ...

▷ gauge sector: 2-loop, 3-loop, 4-loop β function

$\overline{\text{MS}}$ scheme vs. RG scheme (∼10, 30, 50 % variation (?))

χSB Critical Coupling



▷ SU(3) "conformal phase" for

$$N_{f,cr} = 10.0 \pm 0.29(\text{fermion})\,^{+1.55}_{-0.63}(\text{gluon}) \lesssim N_f < 16.5$$

(HG,JAECKEL'05)

Lessons to be learned for “real QCD”

- fermionic screening is rather weak
- fermionic truncation (surprisingly) stable in χ symmetric phase
- phase boundary detectable with fermionic “derivative expansion”
- “real QCD” requires nonperturbative estimate of β_{g^2}

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$$\Gamma_k = \int \frac{Z_F}{4} F_z^{\mu\nu} F_{\mu\nu}^z \underbrace{+ \quad \dots}_{\uparrow}$$

RG Flow for Gluodynamics

RG Flow of Gluodynamics

▷ Operator expansion with the background-field method

(Reuter, Wetterich'94; Freire, Litim, Pawlowski'00)

$$\Gamma_k[A] = \int d^d x \; W_k(F^2), \quad F^2 \equiv F^a_{\mu\nu} F^a_{\mu\nu}$$

$$W_k(F^2) = \frac{Z_F}{4} F^2 + \frac{W_2}{2!4^2} (F^2)^2 + \frac{W_3}{3!4^3} (F^2)^3 + \dots$$

(HG'02)

▷ running coupling: (Abbott'82)

$$g^2 = Z_F^{-1} \bar{g}^2$$

▷ β function:

$$\partial_t g^2 \equiv \beta_{g^2} = -\frac{22 N_c}{3} \frac{g^4}{(4\pi)^2} \dots$$

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1	-29.3333
2	-357.83
3	-191.32
4	15499.6
5	$-1.88776 \cdot 10^6$
6	$1.65315 \cdot 10^7$
7	$2.79324 \cdot 10^9$
8	$-1.37622 \cdot 10^{11}$
9	$-4.21715 \cdot 10^{12}$
10	$8.60663 \cdot 10^{14}$
11	$-8.05611 \cdot 10^{16}$
12	$5.21052 \cdot 10^{19}$
13	$-6.30043 \cdot 10^{22}$
14	$9.35648 \cdot 10^{25}$
15	$-1.78717 \cdot 10^{29}$
16	$4.35314 \cdot 10^{32}$
17	$-1.33397 \cdot 10^{36}$
18	$5.08021 \cdot 10^{39}$
19	$-2.37794 \cdot 10^{43}$
20	$1.35433 \cdot 10^{47}$

Running Coupling

(HG'02)

(BRAUN,HG'05)

cf. Landau gauge:

(V.SMEKAL,ALKOFER,HAUCK'97)

(LANGFELD,REINHARDT,GATTNAR'01)

(LERCHE,V.SMEKAL'02)

(FISCHER,ALKOFER'02)

(ZWANZIGER'02)

(PAWLOWSKI ET AL.'03)

(FISCHER, HG'04)

(OLIVEIRA,SILVA'04)

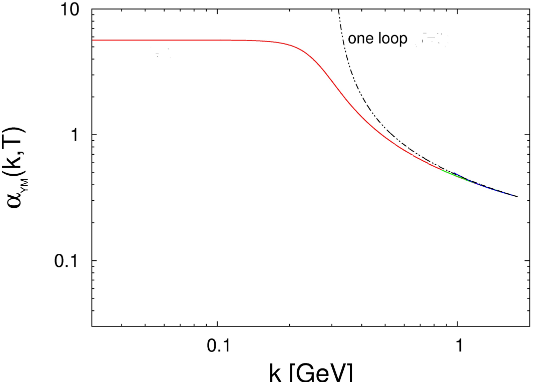
(BLOCK,CUCCHIERI,LANGFELD,MENDES'04)

(SCHLEIFENBAU,LEDERER,REINHARDT'06)

(EPPLE,REINHARDT,SCHLEIFENBAUM'07)

(MAAS'07)

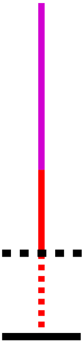
(CUCCHIERI,MENDES,OLIVEIRA,SILVA'07)



IR fixed point: α_*

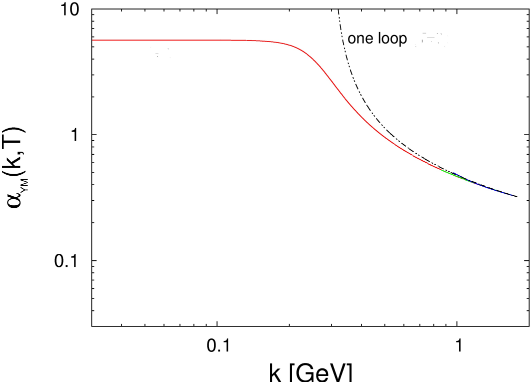
Running Coupling

- mass gap and threshold behavior:



(HG'02)

(BRAUN,HG'05)

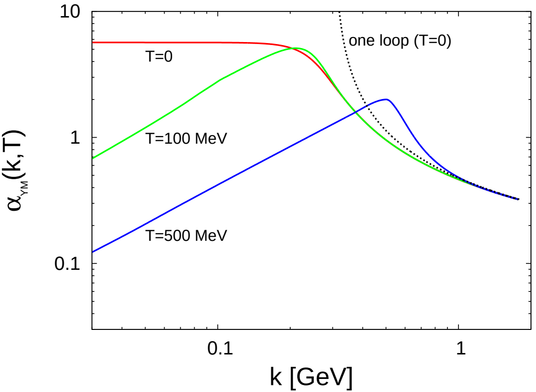


IR fixed point α_* compatible with mass gap

Running Gauge Coupling at finite T

(HG'02)

(BRAUN,HG'05)



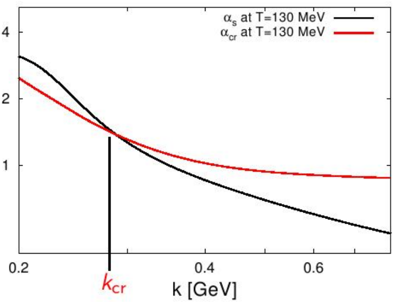
- $T/k \to \infty$: strongly interacting 3D theory

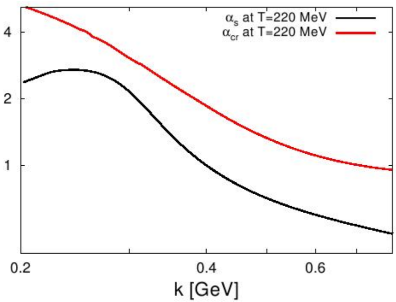
$$\alpha \to \frac{k}{T}\alpha_{3D}, \quad \alpha_{3D} \to \alpha_{3D,*} \simeq 2.7$$

cf. lattice: (CUCCHIERI,MAAS,MENDES'07)

Chiral Phase Transition

▷ $\alpha(k, T)$ vs. $\alpha_{\rm cr}(T/k)$





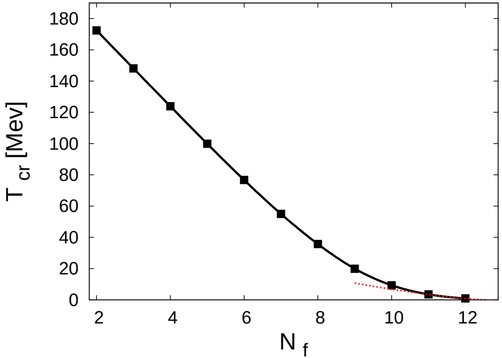
$\Longrightarrow$ χSB triggered by $\alpha_{\rm s}$

single input: $\alpha_{\rm s}(m_\tau) = 0.322$

T_c [MeV]	RG (Braun,HG'05)
$N_{\rm f}$=2	172 ± 37
$N_{\rm f}$=3	148 ± 32

T_c [MeV]	Lattice (BI) (Chen et al.'06)	Lattice (W) (Aoki et al.'06)
$N_{\rm f}$=2+1	192(7)(4)	151(3)(3)

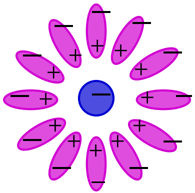
Chiral Phase Boundary $T - N_f$



(Braun,HG'05,'06)

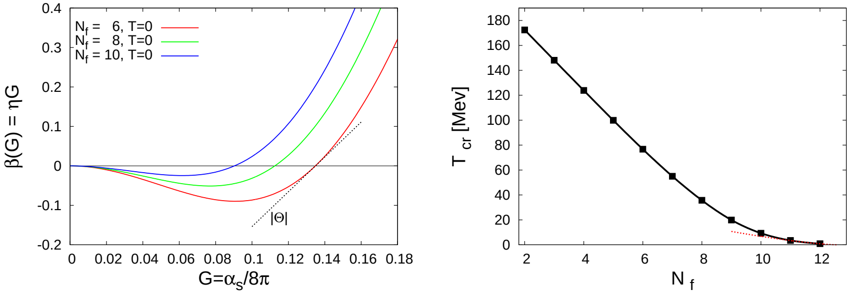
- ▷ small N_f: fermionic screening, $\beta_{\text{quark}} \simeq \frac{2}{3} N_f \frac{g^4}{8\pi^2}$
- ▷ critical flavor number:

$$N_f^{cr} \simeq 12$$



(cf. Appelquist et al.'96; Miranski,Yamawaki'96; HG,Jaeckel'05)

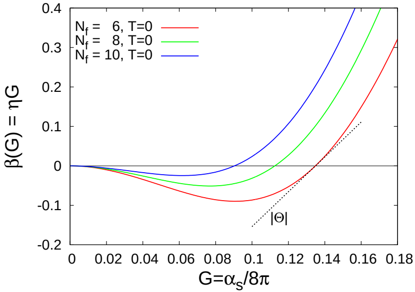
Chiral Phase Boundary $T - N_f$

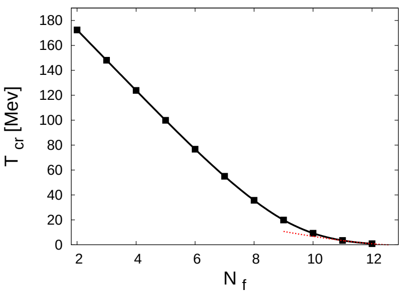


▷ fixed-point regime: critical exponent Θ

$$\beta_{g^2} \simeq -\Theta\,(g^2 - g_*^2)$$

Chiral Phase Boundary $T - N_f$





▷ fixed-point regime: critical exponent Θ

$$\beta_{g^2} \simeq -\Theta \, (g^2 - g_*^2)$$

▷ shape of the phase boundary for $N_f \simeq N_f^{cr}$: (Braun,HG'05,'06)

$$T_{cr} \sim k_0 \, |N_f - N_f^{cr}|^{\frac{1}{|\Theta|}}, \quad \Theta \simeq -0.71$$

Conclusions

▷ "conformal phase" in many-flavor QCD:

$$N_{\rm f,cr} \simeq 10 - 12 < N_{\rm f} < 16.5 \text{ for SU(3)}$$

...applications to walking technicolor

(Dietrich et al.'06, Terao et al.'07)

▷ relation among universal aspects:

shape of the phase boundary $\Longleftrightarrow$ IR critical exponent

▷ functional RG for $\Gamma[\phi]$

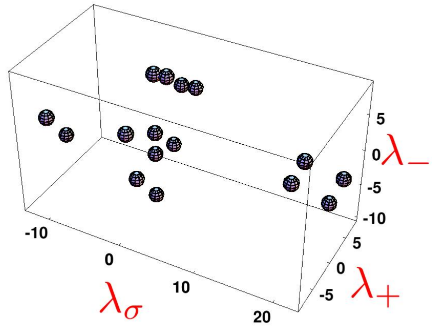
- systematic and consistent expansion schemes for QCD
- chiral symmetry ✓
- calculations "from first principles"

RG Flow of the Chiral Sector

- ▷ 2 fixed points per $\partial_t \lambda$

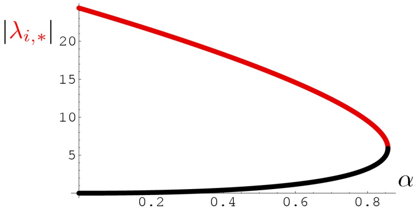
$\Longrightarrow$ $2^4 = 16$ fixed points

- ▷ in general: 2^n FP's for $n = \#$ of λ's



- ▷ fixed-point annihilation

e.g., $N_c = N_f = 3$



Chiral Criticality at Finite Temperature

▷ quark modes:

$$m_T^2 = m_f^2 + (2\pi T(n + \frac{1}{2}))^2$$

$\Longrightarrow$ T-dependent

critical coupling:

$$\alpha_{cr}(T) \gtrsim \alpha_{cr} \simeq 0.85$$

(BRAUN,HG'05)