Effective potentials, thermodynamics, and twisted mass

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Two powerful non-perturbative tools

- lattice gauge theory
- effective chiral Lagrangians

Combining them can give qualitative insight

- clarifies lattice distortions of chiral symmetry
  - Aoki phase
- stimulates development of new lattice actions
  - domain wall, overlap
- exposes the evils of rooted determinants

Thermal QCD

- lattice gives quantitative information on deconfinement
- chiral symmetry restoration coincides with deconfinement
- a playground for understanding lattice chiral symmetry

This talk is on lattice artifacts

- interplay of twisted mass Wilson fermions with deconfinement
- intricate phase structure expected
- arXiv:0706.1207; follow up to hep-lat/9608024

Why care about artifacts?

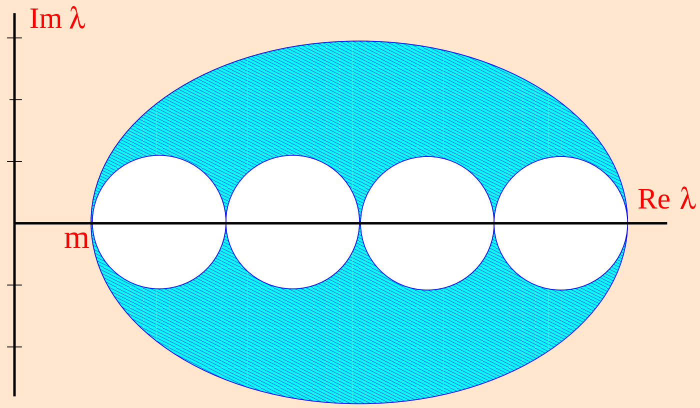
- understanding lattice limitations
  - failure of rooting
- understanding chiral symmetry
  - how does the anomaly work on the lattice
  - how are masses of confined constituents defined
- gain insight into why chiral gauge theories are so hard
- an amusing non-trivial phase structure

Brief review of Wilson fermions

- lattice makes things periodic in momenta
  - $p_\mu \longrightarrow \frac{1}{a}\sin(p_\mu a)$
  - doublers at $p_\mu \sim \pi/a$

Wilson gives doublers a momentum dependent mass

- free case $\qquad D_W(p) = 1/a + \frac{2K}{a}\sum_\mu \left[i\gamma_\mu \sin(p_\mu a) - \cos(p_\mu a)\right]$
  - fermion mass $m \sim (1/2K - 4)/a$ vanishes at $K = K_c = 1/8$
  - eigenvalues on “nested circles”



Gauge fields on links as usual

- gauge action a sum over plaquettes
  - $S_g = \frac{\beta}{3} \sum_p \mathrm{Re\ Tr}\ U_p$
- dynamics fill in the eigenvalue pattern
- real eigenvalues related to topology

Parameters β and K related to bare coupling and quark mass

- $\beta \sim 6/g_0^2$
- $(1/K - 1/K_c) \sim m_q a$
  - $K = 0$ corresponds to infinite quark mass
  - continuum limit: $K \to K_c + O(m_q a)$

Concentrate on two flavor QCD

- 3 flavors somewhat more complicated
- restrict discussion to degenerate quarks
- only consider $\Theta_{CP} = 0$

Physical parameters

- Λ_{qcd}
  - sets the scale, use it for units
  - coupling g absorbed via asymptotic freedom
- quark mass $\frac{m}{\Lambda_{qcd}}$
- temperature $\frac{T}{\Lambda_{qcd}}$
- ultraviolet regulator adds a cutoff scale $a\Lambda_{qcd}$, “lattice spacing”

Lattice parameters with Wilson fermions

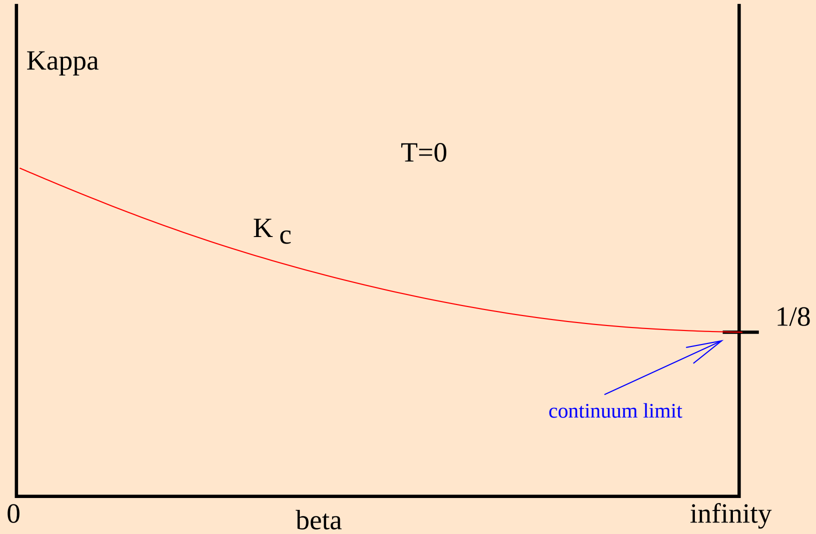
- coupling $\beta = 6/g_0^2$
- hopping parameter K
- temporal extent N_t
- lattice spacing a sets scale

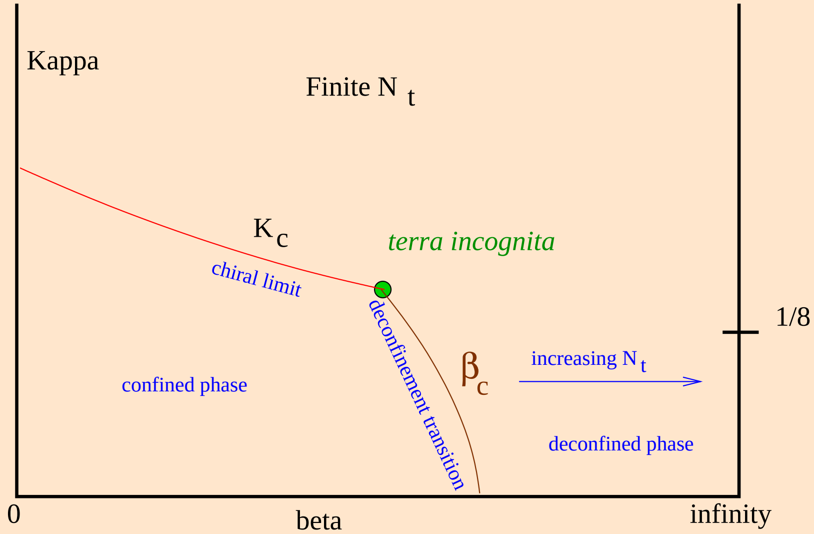
Non-linear mapping $\left\{ \frac{m}{\Lambda_{qcd}}, \frac{T}{\Lambda_{qcd}}, a\Lambda_{qcd} \right\} \longleftrightarrow \{\beta, K, N_t\}$

- well understood

Continuum limit $a\Lambda_{qcd} \to 0$

- $\beta = 6/g_0^2 \to \infty$ at a rate tied to Λ_{qcd}
- $K \to K_c \to 1/8$ tied to quark mass m
- $N_t = 1/aT \to \infty$



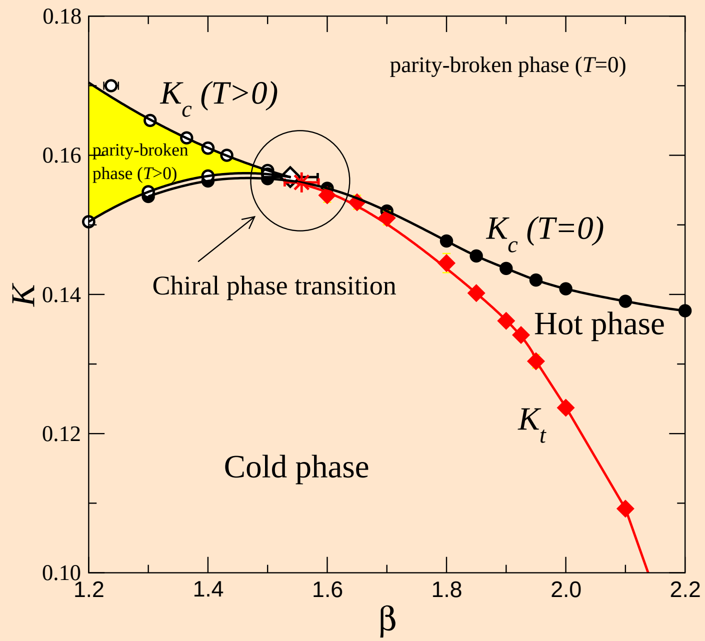


Lattice artifacts

- introduce further structure
- the subject of this talk

CP-PACS (hep-lat/0610038)

- $N_t = 4$

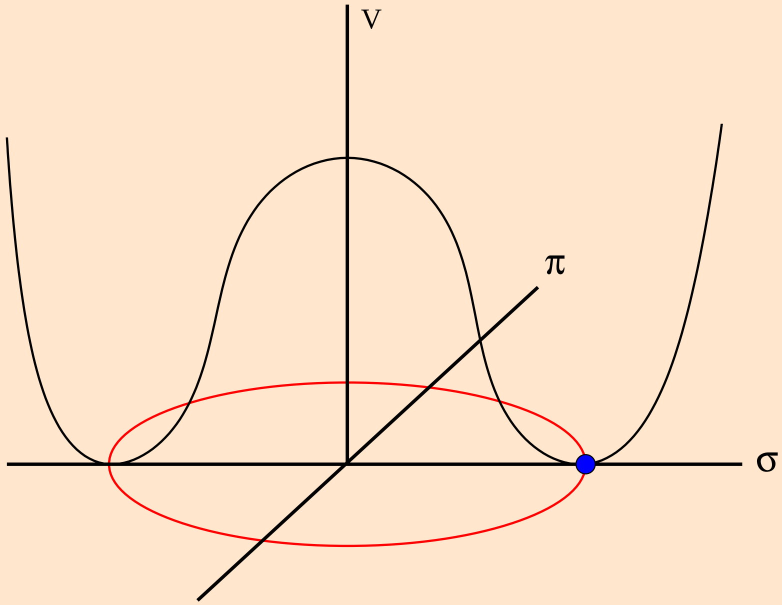


Some recent preliminary APE results (hep-lat/0610112)

- $N_t = 8$ including twisted mass
- more in Regensburg

Effective chiral Lagrangians

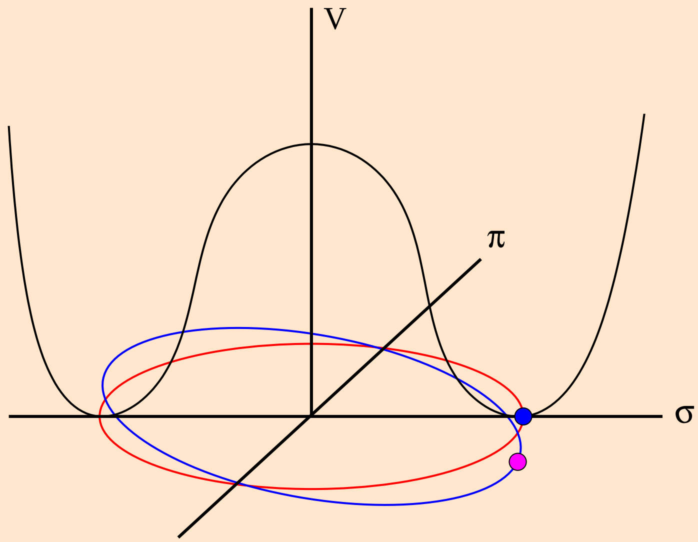
- $\vec{\pi} \sim i\overline{\psi}\gamma_5\vec{\tau}\psi$
- $\sigma \sim \overline{\psi}\psi$
- $V_0 = \lambda(\sigma^2 + \vec{\pi}^2 - v^2)^2$



- chiral symmetry
  - $O(4)$ rotations of $\Sigma = (\sigma, \vec{\pi})$
  - V_0 invariant
- vacuum spontaneously breaks the chiral symmetry
  - $\langle\sigma\rangle \neq 0$ when $v^2 > 0$
  - pions are Goldstone bosons

Mass term $V_1 = -m\sigma$

- breaks chiral symmetry
- tilts potential to select unique vacuum
- $\langle\sigma\rangle > 0 \qquad \langle\vec{\pi}\rangle = 0$
- $m_\pi^2 \sim m$



Chiral rotation

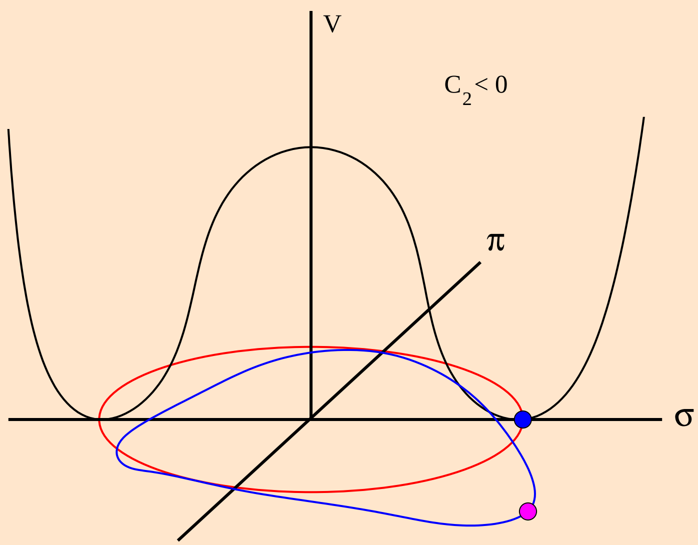
- equivalent physics with $m\sigma \to m\cos(\theta)\sigma + m\sin(\theta)\pi_3$
- lattice artifacts will break this $\to$ "twisted mass"

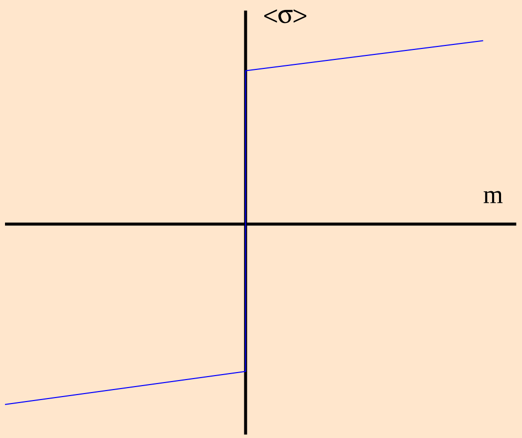
Lattice artifacts

Break chiral symmetry

- lowest correction additively renormalizes mass
- $K_c \to K_c(\beta)$
- $m \to (1/K - 1/K_c(\beta))$

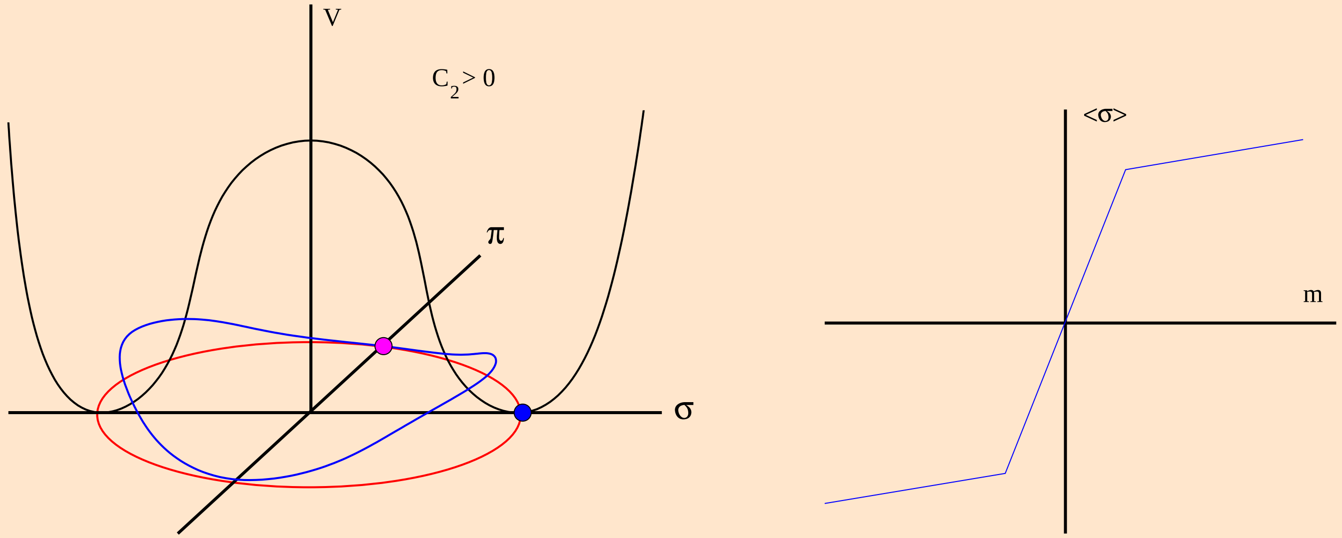
- model next correction with $c_2\sigma^2$ (MC; Sharpe and Singleton)
  - sign of c_2 can depend on gauge action
  - $c_2 < 0$ chiral transition goes first order
  - no exact Goldstone bosons

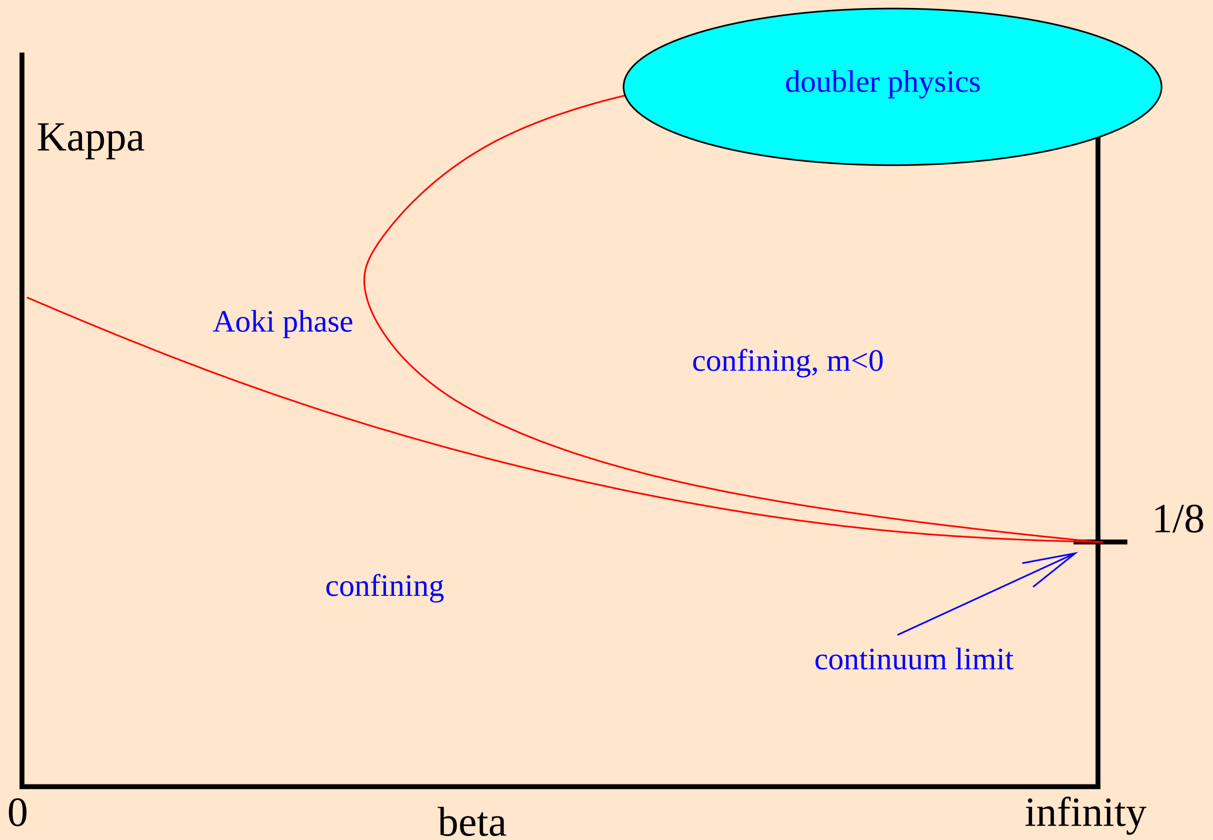




If $c_2 > 0$ chiral transition splits into two second order transitions

- separated by phase with $\langle \vec{\pi} \rangle \neq 0$
  - breaks parity and flavor spontaneously
  - two Goldstone bosons from flavor breaking
  - third massless pion only at critical point
- the “Aoki phase”



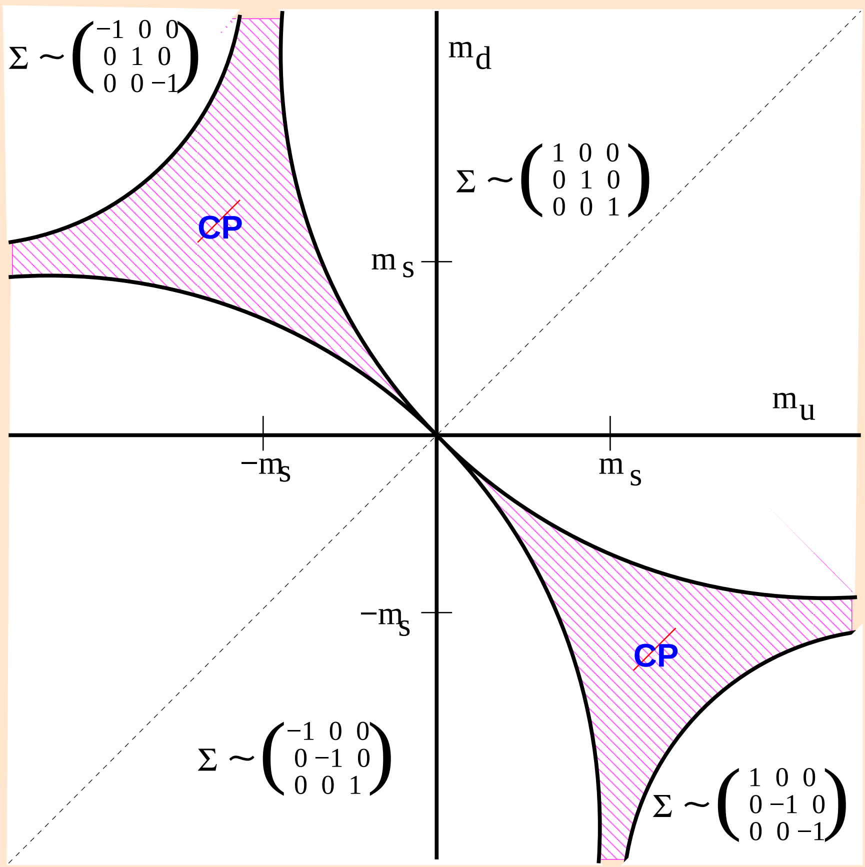


Add c_2 term to the effective model

- $V(\vec{\pi}, \sigma, L) = \lambda(\sigma^2 + \vec{\pi}^2 - v^2)^2 - (1/K - 1/K_c(\beta))\sigma + c_2\sigma^2$

Comment

- Aoki phase survives continuum limit if $m_u \neq m_d$
  - at fixed $m_s \neq 0$:



Twisted Mass

The c_2 term breaks the equivalence of different chiral directions

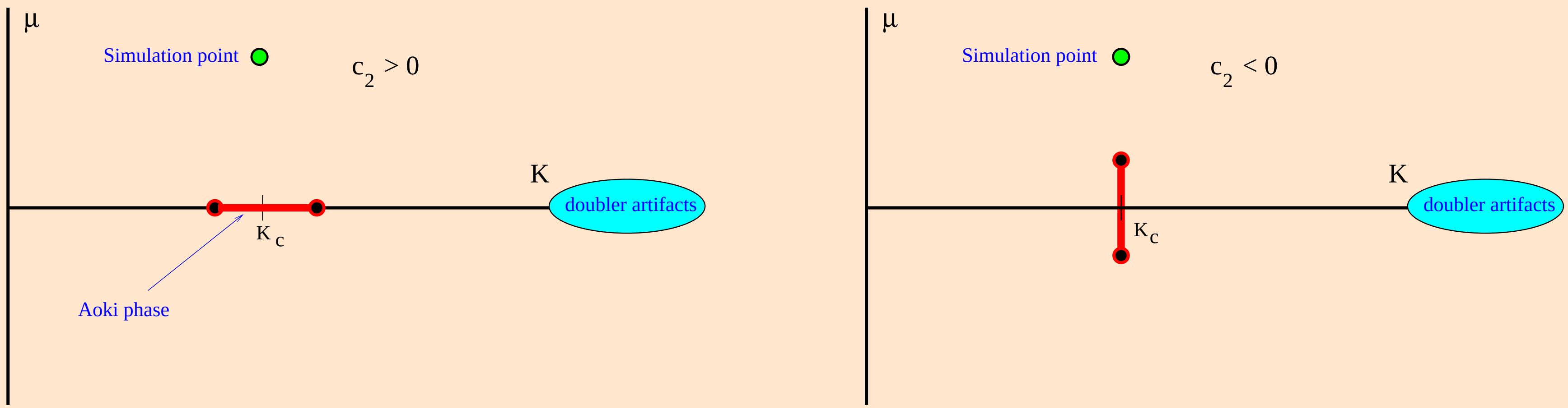
- no longer equivalent physics with $m\sigma \to m\cos(\theta)\sigma + m\sin(\theta)\pi_3$
- equivalent to giving up and down quarks opposite phases
  - $m\overline{\psi}\psi \to \frac{1}{2}(m\overline{\psi}_L\psi_R + m^*\overline{\psi}_R\psi_L)$
  - $m_u \to e^{i\theta}m_u \qquad m_d \to e^{-i\theta}m_d$
- on the lattice add new mass term
  - $\mu\, i\overline{\psi}\tau_3\gamma_5\psi \sim \mu\pi_3$

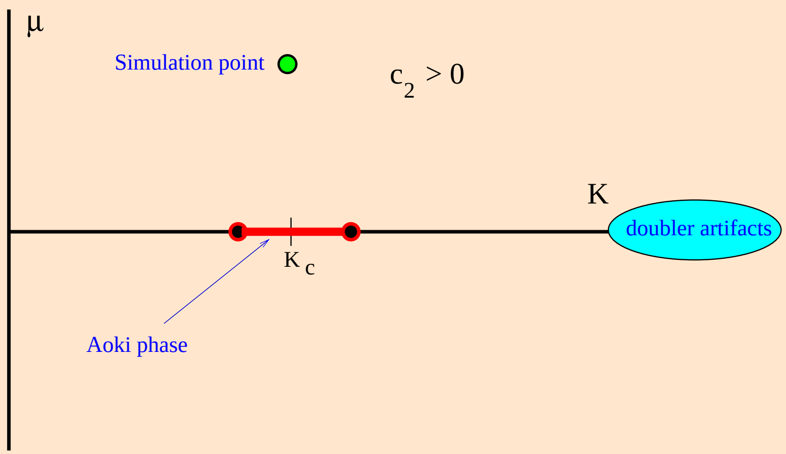
$$V(\vec{\pi},\sigma) = \lambda(\sigma^2 + \vec{\pi}^2 - v^2)^2 - (1/K - 1/K_c(\beta))\sigma + c_2\sigma^2 - \mu\pi_3$$

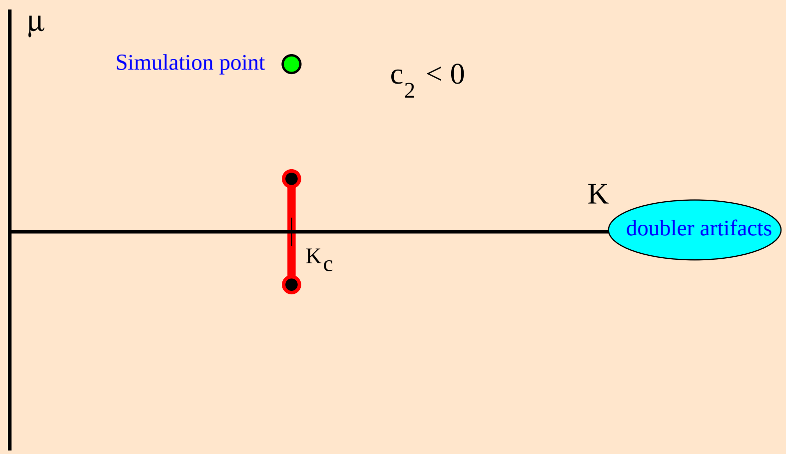
“Magnetic field” conjugate to the Aoki phase order parameter

Motivations

- $O(a)$ lattice artifacts cancel
- fermion determinant positive
- faster than overlap or domain wall
- avoid diseases of staggered quarks
- allows continuation around Aoki phase







Difficulties

- need to know K_c
- odd N_f?
  - $m \leftrightarrow -m$ symmetry broken by anomalies
- mass needs to be larger than c_2 artifacts
- chiral symmetry still not exact

Deconfinement

Wilson-Polyakov line

- $L(\vec{x}) = \mathrm{Re}\ \mathrm{Tr}\ \mathcal{T} \exp(i \int_0^{1/T} A_0(\vec{x}) dt)$ continuum
- $L(\vec{x}) = \mathrm{Re}\ \mathrm{Tr}\ \mathcal{T} \prod_t U_{t+1,t}(\vec{x})$ lattice
- symmetry of pure gauge theory under $L \to e^{2\pi i/3} L$
- $\langle L \rangle$ vanishes at low temperature and $m = \infty$
- symmetry broken by quarks when $m < \infty$

Quark condensate $\langle \overline{\psi} \psi \rangle \sim \langle \sigma \rangle$

- vanishes at high temperature and $m = 0$
- $V(\vec{\pi}, \sigma)$ evolves to a single minimum.

Both $\langle L\rangle$ and $\langle\overline{\psi}\psi\rangle$ show rapid change over a small region of T

- first order for large m
  - like 3 state Potts model
- probably second order for $m = 0$
  - like an $SO(4)$ sigma model
  - first order not ruled out (D'Elia, Di Giacomo, Pica)
- either crossover or phase transition for intermediate m

Effective model for the Wilson line (Svetitsky, Yaffe, Gocksch, Ogilvie, Pisarski ...)

- complex field $L(x)$

$$V(L) = \alpha_1 |L|^4 + \alpha_2 (T_c - T)|L|^2 - \alpha_3 \mathrm{Re}(L^3) - \alpha_4 \mathrm{Re} L$$

- α_2 drives the transition
- α_3 term reduces symmetry to Z_3
  - drives transition towards first order
- α_4 term models fermion breaking of Z_3
  - can soften transition to a crossover

Changing to lattice parameters $T_c - T \to \beta_c(K, N_t) - \beta$

$$V(L) = \alpha_1 |L|^4 + \alpha_2 (\beta_c(K, N_t) - \beta)|L|^2 - \alpha_3 \mathrm{Re}(L^3) - \alpha_4 \mathrm{Re} L$$

Merging the models

(Mocsy, Sannino, Tuominin, Ratti, Thaler, Roessner, Weise, ...)

$$
\begin{aligned}
V(\vec{\pi},\sigma,L) = {} & \lambda(\sigma^2+\vec{\pi}^2-v^2)^2-(1/K-1/K_c(\beta))\sigma+c_2\sigma^2-\mu\pi_3 \\
& +\alpha_1|L|^4+\alpha_2(\beta_c(K)-\beta)|L|^2-\alpha_3\mathrm{Re}(L^3)-\alpha_4\mathrm{Re}L \\
& +\alpha_5|L|^2(\sigma^2+\vec{\pi}^2)
\end{aligned}
$$

The α_5 term couples the chiral fields and the loop field

- jump in $|L|$ can turn off the chiral symmetry breaking

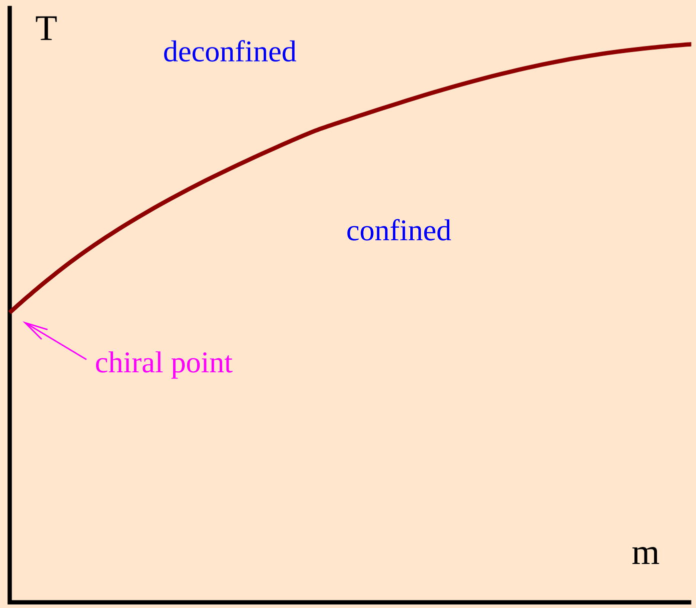
Two unknown functions: $K_c(\beta)$ and $\beta_c(K)$

- for simplicity ignore K_c dependence on N_t before deconfinement

Increasing quark mass increases deconfinement temperature

Continuum physics independent of twisting

- an "inverted umbrella" or "conical" structure in m, μ, T space
- symmetric under rotations about T axis
  - possibly a crossover



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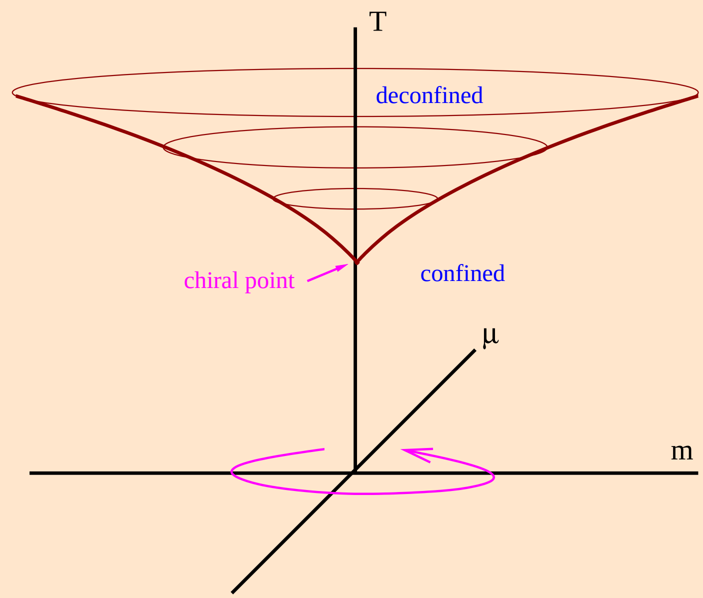
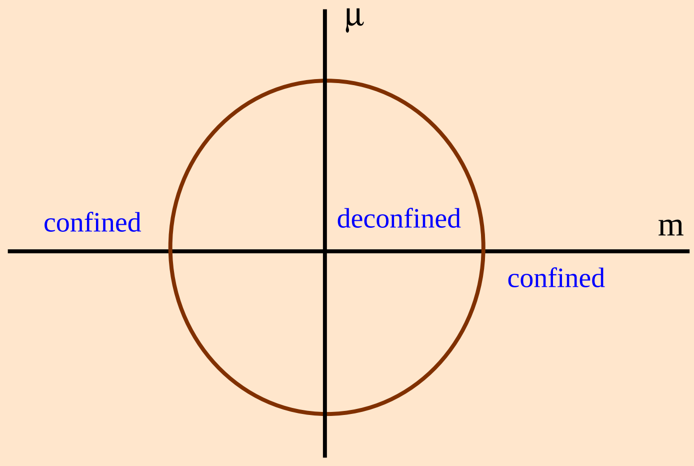


Consider an intermediate temperature where

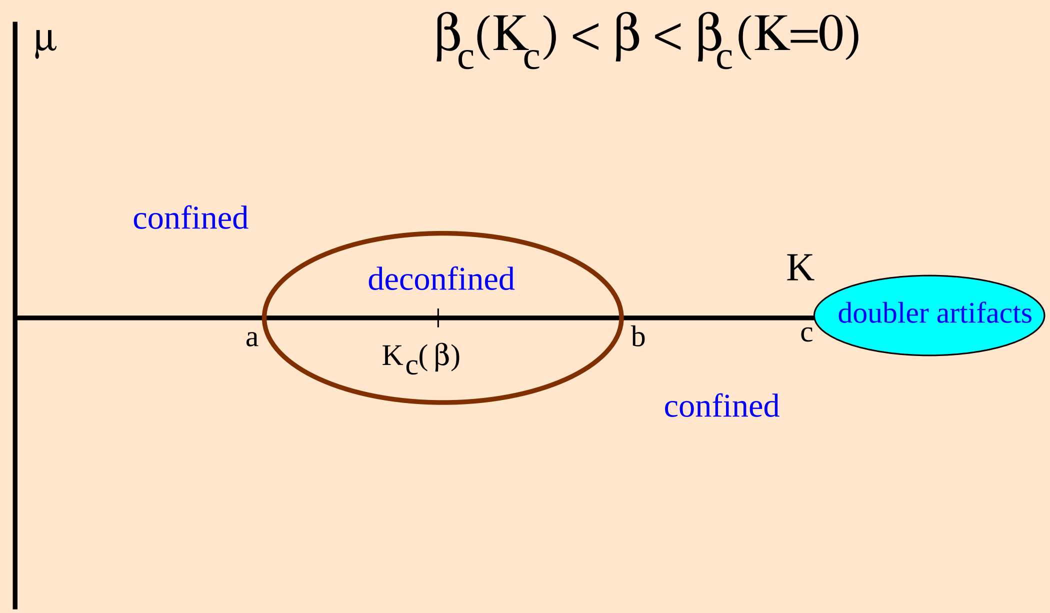
- massless theory deconfined
- massive one still confined
- horizontal slice through the "cone"

Continuum physics independent of twisting

- $m \leftrightarrow -m$ connected



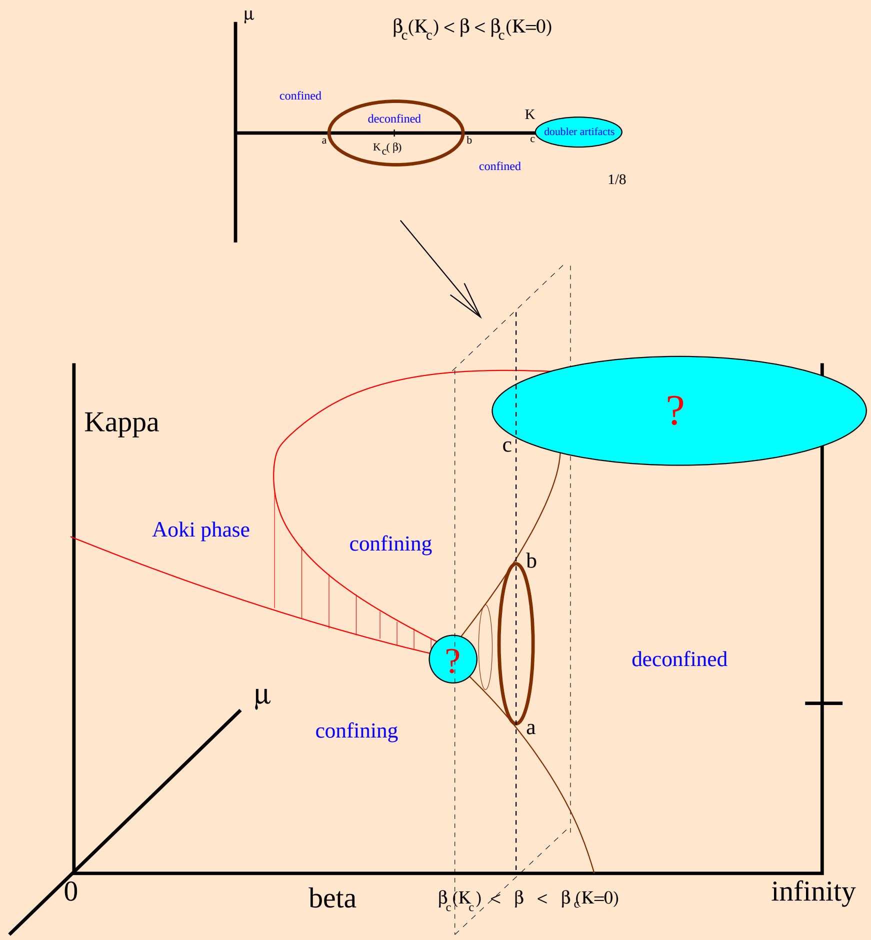
Lattice distorts the equivalent physics circle



- Aoki phase washed out in deconfined region
  - $V(\sigma, \pi)$ has a simple and unique minimum

Deconfinement becomes a "conical" structure in β, K, μ space

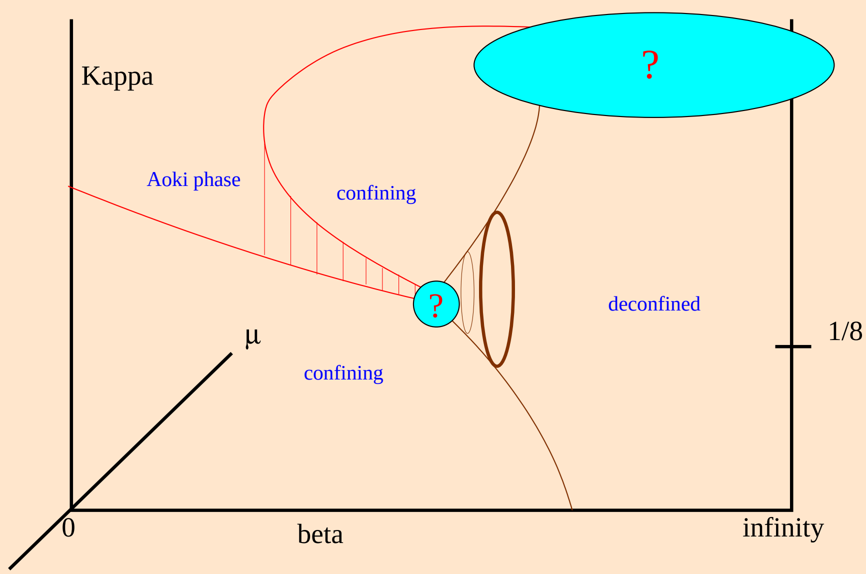
- at fixed β, deconfinement at constant $\sqrt{c\mu^2 + (1/K - 1/K_c)^2}$



Conclusions

Complex phase structure tied to lattice artifacts

- should study $K > K_c$ in more detail
  - second deconfinement line expected
- twisted mass connects phases
  - at fixed β: elliptical structure in μ, K plane



Questions

$N_f = 3$?

- parity broken phase becomes physical: $m < 0 \Rightarrow \Theta = \pi$
- possible twists in λ_3 and λ_8 directions
  - i.e: $m_u \sim e^{2\pi i/3}$, $m_d \sim e^{-2\pi i/3}$, $m_s \sim 1$

$N_f = 1$?

- no chiral symmetry
  - quark condensate loses meaning
- critical kappa and zero quark mass decoupled
- parity broken phase at sufficiently negative mass
- nothing to twist?