Thermodynamics of the $O(N)$ sigma model in the $1/N$ expansion

J. O. Andersen[1] and T. Brauner[2]

[1]Department of Physics
Norwegian University of Science and Technology
Trondheim

[2]Institute for Theoretical Physics
Johann Wolfgang Goethe University
Frankfurt am Main

QCD in Extreme Conditions 2007

Outline

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Introduction

- The general framework: study of the QCD phase diagram.
- Low temperature and chemical potential: only LE DOFs excited.
- Simplest possible approximation: include only the light pseudoscalar mesons, i.e., the Goldstone bosons of the spontaneously broken $SU_L(N_f) \times SU_R(N_f)$ chiral symmetry.
- In particular for $N_f = 2$ make use of the isomorphism $SU(2) \times SU(2) \simeq SO(4)$ and study the $O(4)$ model.
- A special case of the general $O(N)$ model; applications besides low-energy QCD also e.g. in the electroweak interactions (Higgs mechanism), or in condensed matter physics (spin models).
- At large N, a $1/N$ expansion possible as a substitute for the conventional perturbation theory.
- Nonperturbative technique which preserves, even at the leading order, "much more of the nonlinear structure of the exact theory than does ordinary lowest-order perturbation theory".

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- $O(N)$ model studied in the 1PI-1/N expansion since long time ago.

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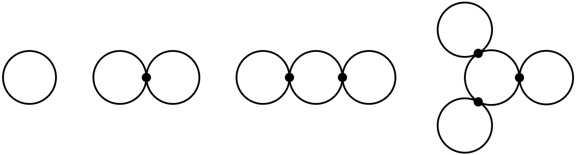
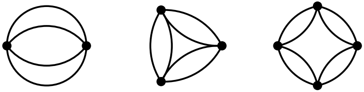
$1/N$ expansion continued

- Generalize the $O(4)$ model to arbitrary N by a suitable redefinition of the couplings.
- Perform the (nonperturbative) resummation to a given order in $1/N$ and set $N = 4$ afterwards. Hope that 4 is large enough.
- Some leading-order, $O(N)$, contributions to the pressure:



- Some next-to-leading-order, $O(1)$, contributions to the pressure:



Auxiliary field technique

- A real scalar N-vector, $\phi = (\pi_1, \pi_2, \ldots, \pi_{N-1}, \sigma)$.
- Include isospin chemical potential for the pair $(\pi_1, \pi_2) \sim (\pi^+, \pi^-)$.
- Include explicit chiral symmetry breaking (quark mass) in terms of a source for $\phi_N \sim \sigma$.
- Define all the couplings so that the action scales naturally with N.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_i)^2 + \frac{\lambda_b}{8N}(\phi_i\phi_i - Nf_{\pi,b}^2)^2$$

- Introduce a new auxiliary field α and add pure Gaussian integral over α.

$$\Delta\mathcal{L} = \frac{N}{2\lambda_b}\left[\alpha - \frac{i\lambda_b}{2N}(\phi_i\phi_i - Nf_{\pi,b}^2)\right]^2$$

- Integrate over the non-condensing fields $\pi_3, \ldots, \pi_{N-1}$.

$$\mathcal{L} = \frac{1}{2}(N-3)\,Tr\,log\left(-\partial^2 - i\alpha\right) + \frac{1}{2}\sum_{i=1,2,N}\left[(\partial_\mu\phi_i)^2 - i\alpha\phi_i^2\right] +$$

$$+\frac{i}{2}Nf_{\pi,b}^2\alpha + \frac{N}{2\lambda_b}\alpha^2 - \sqrt{N}H\phi_N - i\mu\delta_{\mu 0}(\phi_1\partial_\mu\phi_2 - \phi_2\partial_\mu\phi_1) - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2)$$

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- Introduce the condensates $\bar{\rho} = \frac{1}{\sqrt{N}}\langle\phi_1\rangle$, $\bar{\sigma} = \frac{1}{\sqrt{N}}\langle\phi_N\rangle$, $m^2 = -i\langle\alpha\rangle$, and shift the fields.

$$\phi_1 = \sqrt{N}\bar{\rho} + \tilde{\pi}_1, \quad \phi_N = \sqrt{N}\bar{\sigma} + \tilde{\sigma}, \quad \alpha = im^2 + \frac{\tilde{\alpha}}{\sqrt{N}}$$

- Expand the action (thermodynamic potential) in powers of $1/N$.
- At NLO, the path integral over the field fluctuations is Gaussian and hence exactly calculable.

$$\frac{S_{eff}^{NLO}}{\beta V} = \frac{1}{2}(N-3)\sum\!\!\!\!\!\!\int_P \log(P^2+m^2) - NH\bar{\sigma} - \frac{Nm^4}{2\lambda_b} + $$
$$+\frac{1}{2}Nm^2(\bar{\sigma}^2 - f_{\pi,b}^2) + \frac{1}{2}(m^2-\mu^2)N\bar{\rho}^2 + \frac{1}{2}\sum\!\!\!\!\!\!\int_P \chi^T\mathcal{D}^{-1}\chi^*$$

$$\mathcal{D}^{-1} = \begin{pmatrix} \frac{1}{2}\Pi(P,m) + \frac{1}{\lambda_b} & -i\bar{\sigma} & -i\bar{\rho} & 0 \\ -i\bar{\sigma} & P^2+m^2 & 0 & 0 \\ -i\bar{\rho} & 0 & P^2+m^2-\mu^2 & -2\mu\omega \\ 0 & 0 & +2\mu\omega & P^2+m^2-\mu^2 \end{pmatrix}, \quad \chi = \begin{pmatrix} \tilde{\alpha} \\ \tilde{\sigma} \\ \tilde{\pi}_1 \\ \pi_2 \end{pmatrix}$$

$$\Pi(P,m) = \sum\!\!\!\!\!\!\int_Q \frac{1}{Q^2+m^2}\frac{1}{(P+Q)^2+m^2}$$

Inverse propagators

For simplicity, only consider the case $\mu = 0 \Rightarrow$ mixing only in the $(\tilde{\alpha}, \tilde{\sigma})$ block, separate propagation of degenerate pions.

- Inverse propagator of $\tilde{\alpha}$.

$$D_{\tilde{\alpha}}^{-1}(P,m) = \frac{1}{2}\Pi(P,m) + \frac{1}{\lambda_b} + \frac{\bar{\sigma}^2}{P^2+m^2} \xrightarrow{\lambda_b \to 0} \frac{1}{\lambda_b}$$

- Inverse propagator of $\tilde{\sigma}$.

$$D_{\tilde{\sigma}}^{-1}(P,m) = P^2 + m^2 + \frac{\bar{\sigma}^2}{\frac{1}{2}\Pi(P,m) + \frac{1}{\lambda_b}} \xrightarrow{\lambda_b \to 0} P^2 + m^2 + \lambda_b \bar{\sigma}^2$$

- Both $\tilde{\sigma}$ and $\tilde{\alpha}$ describe the propagation of the same mode.

$$D_{\tilde{\alpha}}^{-1} = D_{\tilde{\sigma}}^{-1} \frac{\frac{1}{2}\Pi + \frac{1}{\lambda_b}}{P^2+m^2}$$

Those are all results of the resummation of the NLO in the $1/N$ expansion. In the $\lambda_b \to 0$ limit, we recover perturbative results.

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LO effective potential

- At LO, the only quantum contribution to the thermodynamic potential comes from the $N-3$ "pions" $\pi_3, \cdots, \pi_{N-1}$.

$$V_{LO} = \frac{m^2}{2}(f_{\pi,b}^2 - \bar{\sigma}^2 - \bar{\rho}^2) + \frac{m^4}{2\lambda_b} + \frac{1}{2}\mu^2\bar{\rho}^2 + H\bar{\sigma} - \frac{1}{2}\sum\!\!\!\!\!\!\!\int_P \log(P^2 + m^2)$$
$$= \frac{m^2}{2}(f_\pi^2 - \bar{\sigma}^2 - \bar{\rho}^2) + \frac{T^4}{64\pi^2}J_0(\beta m) + \frac{m^4}{64\pi^2}\left(\frac{32\pi^2}{\lambda} + \log\frac{Q^2}{m^2} + \frac{1}{2}\right) + \frac{1}{2}\mu^2\bar{\rho}^2 + H\bar{\sigma}$$

where $J_0(\beta m) = \frac{32}{3T^4}\int_0^\infty dp\,\frac{p^4}{\omega_p}n(\omega_p)$ and $\omega_p = \sqrt{p^2 + m^2}$

- LO renormalization by redefinition of the parameters.

$$f_\pi^2 = f_{\pi,b}^2 - \frac{\Lambda^2}{16\pi^2}, \qquad \frac{32\pi^2}{\lambda} = \frac{32\pi^2}{\lambda_b} + \log\frac{\Lambda^2}{Q^2}$$

- LO β-function: $\beta(\lambda) = \frac{\lambda^2}{16\pi^2}$.

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LO gap equations

- Solve the LO gap equations.

$$\boxed{G = 16\pi^2 f_\pi^2, \quad H = m^2\bar{\sigma}, \quad (m^2 - \mu^2)\bar{\rho} = 0}$$

$$G = 16\pi^2(\bar{\sigma}^2 + \bar{\rho}^2) + T^2 J_1(\beta m) - m^2 \log\frac{Q^2}{m^2} - \frac{32\pi^2 m^2}{\lambda}$$

$$J_1(\beta m) = \frac{8}{T^2}\int_0^\infty dp\, \frac{p^2}{\omega_p} n(\omega_p)$$

- Chiral limit: $H = 0$.
  - Chiral-symmetry-breaking phase, just the $\bar{\sigma}$ condensate.
  - Pion-condensation phase, just the $\bar{\rho}$ condensate.
  - Normal phase
- Physical point: $H \neq 0$.
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Chiral limit

The chiral-symmetry-breaking phase is never energetically favorable, except at $\mu = 0$ where it is degenerate with pion condensation.

- Normal phase: $\tilde{\sigma}$ is degenerate with the pions. Their common mass is given implicitly by

$$16\pi^2 f_\pi^2 = T^2 J_1(\beta m) - m^2 \log \frac{Q^2}{m^2} - \frac{32\pi^2 m^2}{\lambda}$$

- Nonzero m^2 only for $T \geq T_c^{(m)} = \sqrt{12} f_\pi$.
- Above $T_c^{(m)}$, m^2 increases with T until it reaches the value $m^2 = \mu^2$ at the critical temperature for pion condensation.

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$$16\pi^2 f_\pi^2 = T^2 J_1(\beta m) - m^2 \log\frac{Q^2}{m^2} - \frac{32\pi^2 m^2}{\lambda}$$

- Nonzero m^2 only for $T \geq T_c^{(m)} = \sqrt{12} f_\pi$.
- Above $T_c^{(m)}$, m^2 increases with T until it reaches the value $m^2 = \mu^2$ at the critical temperature for pion condensation.

- Pion-condensation phase: $m^2 = \mu^2$, pion condensate $\bar{\rho}$ is given explicitly by

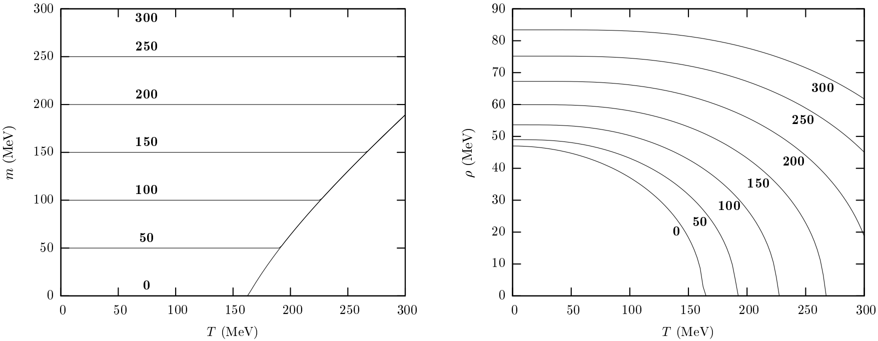
$$16\pi^2 f_\pi^2 = 16\pi^2\bar{\rho}^2 + T^2 J_1(\beta\mu) - \mu^2 \log\frac{Q^2}{\mu^2} - \frac{32\pi^2\mu^2}{\lambda}$$

- Pion condensate grows with chemical potential and decreases with temperature.
- Critical temperature for pion condensation increases with chemical potential.

Numerical results

Analytical expression for the chiral condensate $\bar{\sigma}$ (or $\bar{\rho}$) at $\mu = 0$:

$$\bar{\sigma}^2 = f_\pi^2 - \frac{T^2}{12}$$



Physical point

- $\bar{\sigma}$ (as well as m^2) is always nonzero $\Rightarrow$ it is not an order parameter for spontaneous symmetry breaking.
- In the normal phase, m^2 solves the implicit equation

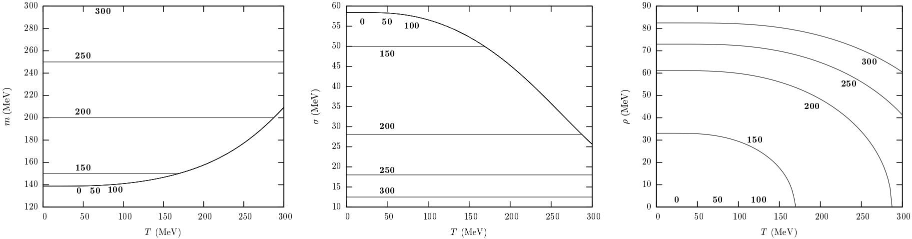
$$16\pi^2 f_\pi^2 = 16\pi^2 \left(\frac{H}{m^2}\right)^2 + T^2 J_1(\beta m) - m^2 \log \frac{Q^2}{m^2} - \frac{32\pi^2 m^2}{\lambda}$$

- In the pion-condensation phase, $\bar{\rho}$ solves the implicit equation

$$16\pi^2 f_\pi^2 = 16\pi^2 \left[\left(\frac{H}{\mu^2}\right)^2 + \bar{\rho}^2\right] + T^2 J_1(\beta\mu) - \mu^2 \log \frac{Q^2}{\mu^2} - \frac{32\pi^2 \mu^2}{\lambda}$$

- At critical chemical potential for pion condensation, $m^2 = \mu^2$ and $\bar{\rho} = 0$.
- At fixed temperature, pion condensation occurs at chemical potential equal to the renormalized pion mass at $\mu = 0$.

Numerical results



- Parameter set:

$$\lambda(Q = 100\,\text{MeV}) = 30, \quad f_\pi = 47\,\text{MeV}, \quad H = (104\,\text{MeV})^3$$

- Chiral-symmetry-breaking parameter H chosen in order to reproduce the (LO) renormalized pion mass $m_\pi = 138\,\text{MeV}$.
  Warringa, hep-ph/0604105 (PhD thesis)

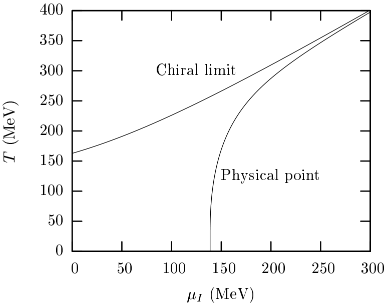
Phase diagram

The phase boundary is given by the solution of the implicit equation

$$16\pi^2 f_\pi^2 = \frac{16\pi^2 H^2}{\mu^4} + T^2 J_1(\beta\mu) - \mu^2 \log\frac{Q^2}{\mu^2} - \frac{32\pi^2\mu^2}{\lambda}$$

In the weak-coupling limit, we get an analytic expression for the critical temperature.

$$T_c^2 = 12\left(f_\pi^2 + \frac{2\mu^2}{\lambda}\right)$$



At large chemical potential, the effects of explicit chiral symmetry breaking are negligible.

Outline

NLO effective potential

- NLO expression for the effective potential (pressure):

$$V_{NLO} = \sum\!\!\!\!\!\!\int_P \log(P^2+m^2) - \frac{1}{2}\sum\!\!\!\!\!\!\int_P \log[(P^2+M^2)^2+(2\mu\omega)^2] - \frac{1}{2}\sum\!\!\!\!\!\!\int_P \log J(P,m)$$

$$\frac{1}{32\pi^2}J(P,m) = \frac{1}{2}\Pi(P,m) + \frac{1}{\lambda_b} + \frac{\bar{\sigma}^2}{P^2+m^2} + \frac{\bar{\rho}^2(P^2+M^2)}{(P^2+M^2)^2+(2\mu\omega)^2}$$

- The correction due to the finite chemical potential of the free pion gas is “analytically” calculable.

$$\frac{T^4}{64\pi^2}\left[K_0^+(\beta m,\beta\mu)+K_0^-(\beta m,\beta\mu)-2J_0(\beta m)\right], \quad K_0^\pm(\beta m,\beta\mu) = \frac{32}{3T^4}\int_0^\infty dp\,\frac{p^4}{\omega_p}n(\omega_p\pm\mu)$$

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Extraction of divergences

- Expand the log in V_{NLO} in powers of momentum.

$$\log J = \log\left(\alpha + \log\frac{\Lambda^2}{P^2}\right) + \frac{2}{P^2}\left(m^2 + \frac{G-2m^2}{\alpha+\log\frac{\Lambda^2}{P^2}}\right) - \frac{2}{P^4}\left[2m^4 + \frac{3m^2(G-\frac{3}{2}m^2)}{\alpha+\log\frac{\Lambda^2}{P^2}} + \frac{(G-2m^2)^2}{\left(\alpha+\log\frac{\Lambda^2}{P^2}\right)^2}\right] +$$

+UV-finite terms

$$G = 16\pi^2(\bar{\sigma}^2+\bar{\rho}^2) + T^2 J_1(\beta m) - m^2\log\frac{Q^2}{m^2} - \frac{32\pi^2 m^2}{\lambda}, \quad \alpha = 1 + \frac{32\pi^2}{\lambda_b}$$

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- Quartic UV-divergence, independent of $m, \bar{\sigma}, \bar{\rho}$.
- Quadratic UV-divergence.
- Logarithmic UV-divergence. Also IR divergent: This IR divergence is absent in the full expression for $J(P, m)$, but for sake of numerical computation must be conveniently regulated.

NLO renormalization

- Divergent part of the effective potential after integration using momentum cutoff Λ:

$$V_{NLO}^{div} = \frac{1}{16\pi^2}\left[(G-2m^2)\Lambda^2 e^{\alpha}\mathrm{li}\left(\frac{1}{e^{\alpha}}\right) - m^2\Lambda^2 + 2m^4\log\frac{\Lambda^2}{m^2} - 3m^2(G-\frac{3}{2}m^2)\log\alpha + \frac{(G-2m^2)^2}{\alpha}\right]$$

- The divergences only T-independent after using the LO gap equation $G = 16\pi^2 f_\pi^2$! $\Rightarrow$ it was reported that the effective potential is only renormalizable at the minimum.

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- The NLO renormalization prescription is then

$$f_\pi^2 = f_{\pi,b}^2 + \frac{\Lambda^2}{16\pi^2}\left\{1+\frac{2}{N}\left[1+2e^{\alpha}\mathrm{li}\left(\frac{1}{e^{\alpha}}\right)\right]\right\}$$

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- Only the LO gap equation for $\langle\alpha\rangle = im^2$ was used!
- When this is treated as merely a constraint to eliminate m^2 in favor of $\bar{\sigma}, \bar{\rho}$, we get an effective potential of $\bar{\sigma}, \bar{\rho}$ solely, which is renormalizable for any value of the classical fields.

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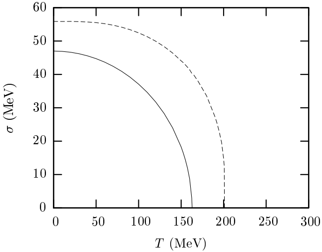
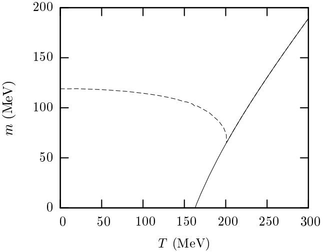
Chiral limit

- The chiral condensate increases significantly. Also, the critical temperature for symmetry restoration increases by about 25%.

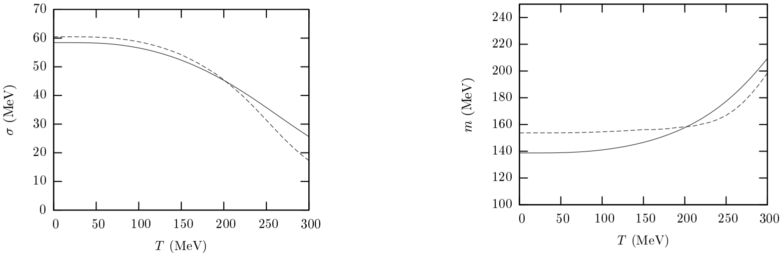




- The parameter m acquires nonzero value even in the symmetry-broken phase. This is no contradiction with the Goldstone theorem, since at NLO m has no longer the interpretation as the pion mass.

Physical point: temperature axis

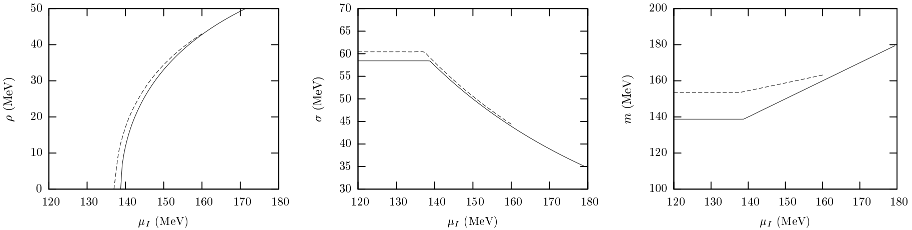
- Both the chiral condensate and the mass parameter change rather decently $\Rightarrow$ $1/N$ is a reasonable expansion parameter even for $N = 4$.



- Chiral condensate at $T = 0$ slightly increases $\Rightarrow$ the physical pion mass slightly reduces.

Physical point: chemical potential axis

- The change in the critical chemical potential for pion condensation is in accord with the correction of the physical pion mass.

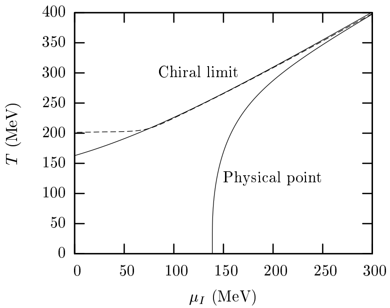


- Minimum of the NLO effective potential was only found for not-too-large chemical potential. For larger μ the effective potential seems to acquire nonzero imaginary part.

NLO phase diagram

- So far, only the NLO correction to the phase diagram in the chiral limit.
- The NLO critical temperature stays almost constant up to chemical potential about 75 MeV, where it merges with the LO curve.

Preliminary!



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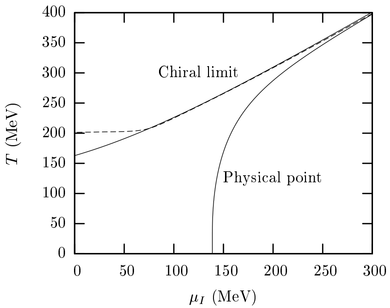

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Summary

- To determine the NLO contribution to the pressure, it is only necessary to solve the gap equation at the leading order. We have extended previous calculations to nonzero chemical potential.
- We have shown that the NLO effective action in the $1/N$ expansion can be consistently renormalized for all values of the classical field.
- We have renormalized and solved the NLO gap equation, and determined the NLO phase diagram (so far, in the chiral limit).
- The NLO corrections to the pressure are decent, less than $1/N$ $\Rightarrow$ $1/N$ is a good expansion parameter even for $N = 4$.
- However, the chiral and pion condensates may display rather large deviations from the LO values, especially in the chiral limit.

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