Magnetic field calculation and design optimization of the wigglers for CLIC dumping ring

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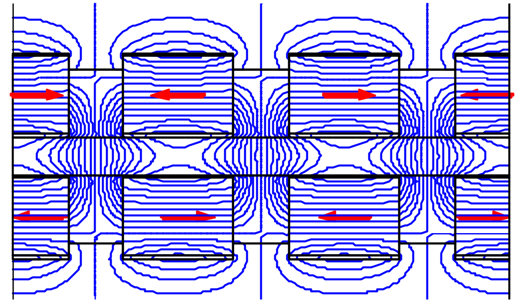
During wiggler optimization we took into account the following considerations:

- ***Ferromagnetic pole saturation should be absent.***
- ***A integral of the square field strength within a half wiggler period should be maximum.***
- ***The value of wiggler period must be minimized.***
- ***The wiggler design should be simple in its construction, adjustment and maintenance.***
- ***The cost efficiency factor is taken into account in the selection of the wiggler design because of their great number in the dumping ring.***

The magnetic potential and field strength in working aperture of wigglers

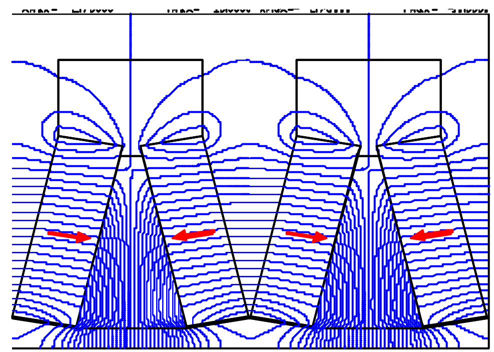


Hybrid wiggler with rectangular poles

$$y\left(x,y\right)=\sum_{m=1}^{\infty}B_m\frac{l}{2pm}\sin\frac{2pmx}{l}\,sh\,\frac{2pmy}{l} \qquad \textbf{(1)}$$

$$B_y\left(x,y\right)=\sum_{m=1}^{\infty}B_m\sin\frac{2pmx}{l}\,ch\,\frac{2pmy}{l} \qquad \textbf{(2)}$$

$$B_x\left(x,y\right)=\sum_{m=1}^{\infty}B_m\cos\frac{2pmx}{l}\,sh\,\frac{2pmy}{l} \qquad \textbf{(3)}$$



Hybrid wiggler with wedge-shaped pole

Expend into Fourier- series we get:

$$y\left(x,h\right)=\frac{4P\,l}{2p^{2}l_{m}}\left(\sin\frac{2pl_{m}}{l}\sin\frac{2px}{l}+\frac{1}{3^{2}}\sin\frac{6pl_{m}}{l}\sin\frac{6px}{l}+\frac{1}{5^{2}}\sin\frac{10pl_{m}}{l}\sin\frac{10px}{l}+\ldots.\right) \qquad (4)$$

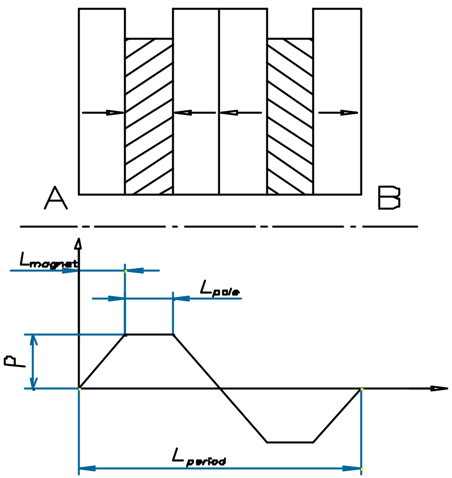
If we choose $l_p=l_m=?/6$ the potential at the pole surface will be equal to $P=H_m?/6$ and the magnetic potential distribution is practically determined by the first harmonics since the third harmonics amplitude at the selected parameters turns to be equal to zero.

$$y\left(x,h\right)=\frac{2H_{m}\,l}{p^{2}}\sin\frac{p}{3}\sin\frac{2px}{l} \qquad (5)$$

From equations 1 and 5 one determines the first harmonics amplitude of the magnetic potential.

$$B_{1}=\frac{4H_{m}\sin\dfrac{p}{3}}{p\;sh\,\dfrac{2ph}{l}}\approx\frac{1.1H_{m}}{sh\,\dfrac{2ph}{l}} \qquad (6)$$

The magnetic potential distribution along the upper half of the wiggler



The field value at any point of the working aperture one can find out from equation 2 and 3

$$B_y = \frac{1.1H_m}{sh\frac{2ph}{l}} \sin\frac{2px}{l} ch\frac{2py}{l} \qquad \text{(7)}$$

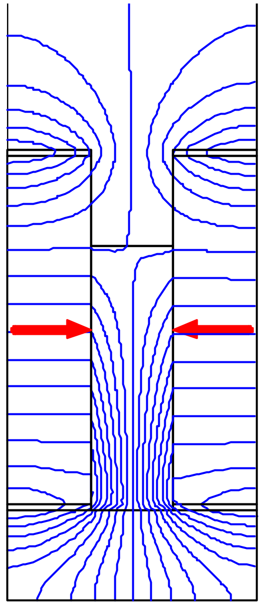
$$B_x = \frac{1.1H_m}{sh\frac{2ph}{l}} \cos\frac{2px}{l} sh\frac{2py}{l} \qquad \text{(8)}$$

Knowing the field value at any point of the working aperture, we can easily find out the value of the magnetic flux outgoing out of the ferromagnetic poles

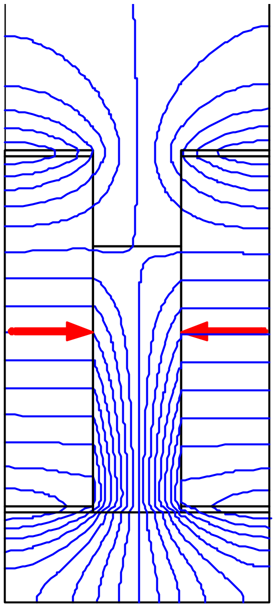
$$\Phi_y = \frac{1.1H_m}{sh\frac{2ph}{l}} \int_0^{\frac{l}{2}} \sin\frac{2px}{l} dx = \frac{B_1 l}{p} \qquad \text{(9)}$$

$$\Phi_x = 2B_1 \int_0^h sh\frac{2py}{l} dy = B_1\frac{l}{p}\left(ch\frac{2ph}{l} - 1\right) = \Phi_y\left(ch\frac{2ph}{l} - 1\right) \qquad \text{(10)}$$

$$\Phi_p = \Phi_y + \Phi_x = \Phi_y ch\frac{2ph}{l} \qquad \text{(11)}$$



The magnetic induction at the pole tip is equal to Bp=Fp/lp. For the ferromagnetic poles made of a soft pure iron, the maximum value of magnetic induction, as a rule, does not exceed 18-19 kGs and for poles made of the annealed vanadium permendur 21-22 kGs. An excess of the pole magnetic induction value over these values leads to overconsumption of magnets because of the fact that the differential permeability of the ferromagnetic material under these induction values tuns to be close to unity and further increase in the volume of permanent magnets does not make any practical influence on the wiggler field amplitude.



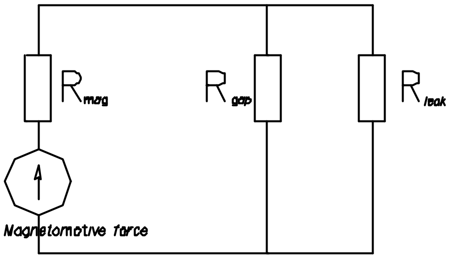
Let us evaluate the value of a magnetic flux F_{side} outgoing from the pole surfaces not adjoining to the working aperture. To this end, let us use the equation (10) choosing value much greater than the vertical gap. For estimation, it is enough to select h=?. The value of the flux outgoing from the unit length of the side surface will be equal to

$$\Phi'_x = \frac{1.1 H_m l}{p\, sh \dfrac{2p\, l}{l}} \left(ch \frac{2p\, l}{l} - 1 \right) \approx \frac{1.1 H_m l}{p} \qquad \textbf{(12)}$$

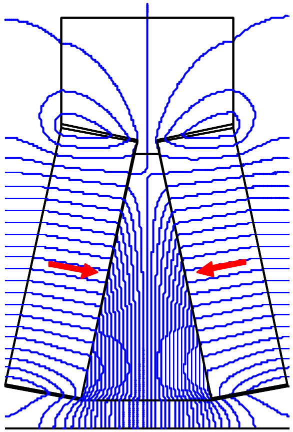
Because of the presence of the substantial value of F_{side}, the total scattering flux is several times larger than the value of the useful flux passing through the wiggler working aperture

From the wiggler magnetic circuit diagram one can see that the scattering losses lead to a substantial increase in the volume of magnets required for obtaining the given amplitude of the field. It is obvious that the choice of the wedge-shaped poles instead of the rectangular-shape poles results in a substantial decrease in the scattering flux since in this case the pole side surfaces are twice reduced and the upper surfaces just vanishes.

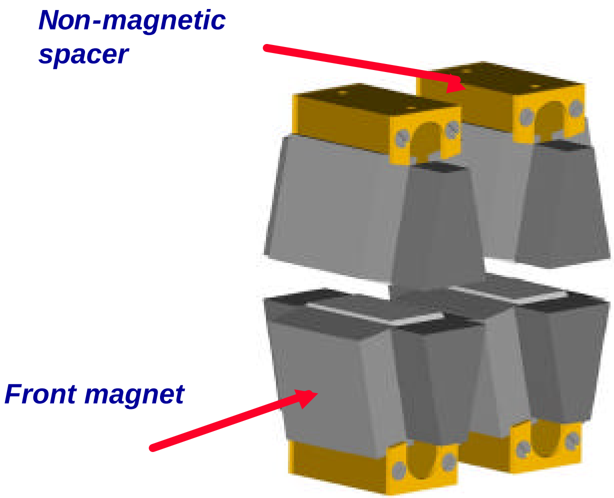
Following the above said considerations we choose the wedge-shaped pole design of the wiggler. From our viewpoint this design version has one additional and even more important advantage than a reduction of the magnet volume compared to the commonly accepted design of the hybrid wiggler, namely the absence in practice of the electromagnetic coupling between the adjacent poles. Therefore, if for the commonly accepted wiggler design the magnetization error in one magnet affects the pole amplitude value within the limit of six poles, in the given design, only within the limit of one pole. This feature simplifies drastically the adjustment procedure and enables one to reduce the requirements to the magnetization spread of the magnets with no deterioration of the wiggler magnetic parameters.

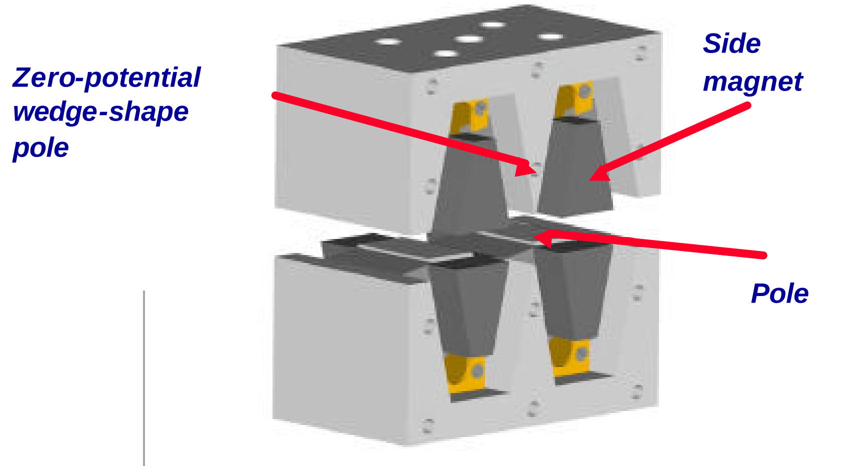


Magnetic circuit diagram

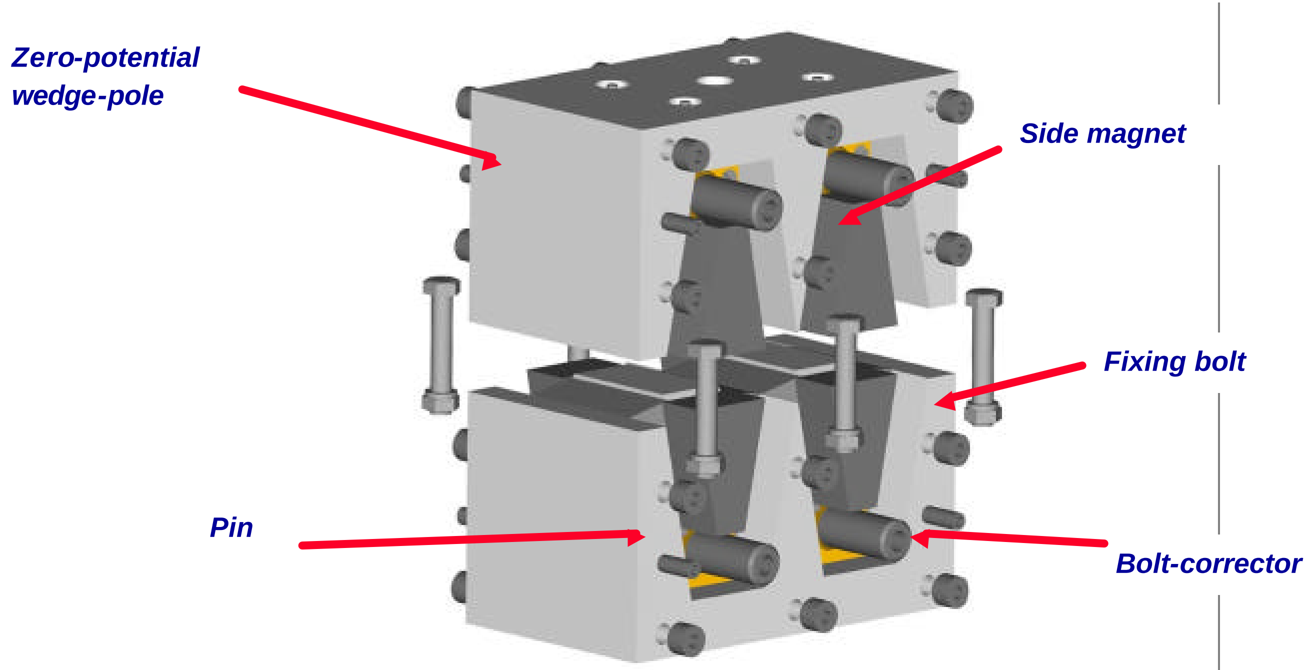


This approach was used in selection of the optimized design of a dumping wiggler for the PETRA-3 ring

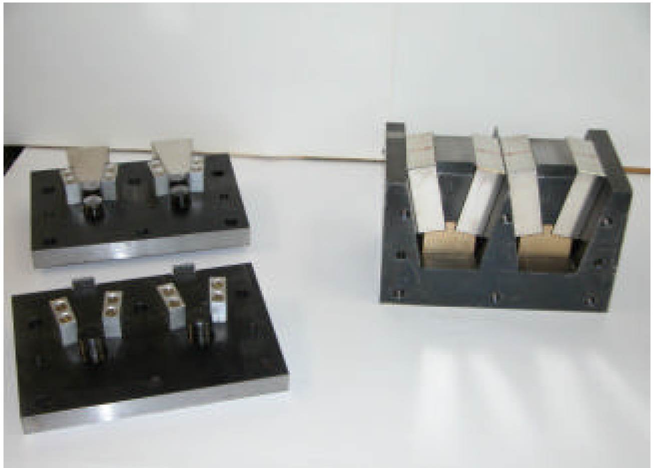


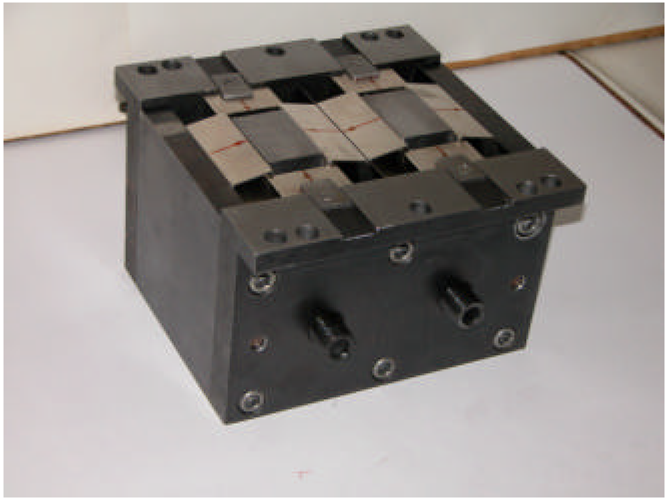


Schematic view of the magnetic field adjustment



Photos show the short (one period) model of a wiggler recently manufactured at BINP





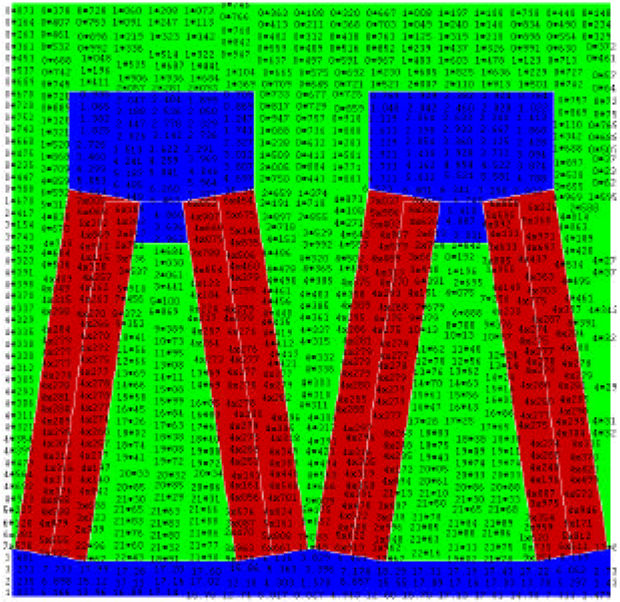
Wiggler parameters for PETRA-3

1. Period	*- 20 cm*
2. Vertical gap	*- 2.4 cm*
3. Field amplitude	*- 15.2 kOe*
4. Tuning range of the field amplitude	*- 0.5 kOe*
5. Wiggler length	*- 4 m*

Magnetic calculations of dumping wiggles for CLIC ring

All magnetic field calculations were made with 3-D Mermaid code, written by Andrey Dubrovin from BINP. The main feature of this code is very fast algorithm of the magnetic field calculations. For example a task with a mesh size about 5 million nodes is calculated for 1.5-2 hours with a precision 10-4.

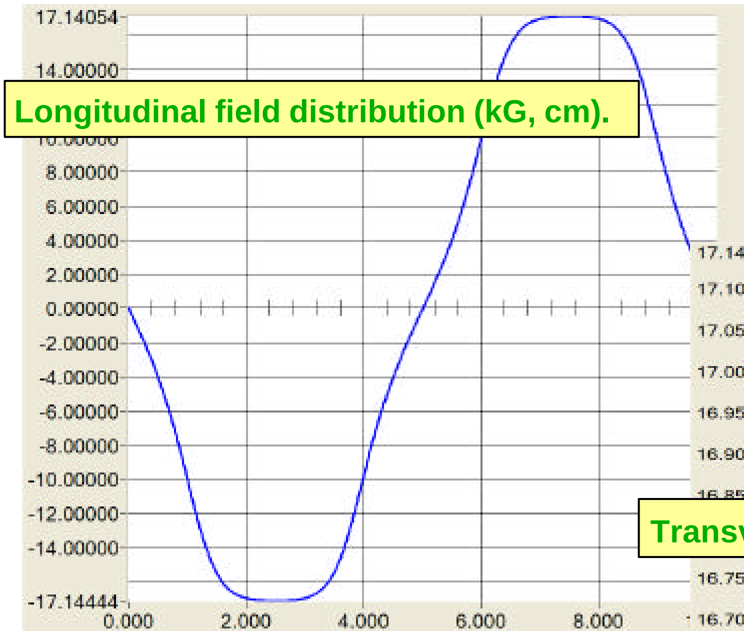
It is impossible to increase the wiggler field amplitude keeping just the same vertical gap since the magnetic induction in the pole tips achieves its maximum value determined earlier for permendure as 21-22 kGs. A decrease in the period with the same field amplitude turns to be possible only in the case of a substantial over-expenditure of magnets when H_m tends to H_{cb} value. So, for example, by a twice increase the volume of the magnetic material in an optimized design of the wiggler for CLIC we will decrease the wiggler period only by 10%. At H_m=H_{cb}, the wiggler period will attain its maximum.



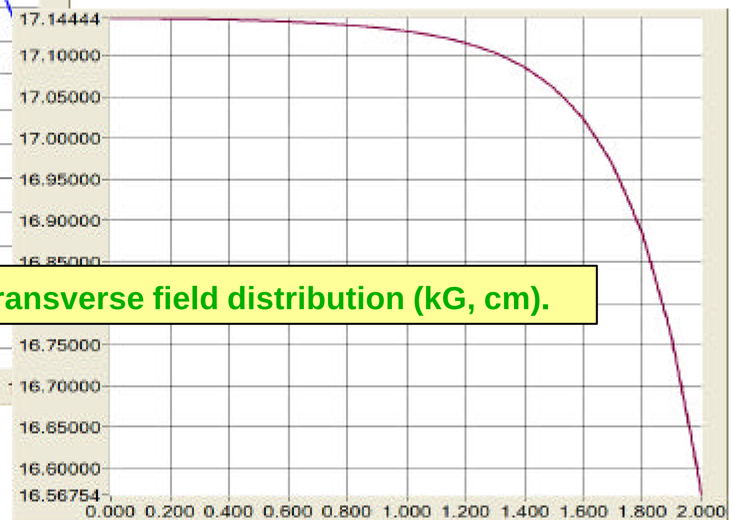
Magnetic induction inside iron pole

Transverse and longitudinal magnetic field distribution in CLIC wiggler

Bmax = 17.1 kG

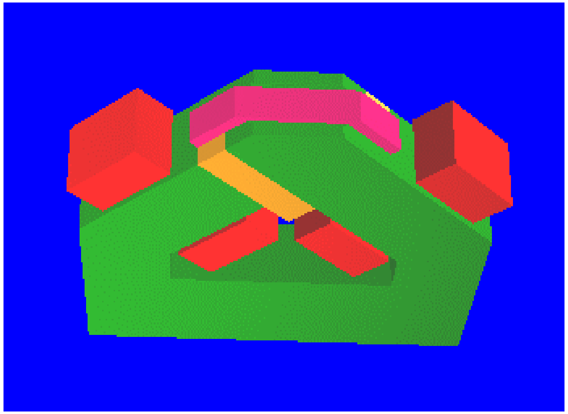


Longitudinal field distribution (kG, cm).

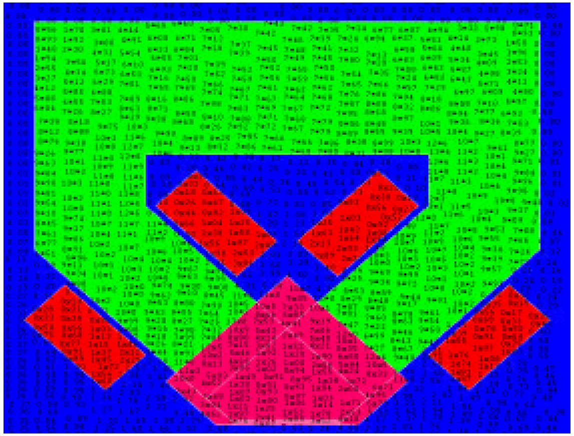


Transverse field distribution (kG, cm).

Equipotential bus wiggler

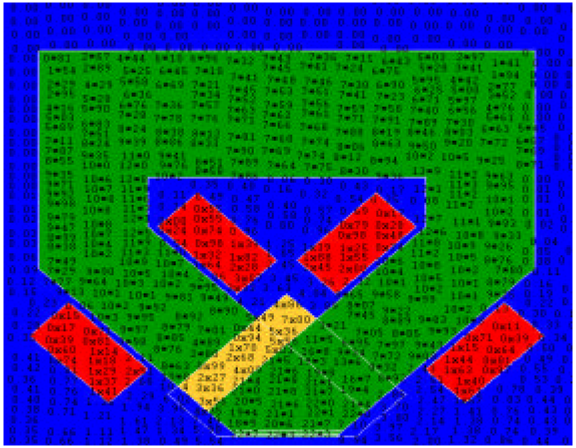


A substantial decrease in the period with the simultaneous decrease in the magnetic material volume can be achieved if we choose the wiggler design proposed by K.Halbach in 1985

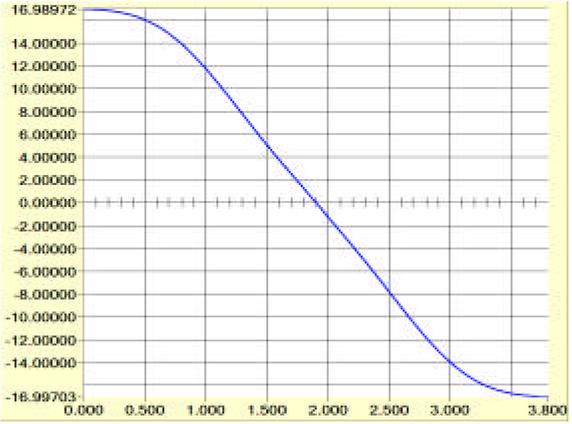


The magnets in such a wiggler design are used only for decreasing the scattering flows between the adjacent poles (magnetic insulators). In this design, the maximum amplitude of the field is achieved at Hm=Hcb

The minimum period at the same vertical gap and the field amplitude will always be lower than that in the hybrid wiggler due to H_m=H_{cb}. In this case, the minimum period is equal to ***7.6 cm*** *compared to* ***10 cm****.*



This wiggler design has one interesting special feature, namely it allows to change the wiggler field amplitude without the vertical gap variation. For this purpose it is enough to change only a current in the electromagnetic coils. Several years ago, such design of the undulator was realized at BINP for the Compact FEL in KAERI (South Korea) [2]. The field tuning range, at which lasering was occurred, is ***4.5-6.8 kGs***



Conclusions

? Two different types of wiggler designs have been suggested for CLIC dumping ring:

1. Wedge-shape hybrid wiggler

2. Equipotential bus wiggler

? Wedge-shape hybrid wiggler design may have two modifications. One of them when the synchrotron radiation from all poles summarize in one point and another version when the maximum synchrotron radiation from each pole is directed in lateral side and for this reason it doesn't accumulate in one point