Linear and non-linear modeling of the modified DAΦNE wiggler

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Contributors

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Ref: "The modified wiggler of the DAFNE Main Rings"
DAFNE Technical Note MM-34

"The Wiggler Transfer Matrix"
DAFNE Technical Note L-34

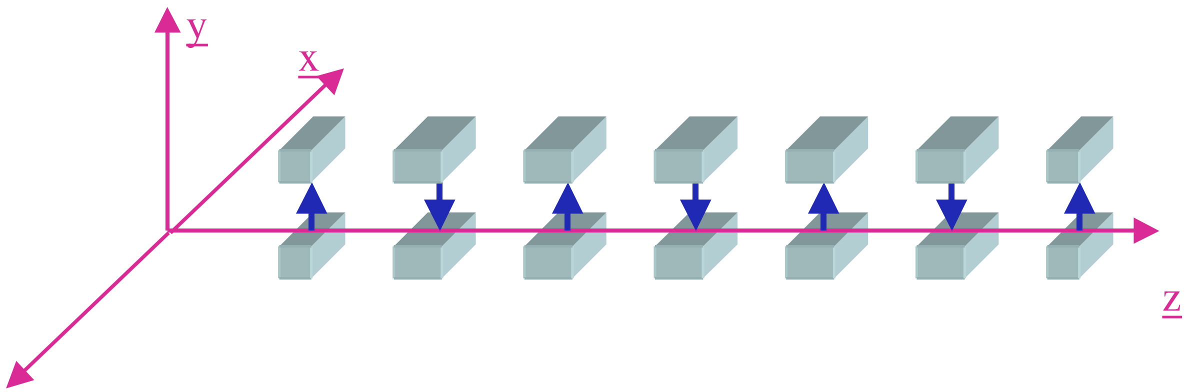
"Hard-Edge Model of the DAFNE Wigglers"
DAFNE Technical Note L-35

http://www.lnf.infn.it/acceleratori/dafne/technotes.html

Wiggler full map

Obtained by fitting the transverse measurements of the vertical field using a polynomial expansion of the field around the wiggler axis

$$B(z,x) = B(z,0) + \sum_{1}^{\infty} \frac{1}{n!} \left(\frac{\partial^n B}{\partial x^n}\right)_0 x^n$$



B = 1.8 T
E = .51 GeV

Due to the symmetry in the wiggler

$$B_x(x,0,z) = B_z(x,0,z) = 0$$

Relying on magnetic measurements

The transverse behaviour of the vertical field in the region of the beam trajectory (±12 mm) is accurately fitted by a 2th order polynomial interpolating the points measured at intervals of 1 cm in the range ± 30 mm

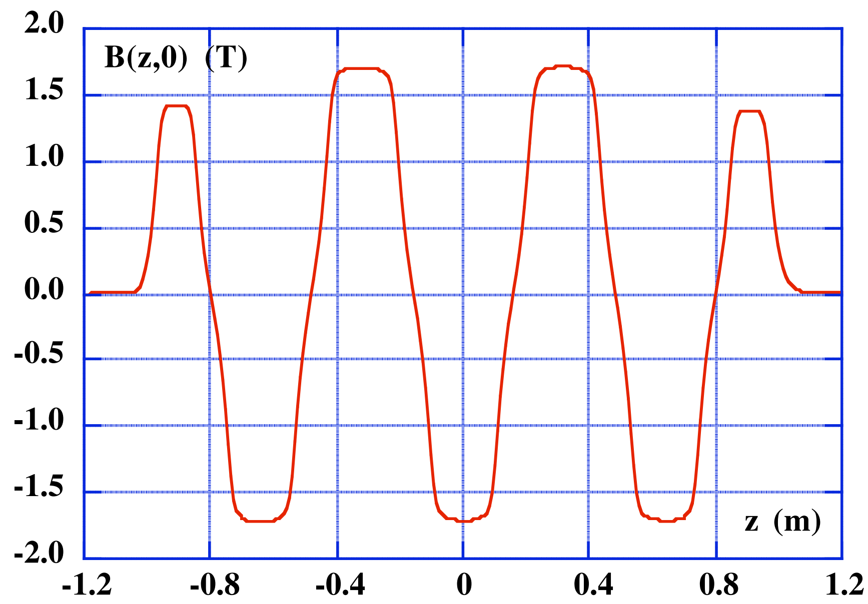
$$B_y(x,0,z) = b_0(z) + b_1(z)x + b_2(z)x^2$$

while the longitudinal dependence of the b_i coefficients requires a 4th order polynomial approximation

$$b_0(z) = b_{00i} + b_{01i}z + b_{02i}z^2 + b_{03i}z^3 + b_{04i}z^4$$

$$b_1(z) = b_{10i} + b_{11i}z + b_{12i}z^2 + b_{13i}z^3 + b_{14i}z^4$$

$$b_2(z) = b_{20i} + b_{21i}z + b_{22i}z^2 + b_{23i}z^3 + b_{24i}z^4$$



the horizontal and longitudinal field components for $y \neq 0$ can be obtained, in the linear approximation, from the curl theorem

$$B_x(x,y,z) \approx \frac{\partial B_x}{\partial y} y = \frac{\partial B_y}{\partial x} y = [b_1(z) + 2b_2(z)x]y$$

$$B_z(x,y,z) \approx \frac{\partial B_z}{\partial y} y = \frac{\partial B_y}{\partial z} y = [\frac{\partial b_0(z)}{\partial z} + \frac{\partial b_1(z)}{\partial z} x + \frac{\partial b_2(z)}{\partial z} x^2]y$$

Trajectory in the wiggler

The equations of motion for a test particle going through the wiggler can be written according to the expression for the Lorentz force :

$$\vec{F} = e\vec{v} \times \vec{B}$$

$$\frac{\partial^2 x}{\partial t^2} = a\left(\frac{\partial y}{\partial t}B_z - \frac{\partial z}{\partial t}B_y\right)$$

$$\frac{\partial^2 y}{\partial t^2} = a\left(\frac{\partial z}{\partial t}B_x - \frac{\partial x}{\partial t}B_z\right) \qquad a = \frac{e}{m_0\gamma}$$

$$\frac{\partial^2 z}{\partial t^2} = a\left(\frac{\partial x}{\partial t}B_y - \frac{\partial y}{\partial t}B_x\right)$$

- The equations of motion have been solved numerically using the previous field approximations.

- A vanishing field integral along the particle trajectory has been obtained by finding the correct current in the terminal pole windings, by means of a linear interpolation between two maps taken at different terminal currents.

- The wiggler transfer matrix **M** is obtained by particle tracking as a function of small variation in its initial conditions:

  $x, x', \Delta E/E, y, y'$

- The **M** matrix is asymmetric due to the asymmetry in the wiggler end poles

$$\mathbf{M}_{B(SXT)\to A} = \begin{pmatrix} 1.13395 & 2.28576 & 0.00031 & 0.00767 & -0.00044 \\ 0.07580 & 1.03514 & -0.00049 & 0.00360 & -0.00052 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.18381 & 1.17653 \\ 0 & 0 & 0 & -0.83261 & -0.11092 \end{pmatrix}$$

$$\mathbf{M}_{A\to B(SXT)} = \begin{pmatrix} 1.03489 & 2.28369 & -0.00149 & -0.00044 & 0.00030 \\ 0.07202 & 1.12119 & -0.00068 & -0.00035 & 0.00501 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.11151 & 1.17685 \\ 0 & 0 & 0 & -0.83276 & -0.17886 \end{pmatrix}$$

- The **M** matrix is not unitary and not symplectic!

Horizontal beam trajectories **x** in the wiggler

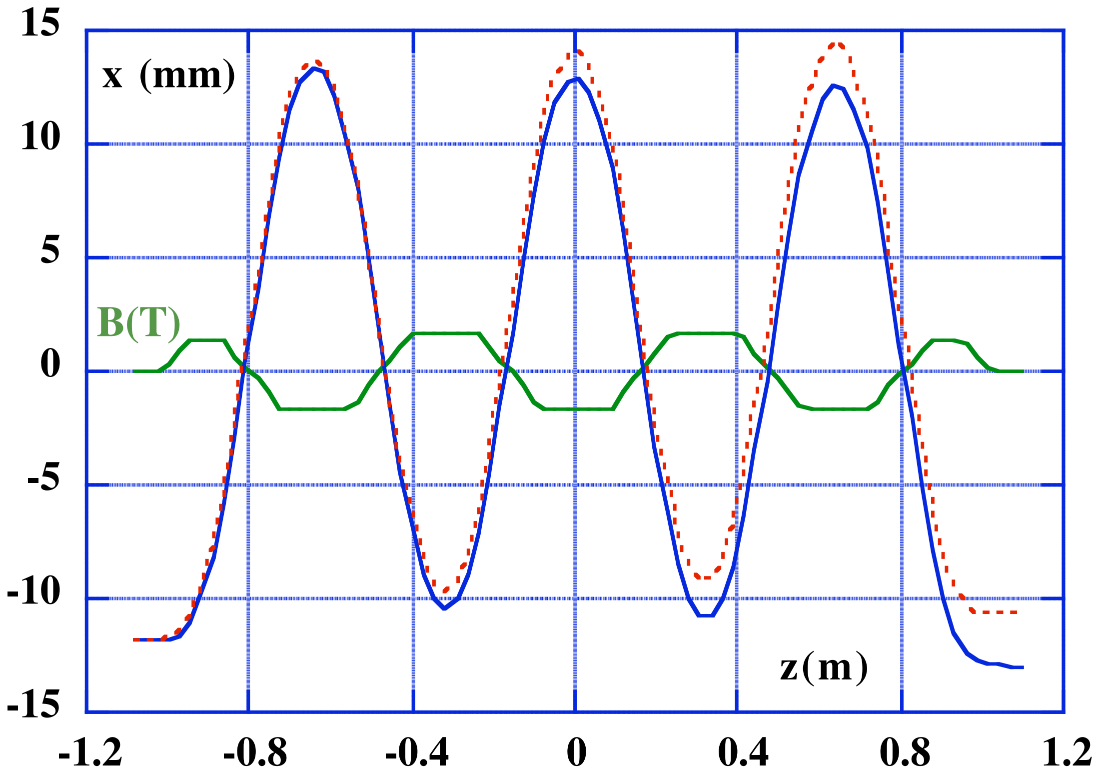
initial conditions:

$x_0 = -11.8$ mm

$x_0' = y_0 = y_0' = 0$

blue line **x** $B_{(SXT)}$ --> A

red line **x** A --> $B_{(SXT)}$



$l_{wig} - l_{str} = 6.62\ mm$

$|\boldsymbol{x}_f - \boldsymbol{x}_i| \sim 1.2\ mm$

Vertical beam trajectories **y** in the wiggler

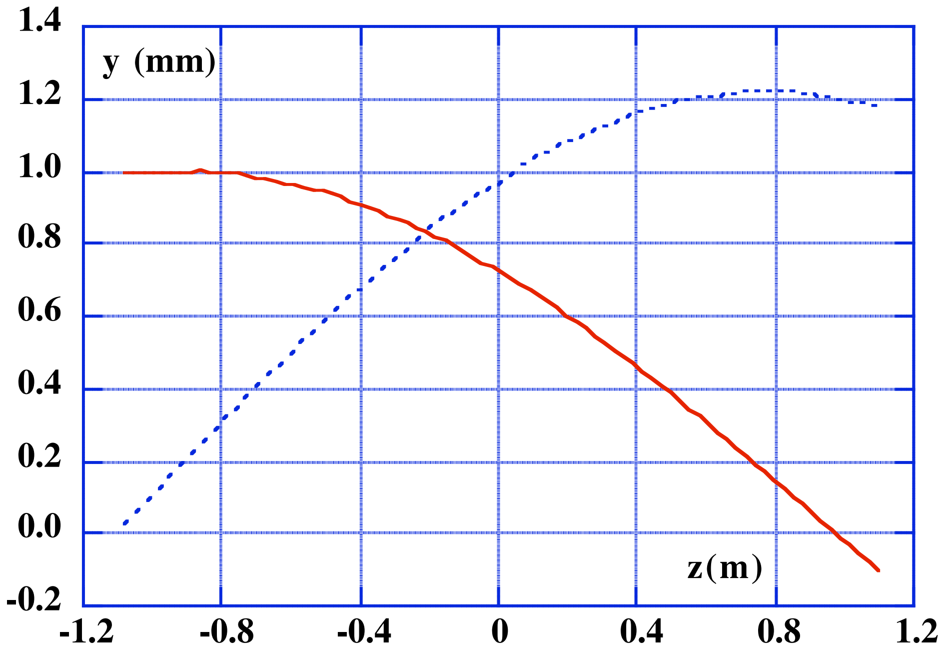
initial conditions:

$y_0 = 1$ mm red line

$x_0' = x_0 = y_0' = 0$

$y_0' = 1$ mrd blue line

$x_0' = x_0 = y_0 = 0$



the wiggler vertical focusing effect is evident

Multipole coefficients

- The effects of high order terms in the tranverse field expansion on the beam dynamics are described by the multipole coefficients

- The multipole coefficients can be obtained from the particle trajectory and the field derivatives on the wiggler axis

$$K_n = \frac{1}{B\rho}\int \frac{\partial^n B(z,x)}{\partial x^n} dz$$

- The general expression for the field derivatives along the wiggling trajectory is

$$\frac{\partial^k B(z,x)}{\partial x^k} = \left(\frac{\partial^k B}{\partial x^k}\right)_0 + \sum_{k+1}^{\infty} \frac{1}{(n-k)!}\left(\frac{\partial^n B}{\partial x^n}\right)_0 x^{(n-k)}$$

Multipole coefficient explicit expression

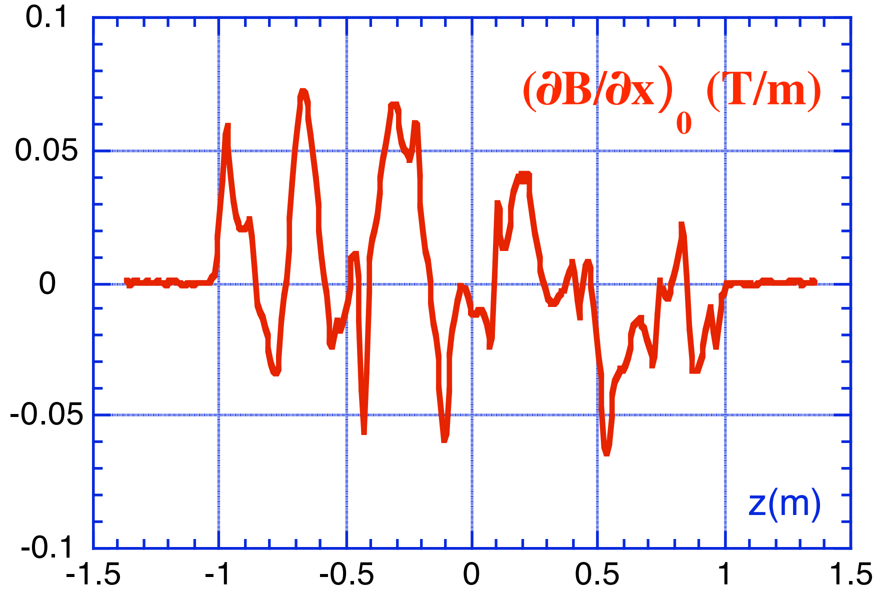
$$K_1 = \frac{1}{B\rho}\left[\int \frac{\partial B}{\partial x} dz + \int \frac{\partial^2 B}{\partial x^2} x dz + \frac{1}{2}\int \frac{\partial^3 B}{\partial x^3} x^2 dz + \frac{1}{6}\int \frac{\partial^4 B}{\partial x^4} x^3 dz\right]$$

$$K_2 = \frac{1}{B\rho}\left[\int \frac{\partial^2 B}{\partial x^2} dz + \int \frac{\partial^3 B}{\partial x^3} x dz + \frac{1}{2}\int \frac{\partial^4 B}{\partial x^4} x^2 dz\right]$$

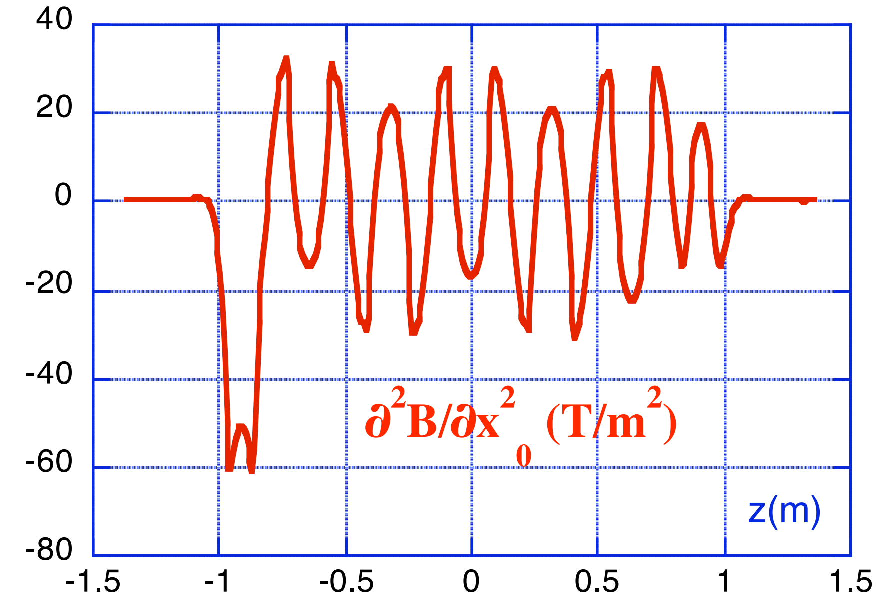
$$K_3 = \frac{1}{B\rho}\left[\int \frac{\partial^3 B}{\partial x^3} dz + \int \frac{\partial^4 B}{\partial x^4} x dz\right]$$

$$K_4 = \frac{1}{B\rho}\int \frac{\partial^4 B}{\partial x^4} dz$$

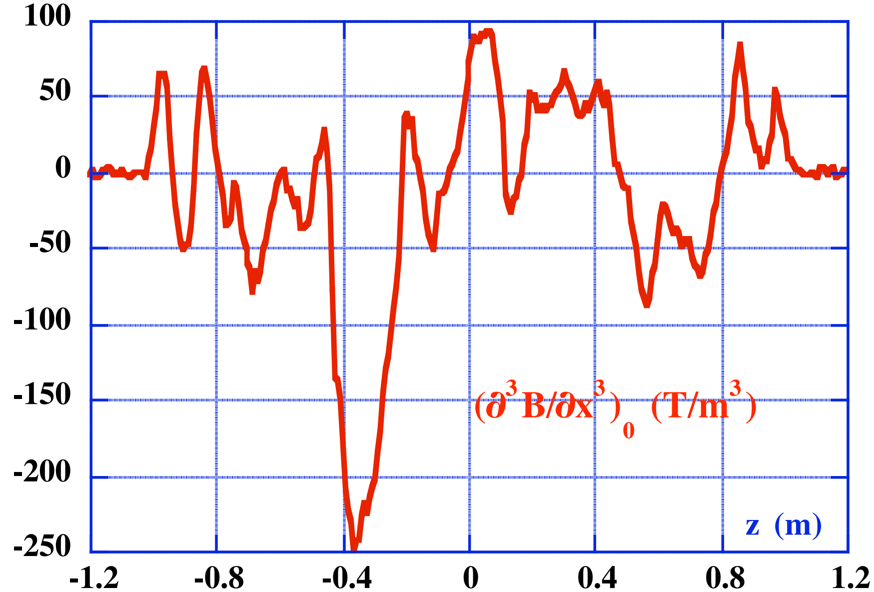
Where all derivatives are taken on the wiggler axis and x is the beam trajectory



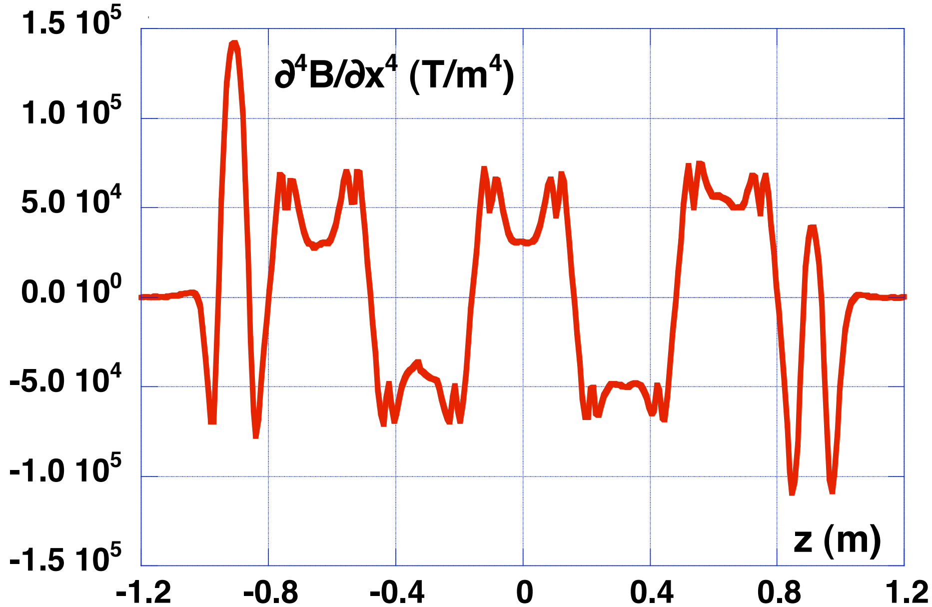
0.1
0.05
0
-0.05
-0.1
-1.5
-1
-0.5
0
0.5
1
1.5
$(\partial B/\partial x)_0$ (T/m)
z(m)



40
20
0
-20
-40
-60
-80
-1.5
-1
-0.5
0
0.5
1
1.5
$\partial^2 B/\partial x^2{}_0$ (T/m^2)
z(m)



100
50
0
-50
-100
-150
-200
-250
(∂3B/∂x3)0 (T/m3)
z (m)
-1.2
-0.8
-0.4
0
0.4
0.8
1.2



1.5 10^5
1.0 10^5
5.0 10^4
0.0 10^0
-5.0 10^4
-1.0 10^5
-1.5 10^5
∂4B/∂x4 (T/m4)
z (m)
-1.2
-0.8
-0.4
0
0.4
0.8
1.2

Multipole coefficient evaluation

	Term.B (SXT)	First pole	Second pole	Third pole	Fourth pole	Fifth pole	Term.A	Full wiggler
K_1^{MAD} (m^{-1})	0.043	0.012	0.004	0.005	0.003	-0.003	0.000	0.064
K_2^{MAD} (m^{-2})	-4.7	2.0	-1.1	1.8	-1.5	1.4	-0.4	-2.6
K_3^{MAD} (m^{-3})	-30	70	38	78	74	84	45	358
K_4^{MAD} (m^{-4})	$2.8 \cdot 10^3$	$8.3 \cdot 10^3$	$-9.1 \cdot 10^3$	$8.3 \cdot 10^3$	$-9.4 \cdot 10^3$	$10.0 \cdot 10^3$	$-5.0 \cdot 10^3$	$5.8 \cdot 10^3$

- After the wiggler modification the main contribution to the 1st order multipole comes from the terminal pole B where the trajectory has a ~12 mm offset in the additional 2nd order multipole introduced to improve chromaticity correction
- 2nd order multipole comes mainly from the second derivative of the field and is larger in the end pole B
- 3rd order multipole comes from the 4th order term in the field expansion combined with the wiggling trajectory

Wiggler linear model

- Knowing:
  - the trajectory on the horizontal symmetry plane:
    amplitude
    deflection angle
    length
  - the wiggler pole transfer matrix

- Each wiggler pole having a L_p length is described as a hard edge dipole and 2 drift sections

$$L_p = L_{dip} + 2L_{drift}$$

- L_p is assumed equal to the integrated particle path length over the pole

$$L_p = \int_{pole} x \; dz$$

Linear model parameters

the linear wiggler model is determined by setting the parameters:

θ	dipole deflection angle
e_1 e_2	dipole entrance and exit angle
f_{edge}	dipole edge focusing parameter
L_{drift}	length of the drift section

For each wiggler pole:

$$\vartheta = \frac{1}{B\rho}\int B_y dz$$

e_1 = $e_2 = \theta/2$ for inner poles

$e_1 = 0$ $e_2 = \theta$ for terminal poles

f_{edge} and L_{drift} have been set in order to reproduce the vertical block of the wiggler pole transfer matrix

	Term.B (SXT)	Inner pole	Term. A
L_{drift} (m)	.1368	.2355	.1368
L_{pole} (m)	.2	.32	.2
θ (rad)	.1196	.2375	.1167
f_{edge}	.384	.317	.384

wiggler non-linear model

non-linear terms are included adding at both sides of each hard edge bend a thin lens providing the previously described

$K_i \quad i = 1..4$

multipole terms

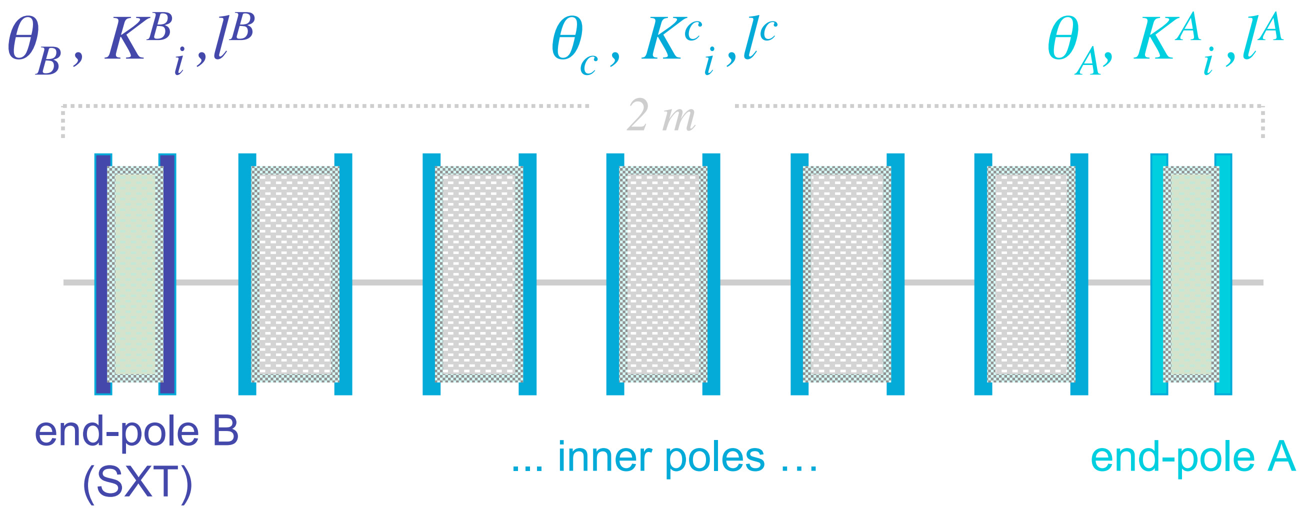
For the inner poles each K_i has been taken equal to the average over the values obtained for the five poles

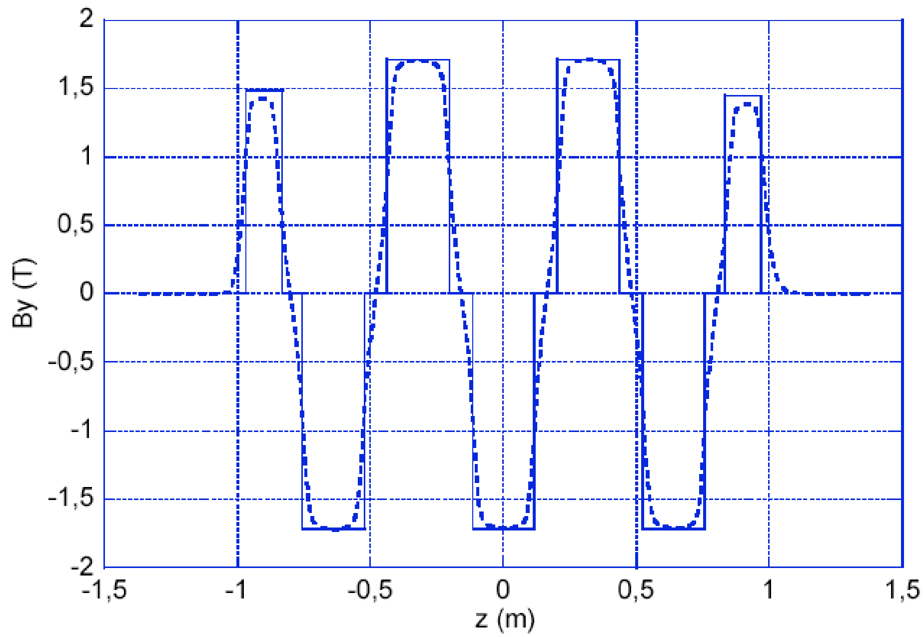
Further focusing effects

arise from the 1st order multipole added on the terminal B in combination with the horizontal orbit

$$\Delta K_1 \sim \Delta K_2 \, \Delta x$$

$$\Delta K_1 \sim .007 \text{ m}^{-1} \qquad \text{for } \Delta x \sim 1.5 \text{ mm}$$





The wiggler vertical field, dashed line, together with the hard edge model, full line.

Conclusions

Magnetic measurements are fitted in order to get the magnetic field polynomial expansion around the wiggling trajectory.

Wiggler full map has been used to define a test particle trajectory inside the wiggler

Studying the particle trajectory as a function of initial conditions the wiggler first order transfer matrix has been obtained for the whole wiggler as well as for the single poles

Knowing the particle trajectory and the field derivatives on the symmetry plane the K_n multipole coefficient are evaluated

All these elements have been used to build a wiggler model, to be used within the MAD simulation code