REVIEW OF WIGGLER PARAMETERS

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Content

- Storage ring lattice and damping wiggler parameters
- Damping wigglers optimization
- Few examples of possible design of damping wiggler
- Conclusions

Lattice function optimization

Wiggler influences the beam parameters in three ways:

- **Linear effects: tune shift and beta distortion.**
- **Change of radiation integral.**
- **Nonlinear effects.**

Linear effect

For small transverse field roll-off wiggler mainly distorts the vertical optic.
For sin-like wiggler field approximation the vertical tune shift is

$$\Delta n_y = \frac{\bar{b}_y L_w}{8 p r_w^2}$$

For FODO cell one can minimize
the average beta function as

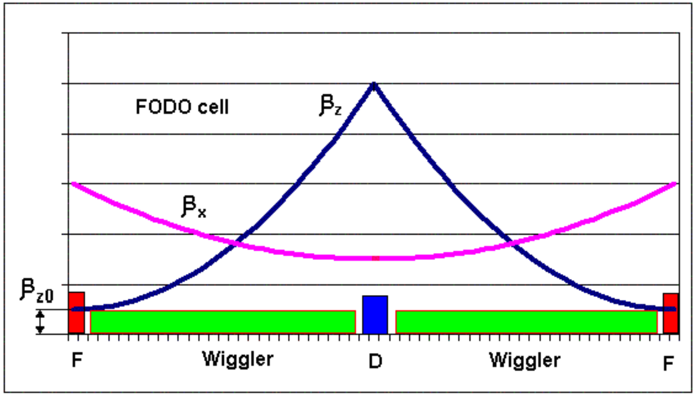
$$\bar{b}_{y\,\min} = \frac{2}{\sqrt{3}} l_w$$

where l_w is the wiggler length and

$$b_{y0} = \frac{1}{\sqrt{3}} l_w$$

The minimum tune shift is

$$\Delta n_{y\,\min} = \frac{1}{4\sqrt{3}p} n_w \frac{l_w^2}{r_w^2}$$



? Increasing the wiggler number n_w and small wiggler length l_w is preferable.

Mini-Workshop on Wiggler Optimization for Emittance Control
INFN-LNF, Frascati 21-22 February 2005

Radiation integrals (damping)

Damping integral:

$$I_2 = \int_M \frac{ds}{r^2}$$

wiggler

?

$$i_2 = \frac{1}{2} h_w^2 L_w$$

where *h*$_w$ is the peak curvature and *L*$_w$ is the total wiggler length.

? For higher damping increasing of the wiggler field is desirable.

For several harmonics wiggler field

$$B_y(s) = \sum_k B_k \sin\left(\frac{2pk}{l_w} \cdot s\right)$$

$$i_2 \propto \sum_k B_k^2$$

Radiation integrals (energy spread and partition numbers)

? $$I_3 = \int_M \frac{ds}{\left| r^3 \right|}$$ wiggler ? $$i_3 = \frac{4}{3p} h_w^3 L_w$$

Wigglers increase the energy spread but the **effect is small** $\propto 1/r_w^3$

? $$I_4 = \int_M \frac{1-2n}{r^3} h ds$$ wiggler ? $$i_4 = -\frac{1}{32p^2} L_w \left(\frac{l_w}{r_w^2} \right)^2$$

Effect is negligible $\propto 1/r_w^4$

Radiation integrals (horizontal emittance)

Wiggler dispersion
¯ derivative

$$I_5 = \int_{mag} \frac{H(s)}{\left|r^3(s)\right|} ds$$

wiggler
?

$$i_5 = \frac{8}{15} N_w q_w \left[\frac{1}{\bar{b}_x} \left(5\frac{h_0^2}{r_w^2} + 9q_w^4 \right) + \bar{b}_x \frac{q_w^2}{r_w^2} \right] \approx \frac{8}{15} N_w \bar{b}_x \frac{q_w^3}{r_w^2}$$

$$\boxed{H(s) = g_x h_x^2 + 2a_x h_x h_x' + b_x h_x'^2}$$

Residue ring - dispersion

- Wiggler dispersion

$$\boxed{q_m = \frac{l_w}{2p\, r_w}}$$

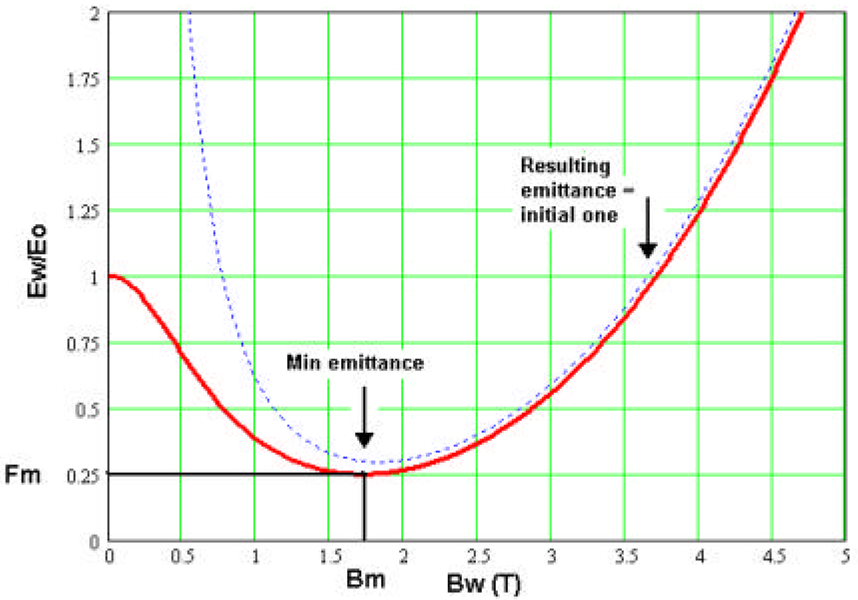
? **Residue dispersion control:** $h_0 << \frac{\bar{b}_x \cdot q_m}{\sqrt{5}}$

? **Beta-x optimization for FODO cell** $\bar{b}_{x\,\min} = l_w / \sqrt{3}$ $i_{5\,\min} = \frac{2}{15\sqrt{3}p^3} \cdot L_w \cdot l_w \cdot \frac{l_w^2}{r_w^5}$

? **To reduce *i*$_5$ it is necessary (a) reduce the cell length, (b) reduce the period length, (c) increase the peak field but to some extend.**

Emittance minimization

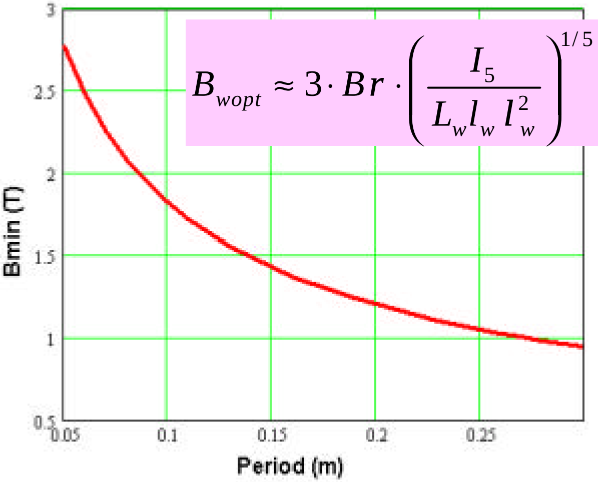
$$\frac{\varepsilon_w}{\varepsilon_0} = C_q \frac{\gamma^2}{J_x} \frac{1+i_5/I_5}{1+i_2/I_2}$$

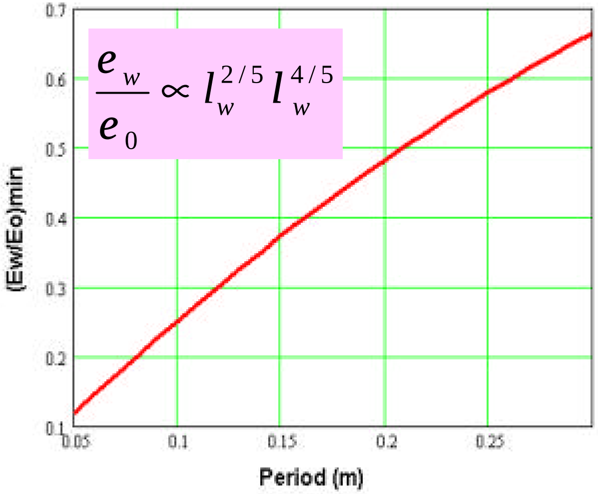


? Resulting emittance has minimum for particular peak field.

Optimum peak field and period length

- ♦Spurious ring dispersion is zero.
- ¨ FODO cell with minimized horizontal beta.
- ¨ Wiggler dominating damping.
- ¨ Sine-like wiggler model.


$$B_{wopt} \approx 3\cdot Br\cdot\left(\frac{I_5}{L_w l_w l_w^2}\right)^{1/5}$$


$$\frac{e_w}{e_0} \propto l_w^{2/5} l_w^{4/5}$$

? To reduce the resulting emittance one have to reduce the wiggler period and increase the peak field.

Wiggler nonlinearity

The main effect is due to the wiggler magnets edge field producing strong vertical cubic nonlinearity.

$$\Delta H = \frac{1}{24} n(s) y^3 \qquad ? \qquad (n \cdot l) = \frac{B''' \cdot l_w}{Br} = \frac{8p^2}{l_w r_w^2}$$

and relevant amplitude-dependent tune shift is given by

$$\Delta n_y (J_y) = \left(\frac{p \cdot L_w \bar{b}_y^2}{l_w^2 r_w^2} \right) \cdot J_y$$

? Reducing of the wiggler period results in the enhancement of the vertical cubic nonlinearity.

Damping wiggler parameters

- Damping effect depends on the wiggler length and squared field amplitude.

- To minimize the resulting emittance the shortest period length and the high field are desired. But very high field provides the emittance blow-up.

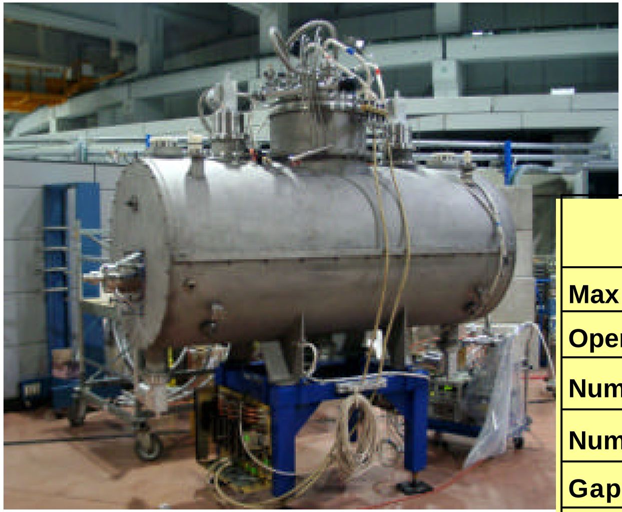
- Short period length yields increasing of the vertical cubic nonlinearity.

For VEPP-3 optical klystron undulator (1983) and for the VEPP-4M dipole wiggler it was controlled by properly placed octupole magnets.

- Transverse nonlinearity can be kept small ($<0.5\ldots1\times10^{-3}$ at ±1 cm) by proper pole design.

Odd number transverse multipoles (sextupole, …) integral values are cancelled because of wiggler periodicity.

Superconducting wiggler

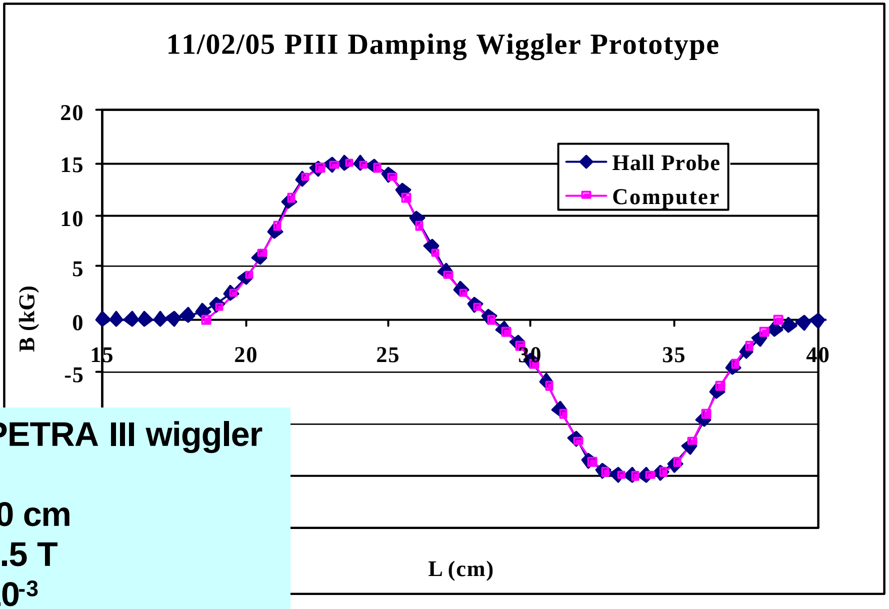


3.5 TESLA SUPERCONDUCTING WIGGLER for ST (TRIESTE, Italy), 2002

Parameter	Units	Value
Max magnetic field	T	3.62
Operating magnetic field	T	3.5
Number of base poles		45
Number of additional poles		4
Gap	mm	16.5
Pole length (period)	mm	32 (64)
Energy content	kJ	240

Permanent magnet wiggler

Field measured and computed ?



Main parameters of the PETRA III wiggler

Period:	20 cm
Field amplitude:	1.5 T
Field quality @ 1 cm:	10^{-3}
Total length:	80 m
Total radiation power:	887 kW

Permanent magnet wiggler

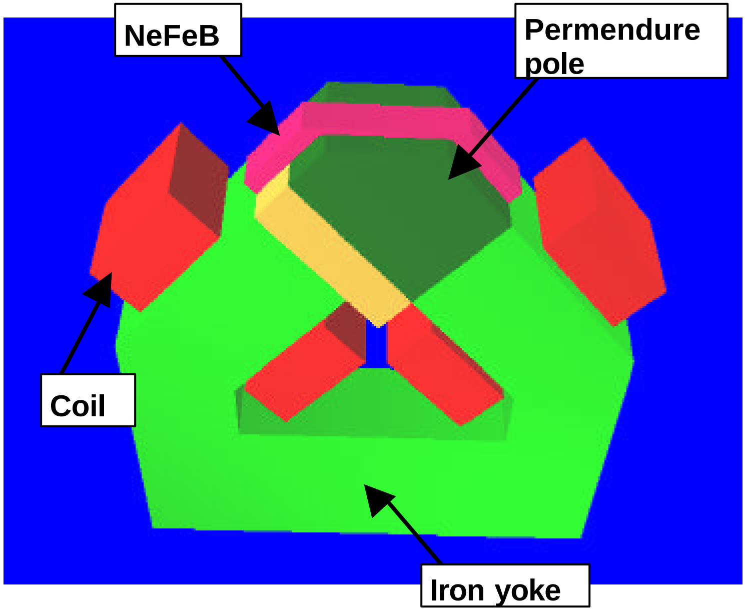
Electromagnet wiggler

Usual electromagnet wigglers can not be used as damping wigglers because it is difficult to achieve high field with small period.

However combined permanent/electromagnet devices (equipotential bus wigglers, K.Halbach) can show good damping parameters.

g = 6 mm
l_w = 25 mm
Bm = 0.45…0.7 T
L = 2 m
DB/B < 5x10^{-4} at 1 cm.

FEL undulator for KAERI (1999).



g = 12 mm
l_w = 76 mm
Bm = 1.7 T

? Proposal for damping wiggler (2005)

Conclusions

♦ Superconducting devices seem to be most effective as damping wigglers.
The field up to 3.5-4 T can be achieved for 60-70 mm period and 15-20 mm gap.
They are very expensive and require complicated cryogenic equipment.

¨ Permanent magnet devices can provide 1.5-2 T in gap 20-10 mm for period ~10…15 cm.
Such wigglers 4-5 cheaper compare to the superconducting ones and rather reliable.

¨ Equipotential bus wigglers reach same parameters as permanent magnets wigglers and even better for approximately same price.
They need power supply system. They effective for small gap.
They allow to change amplitude of magnetic field in the range ±25%.