Requirements & Studies for CLIC

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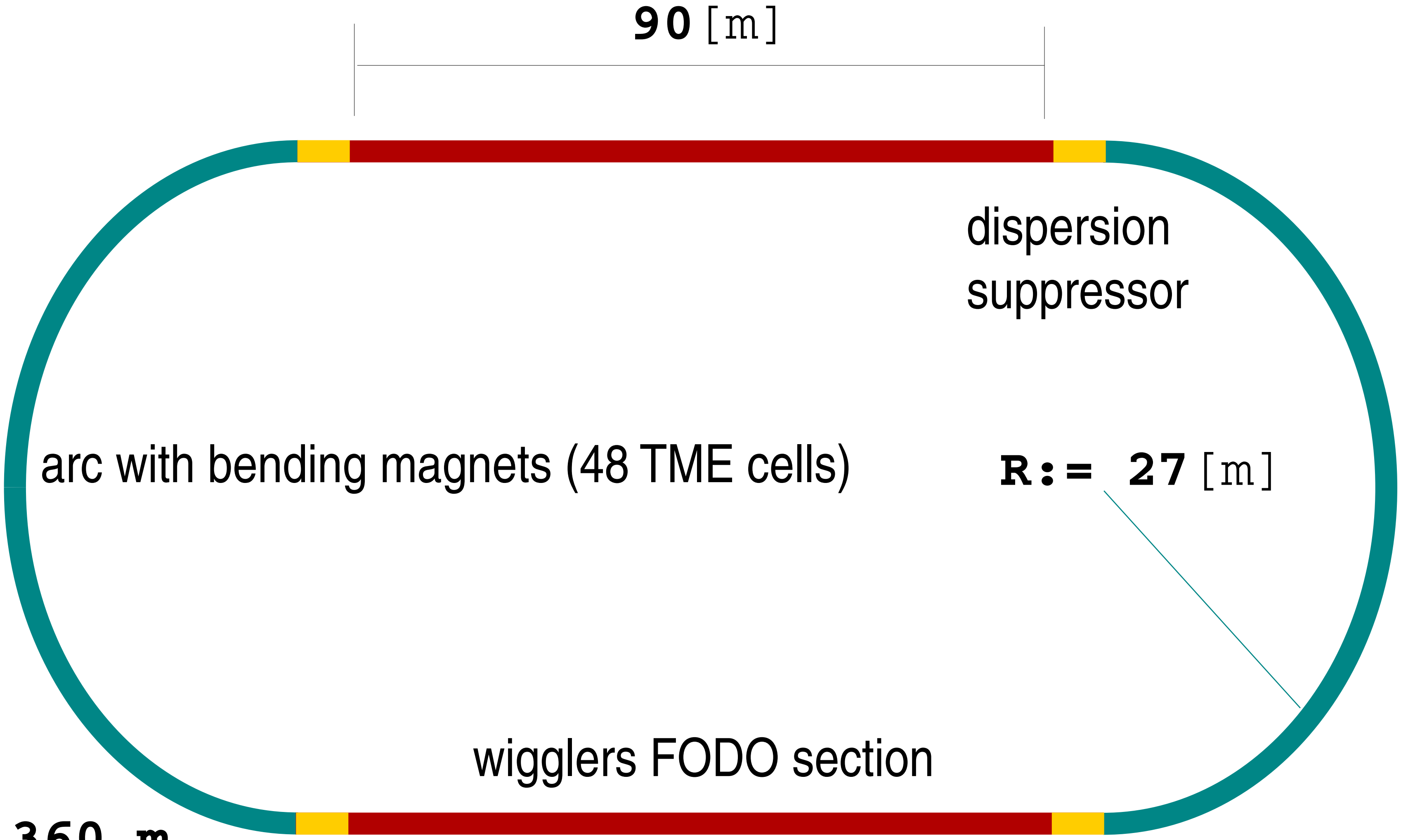
Mini-Workshop on Wiggler Optimization
For Emittance Control

INFN-LNF, Frascati 21-22 February 2005

layout of the CLIC positron damping ring



C:= 360 m
E:= 2.424 GeV

76 wigglers in the ring
length of each wiggler is 2m

damping time

$$\tau_x = \frac{3C}{r_e c\gamma^3 I_2 J_x} = \frac{C(2.89\times 10^{12} kG)}{|B_a|\gamma^2 c(J_{x0}+F_w)}$$

$$\tau_y = \frac{3C}{r_e c\gamma^3 I_2 J_y} = \frac{C(2.89\times 10^{12} kG)}{|B_a|\gamma^2 c J_{y0}}$$

$$\tau_t = \frac{3C}{r_e c\gamma^3 I_2 J_t} = \frac{C(2.89\times 10^{12} kG)}{|B_a|\gamma^2 c(4-J_{x0}-J_{y0}+2F_w)}$$

B_a strength of magnetic field of bending magnet

B_w strength of magnetic field of wiggler

L_w length of wigglers section

C ring circumference

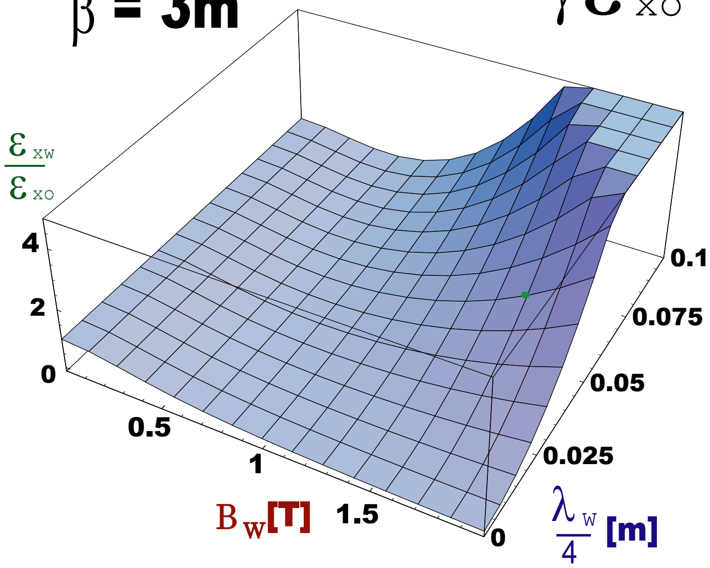
the factor F_w represents the relative damping in the wiggler compared to the arcs

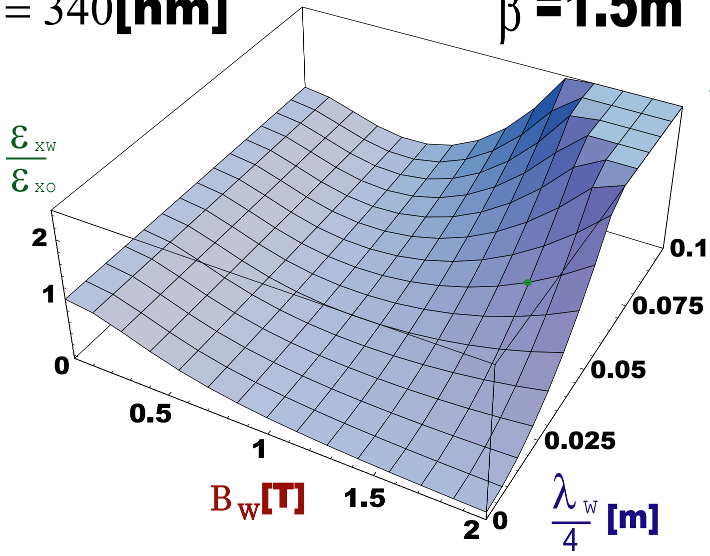
$$F_w = \frac{L_w B_w^2}{4\pi(B\rho)|B_a|}$$

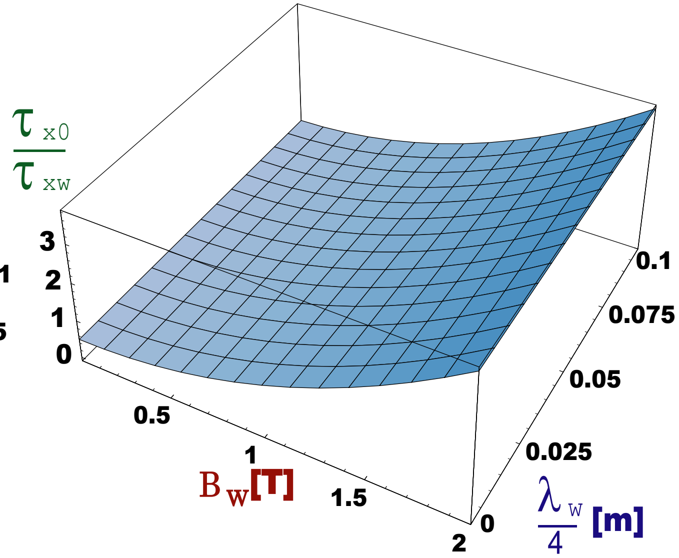
wiggler influence on emittance without action of IBS

$$\gamma\varepsilon_x = \frac{C_q\gamma^3 I_5}{J_x I_2} = \frac{\varepsilon_r C_q\gamma^3}{12(J_{x0}+F_w)}\left[\frac{\theta^3}{\sqrt{15}} + \frac{F_w|B^3{}_w|\lambda_w^2\langle\beta_x\rangle}{16(B\rho)^3}\right]$$

β = 3m $\quad$ $\gamma\varepsilon_{xo}$ = 340[nm] $\quad$ β =1.5m



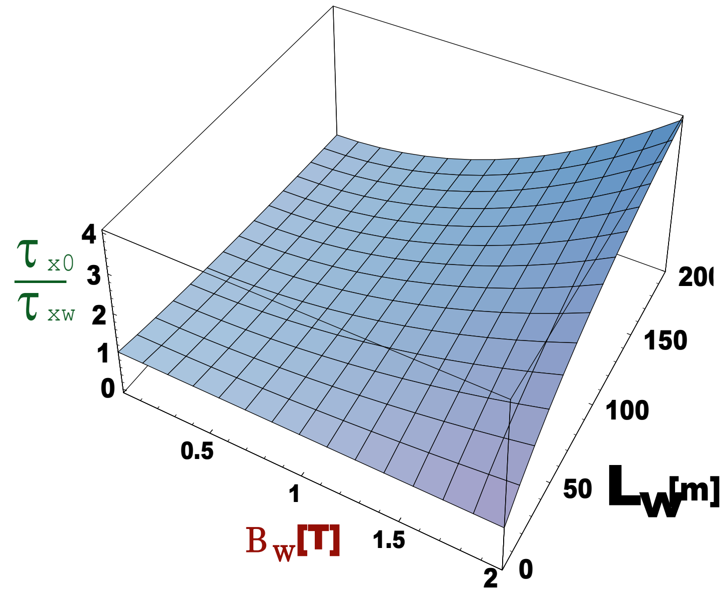


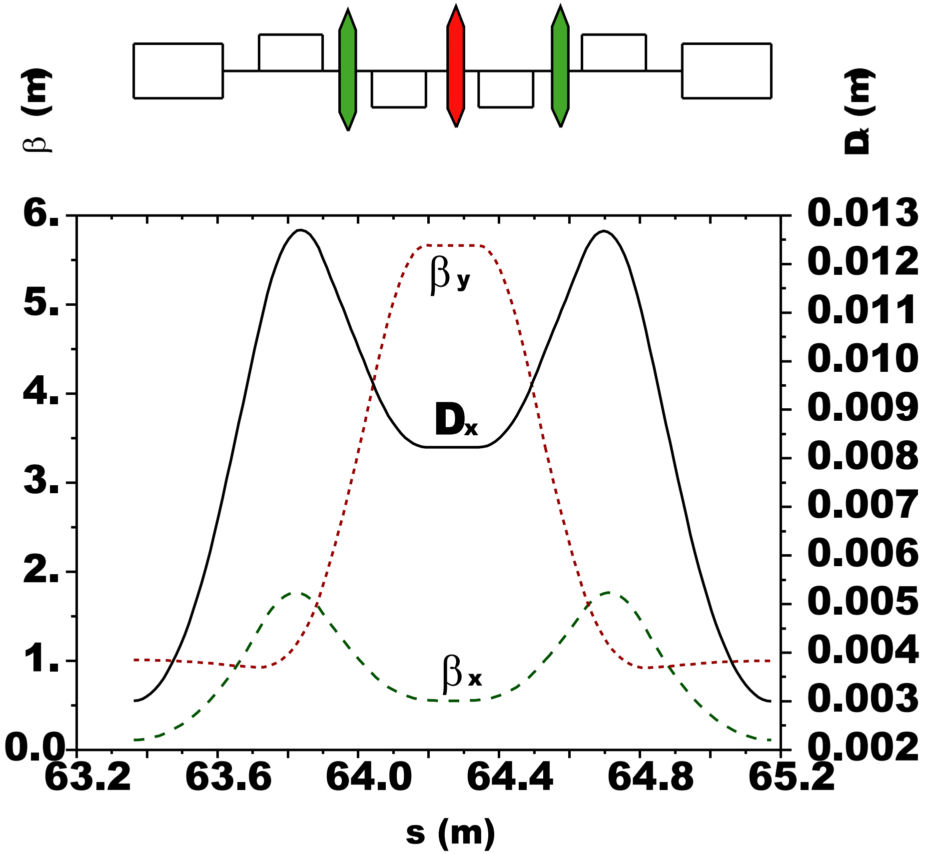


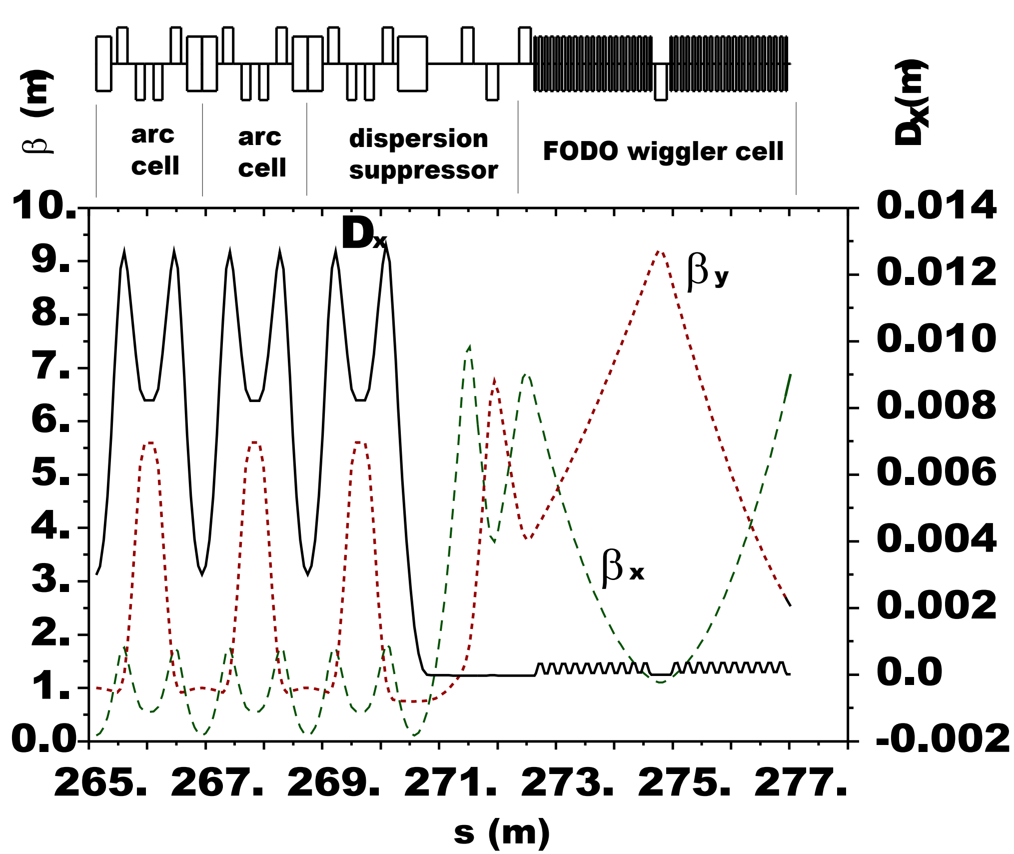
$$\frac{\varepsilon_{xw}}{\varepsilon_{x0}} = \frac{1 + \frac{5}{6\pi}\, G_a\, \frac{\langle\beta_x\rangle}{\rho}\, \frac{E^2}{\varepsilon_{x0}}\, (\frac{\rho}{\rho_w})^2 N_w\, N_p \Theta_w^3}{1 + \frac{2}{\pi}\, \frac{\rho}{\rho_w}\, N_w\, N_p \Theta_w}$$

$$\frac{\tau}{\tau_w} = \frac{1 + \frac{\rho L_w}{2\pi}\left(\frac{B_w}{3.356[\frac{Tm}{GeV}]E}\right)^2}{1 + \frac{L_w}{L_a}}$$

$$\beta_x \le \frac{384}{275}\sqrt{3}\, \frac{(mc^2)^3}{\hbar c e^3}\, \frac{16\varepsilon_{x0}E}{\lambda_w^2 B_w^3},$$




β (m)
Dx (m)
6.
5.
4.
3.
2.
1.
0.0
0.013
0.012
0.011
0.010
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
63.2
63.6
64.0
64.4
64.8
65.2
s (m)
βy
Dx
βx


β (m)
Dx (m)
arc cell
arc cell
dispersion suppressor
FODO wiggler cell
10.
9.
8.
7.
6.
5.
4.
3.
2.
1.
0.0
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.0
-0.002
265.
267.
269.
271.
273.
275.
277.
s (m)
Dx
βy
βx

The horizontal ε_x, vertical ε_y and longitudinal ε_t emittances evolve with time according to a set of three differential equations:

$$\dot{\varepsilon}_x = -\frac{2}{\tau_x}(\varepsilon_x - \varepsilon_{x0}) + \frac{2\varepsilon_x}{T_x(\varepsilon_x, \varepsilon_y, \varepsilon_t)} \quad (1)$$

$$\dot{\varepsilon}_y = -\frac{2}{\tau_y}(\varepsilon_y - \varepsilon_{y0}) + \frac{2\varepsilon_y}{T_y(\varepsilon_x, \varepsilon_y, \varepsilon_t)} \quad (2)$$

$$\dot{\varepsilon}_t = -\frac{2}{\tau_t}(\varepsilon_t - \varepsilon_{t0}) + \frac{2\varepsilon_t}{T_t(\varepsilon_x, \varepsilon_y, \varepsilon_t)} \quad (3)$$

where τ_x, τ_y, τ_t are the radiation damping times of the betatron (xy) and synchrotron (t) oscillations respectively. ε_{x0}, ε_{y0}, ε_{t0} are equilibrium emittances determined by radiation damping and quantum excitation in the absence of IBS and $T_\mu(\varepsilon_x, \varepsilon_y, \varepsilon_t)$, $\mu \in \{x, y, t\}$ are intrabeam scattering gtowth times which are non-linear functions of emittances.

The equilibrium emittances follow from equation

$$\dot{\varepsilon}_x = \dot{\varepsilon}_y = \dot{\varepsilon}_t = 0$$

Intra beam scatterin (Bjorken and Mtingwa formalism in the case of ultrarelativistic beams)

Intra-beam scattering is the multiple small angle Coulomb interaction of charged particles within bunched or coasting beams [13]. This effect causes both longitudinally and transversely beam diffusion, that leads to the growth of the momentum spread and one or both transverse beam dimensions.

The growth rates are:

$$\frac{1}{\tau_x} = \frac{A}{L}\int_0^L \left[\left(2a - \frac{\beta_x}{\varepsilon_x} - \frac{\beta_y}{\varepsilon_y} + a_2\right) G_1(s) + \left(b_1 - 3\frac{\beta_x}{\varepsilon_x}\frac{\beta_y}{\varepsilon_y} + b_2\right) G_2(s)\right] \Delta(s)\, ds$$

$$\frac{1}{\tau_y} = \frac{A}{L}\int_0^L \frac{\beta_y}{\varepsilon_y}\left[\left(2\frac{\beta_y}{\varepsilon_y} - \frac{\beta_x}{\varepsilon_x} - a\right) G_1(s) + \left(b_1 - 3c\frac{\beta_y}{\varepsilon_y}\right) G_2(s)\right] ds$$

$$\frac{1}{\tau_\delta} = \frac{A}{L}\frac{m\gamma^2}{\sigma_\delta^2}\int_0^L \left[\left(2a - \frac{\beta_x}{\varepsilon_x} - \frac{\beta_y}{\varepsilon_y}\right) G_1(s) + \left(b - 2\frac{\beta_x}{\varepsilon_x}\frac{\beta_y}{\varepsilon_y}\right) G_2(s)\right] ds$$

in which

$$G_1(s) = \int_0^\infty \frac{\lambda\sqrt{\lambda}\, d\lambda}{\left(\lambda^3 + a_1\lambda^2 + b_1\lambda + c\right)^{3/2}}, \qquad G_2(s) = \int_0^\infty \frac{\sqrt{\lambda}\, d\lambda}{\left(\lambda^3 + a_1\lambda^2 + b_1\lambda + c\right)^{3/2}}$$

are the azimuthal dependent scattering integrals and

$$a = \gamma^2\left(\frac{D_x^2}{\beta_x\varepsilon_x} + \Phi^2\frac{\beta_x}{\varepsilon_x} + \frac{m}{\sigma_\delta^2}\right), \qquad b = \gamma^2\left(\frac{\beta_x}{\varepsilon_x} + \frac{\beta_y}{\varepsilon_y}\right)\left(\frac{D_x^2}{\beta_x\varepsilon_x} + \frac{m}{\sigma_\delta^2}\right) + \gamma^2\Phi^2\frac{\beta_x}{\varepsilon_x}\frac{\beta_y}{\varepsilon_y}$$

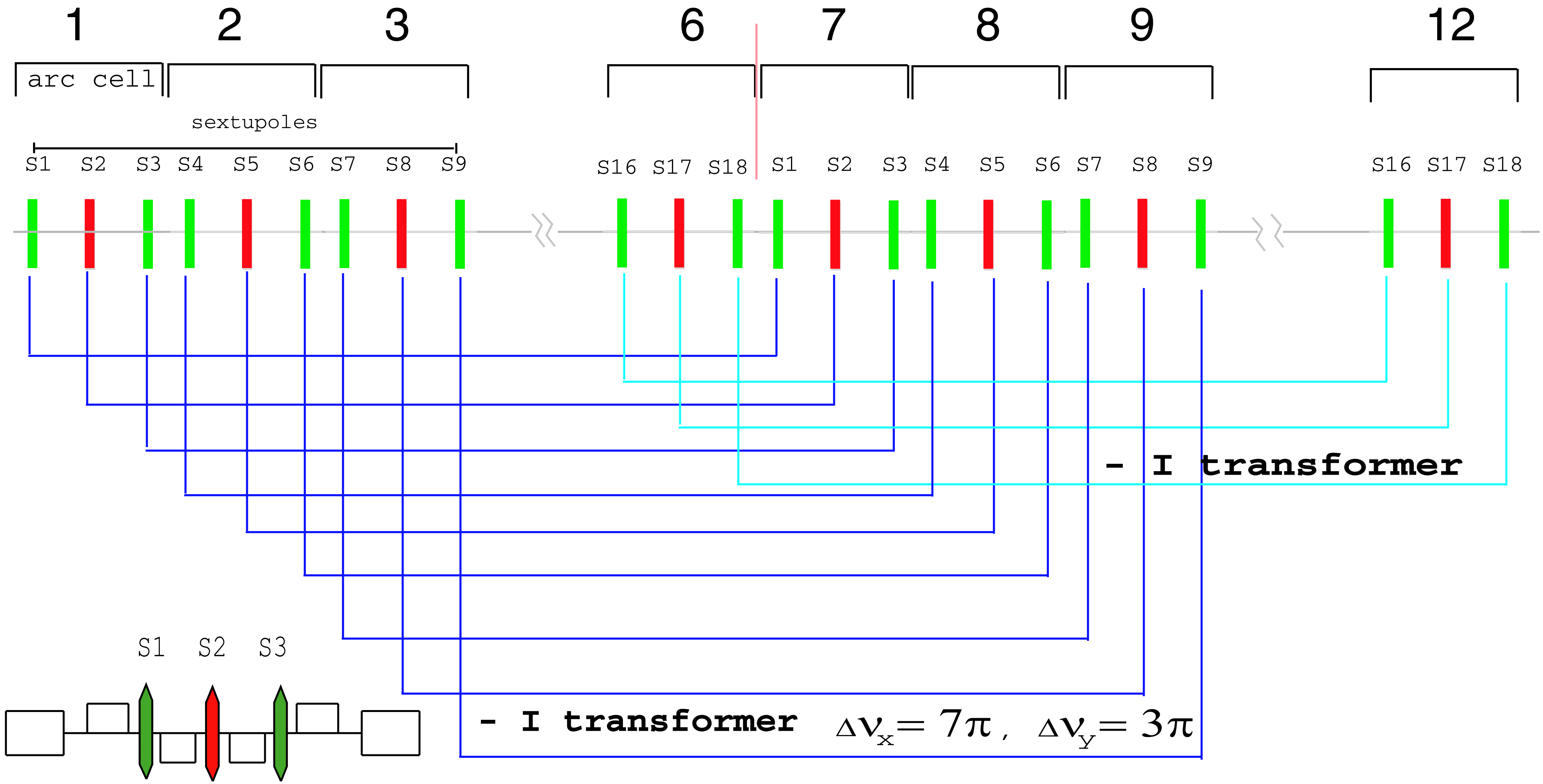
$$c = \gamma^2\frac{\beta_x}{\varepsilon_x}\frac{\beta_y}{\varepsilon_y}\left(\frac{D_x^2}{\beta_x\varepsilon_x} + \frac{m}{\sigma_\delta^2}\right) \quad \Delta = \gamma^2\left(\frac{D_x^2}{\beta_x\varepsilon_x} + \frac{\beta_x}{\varepsilon_x}\Phi^2\right) \quad a_1 = a + \frac{\beta_x}{\varepsilon_x} + \frac{\beta_y}{\varepsilon_y}, \quad b_1 = b + \frac{\beta_x}{\varepsilon_x}\frac{\beta_y}{\varepsilon_y}$$

$$a_2 = \frac{\beta_x}{\varepsilon_x}\left(6\gamma^2\Phi^2\frac{\beta_x}{\varepsilon_x} - a + 2\frac{\beta_x}{\varepsilon_x} - \frac{\beta_y}{\varepsilon_y}\right)\frac{1}{\Delta}, \qquad b_2 = \frac{\beta_x}{\varepsilon_x}\left(6\gamma^2\Phi^2\frac{\beta_x}{\varepsilon_x}\frac{\beta_y}{\varepsilon_y} + b_1 - 3a\frac{\beta_y}{\varepsilon_y}\right)\frac{1}{\Delta}$$

$$a = \begin{cases} \dfrac{L_{\mathrm{IBS}} N_b \tilde{r}_0^2 c_0}{8\pi\beta^3\gamma^4\varepsilon_x\varepsilon_y\sigma_s\sigma_\delta} & \text{for bunched beam} \\ \dfrac{L_{\mathrm{IBS}} N \tilde{r}_0^2 c_0}{4\sqrt{\pi}\beta^3\gamma^4\varepsilon_x\varepsilon_y L\sigma_\delta} & \text{for coasting beam} \end{cases} \quad \text{and} \quad \Phi = D_x' + \frac{\alpha_x}{\beta_x}D_x \qquad \tilde{r}_0 = r_0 Z^2/A$$

L is the period length, N_b the number of particles per bunch, *N* the total number of particles, c_0 the velocity of light, β relativistic factor, γ normalized beam energy, the *m* is 1 for coasting beams and *2* for bunched beams, *A* atomic mass, *Z* atomic number.

second-order achromat



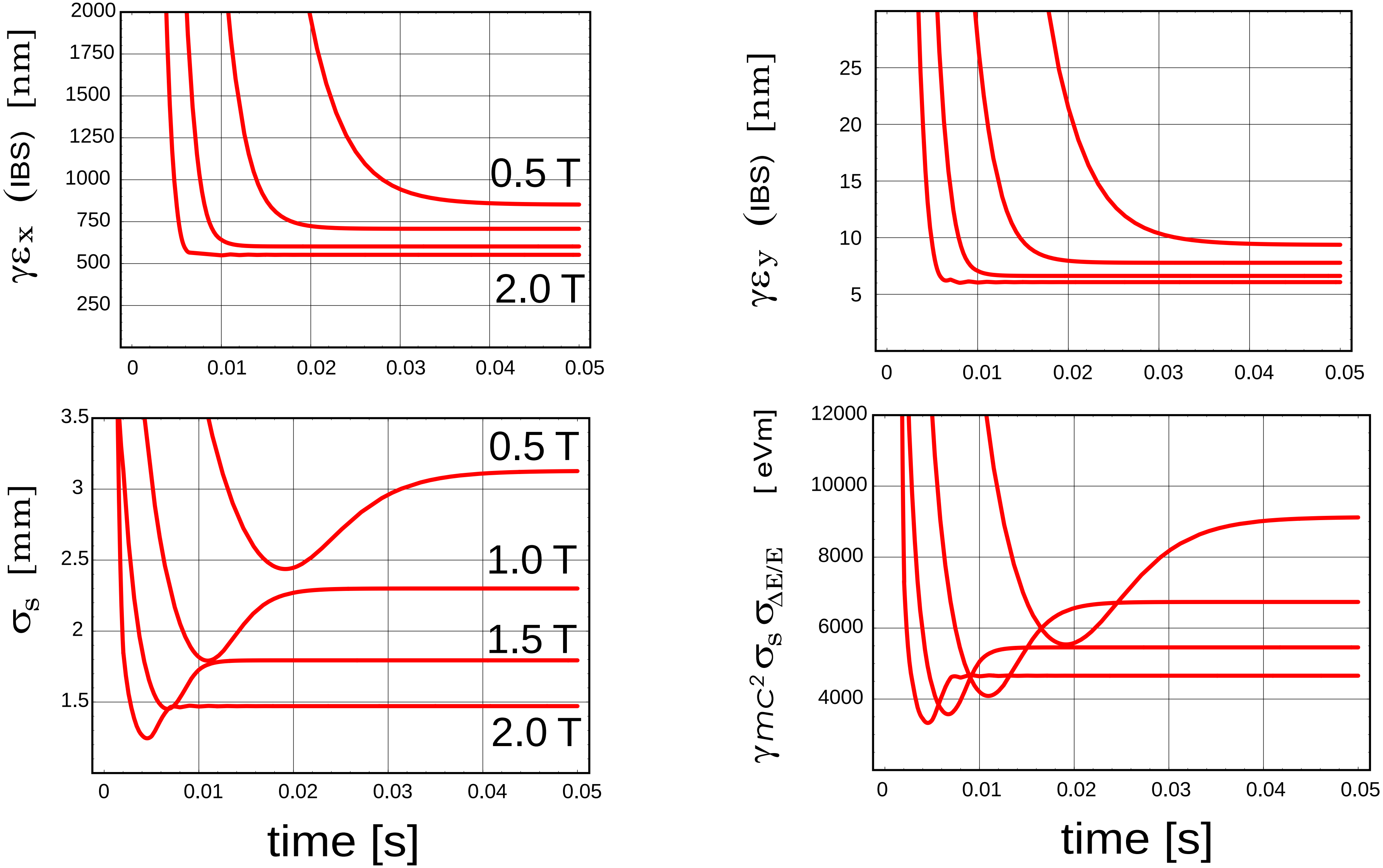
$\Delta\nu_x = 0.5833$, $\Delta\nu_y = 0.2500$ per cell

$\Delta\nu_x = 7\pi$, $\Delta\nu_y = 3\pi$ per 6 cells

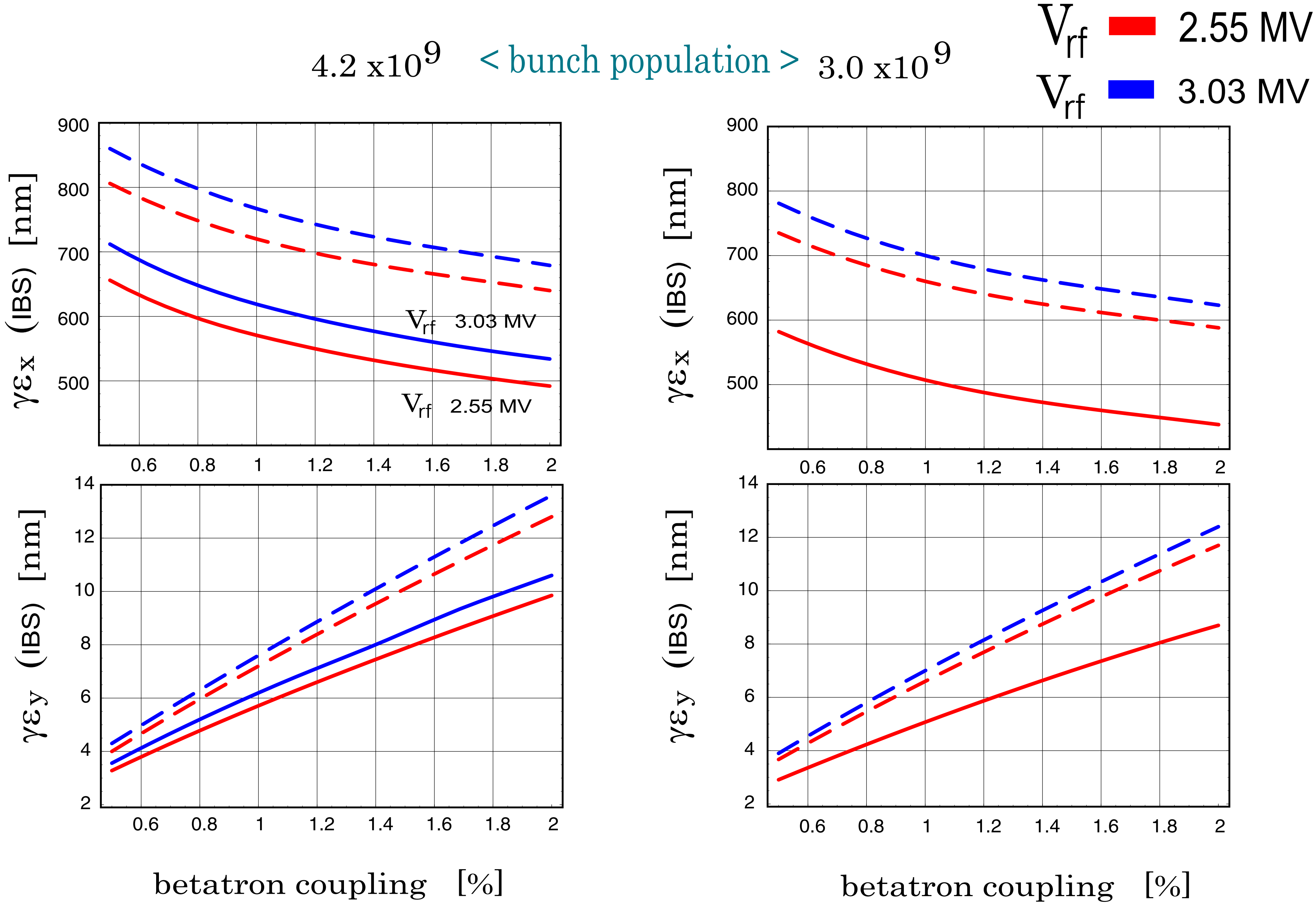
$$\int_0^L F_p\, e^{\pm i\psi} ds = 0 \quad \text{and} \quad \int_0^L F_p\, e^{\pm 3i\psi} ds = 0$$

$$\int_0^{12\times cells} F_p\, e^{\pm i\psi} ds = 0 \quad \text{and} \quad \int_0^{12\times cells} F_p\, e^{\pm 3i\psi} ds = 0$$

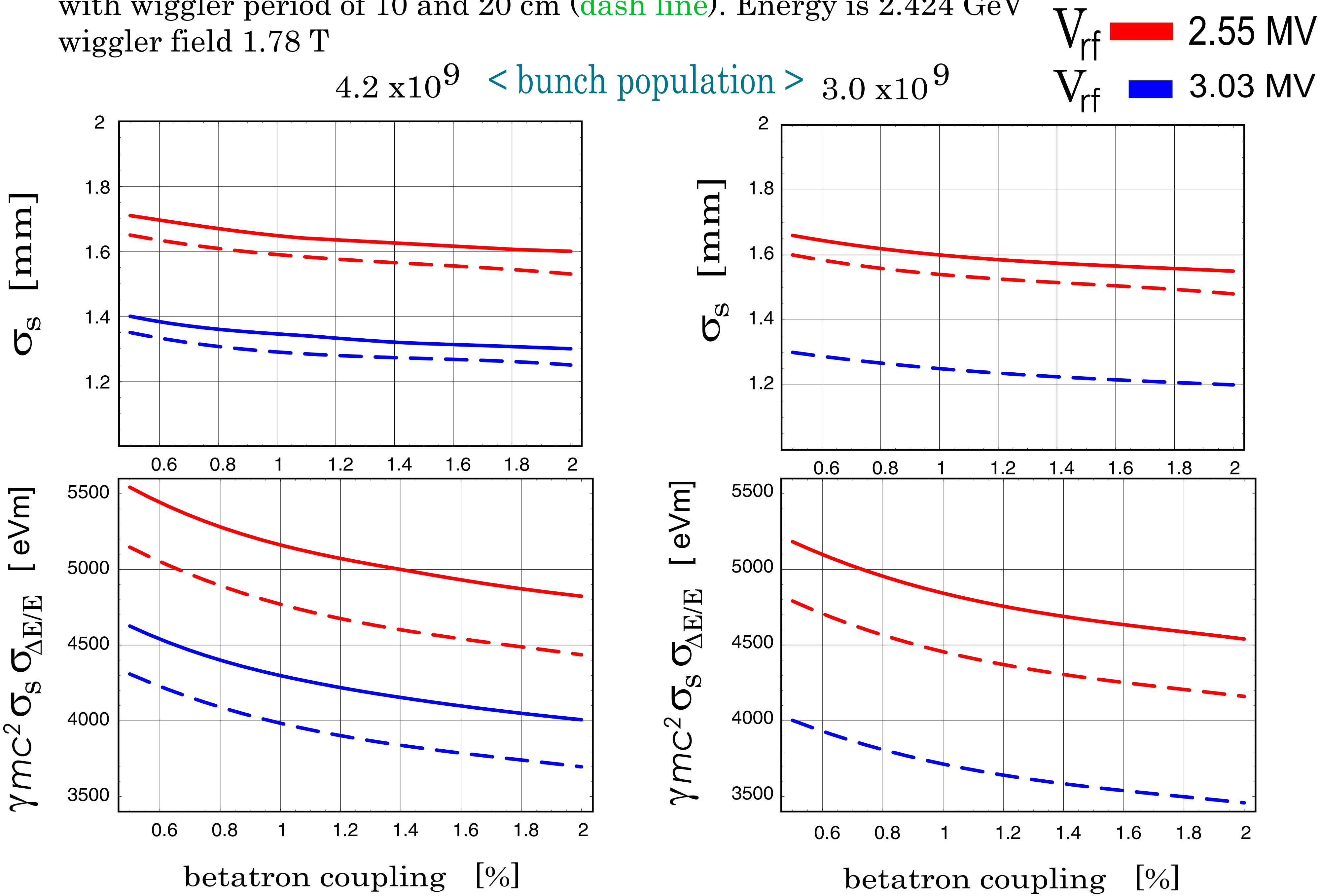
Emittance & bunch length evolution in CLIC_DR with wiggler period of 10 cm at wiggler field of 0.5, 1.0, 1.5 and 2.0 T. Energy is 2.424 GeV, betatron coupling 1.1 %, bunch population 4.2×10^9, RF phase is constant.



Transverse normalized emittances in CLIC_DR with wiggler period of 10 and 20 cm (dash line).
Energy is 2.424 GeV, wiggler field 1.78 T



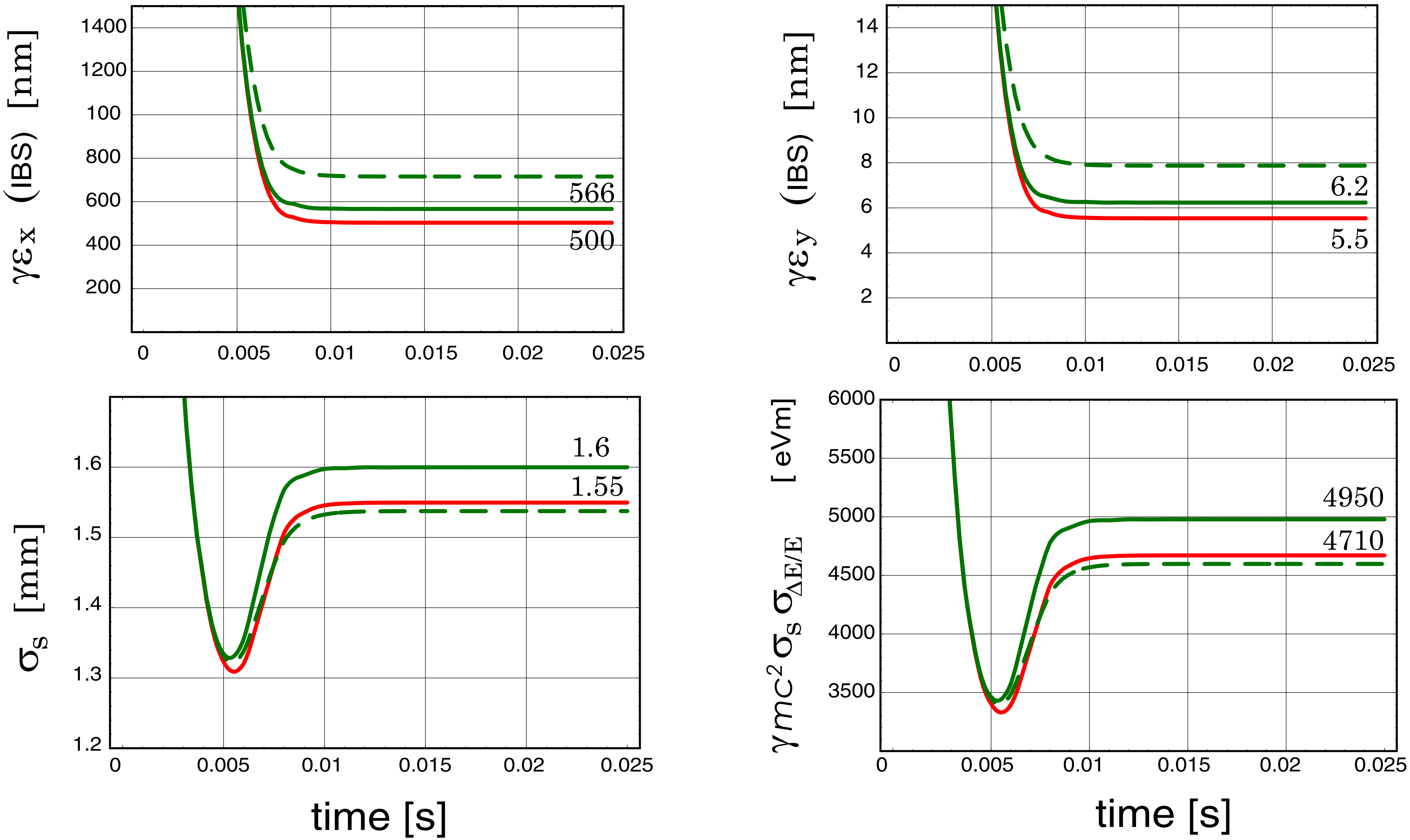
Longitudinal lormalized emittance and rms bunch length in CLIC_DR with wiggler period of 10 and 20 cm (dash line). Energy is 2.424 GeV wiggler field 1.78 T

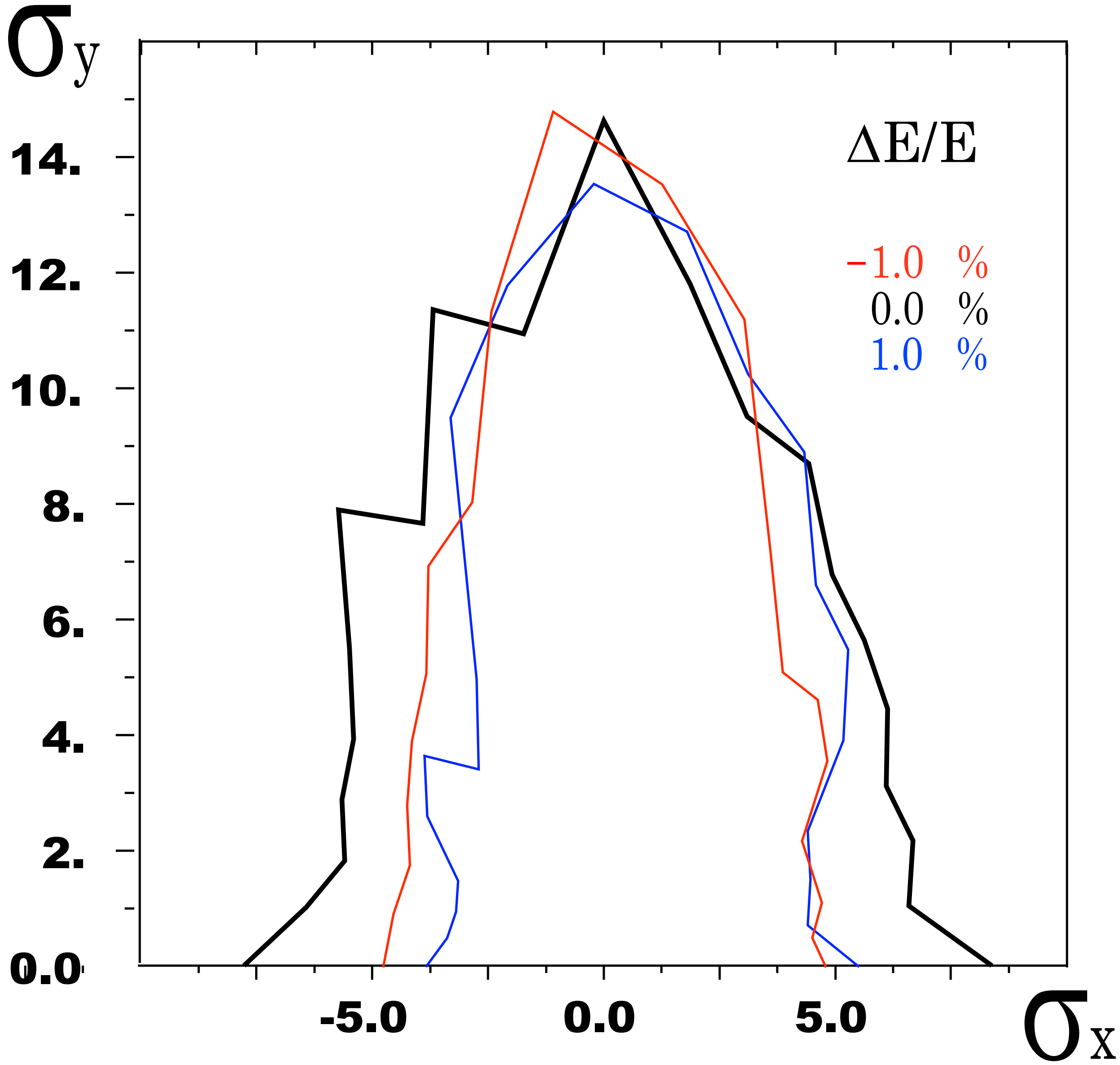


Emittance & bunch length evolution in CLIC_DR with wiggler period of 10 and 20 cm (dash line). Energy is 2.424 GeV, betatron coupling 1.1 %, RF volage 2.6 MV, wiggler field 1.78 T.

red line is bunch population of 3.0×10^9

green line is bunch population of 4.2×10^9





σy
14.
12.
10.
8.
6.
4.
2.
0.0
-5.0
0.0
5.0
σx
ΔE/E
-1.0 %
0.0 %
1.0 %

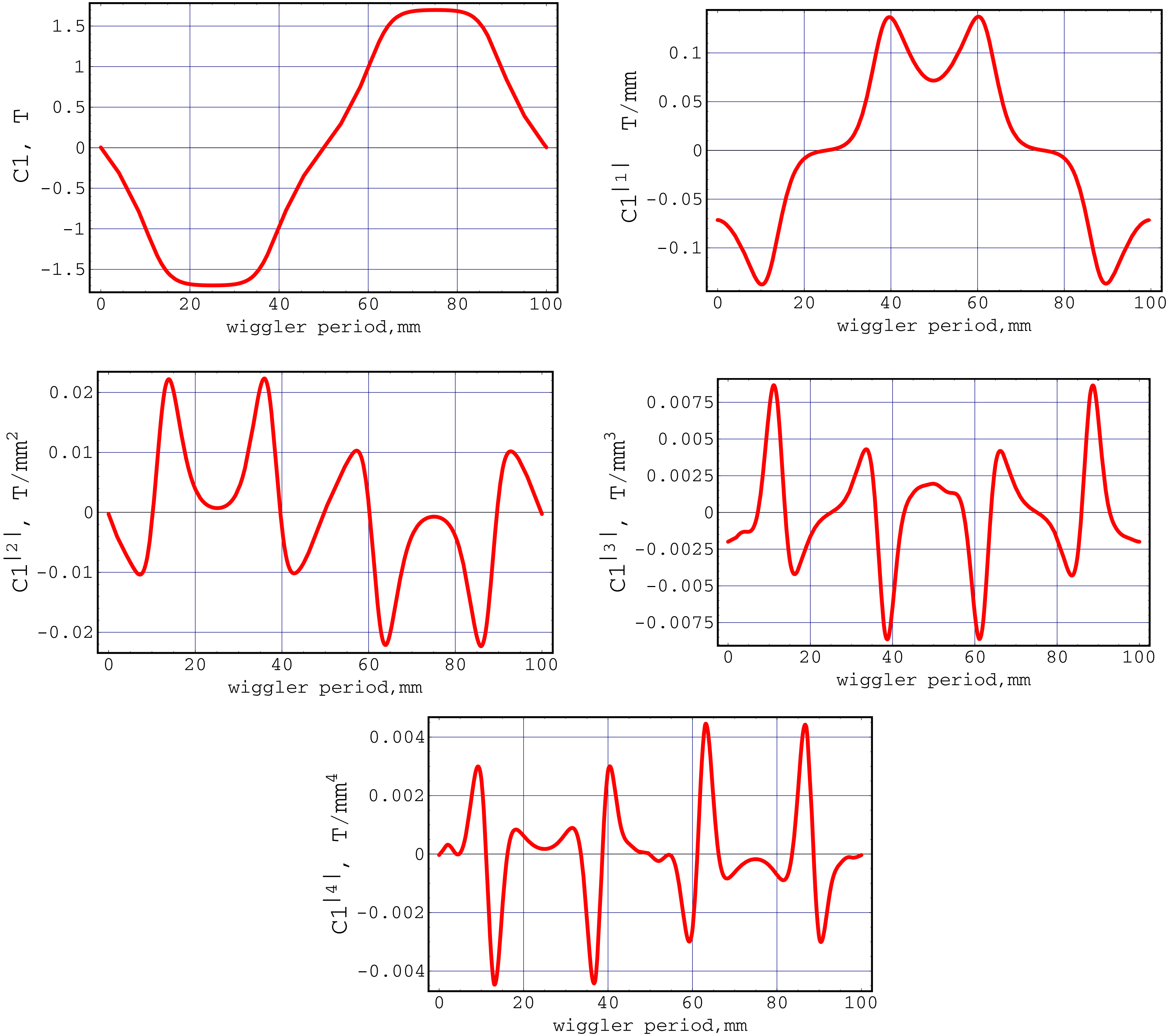
$$B_\rho(\rho = R, \phi, z) = \sum_{m=0}^{\infty} B_m(R,z)\sin(m\phi)$$

$$\tilde{B}_{m,p} = \frac{1}{\lambda_w}\int_0^{\lambda_w} dz e^{-i2\pi pz/\lambda_w} B_m(R,z)$$

$$C_m(z) \equiv C_m^{[0]}(z)$$

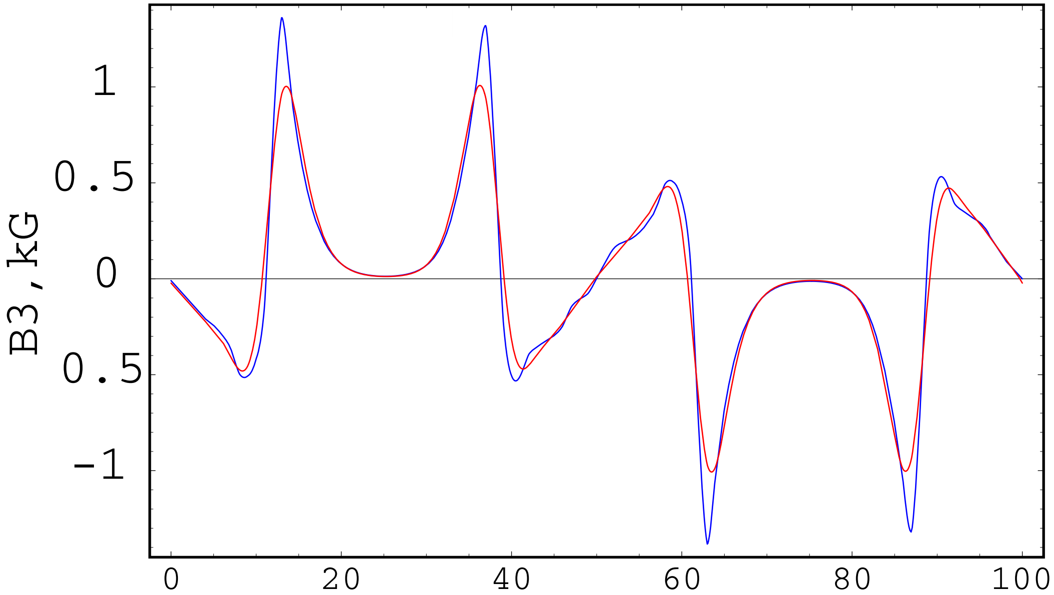
$$C_m^{[k]}(z) = \frac{1}{2^m m!}\sum_{p=-\infty}^{\infty} i^k \frac{(2\pi p/\lambda_w)^{m+k-1}}{I'_m(2\pi pR/\lambda_w)} e^{i2\pi pz/\lambda_w}\tilde{B}_{m,p}$$

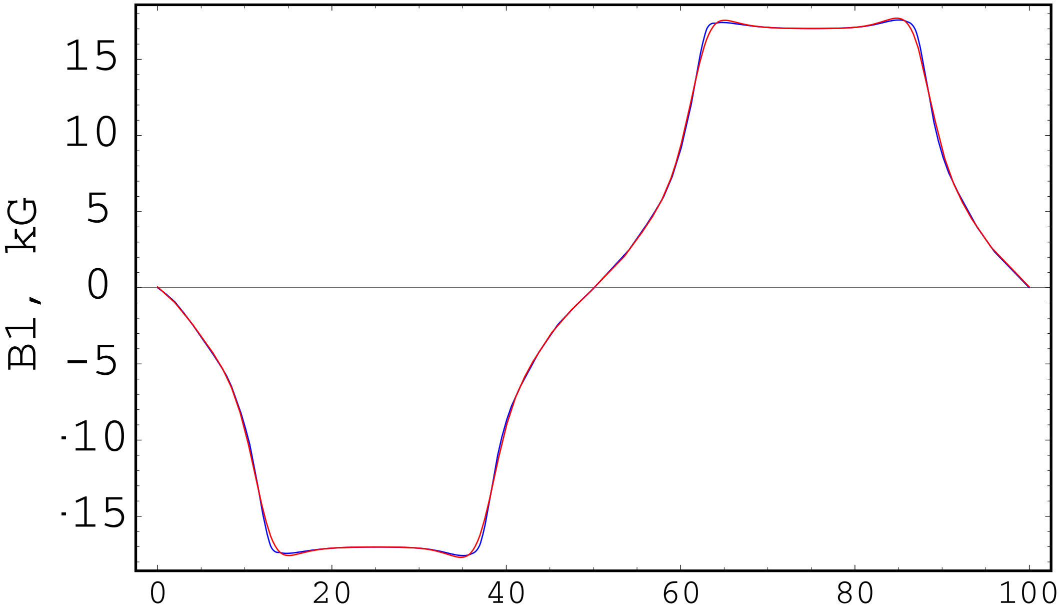
$$B_\rho = \underbrace{\left(C_1 - \frac{3R^2}{8}C_1^{[2]} + \frac{5R^4}{192}C_1^{[4]}\right)}_{B_1(R,z)}\sin\phi + \underbrace{\left(3C_3R^2 - \frac{5R^4}{16}C_3^{[2]}\right)}_{B_3(R,z)}\sin 3\phi + \underbrace{5C_5R^4}_{B_5(R,z)}\sin 5\phi$$

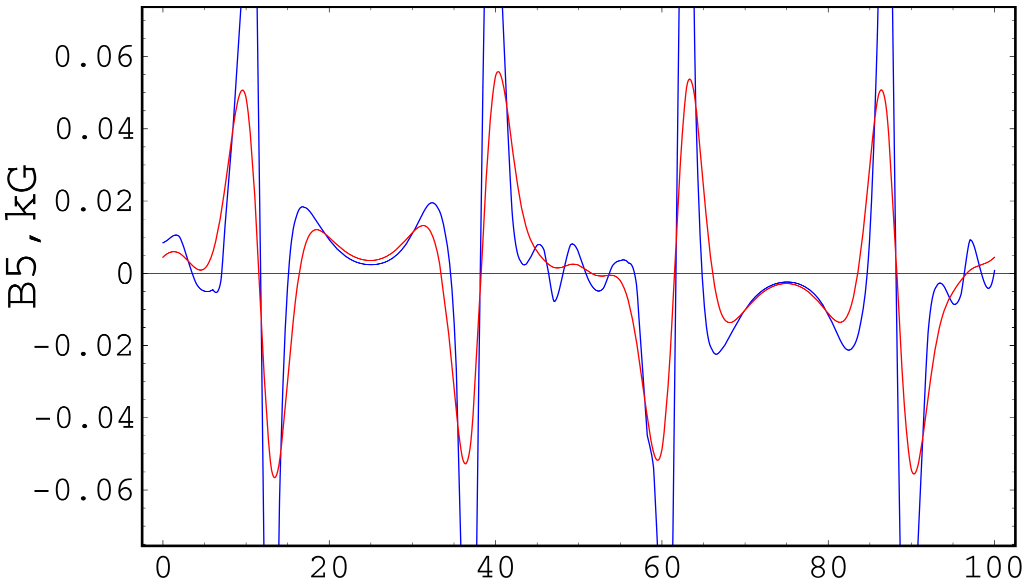

C1, T
1.5
1
0.5
0
-0.5
-1
-1.5
0
20
40
60
80
100
wiggler period,mm
C1|1| T/mm
0.1
0.05
0
-0.05
-0.1
wiggler period,mm
C1|2|, T/mm2
0.02
0.01
0
-0.01
-0.02
wiggler period,mm
C1|3|, T/mm3
0.0075
0.005
0.0025
0
-0.0025
-0.005
-0.0075
wiggler period,mm
C1|4|, T/mm4
0.004
0.002
0
-0.002
-0.004
wiggler period,mm

$$B_\rho = \underbrace{\left(C_1 - \frac{3R^2}{8}C_1^{[2]} + \frac{5R^4}{192}C_1^{[4]}\right)}_{B_1(R,z)} \sin\phi + \underbrace{\left(3C_3R^2 - \frac{5R^4}{16}C_3^{[2]}\right)}_{B_3(R,z)} \sin 3\phi + \underbrace{5C_5R^4}_{B_5(R,z)} \sin 5\phi$$

$$B_\rho(\rho = R, \phi, z) = \sum_{m=0}^{\infty} B_m(R,z)\sin(m\phi)$$







$$
\begin{aligned}
B_x &= -\left(\frac{1}{4}C_1^{[2]} - 6C_3\right)xy + \left(\frac{1}{48}C_1^{[4]} - \frac{3}{4}C_3^{[2]} + 20C_5\right)x^3y + \left(\frac{1}{48}C_1^{[4]} - \frac{1}{4}C_3^{[2]} - 20C_5\right)xy^3, \\
B_y &= C_1 - \left(\frac{1}{8}C_1^{[2]} - 3C_3\right)x^2 - \left(\frac{3}{8}C_1^{[2]} + 3C_3\right)y^2 + \left(\frac{1}{192}C_1^{[4]} - \frac{3}{16}C_3^{[2]} + 5C_5\right)x^4, \\
&+ \left(\frac{1}{32}C_1^{[4]} - \frac{3}{8}C_3^{[2]} - 30C_5\right)x^2y^2 + \left(\frac{5}{192}C_1^{[4]} - \frac{5}{16}C_3^{[2]} + 5C_5\right)y^4 \\
B_z &= yC_1^{[1]} - \left(\frac{1}{8}C_1^{[3]} - 3C_3^{[1]}\right)x^2y - \left(\frac{1}{8}C_1^{[3]} - C_3^{[1]}\right)y^3.
\end{aligned}
$$

x=3.5, y=3.5