Dynamic Aperture in Damping Rings with Realistic Wigglers

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Outline

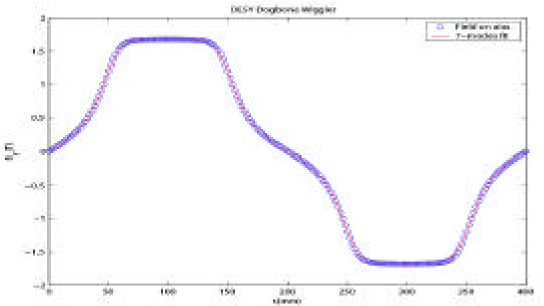
- A simple and physical presentation of three-dimensional magnetic field of wiggler
  - Intrinsic nonlinear field
  - fields of finite width poles
  - DESY dogbone wiggler
- Hybrid symplectic integrators
- Damping rings based on the non-interlaced sextupoles
  - Scaling of dynamic aperture
  - Design lattices: compact damping ring & dogbone
  - Dynamic effects due to wigglers
  - Specification of wiggler magnets
- Conclusion

Intrinsic Field of Wiggler

Magnetic field: (B_x=0)

$$B_y = \sum_{n=1}^{N_s} B_y^{(n)} \cosh[(2n-1)k_w y] \sin[(2n-1)k_w s + f_y^{(n)}],$$

$$B_s = \sum_{n=1}^{N_s} B_y^{(n)} \sinh[(2n-1)k_w y] \cos[(2n-1)k_w s + f_y^{(n)}]$$



where $k_w = 2p / l_w$ and λ_w is the period of wiggler. Each mode independently satisfies:

$$\nabla \cdot \vec{B} = 0, \nabla \times \vec{B} = 0$$

Its vector potential: (A_y=A_z=0)

$$A_x = \sum_{n=1}^{N_s} -\frac{B_y^{(n)}}{(2n-1)k_w} \cosh[(2n-1)k_w y] \cos[(2n-1)k_w s + f_y^{(n)}]$$

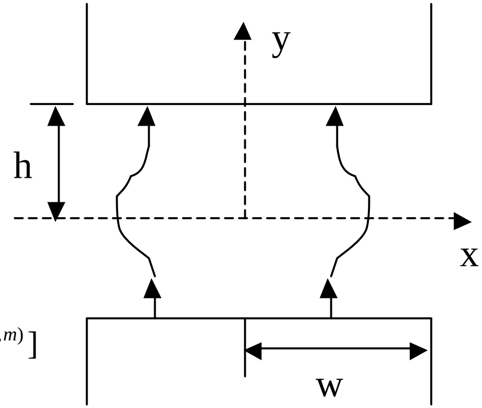
Note it does not depend on coordinate x. This makes Hamiltonian:

$$H_2 = -\frac{e}{(1+d)cp_0}(p_x A_x + p_y A_y)$$

exactly solvable.

Field from Finite Width Poles

Magnetic field [Halbach]: ($B_x(x,h,s)=0$)



$$B_x = \sum_{n=1}^{N_{sy}} \sum_{m=1}^{N_y} B_x^{(n,m)} \sinh(k_x^{(n,m)} x) \sin(m k_y y) \sin[(2n-1) k_w s + f_x^{(n,m)}],$$

$$B_y = \sum_{n=1}^{N_{sy}} \sum_{m=1}^{N_y} (m k_y B_x^{(n,m)} / k_x^{(n,m)}) \cosh(k_x^{(n,m)} x) \cos(m k_y y) \sin[(2n-1) k_w s + f_x^{(n,m)}],$$

$$B_s = \sum_{n=1}^{N_{sy}} \sum_{m=1}^{N_y} ((2n-1) k_w B_x^{(n,m)} / k_x^{(n,m)}) \cosh(k_x^{(n,m)} x) \sin(m k_y y) \cos[(2n-1) k_w s + f_x^{(n,m)}]$$

where $k_y = p / h, k_x^{(n,m)} = \sqrt{(2n-1)^2 k_x^2 + m^2 k_y^2}$ and h is height of magnetic material.

$B_x^{(n,m)} \sim 1/\cosh[k_x^{(n,m)} w]$.

Its vector potential in the axial gauge: $A_s=0$

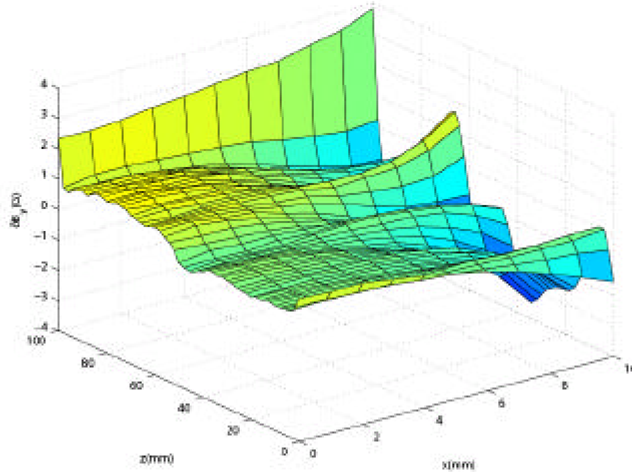
$$A_x = \sum_{n=1}^{N_{sy}} \sum_{n=1}^{N_y} (-m k_y B_x^{(n,m)} / k_x^{(n,m)} (2n-1) k_w) \cosh h(k_x^{(n,m)} x) \cos(m k_y y) \cos[(2n-1) k_w s + f_x^{(n,m)}],$$

$$A_y = \sum_{n=1}^{N_{sy}} \sum_{n=1}^{N_y} (B_x^{(n,m)} / (2n-1) k_w) \sinh(k_x^{(n,m)} x) \sin(m k_y y) \cos[(2n-1) k_w s + f_x^{(n,m)}]$$

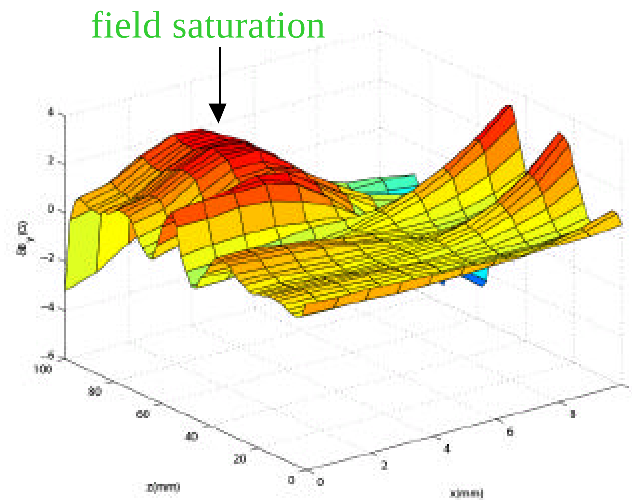
Klaus Halbach, FEB A95, LBL, 1995

DESY Dogbone Wiggler (λ_w=0.4m, w=60mm)

Field map of a quarter period includes B_x, B_y, B_s on grid of 11 x 11 x 51 with cubic dimension: 1mm x 1mm x 2mm. The residual of the fitting is a few Gausses as shown:



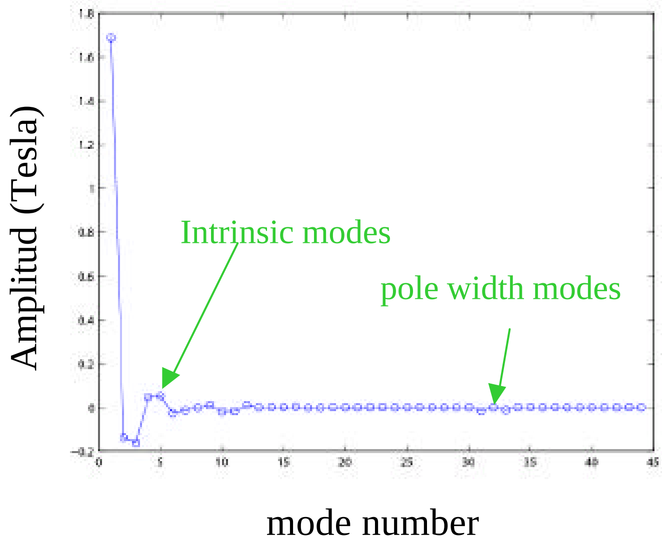
y = 0 mm



y=6 mm

Fixed parameters: λ_w=0.4m, h=0.025m and fitting parameters: N_s=30, N_{sy}=7, N_y=2 (44 modes or 88 parameters)

Amplitudes of Modes



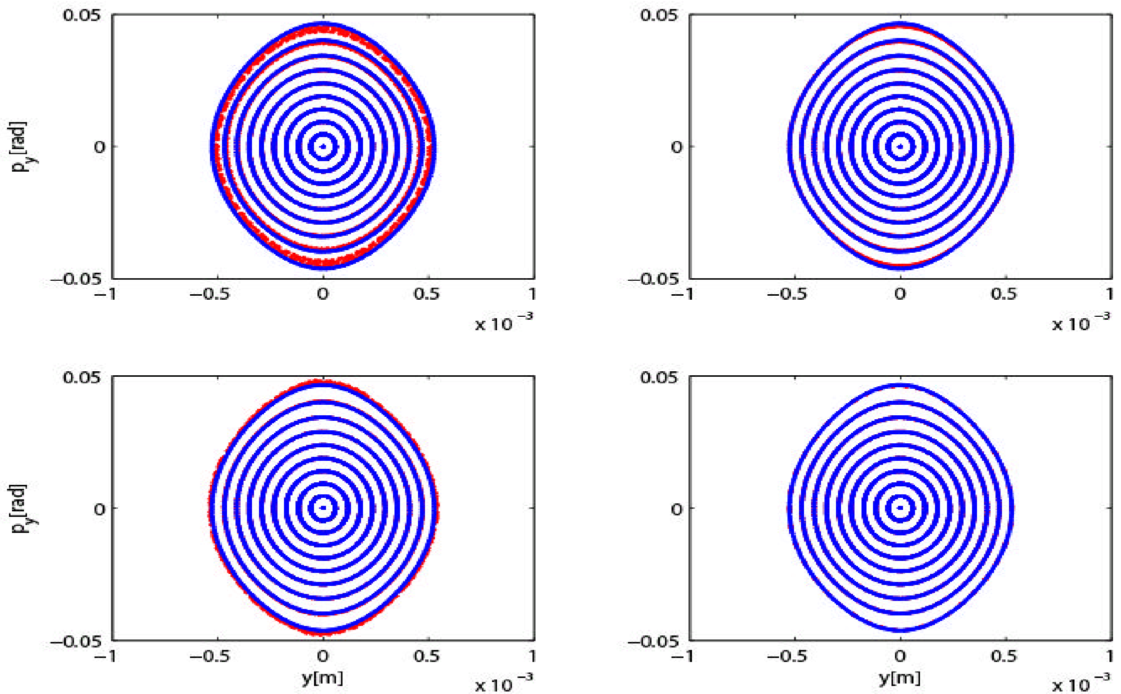
Large terms are those intrinsic ones and the pole width modes are small perturbation.

- Relative fewer modes in the fitting
- Modes has physical meaning and simple relation to wiggler parameters: λ_w, h, and w.
- Easily used to make specification of field quality of the wiggler magnet
- Modes due to saturation are not included

Symplectic Conditions in Hamiltonian System

6th order

8th order



Increment:
$5\sigma_y$

Taylor map (Zlib)

Mix-variable generating function (Zlib)

element-by-element tracking (LEGO)

Hamiltonian for Wiggler in Cartesian Coordinate

Hamitonians:

$$H(x, p_x, y, p_y, d, l; s) = -\sqrt{(1+d)^2 - (p_x - a_x)^2 - (p_y - a_y)^2} - a_s$$

where

$$\vec{a} = \frac{e}{cp_0}\vec{A}(x, y, s)$$

small-angle approximation

$$H = -(1+d) + \frac{1}{2(1+d)}[(p_x - a_x)^2 + (p_y - a_y)^2] - a_s$$

This Hamiltonian is used in LEGO

Hybrid Integrators for S-Dependent Hamiltonian

Separate H(s) into three exactly "solvable" parts:

$$H_0 = -(1+d) + \frac{1}{2(1+d)}(p_x^2 + p_y^2),$$ ← drift (explicit)

$$H_1 = \frac{1}{2(1+d)}(a_x^2 + a_y^2) - a_s,$$ ← kick (explicit)

$$H_2 = -\frac{1}{(1+d)}(p_x a_x + p_y a_y)$$ ← generating function

second-order integrator:

$$e^{-\int :H(s):ds} = \prod_{i=1}^{n}[e^{-\frac{:H_0:}{2}\Delta s} e^{-\frac{:H_1:}{2}\Delta s} e^{-:H_2:\Delta s} e^{-\frac{:H_1:}{2}\Delta s} e^{-\frac{:H_0:}{2}\Delta s} + O(\Delta s)^3]$$

- Can be easily shown using the Baker-Cambell-Hausdorf formula
- Becomes the exact solution at the limit of infinite number of segments
- Preserves symplectic condition during the integration

Mixed Variable Generating Function

Using generating function:

$$F_2 = \sum_{i=1}^{3} q_i \bar{p}_i + H(q, \bar{p}) \Delta s$$

To make a canonical transformation:

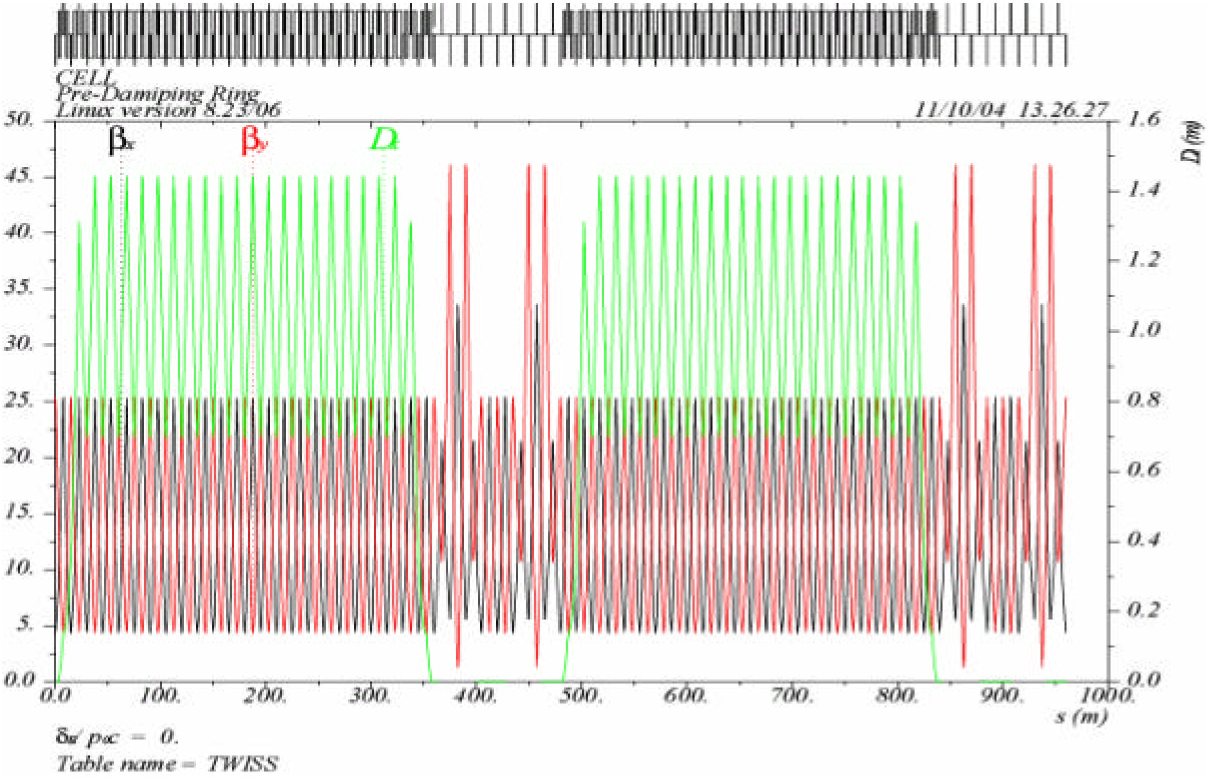
$$\bar{q}_i = \frac{\partial F_2}{\partial \bar{p}_i} = q_i + \frac{\partial H(q, \bar{p})}{\partial \bar{p}_i} \Delta s,$$

$$p_i = \frac{\partial F_2}{\partial q_i} = \bar{p}_i + \frac{\partial H(q, \bar{p})}{\partial q_i} \Delta s$$

The first order perturbation recovers Hamiltonian equation

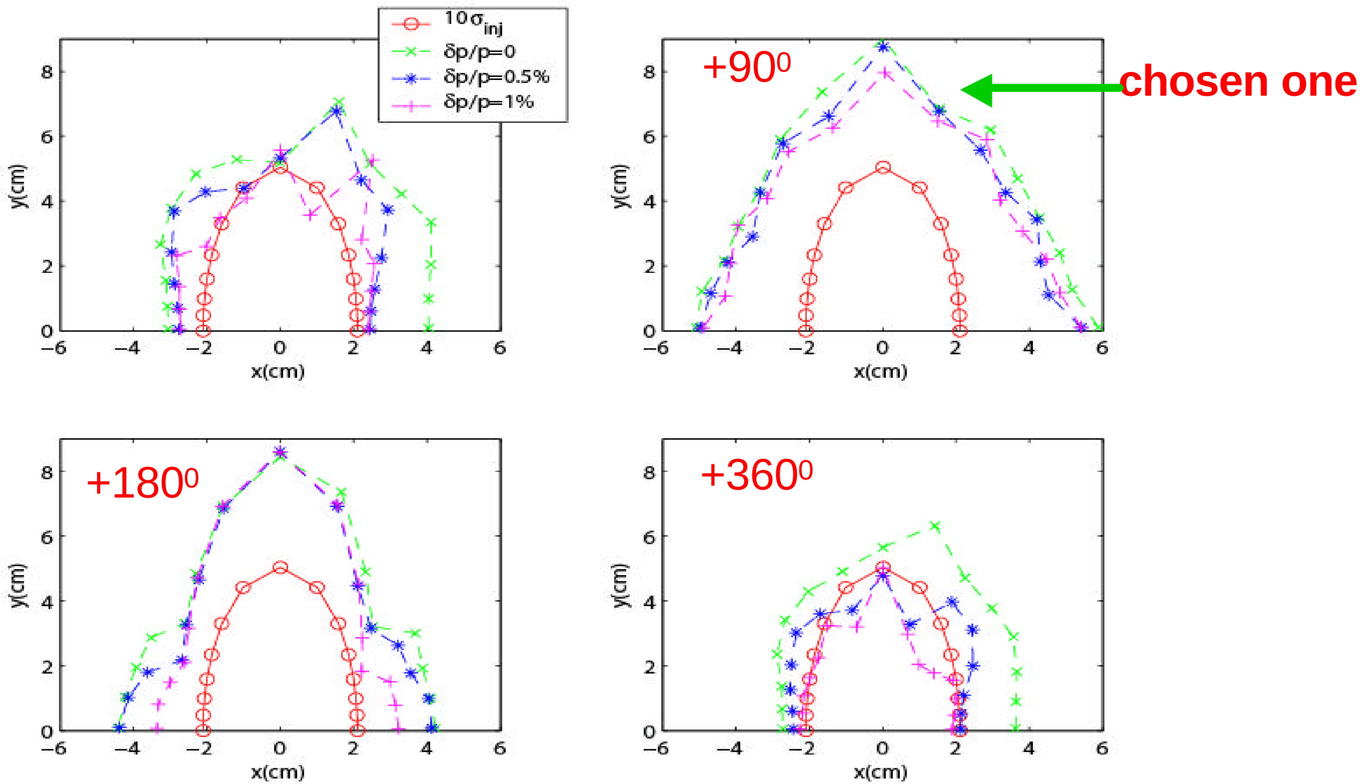
In general, these equations have to be solved numerically. But for special form of H, such as H_2 which has linear dependency to p_x and p_y, they are solvable analytically. In this case, we retain all the nice properties of explicit integrators. For example, use differential algebra to obtain high-order maps.

Lattice of a Simple Ring
with 90^0 FODO Cells



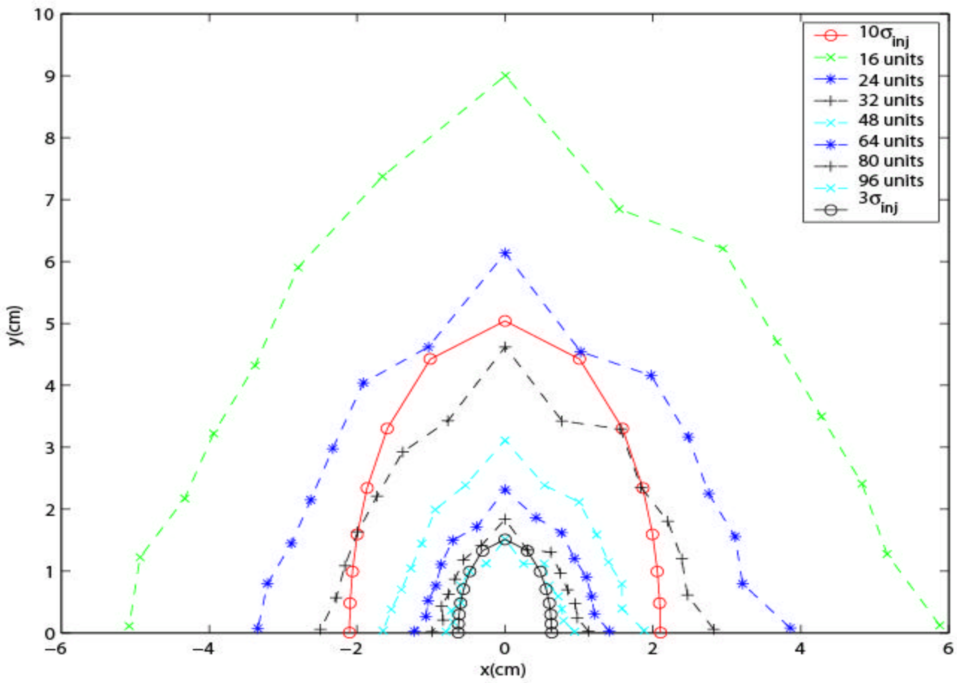
Two families of interlaced sextupoles: SF&SD

Optimizing Dynamic Aperture with the Phase Advance In Straights



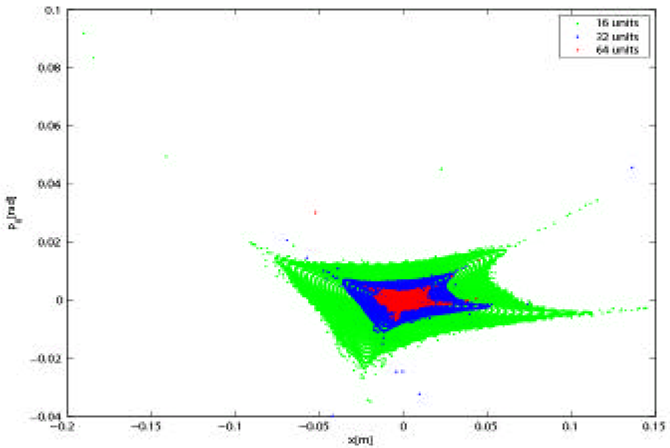
The best phase advances in straight section are nearly integer of 360^0, which maximizes the symmetry of the ring.

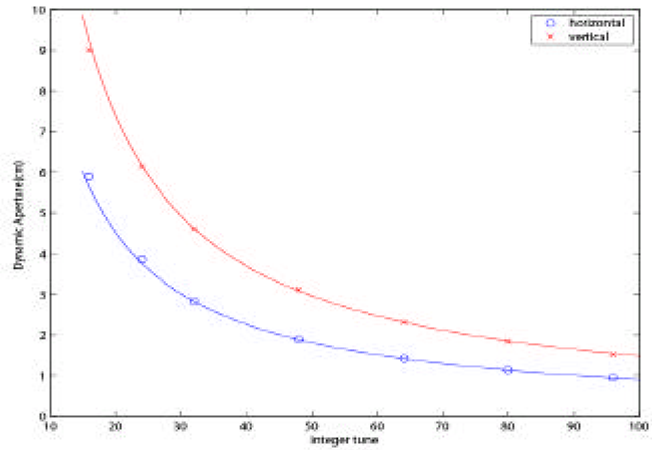
Dynamic Aperture v.s. Strength of Sextupoles



Dynamic aperture scales inversely proportional to the strength of the sextupoles! It is not so bad and it can be worse.

Scaling of Dynamic Aperture





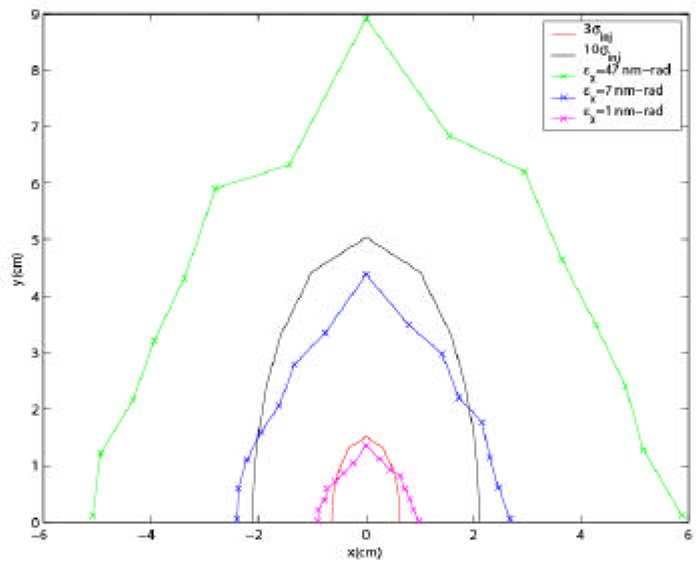
scaling of phase space

solid lines are inverse curves

Dynamic aperture is determined by the location of fix points In phase space when a single resonance dominates the system. Perturbation theory can be used to explain this scaling property of the dynamic aperture.

Reduce Emittance by Enlarging the Ring While Keeping the Cell Structure

Simulation of actual lattices:



40 cells -> 80 cells -> 160 cells,
e_x=47 nm -> 7 nm -> 1 nm
C=960 m -> 1560 m -> 2760 m

Scaling properties:

$$e_x \rightarrow e_x / 10$$

$$q_{dip} \rightarrow q_{dip} / \sqrt[3]{10} = q_{dip} / 2.15$$

$$N_c \rightarrow 2.15 N_c$$

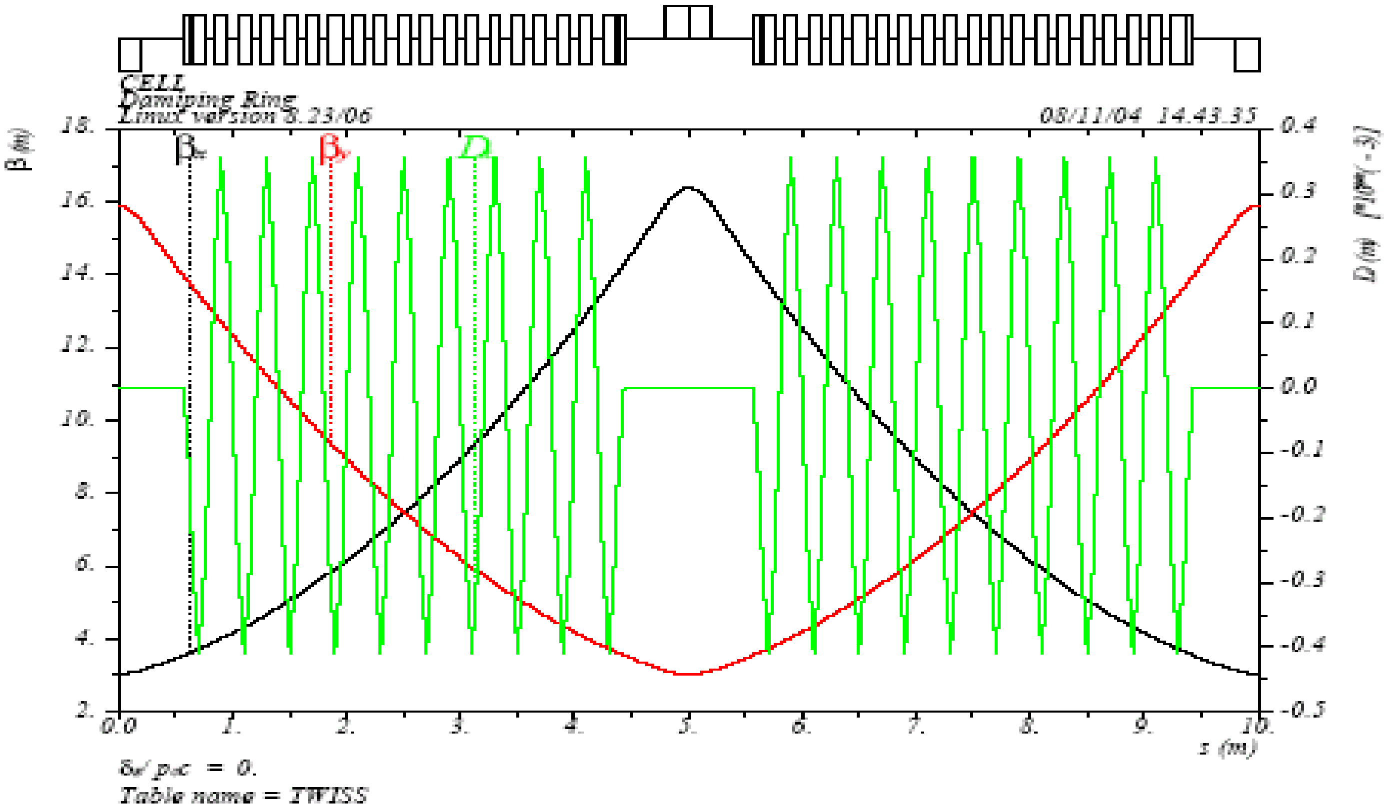
$$r_{dip} \rightarrow 2.15 r_{dip}$$

$$h_x \rightarrow h_x / 2.15$$

$$SF, SD \rightarrow 2.15(SF, SD)$$

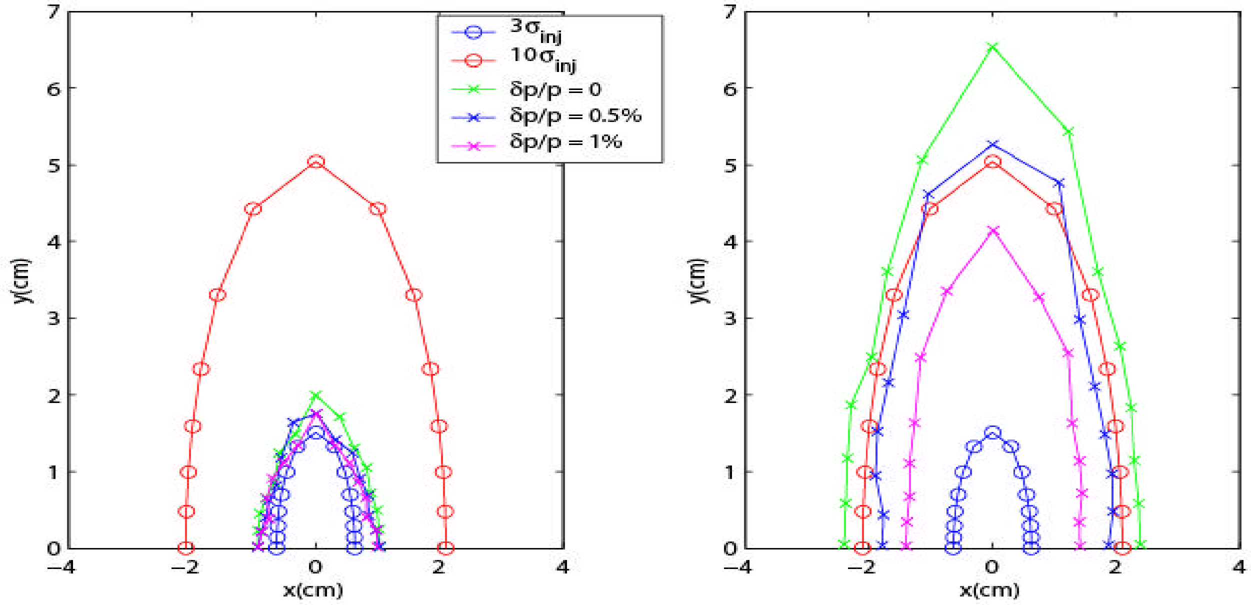
$$DA \rightarrow DA / 2.15$$

Add Wigglers to Reduce the Damping Time



100 meters of wiggler are used to reduce the damping time to 21 ms.

Non-Interlaced Sextupoles to Optimize Dynamic Aperture



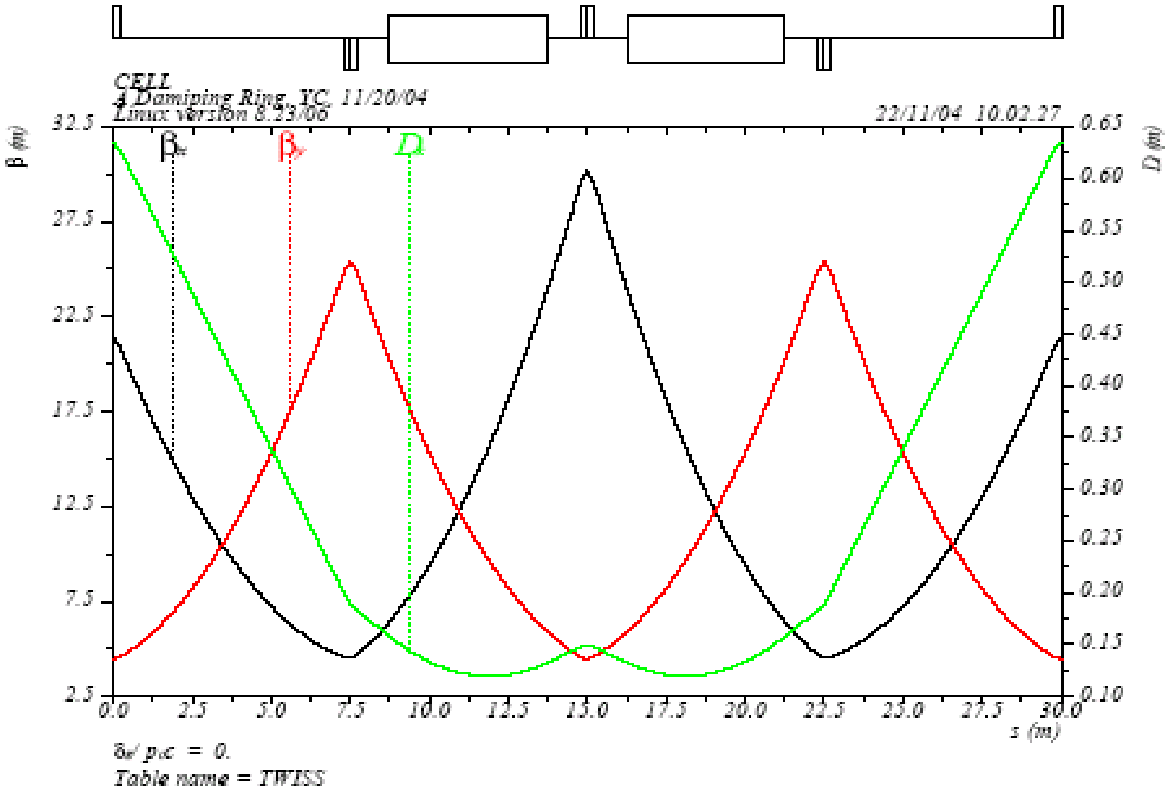
Interlaced sextupoles **non-interlaced sextupoles**

Non-interlaced sextepoles are three times stronger than interlaced ones.

Parameters of Damping Ring
Based on 90^0 FODO cell

Energy E	5 Gev
Circumference C	2820 meter
Horizontal Emittance ε_x	0.3 nm-rad
Damping time τ_x	21 ms
Tunes, ν_x,ν_y,ν_s	47.81, 47.68, 0.016
Momentum Compaction α_c	6.0×10^{-4}
Bunch length σ_z	2.2 cm
Energy spread σ_e/E	1.3×10^{-3}
Energy loss per turn U_0	4.5 Mev
Chromaticity ξ_x,ξ_y	-60, -60

Detuned π Cell: 179^0



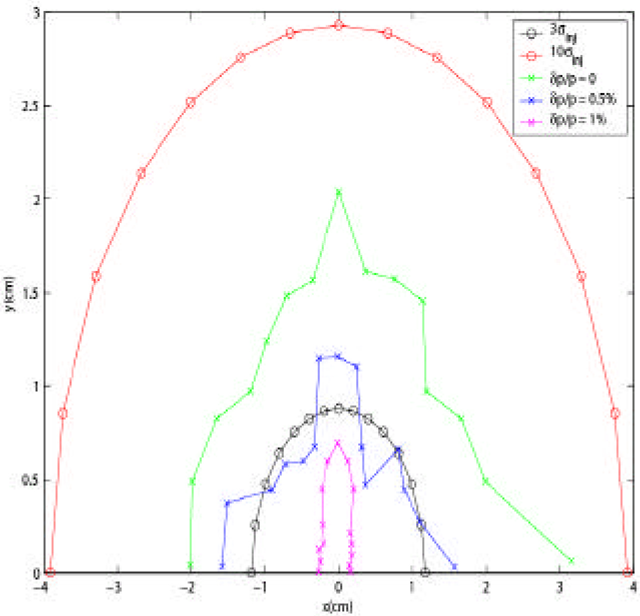
Parameters of a Compact Damping Ring Based on Detuned π cell

Energy E	5 Gev
Circumference C	2820 meter
Horizontal Emittance ε_x	0.49 nm-rad
Damping time τ_x	20 ms
Tunes, ν_x,ν_y,ν_s	47.81, 47.68, 0.021
Momentum Compaction α_c	2.83×10^{-4}
Bunch length σ_z	8.3 mm
Energy spread σ_e/E	1.27×10^{-3}
Energy loss per turn U_0	4.70 Mev
Chromaticity ξ_x,ξ_y	-60, -60

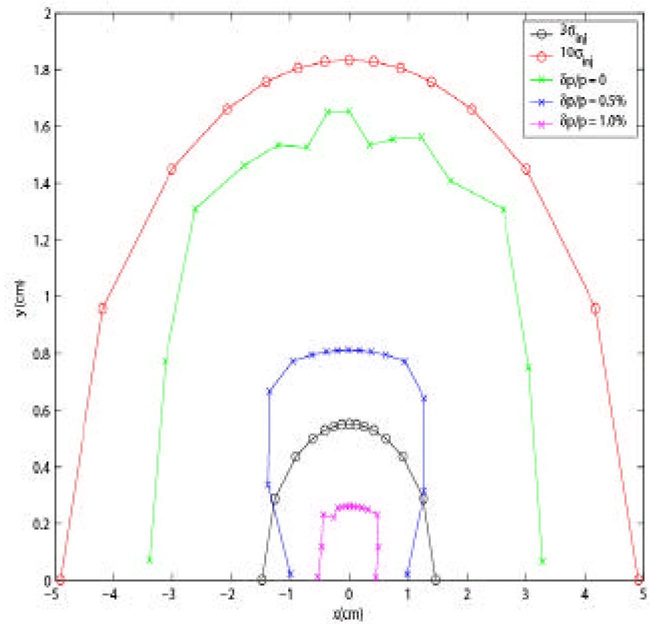
Dogbone Damping Rings

Parameters	DESY	SLAC
Energy E(Gev)	5	5
Circumference (m)	17,000	17,022
Horizontal emittance (nm)	0.50	0.62
Damping time (ms)	28	27
Tunes, ν_x,ν_y,ν_s	76.31, 41.18, 0.071	83.79, 83.64, 0.072
Momentum compaction α_c	1.22×10^{-4}	1.11×10^{-4}
Bunch length σ_z (mm)	6.04	5.90
Energy spread σ_e/E	1.29×10^{-3}	1.30×10^{-3}
Chromaticity ξ_x, ξ_y	-125,-62.5	-105, -106
Energy loss per turn (Mev)	20.4	21.0
Cavity Voltage (MVolt)	25	50

Dynamic Aperture Comparison



DESY dogbone damping ring

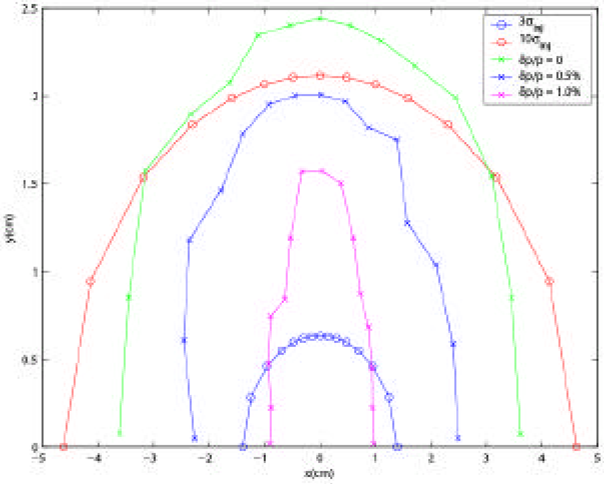


SLAC dogbone damping ring

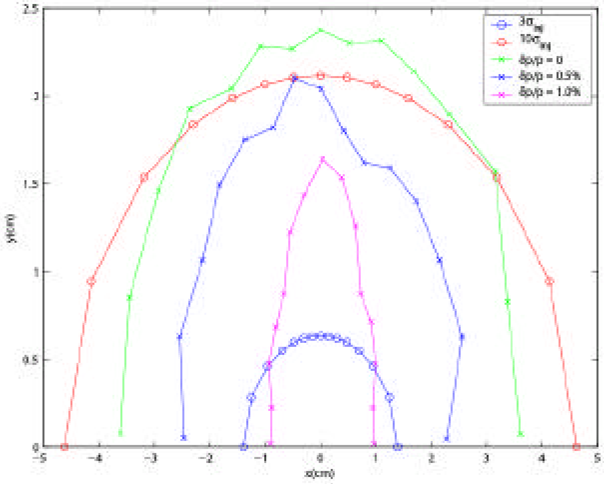
Injected beam: $\varepsilon_x = \varepsilon_y = 1\text{x}10^{-6}$ m-rad, tracked using LEGO with linear wigglers.

Dynamic Apertures with/without an Ideal Wiggler (single mode)

Linear wiggler



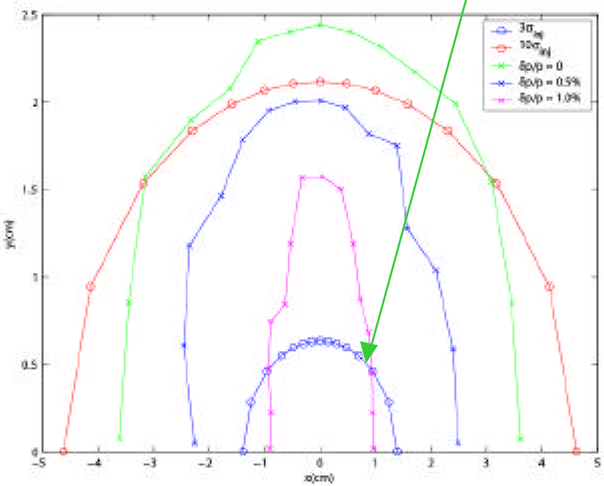
Ideal nonlinear wiggler



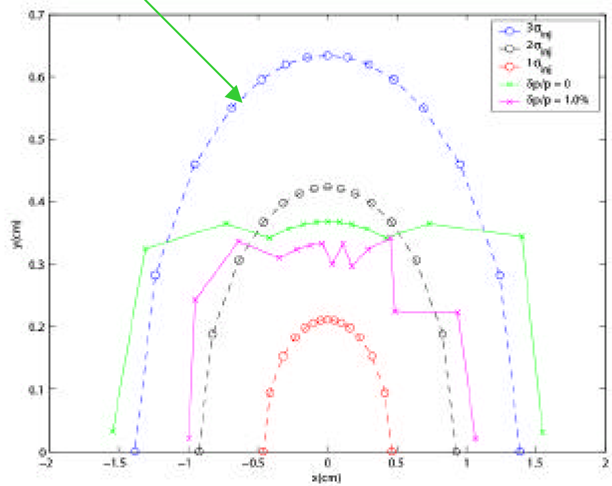
Impact on the dynamic aperture (CDR) is barely noticeable for an ideal but nonlinear wiggler. The problem of wiggler in the lattice is solvable with more engineering effort.

Dynamic Aperture with DESY Dogbone Wigglers

3σ of injected beam



Linear wiggler

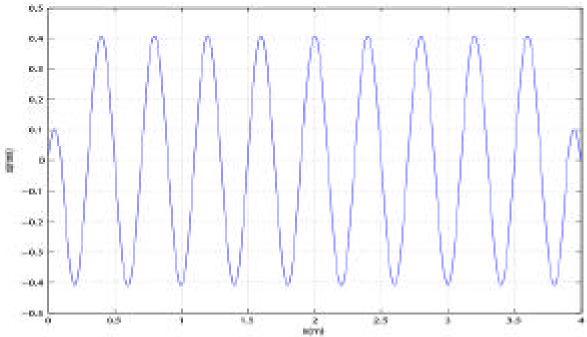


Full nonlinear wiggler

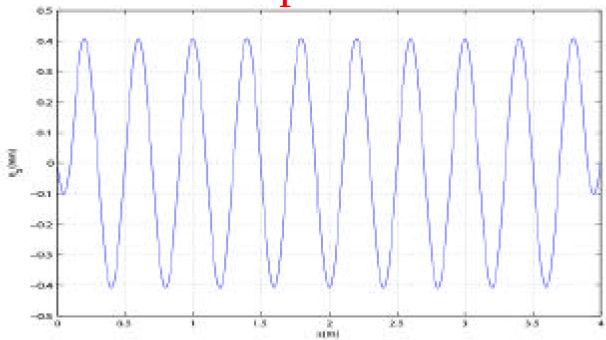
Dynamic aperture is entirely dominated by 24 wigglers in the lattice. They act like physical scrappers.

Other Dynamical Effects of Wigglers

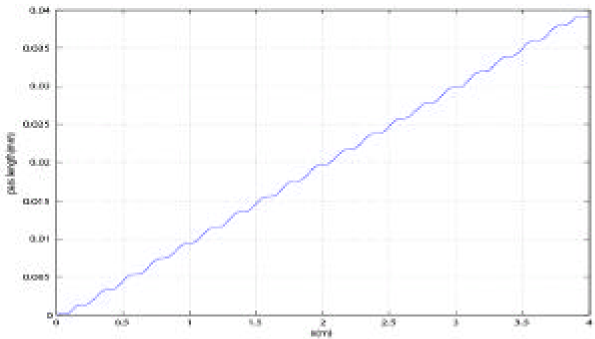
Orbit



Dispersion



Path length



- **Nonlinear end poles are matched for centering the orbit**
- **Path length of wiggled orbit is included as additional circumference**
- **Linear focusing is compensated with two families of quadrupoles**

Tunes vs. Amplitudes (CDR)

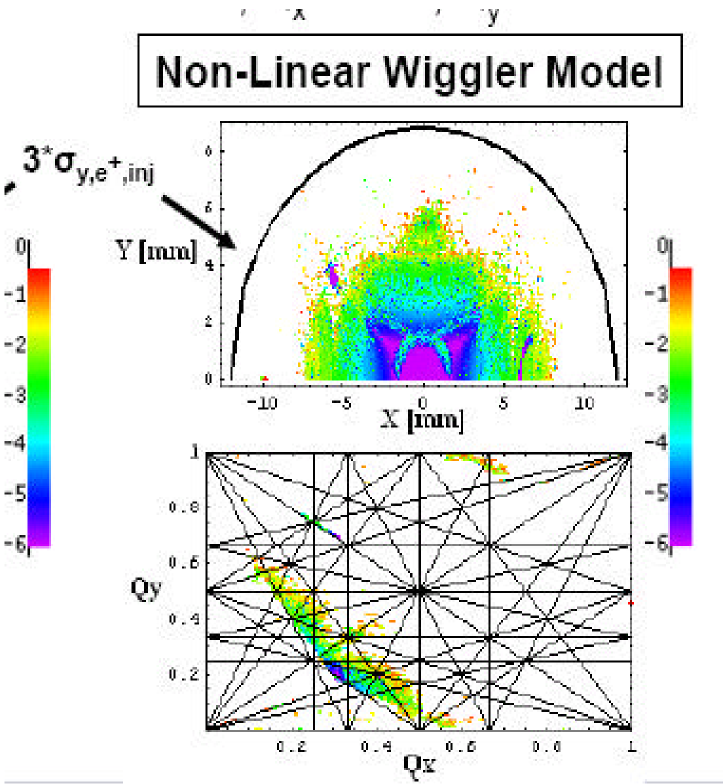
Calculated with nonlinear map and normal form using LEGO & LIELIB:

	Linear Wiggler	Ideal Wiggler	Full Wiggler
$\frac{\partial n_x}{\partial e_x}$	-4903	-4903	-33320
$\frac{\partial n_x}{\partial e_y}, \frac{\partial n_y}{\partial e_x}$	-616	-616	8754
$\frac{\partial n_y}{\partial e_y}$	-1153	-410	-36480

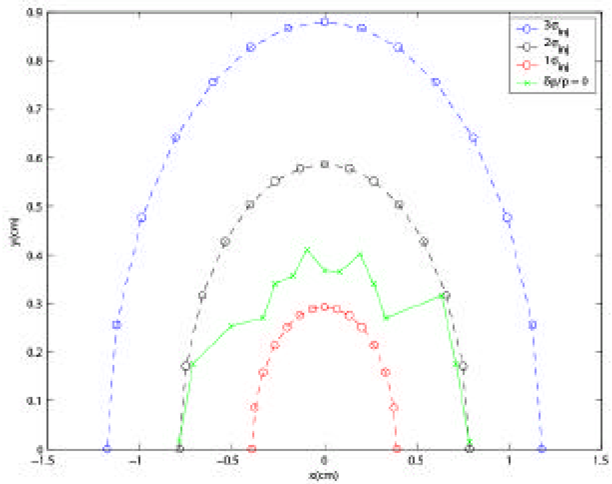
For ideal wiggler:

$$\frac{dn_y}{ds} = \frac{\sin^2 k_w s}{4p(1+d)r_0^2}[b_y(s) + k_w^2 b_y^2(s) J_y + \ldots]$$

Benchmark of Codes Using Dogbone Damping Ring (DESY)



Curtsey of Jeremy Urban



simulated using LEGO

Conclusion

- A simple and physical presentation is introduced to parameterize three-dimensional field of wiggler, including the end poles. The modes in the model have direct relation to the wiggler parameters.
- Hybrid symplectic integrators are developed to integrate through wiggler magnets. They are much simpler than the conventional explicit ones and therefore much fast for tracking. Because the special form of the Hamiltonian, they are still analytically solvable and hence high-order maps can obtained using the traditional differential algebra method.
- Full nonlinear wiggler designed at DESY, degrades the dynamic aperture in the lattices of damping rings designed for the International Linear Collider. However, we demonstrated that the wiggler problem is not a fundamental limitation of the lattices and it is solvable with more engineering efforts.
- Using non-interlaced sextupoles in damping rings is very effective way to increase the dynamic aperture. We believe that the dynamic aperture in these newly designed rings are adequate once the wigglers are improved in terms of field quality.