Wigglers vs. Undulators

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Introduction

Assuming

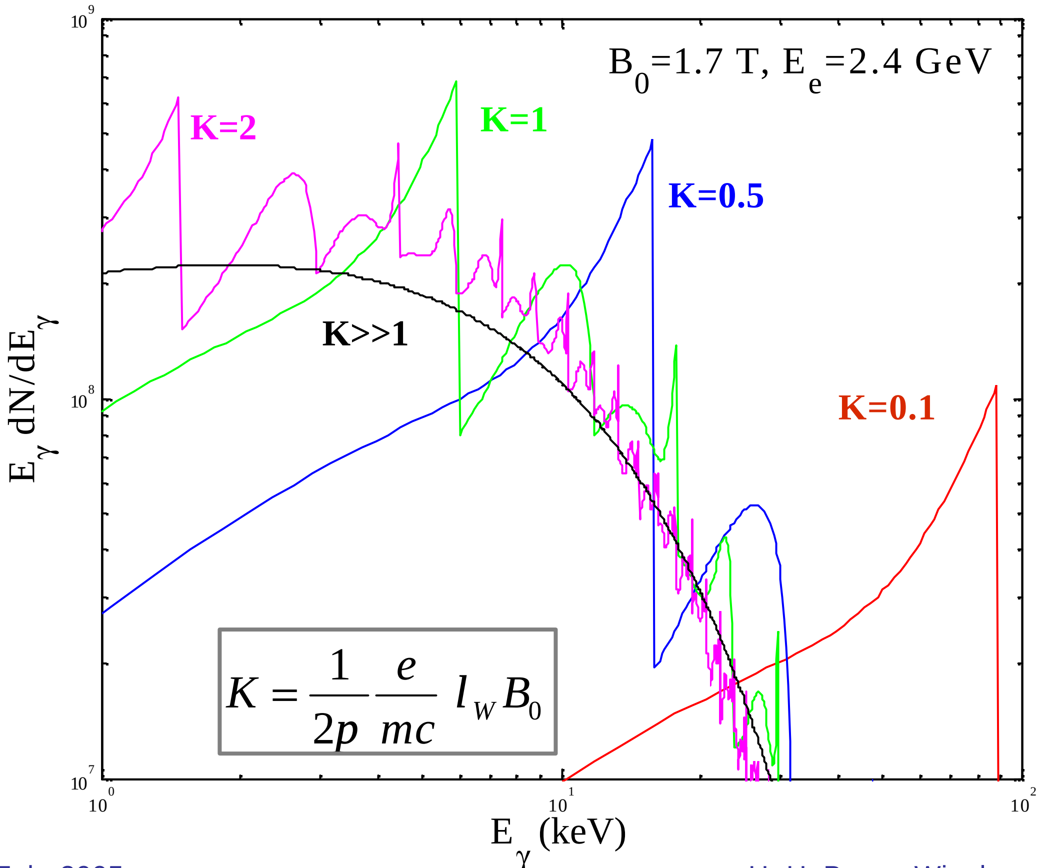
- A ring with damping and emittances dominated by SR from long wigglers
- Wigglers with field $B_Y = B_0 \sin(2\pi z/\lambda_W)$ and $N_W >> 1$
- No technical limitations for choosing B_0 and k_U

What are the best choices of B_0 and k_U to minimise emittances ?

Mean radiated power $P_W = \frac{r_0 c^3 e^2}{3m^3c^6} E_e{}^2 B_0{}^2$ depends only on beam energy and B_0

but photon spectrum and number dependence on λ_W

? damping time independent of λ_W , quantum excitation depends on λ_W



Emittance calculation

$$e_{HN} = \frac{t_H}{4Cm^2c^4g} \oint \dot{N}_{Phot} \left(\left(g_X h_X{}^2 + 2a_X h_X h'_X + b_X h'^{\,2}_X \right) \left\langle E_{Phot}{}^2 \right\rangle + b_X \left\langle E_{Phot}{}^2 x^2 \right\rangle \right) ds$$

$$\approx \frac{\overline{b}_X}{2mc^2 P_W} \left(\frac{K}{g} \overline{\cos^2\left(\frac{2p}{l_W} z \right) \dot{N}_{Phot} \left\langle E_{Phot}{}^2 \right\rangle} + \overline{\dot{N}_{Phot} \left\langle E_{Phot}{}^2 x^2 \right\rangle} \right)$$

$$e_{VN} \approx \frac{\overline{b}_X}{2mc^2 P_W} \overline{\dot{N}_{Phot} \left\langle E_{Phot}{}^2 y^2 \right\rangle}$$

$$\frac{\Delta E}{E} = \sqrt{\frac{\dot{N}_{Phot} \left\langle E_{Phot}{}^2 \right\rangle}{4P_W mc^2 g}}$$

$\dot{N}_{Phot}$, $\left\langle E_{Phot}{}^2 \right\rangle$, $\left\langle E_{Phot}{}^2 ?^2 \right\rangle$, $\left\langle E_{Phot}{}^2 ?^2 \right\rangle$ have to be calculated from $\frac{dN_{Phot}}{dE\, dO}$,

straight forward for $K >> 1$ and $K << 1$, but difficult for $K \approx 1$.

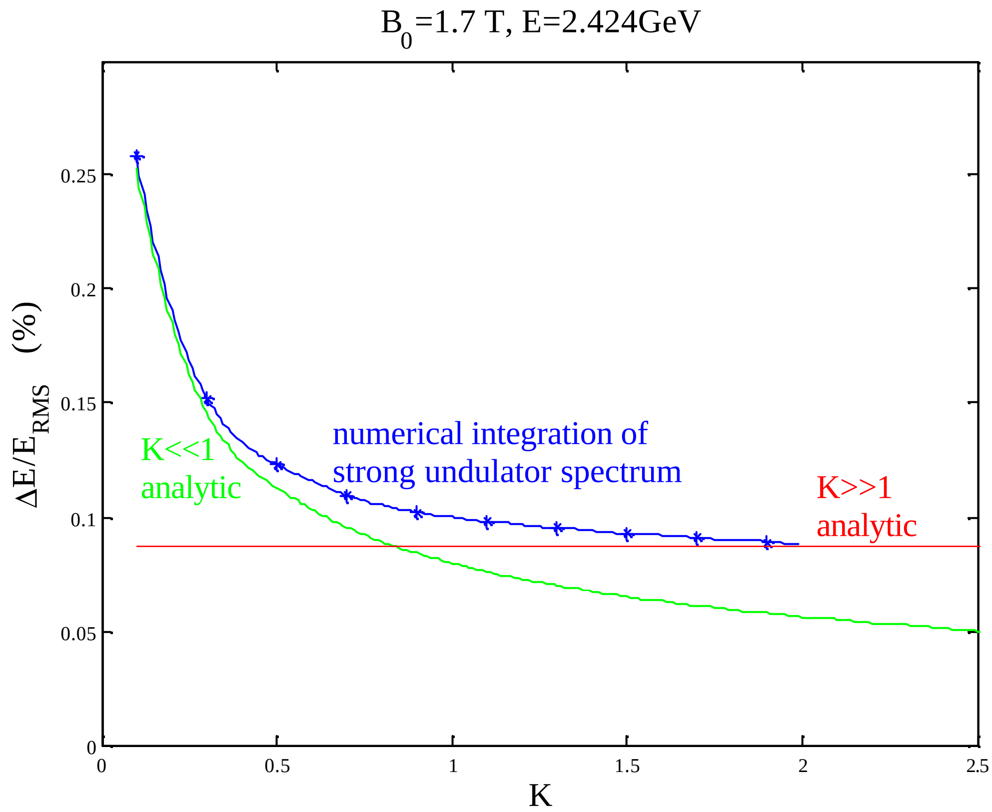
Energy spread

$$K \gg 1$$

$$\left.\frac{\Delta E}{E}\right|_{WIGGLER} = \sqrt{\frac{55\, l_C}{24\sqrt{3}\,p}\,\frac{K\, g}{l_W}}$$

$$K \ll 1$$

$$\left.\frac{\Delta E}{E}\right|_{UNDULATOR} = \sqrt{\frac{7\, l_C}{20}\,\frac{g}{l_W}}$$



Derivation of Undulator regime:
A. Hofmann, SSRL-ACD note 41,1986

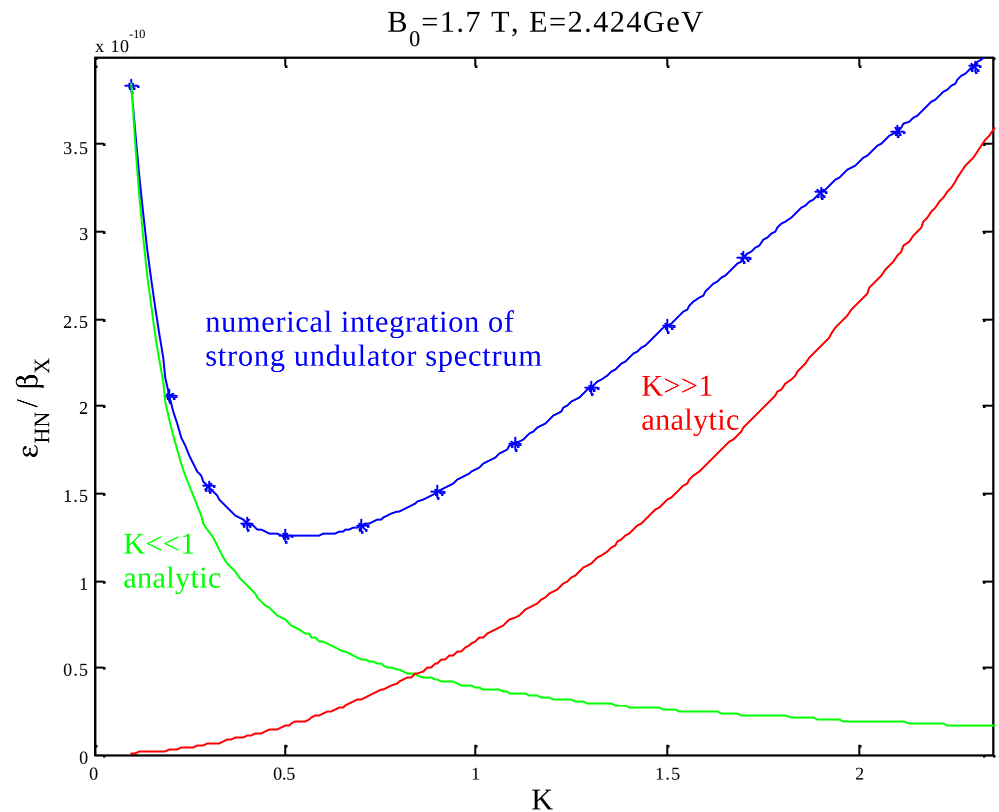
Horizontal Emittance

$K >> 1$

$$e_{HN} = \frac{11}{12\sqrt{3}p} \frac{l_C}{\,} \frac{\bar{b}_X K^3}{l_W}$$

$K << 1$

$$e_{HN} = \frac{l_C}{10} \frac{\bar{b}_X}{l_W}$$



Undulator regime:
Z. Huang and R. Ruth, PRL Vol 80, p. 976, 1998

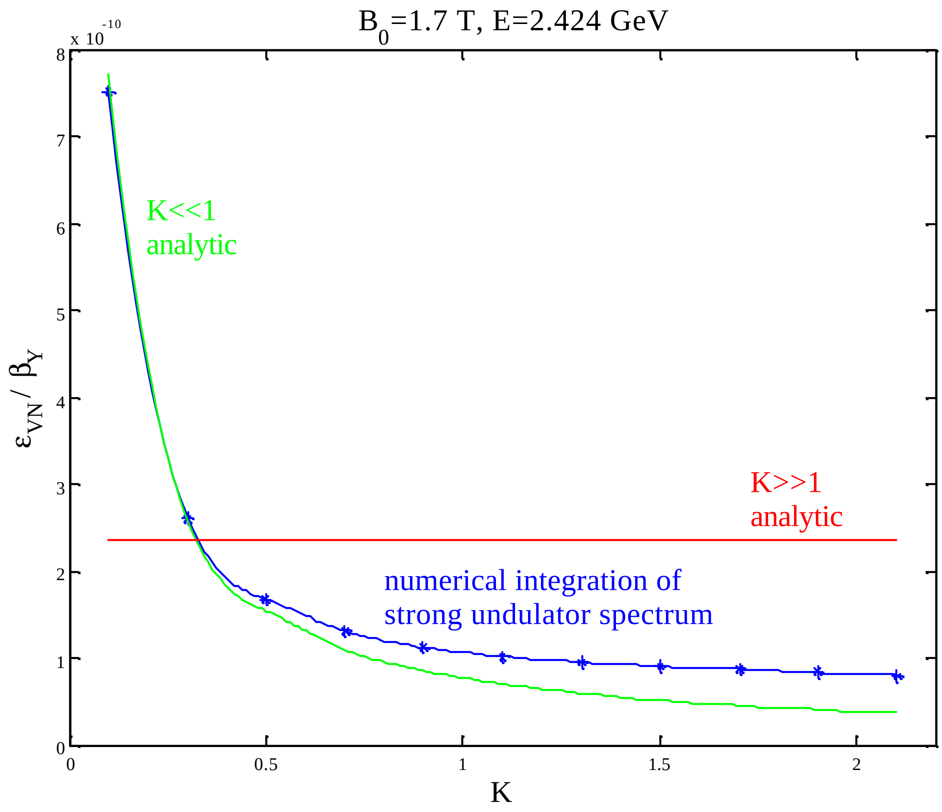
Vertical Emittance

$K >> 1$

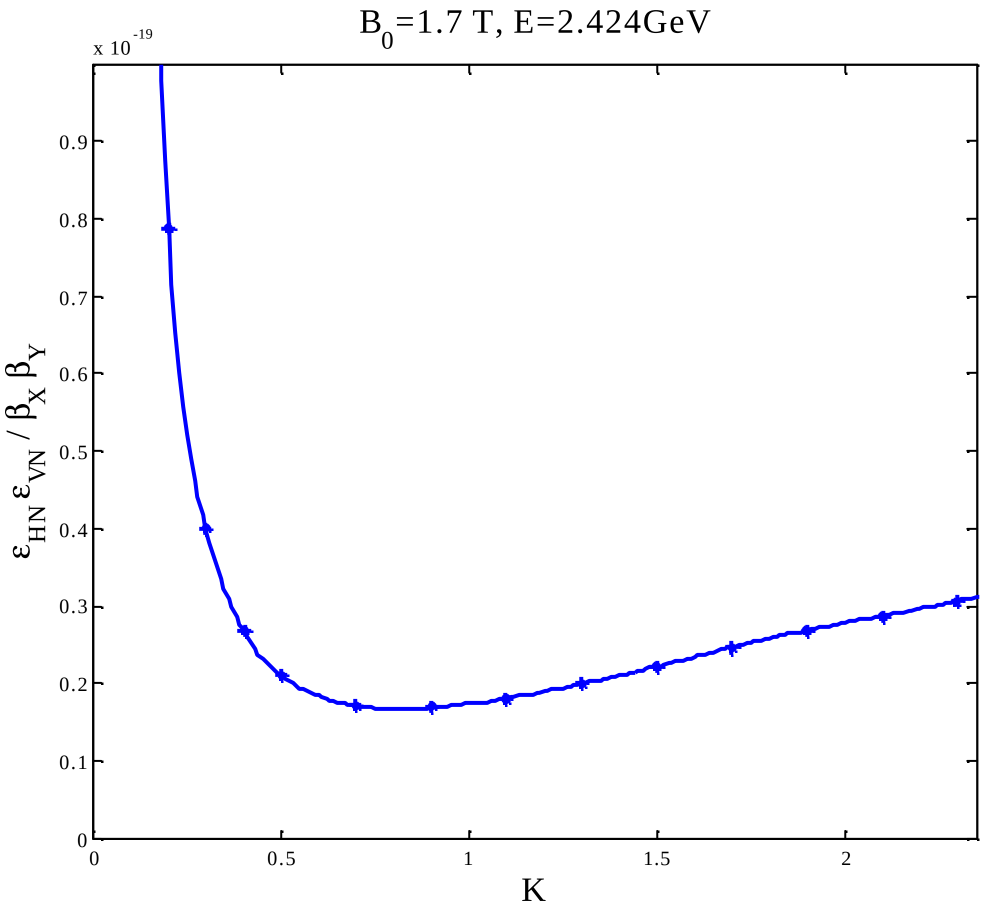
$$e_{VN} = \frac{10\ l_C}{3\sqrt{3}p}\frac{\overline{b}_Y K}{l_W}$$

$K << 1$

$$e_{VN} = \frac{l_C}{5}\frac{\overline{b}_Y}{l_W}$$

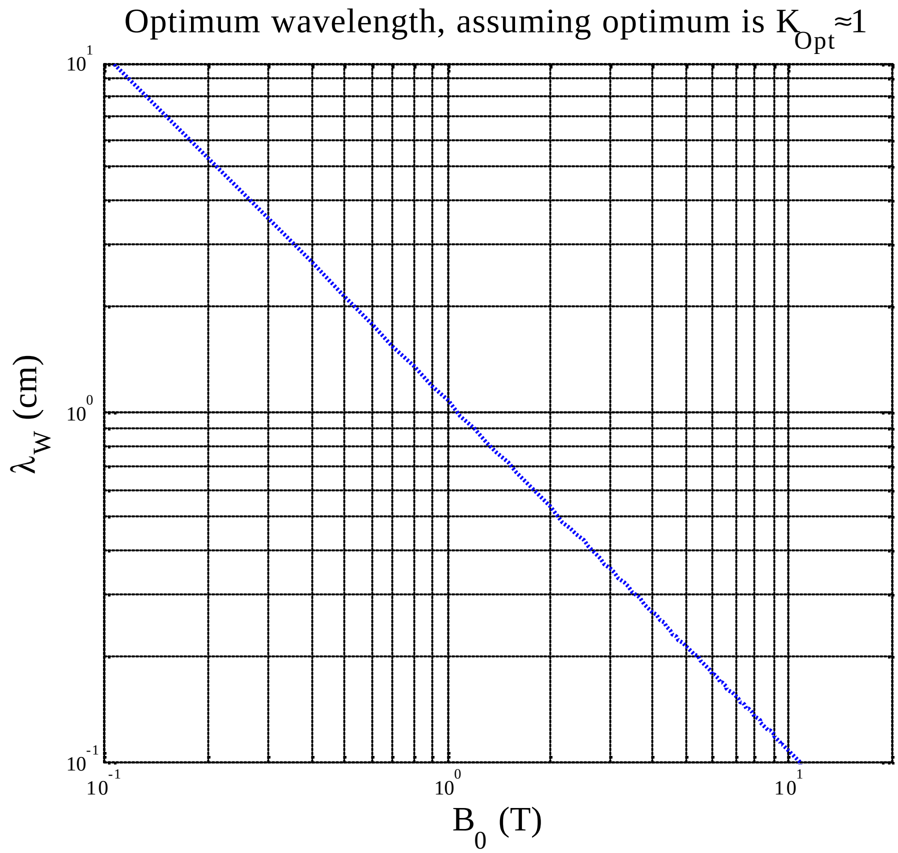


Product ϵ_{HN} ϵ_{VN}



Optimum wiggler wavelength

$$l_{W\ Opt} = \frac{2p\ m\,c}{e}\,\frac{K_{Opt}}{B_0}$$



Summary

- Energy spread increases for small λ_U and is constant for large λ_U
- Vertical emittance increases for small λ_U and is constant for large λ_U
- Horizontal emittance increases for small λ_U and is constant for large λ_U
- Transverse emittance product is minimised for $K \sim 0.5 - 1$
- For fixed B_0, λ_U and $\beta_{X,Y}$ normalised equilibrium emittances are independent of energy