Dark Matter and Extra Dimensions

F.Fucito, A.Lionetto and M.Prisco

INFN sez. Roma 2
Universitá di Roma, Tor Vergata

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Basics

1. SUSY's good properties:
   - cancellation of scalar divergences
   - unification of couplings
   - existence of a stable light particle
   - absence of a cosmological constant
   - solution of the hierarchy problem . . . many more

2. SUSY's bad properties:
   - it is not an exact symmetry
   - breaking it it's difficult: spontaneous vs. explicit (tree level)

SUSY breaking

1. Soft SUSY breaking at low energy
   - structure of divergences
   - generate a gaugino mass but no cosmological constant
   - phen. input (CP violation data) to stop # of params

- But what is the mechanism that gives the low-energy result?

SUSY breaking (Hidden sector)	Flavor-blind $\Longrightarrow$ interactions	MSSM sector (Visible sector)

Three solutions to this problem with similar features: primary breaking happens at a high energy scale with SSB

1. mSUGRA:
   - gravity mediation with tree level Planck suppressed couplings. fine tuning of the form of the superpotential and Khaeler potential
   - neutralino is bino-like
2. GMSB (gauge mediation):
   - mediation by gauge interactions
   - LSP likely to be the gravitino
3. AMSB (anomaly mediation):
   - SUSY breaking transmitted through R-symmetry and scale anomalies
   - some scalars can be tachyonic

Heterotic String

- Extra dimensions are not new (Kaluza-Klein theories)
- String theories need extra dimensions for consistencies (anomaly cancellations)
- A new appealing application: lowering the unification scale
- A first example: the heterotic string (closed strings) or when things go wrong

$$S = \int d^{10}x \left[\frac{1}{g_H^2 \ell_H^8} R + \frac{1}{g_H^2 \ell_H^6} F^2 + \dots \right]$$

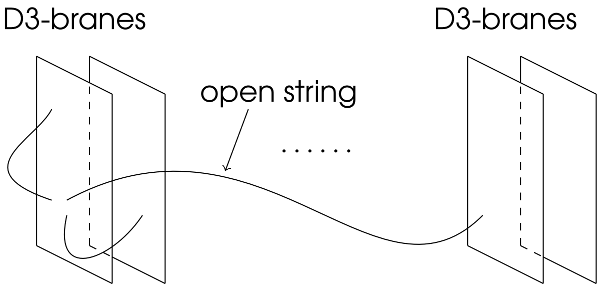
Upon compactification

$$\frac{1}{\ell_{Pl}^2} = \frac{V_6}{g_H^2 \ell_H^8} = \frac{1}{g_H^2} \frac{1}{\ell_H^2} \frac{V_6}{\ell_H^6} = \frac{1}{\ell_H^2} \frac{1}{g_{YM}^2}$$

$$\frac{1}{g_{YM}^2} = \frac{1}{g_H^2} \frac{V_6}{\ell_H^6} \Longrightarrow \frac{g_H^2}{g_{YM}^2} = \frac{V_6}{\ell_H^6}$$

Then $M_{Pl}^2 = g_{YM}^2 M_H^2 \Longrightarrow M_{Pl}^2 \approx M_H^2$. Moreover since $g_H^2 \approx 1 \Longrightarrow V_6 \approx \ell_H^6$:the compactification volume has string size length!!

Type I Strings



Type I Strings

$$S = \int d^{10}x \frac{1}{g_I^2 \ell_I^8} R + \int d^{p+1}x \frac{1}{g_I \ell_I^{p-3}} F^2 + \dots$$

Upon compactification

$$\frac{1}{\ell_{PI}^2} = \frac{V_{||} V_{\perp}}{\ell_I^8} \frac{1}{g_I^2} = \frac{V_{||} V_{\perp}}{\ell_I^8} \frac{1}{g_{YM}^2 \mathcal{V}_{||}^2} = \frac{\ell_I^{p-11}}{g_{YM}^4} \frac{V_{\perp}}{\mathcal{V}_{||}}$$

$$\frac{1}{g_{YM}^2} = \frac{V_{||}}{g_I \ell_I^{p-3}} \Longrightarrow g_I = g_{YM}^2 \mathcal{V}_{||}$$

- Taking $g_I \approx 1$ leads to $\mathcal{V}_{||} = \frac{V_{||}}{\ell_I^{p-3}} \approx 1$ i.e. the longitudinal volume is of the order of the string length.
- On the other side $V_\perp = R_\perp^{9-p}$ is unconstrained. If $\ell_I \approx 1 TeV$ than $R_\perp = 10^8 km, .1 mm$ down to $.1 Fermi$ for $n = 9 - p = 1, 2, 6$ large dimensions.
- So there are values of the compactified transverse dimensions for which string (quantum gravity) effects are as low as the TeV scale.

General scenario

- So we have seen that in some string models we can have large transverse dimensions
- The effects of compactifying, from the field theory point of view, reduce to the presence of massive KK modes with $p_i \equiv n_i/R_\perp$ and $m_n^2 = m_0^2 + \frac{\mathbf{n}\cdot\mathbf{n}}{R_\perp^2}$
- Models that look at phenomenology in this general scenario are called bottom-up. Models that try to build consistent D-branes models (with the standard model field content, no tachyons, fixed moduli etc.) are called top-down. In the following we will discuss a model of the former type, achieving unification at the TeV scale.

- Let $\vec{x} \equiv (x_1, x_2, x_3, x_4)$ and $\vec{y} = (y_1, \ldots, y_\delta), \delta = D - 4$ with $y_i \rightarrow y_i + 2\pi R_\perp$

$$\Phi(x, y) = \sum_{(n_1,\ldots,n_\delta)\in\mathbb{Z}^\delta} \Phi^{(\mathbf{n})}(x) e^{i\mathbf{n}\cdot\mathbf{y}/R_\perp}$$

- Not all MSSM have KK states. For example chiral states, to form a KK mass must appear together with their chiral conjugate mirror. Let η be the # generations with KK states item $\eta = 0$: the KK states of the two Higgs and the KK vector bosons+1 chiral supermultiplet get arranged into two $N = 2$ multiplets

1. The appearance of $N = 2$ multiplets is not surprising
   - Extra dimensions imply extended SUSY
   - $N = 2$ needed for á la Wilson renormalization

2. For $\eta > 0$ the KK excitations of the chiral fermions appear with their mirrors

3. But how it is possible to have $N = 2$ multiplets if zero modes are $N = 1$? And how it is possible to decouple some particles from the others?

- Let's take $\delta = 1$ and $\Phi(x) = \Phi_+(x) + \Phi_-(x)$

$$\Phi_+(x, y) = \sum_{n=0}^{\infty} [\Phi^{(\mathbf{n})}(x) + \Phi^{(-\mathbf{n})}(x)] \cos(ny/R_\perp)$$
$$\Phi_-(x, y) = \sum_{n=0}^{\infty} [\Phi^{(\mathbf{n})}(x) - \Phi^{-(\mathbf{n})}(x)] \sin(ny/R_\perp)$$

- Now we have

$$\Phi_+(x, -y) = +\Phi_+(x, y)$$
$$\Phi_-(x, -y) = -\Phi_-(x, y)$$

- $\Phi_-(x)$ lacks a zero mode so A, λ can be even while the chiral multiplet odd

Compactifying on orbifolds

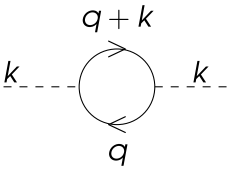
- The condition $y \to -y$ defines and orbifold: S^1 with opposite points identified i.e. $S^1/\mathbb{Z}_2$: $\mathbb{Z}_2 = \{\omega; \omega^2 = 1 : \omega = 1, \omega = \exp(i\pi) = -1\}$
- The orbifold has special points: the fixed points $y^{(A)} = 0, y^{(B)} = \pi R_\perp$

$$\Phi(x, y) = \Phi^{(A)}(x)\delta(y) + \Phi^{(B)}(x)\delta(y - \pi R_\perp)$$

- Fields at orbifold points have no KK tower of states
- Orbifold work for closed strings, for open strings we need orientifold

Early Unification

Let's compute the vacuum polarization

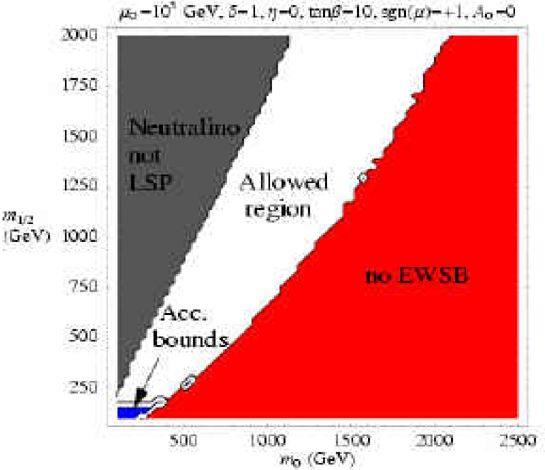


$$\Pi \approx \sum_{n_i=-\infty}^{\infty} \int_0^{\infty} \frac{d^4 q}{(2\pi)^4} \left\{ \frac{-(k+q)\cdot q + 2m_n^2}{(q^2 - m_n^2)[(k+q)^2 - m_n^2]} \right\}$$

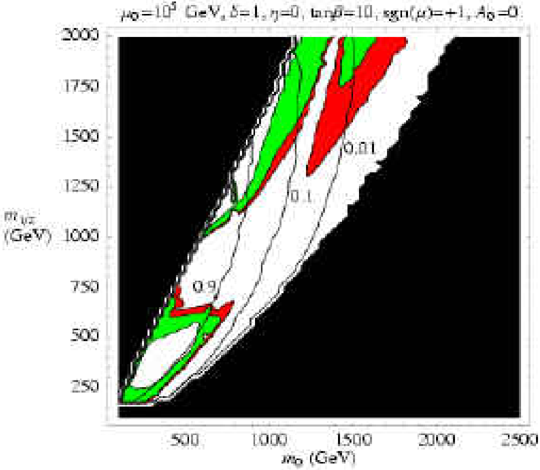
$$\approx \int_0^{\infty} \frac{dt}{t} \left[\vartheta_3 \left(\frac{it}{\pi R_\perp^2} \right) \right]^{\delta}$$

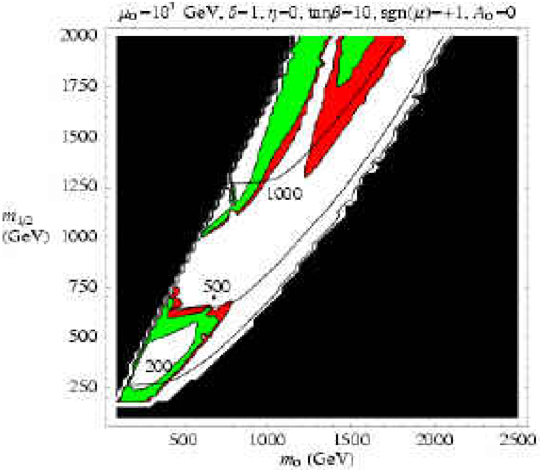
Finally putting $\mu_0 = R_\perp^{-1}$ we find

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(\mu_0) - \frac{b_i - \tilde{b}_i}{2\pi} \ln \frac{\Lambda}{\mu_0} - \frac{\tilde{b}_i X_\delta}{2\pi\delta} \left[\left(\frac{\Lambda}{\mu_0} \right)^\delta - 1 \right]$$



$\mu_0 = 10^8$ GeV, $\delta = 1$, $\eta = 0$, $\tan\beta = 10$, sgn(μ) = +1, $A_0 = 0$
Neutralino not LSP
Allowed region
no EWSB
Acc. bounds
$m_{1/2}$ (GeV)
m_0 (GeV)


$\mu_0=10^5$ GeV, $\delta=1$, $\eta=0$, $\tan\beta=10$, sgn$(\mu)=+1$, $A_0=0$
$m_{1/2}$ (GeV)
m_0 (GeV)
0.9
0.1
0.01



$\mu_0 = 10^7$ GeV, $\delta = 1$, $\eta = 0$, $\tan\beta = 10$, sgn(μ) = +1, $A_0 = 0$
2000
1750
1500
1250
$m_{1/2}$ (GeV)
1000
750
500
250
1000
500
200
500
1000
1500
2000
2500
m_0 (GeV)