# Future precision neutrino-oscillation experiments

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# Neutrino mixing and oscillations: where do we stand ?

### Our knowledge about neutrino masses and mixing

- there exist 3 'light' neutrinos (LEP):  $N_V = 2.984 \pm 0.008$
- limits from direct mass measurements are small (tritium & cosmology):

WMAP:  $\Sigma_{i} m_{i} < 0.7 \text{ eV} (95\% \text{ CL})$ 

- solar and atmospheric neutrino deficit: neutrinos mix (oscillations) → they are massive: m (heaviest v) > ~ 0.05 eV PMNS matrix (3 x 3)
- oscillation parameters: 2 large mixing angles  $\theta_{sol} \sim \theta_{12}$ ,  $\theta_{atm} \sim \theta_{23}$ 2 independent mass splittings:  $\Delta m_{sol}^2 \sim \Delta m_{12}^2$ (masses are small, indeed)  $\Delta m_{atm}^2 \sim \Delta m_{23}^2$

### What we do not know...

- absolute mass values (and why are they small ?)
- why  $\theta_{12}$  and  $\theta_{23}$  angles are large and  $\theta_{13}$  seems very small or null ?
- is mass hierarchy the same as for charged leptons (sign of  $\Delta m_{23}^2$ )
- is there any CP violating phase in the mixing matrix ?

#### LSND effect ? Wait for MiniBoone...

# Our knowledge of the oscillation parameters. Global fits from Maltoni et al. 03, Valle 05, Fogli et al. 06 (all data included, except the very recent MINOS data)





 $\sin^2\theta_{13} < 0.04 \rightarrow \sin^22\theta_{13} < 0.15$  and  $\theta_{13} < 11^{\circ}$ 

### Goals of planned and future neutrino beam experiments:

- observe  $v_{\tau}$  appearance  $\rightarrow$  ...find the body after the murder...
- is there (some) room for a sterile neutrino?  $\rightarrow$  MiniBoone and  $v_{\mu}$  disappearance
- measure L/E dependence  $\rightarrow$  atmospheric and WBB experiments (fixed L)
- accurately measure the two  $\Delta m^2$ ,  $\theta_{12}$  and  $\theta_{23} \rightarrow is \theta_{23}$  exactly  $\pi/4$  ?
- find the value of  $\theta_{13}$  from  $P(v_u v_e) \rightarrow key$  measurement
- show MSW matter effects (without CP violation effects) → mass hierarchy
- show CP violating effects (without matter effects) → the ultimate goal ?
- •...be ready for the unexpected  $! \rightarrow$  experiments may be running for long time...

### this talk: focus on accelerator experiments

### Neutrino mixing matrix and general 3 neutrino oscillation probability

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} \nu_1 \\ e^{i\alpha_2/2} \nu_2 \\ \nu_3 \end{pmatrix}$$

$$P(\nu_{\ell} \to \nu_{\ell'}) = |\sum_{i} U_{\ell i} U_{\ell' i}^{*} e^{-i(m_{i}^{2}/2E)L}|^{2}$$
$$= \sum_{i} |U_{\ell i} U_{\ell' i}^{*}|^{2} + \Re \sum_{i} \sum_{j \neq i} U_{\ell i} U_{\ell' i}^{*} U_{\ell j}^{*} U_{\ell' j} e^{i\frac{|m_{i}^{2} - m_{j}^{2}|L}{2E}}$$

The formula simplifies under the empirical assumptions that:

•  $\Delta m_{atm}^2 >> \Delta m_{sol}^2$ 

• L is comparable to the atmospheric oscillation length (~ 1000 km)

• the angle  $\theta_{13}$  is small

For the special case of  $\nu_{\mu} \rightarrow \nu_{e}$  oscillations, we have:



# Work just done or in progress...



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### Fresh results from MINOS in the NuMi neutrino beam



- low E neutrinos (few GeV):  $\nu_{\mu}$  disappearance experiment
- 4 x10<sup>20</sup> pot/year  $\rightarrow$  2500  $v_{\mu}$  CC/year
- compare Det1-Det2 response vs E  $\rightarrow$  in 2-6 years sensitivity to  $\Delta m^2_{atm}$
- main goal: reduce the errors on  $\Delta m_{23}^2$  and  $sin^2 2\theta_{23}$  as needed for  $sin^2 2\theta_{13}$  measurement



### 1 kton MINOS near detector (Fermilab)



5.4 kton MINOS far detector (Soudan Mine)

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# Example of a $v_{\mu}$ disappearance measurement

• Look for a deficit of  $v_{\mu}$  events at Soudan...







# Numbers of observed and expected events

Data sample	observed	expected	ratio	significance
All CC-like events $(v_{\mu}+\overline{v}_{\mu})$	204	298±15	0.69	<u>4.1</u> σ
$v_{\mu}$ only (<30 GeV)	166	249±14	0.67	<mark>4.0σ</mark>
$\nu_{\mu}$ only (<10 GeV)	92	177±11	0.52	5.0σ

- We observe a 33% deficit of events between 0 and 30 GeV with respect to the no oscillation expectation.
  - Numbers are consistent for  $v_{\mu}+\overline{v_{\mu}}$  sample and for the  $v_{\mu}$ -only sample
- The statistical significance of this effect is 5 standard deviations

# Best-fit spectrum

Oscillation Results for 0.93E20 p.o.t



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## au appearance at LNGS in the CNGS beam



- High energy beam: <E> ~17 GeV: τ appearance search
- 4.5 x10<sup>19</sup> pot/year from the CNGS. In the hypothesis of no oscillation:
- 2600  $v_{\mu}$  CC/year per kton detector mass
- Assuming  $v_{\mu} v_{\tau}$  oscillation, with parameters sin<sup>2</sup>2 $\theta$  =1 and  $\Delta m^2$ =2.5x10<sup>-3</sup> eV<sup>2</sup>: 15  $v_{\tau}$  CC interactions /year per kton
- CNGS beam ready to be commissioned
- A dedicated experiment at LNGS: OPERA

#### The OPERA experiment at LNGS: the rebirth of the emulsion technique

#### see G.Giacomelli's talk

- detector: 1800 ton emulsion/lead bricks (ECC technique) complemented by tracking scintillator planes and two muon spectrometers
- industrial emulsion production and handling
- need huge scanning power/speed: > tens of automatic microscope running in parallel
   @ 20 cm<sup>2</sup>/hour (advances of the technique)



- specialized, single task experiment
- low BG: <1 event (τ track reconstruction)
- low statistics: about 10 events/5 years at nominal CNGS intensity

@ SK parameter values: statistics goes like  $(\Delta m^2)^2$ 

- aim at beam intensity increase
- -installation in progress: technical runs in 2006 with first  $\boldsymbol{\nu}$  events
- huge amount of work for infrastructure: film handling, brick construction, film scanning



#### The OPERA detector installation at the Gran Sasso Hall C



One of the automatic scanning microscopes equipped with a robot for emulsion-film feeding & replacement (Bern University)



Artistic view of the full detector

# Measure $\theta_{13}$

### Simple considerations

 $v_{\mu} \rightarrow v_{e}$  oscillation as a tool to measure  $\theta_{13}$  with accelerator neutrino experiments. Future 'Super-CHOOZ-like' reactor experiments are difficult (and not covered here). Existing or planned atmospheric neutrino detectors can be limited by statistics.

small effect (< 5% from CHOOZ)</li>

- prompt  $v_e$  contamination at % level (accelerator neutrino beams)
- main BG:  $\pi^{\circ}$  production in NC and CC interactions
- additional BG: low energy muons and pions can fake electrons



 $v_e \rightarrow v_{\mu}$  oscillations can solve most of the problems but hard to make  $v_e$  beams (wait for a next generation facilities)

In any case high intensity is a must !

### Need high intensity: future neutrino facilities





### The first Super-Beam: off-axis T2K, from JAERI at Tokai to SK



#### A liquid Argon TPC for T2K:

#### a mid-term goal in a global long term strategy centered on LAr TPC detectors

• Experiments for CP violation: a giant liquid Argon scintillation, Cerenkov and charge imaging experiment,

A.Rubbia, Proc. II Int. Workshop on Neutrinos in Venice, 2003, Italy, hep-ph/0402110

- Ideas for future liquid Argon detectors,
  - A. Ereditato and A.Rubbia, Proc. Third International Workshop on Neutrino-Nucleus Interactions in the Few GeV Region, NUINT04, March 2004, Gran Sasso, Italy, Nucl.Phys.Proc.Suppl.139:301-310,2005, hep-ex/0409034
- Ideas for a next generation liquid Argon TPC detector for neutrino physics and nucleon decay searches,
  - A. Ereditato and A.Rubbia, Proc. Workshop on Physics with a Multi-MW proton source, May 2004, CERN, Switzerland, submitted to SPSC Villars session
- Very massive underground detectors for proton decay searches,

A.Rubbia, Proc. XI Int. Conf. on Calorimetry in H.E.P., CALOR04, Perugia, Italy, March 2004, hep-ph/0407297

• Liquid Argon TPC: mid & long term strategy and on-going R&D,

A.Rubbia, Proc. Int. Conf. on NF and Superbeam, NUFACT04, Osaka, Japan, July 2004

• Present status and prospects of neutrino oscillation experiments,

A. Ereditato, Proc. of the Int. Workshop on Heavy Quarks and Flavors, S.Juan de Puertorico, June 2004, HQ &L, 73-88.

• Liquid Argon TPC: a powerful detector for future neutrino experiments,

A.Ereditato and A. Rubbia, HIF05, La Biodola, Italy, May 2005, hep-ph/0509022

• Neutrino detectors for future experiments,

A.Rubbia, Nucl. Phys. B (Proc. Suppl.) 147 (2005) 103.

• Conceptual Design of a scalable milti-kton superconducting magnetized liquid argon TPC,

A. Ereditato and A. Rubbia, hep-ph/0510131.



#### LAr TPC for the T2K 2km site: detector complex integration

LAI

≈15m

#### Muon Ranger: *Measure high energy tail of neutrino spectrum*

**U**R

Liquid Argon detector: *exclusive final states frozen water inner-target* 

Incoming

neutrino

beam

Water Cerenkov detector: same detector technology as SK ≈1 interaction/spill/kton

Water Cerenkov

= 28 m



#### *Important features provided by the LAr TPC for T2K*

- Fully active, homogeneous, high-resolution device ⇒ fine grain detector and high statistics neutrino interaction studies with bubble chamber accuracy.
- Reconstruction of low momentum hadrons (below Cerenkov threshold), especially recoiling protons.
- Independent measurement of off-axis flux and QE/nonQE event ratio.
- Exclusive measurement of vNC events with clean  $\pi^0$  identification for an independent determination of systematic errors on the NC/CC ratio.
- Measurement of the intrinsic  $v_e$ CC background.
- Large statistical sample of neutrino interactions in the GeV region for the study of quasi-elastic, deep-inelastic and resonance modelling and of nuclear effects.



MC QE event Proton momentum = 490 MeV/c





MC nQE event Pion momentum = 377 MeV/c, Proton momentum = 480 MeV/c

## *T2K* $v_{\mu}$ *disappearance*

### $P(\nu_{\mu} \rightarrow \nu_{x}) \sim 1 - \cos^{4}\theta_{13} \sin^{2}2\theta_{23} \sin^{2}(\Delta m_{23}^{2}L/4E)$



## T2K $v_e$ appearance: measurement of $\theta_{13}$

 $P(\nu_{\mu} \rightarrow \nu_{e}) \sim \frac{\sin^{2}2\theta_{13}}{\sin^{2}\theta_{23}} \sin^{2}(\Delta m_{23}^{2}L/4E)$ 



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### An off-axis experiment in the NuMI beam: NovA

- proposal, approval phase; nominal NuMI beam: 0.4 MW + upgrade?
- if approved: small fraction of the far detector by 2008. Completed by end 2011 (?)
- Start 2010-2011 (?) • far detector: 30 kton @ Ash River (MN) 810 km from Fermilab (12 km, 14 mrad off-axis)
- technique: fine grained calorimeter (liquid scintillator) with fiber/APD R/O
- near detector: same as far, 1 ton fiducial mass
- unlike T2K, NovA is sensitive to matter effects, but is a single task detector (cost ~ \$200 M)





 $v_{a}$  CC event

Sensitivity of NOvA to the measurement of  $sin^2 2\theta_{13}$ 



### Comparison between MINOS, T2K and NovA

Assume 5 years running,  $\Delta m_{23}^2 = 2.5 \times 10^{-3} \text{ eV}^2$ ,  $3\sigma$  evidence for non zero sin<sup>2</sup>  $2\theta_{13}$ :

Experiment	Run	p.o.t.	$3\sigma$ evidence
MINOS	2005-2008	16 x 10 <sup>20</sup>	> 0.080
T2K	2009-2013	50 x 10 <sup>20</sup>	> 0.018
NovA (Booster) NovA (p driver)	2011-2015 ?	32.5 x 10 <sup>20</sup> 125 x 10 <sup>20</sup>	> 0.018 (0.012) > 0.012 (0.006)



# Pin down CP phase and mass hierarchy

### e.g. : detecting CP violating effects

Best method: (in vacuum)

$$_{CP} = \frac{P(\overline{\nu}_e \to \overline{\nu}_\mu) - P(\nu_e \to \nu_\mu)}{P(\overline{\nu}_e \to \overline{\nu}_\mu) + P(\nu_e \to \nu_\mu)} \simeq \frac{\sin 2\theta_{12}}{\sin \theta_{13}} \cdot \sin \delta \cdot \sin \frac{\Delta m_{12}^2 L}{4E}$$

it <u>requires</u>:  $\Delta m_{12}^2$  and  $\sin 2\theta_{12}$  large (LMA solar): OK ! larger effects for long L: 2<sup>nd</sup> oscillation maximum

however...

$$P(
u_{\mu} 
ightarrow 
u_{e}) \propto \sin^{2}2 heta_{13}$$
  $A_{CP} \propto rac{1}{\sin heta_{13}}$ 

A

 $sin^{2}2\theta_{13}$  small: low statistics and large asymmetry  $sin^{2}2\theta_{13}$  large: high statistics and small asymmetry

impact on the detector design

...and:  

$$P(\nu_{\mu} \rightarrow \nu_{e}) \propto \sin^{2} \frac{\Delta m_{23}^{2} L}{4E}$$
oscillations are governed by  $\Delta m_{atm}^{2}$ , L and E  
 $E \approx 5 \text{ GeV} \rightarrow L \approx 3000 \text{ km}$   
flux too low with a conventional LBL beam



2 det: 48m x 50m x 250m each

#### DETECTORS

500-1000 kton Water Cerenkov 'a la SK' (Hyper-K, UNO) are considered as a 'natural' option

Rationale: exploit a well known technique aim at a 'reasonable' cost

However, this is not the only possibility...

#### Water Cerenkov technique

- efficient for 'few' or 1-ring events (QE), small x-section, large detector mass
- good  $\pi^0$  rejection if  $\gamma$  are well separated
- at low energy confusion between  $\mu$  and  $\pi$  tracks
- can go down with energy threshold (5 MeV for 40% coverage) ?
- well established in Japan: success of SK but limited experience elsewhere
- Hyper-K project well advanced: decision around 2012 ?
- PMTs: leadership of Hamamatsu (very large production will be required)
- alternative photo-detectors options unclear: R&D & cost assessment needed
- huge cavern: cost and complexity of excavation works

### A fine grained detector can be alternative/complementary: liquid Argon TPC



### Neutrino detection: LAr TPC vs water Cerenkov

 $\nu_{\mu} + X \rightarrow \mu^{-} + many \ prongs$ 

**ICARUS 50 liters** 

Multi prong event detection not possible with water Cerenkov

#### Super-Kamiokande Run 7436 Event 1405412 99-06-19:18:42:4 Inner: 516 hits, 1018 pE





#### Resid(ns) > 182

Trigger ID: 0x0 D wall: 240.4cm



#### **FIRST K2K EVENT RECORDED BY SUPER-K**

 $v_{\mu} + n \rightarrow \mu^{-} + p$ 





 $v_{\mu} + n \rightarrow \mu^{-} + p$ 

	Water Cerenkov (UNO)	Liquid Argon TPC	
Total mass	650 kton	100 kton	
Cost	≈ 500 M\$	Under evaluation	
$p \rightarrow e \pi^0$ in 10 years	10 <sup>35</sup> years ε = 43%, ≈ 30 BG events	3x10 <sup>34</sup> years ε = 45%, 1 BG event	
$p \rightarrow v K$ in 10 years	2x10 <sup>34</sup> years ε = 8.6%, ≈ 57 BG events	8x10 <sup>34</sup> years ε = 97%, 1 BG event	
$p \rightarrow \mu \pi K$ in 10 years	Νο	8x10 <sup>34</sup> years ε = 98%, 1 BG event	
SN cool off @ 10 kpc	194000 (mostly ⊽ <sub>e</sub> p→ e⁺n)	38500 (all flavors) (64000 if NH-L mixing)	
SN in Andromeda	40 events	7 (12 if NH-L mixing)	
SN burst @ 10 kpc	≈330 v-e elastic scattering	380 $v_e$ CC (flavor sensitive)	
SN relic	Yes	Yes	
Atmospheric neutrinos	60000 events/year	10000 events/year	
Solar neutrinos	E <sub>e</sub> > 7 MeV (central module)	324000 events/year ( $E_e > 5 \text{ MeV}$ )	

Operation of a 100 kton LAr TPC in a future neutrino facility: Super-Beam: 460  $v_{\mu}$  CC per 10<sup>21</sup> 2.2 GeV protons @ L = 130 km Beta-Beam:15000  $v_{e}$  CC per 10<sup>19</sup> <sup>18</sup>Ne decays with  $\gamma$  = 75

### Japanese program phase 2: short L, low E

- intensity up to 4 MW
- detector fiducial mass
- $\sim 0.5$  Mton
- no matter effects: assume mass hierarchy determined elsewhere



Start ~ 2020 ? Major T2K beam upgrade, new Hyper-K detector 1) low energy: low  $\pi^{\circ}$  BG 2) gigantic water Cerenkov: good e ID demanding requirements: 2% syst. from BG subtraction and 2% from data selection

low  $E_{\overline{v}} \rightarrow$  low x-section



### T2KK option: two detectors (270 kton each) in Japan and Korea





#### **Interesting option:**

- sensitivity to matter effects
- systematic errors largely cancel
- solve ambiguities
- build two smaller detectors
- share cost of construction (Japan-Korea collaboration)



### US options: e.g. FERMILAB project (high intensity proton driver)



#### FERMILAB plans for neutrino physics (from P.Oddone)



Proton driver construction might be anticipated if ILC would be delayed

### Neutrino Factories...the ultimate neutrino beam experiments?



#### **Obviously, a great opportunity for neutrino physics!**

- huge neutrino fluxes, increasing with muon energy
- Ev may range from 5 to 30 GeV,  $10^{20}$  muon decays/year
- only two flavors for a given polarity:  $\overline{v}_{\mu}$  and  $v_{e}$  or  $v_{\mu}$  and  $v_{\overline{e}}$
- for a massive, coarse-resolution set-up:  $\mu$  detection easier than *e* ID over  $\pi^{\circ}$  BG (wrong sign muons)
- possible to use large mass detectors already exploited for Super- Beta- Beams (magnetic analysis)
- detector can be simple, but don't forget unexpected, new physics events to be studied in great detail
- in principle 'very' low beam- and detector-BG
- L from ~ 1000 to 8000 km (international enterprise by definition!)
- very complex accelerator facility: R&D needed and being pursued worldwide (EU, USA, Japan)
- the first accelerator stage could be a proton driver for a Super-Beam
- extremely challenging project: target, muon cooling, radiation and environmental issues, cost, etc.

## Tentative layout of a large magnetized GLACIER



### (personal) concluding remarks

- The glory of the massive neutrino! The evidence for neutrino oscillation is today very robust. This has opened the way to precision studies of the mixing matrix with accelerator neutrino experiments, together with future projects on direct mass measurements, double-beta decay, reactor, solar and atmospheric neutrinos. The mass of neutrino is the first (and so far the only) indication of physics beyond the SM.
- Running and planned experiments will contribute to narrow-down the errors on the oscillation parameters and with some chance to prove that the mixing matrix is indeed 3 x 3. The next generation will need high intensity facilities to pin down a non vanishing value of  $\theta_{13}$ . Advanced detector technique will be required to keep BG low for a real improvement of the sensitivities. This physics subject is of outstanding importance 'per se' but also because it will drive future initiatives.
- The detection of matter and of CP violating effects will likely require a further generation of experiments using high intensity (>1 MW) neutrino facilities with more massive detectors. Detector options: a 500-1000 kton water Cerenkov detectors (à la SK), 50-100 kton liquid Argon TPCs, large liquid scintillator detector.
- In addition to the need of large mass, the detectors have to be 'general purpose' (think of tomorrow's physics), must have good energy resolution (measure oscillation parameters), good granularity (to measure channels involving *e*, μ, τ,...) and adequate NC/CC separation for BG suppression. They will need to be as good for astroparticle physics (underground or at shallow depth) and they have to employ cost effective technical solutions/technologies.

### (personal) concluding remarks (cont.)

- Concerning the neutrino beams, a factor ~10 boost in the intensity is required. Super-Beams are the natural approach, based on improved LINAC or Boosters. Synergies are expected with other fields and this can increase the probability of success (funding). Another possibility is given by Beta-Beams (of low and/or high energy) complementary to other approaches (μ appearance). Regardless the neutrino source, the final choice of L/E must come from a global, physics driven optimization of facility and detector. A Neutrino Factory can be considered for a more distant future and constitutes the ultimate neutrino facility with unprecedented features but its construction would represent a huge investment for the entire community. Its main task must be the precision study of CP violation.
- The cost and the complexity of these projects demand a strong worldwide coordinated effort between researchers and agencies, similarly to what occurs in other fields, *e.g.* for collider physics.
   Complementarity of approaches and techniques is mandatory. Choices on projects beyond the experiments presently running or being built must pragmatically take into account (and use, as far as possible) existing facilities and infrastructure (detectors, beams).

• In one sentence: a lot of work (and fun) ahead of us!

# Additional slides

#### e.g. SK atmospheric neutrinos: L/E distribution (for selected high-resolution events) and NC/CC ratio ( $\tau$ or sterile neutrino ?)



→ models alternative to oscillation are disfavored by more than  $3\sigma$ atmospheric neutrino deficit is due to  $v_{\mu} \rightarrow v_{\tau}$  <u>oscillations</u> (not to generic conversion)

### Mass hierarchy from matter oscillations

#### Neutrinos oscillating through matter (MSW effect):

- different behavior of different flavors due to the presence of electrons in the medium
- additional phase contribution to that caused by the non zero mass states.
- asymmetry between neutrinos and antineutrinos even without CP violating phase in the matrix
- the related oscillation length  $L_M$ , unlike  $L_V$  (vacuum), is independent of the energy
- as an example  $L_{\rm M}$  (rock) is  $\sim 10000$  km while  $L_{\rm M}$  (Sun)  $\sim 200$  km

In the limit of  $\Delta m_{sol}^2$  approaching zero (for which there are no CP effects) and of running at the atmospheric oscillation maximum, the asymmetry between neutrinos an antineutrinos equal to

$$\frac{P(\nu_{\mu} \rightarrow \nu_{e}) - P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e})}{P(\nu_{\mu} \rightarrow \nu_{e}) + P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e})} = \frac{2E_{\nu}}{E_{R}} \quad \text{for low } E_{\nu} \quad \text{with} \quad E_{R} = \frac{\Delta m_{atm}^{2}}{2\sqrt{2}G_{F}\rho_{e}} \approx 11 GeV$$
By the measurement of this asymmetry one can determine whether  $\Delta m^{2}_{23}$  is positive or negative (hierarchy)
$$\frac{\sqrt{e}}{\sqrt{\nu}} \sqrt{\frac{\nu_{e}}{\nu_{1}}} \sqrt{\frac{\nu_{e}}{\nu_{1}}}} \sqrt{\frac{\nu_{e}}{\nu_{1}}} \sqrt{\frac{\nu_{e}}{\nu_$$

sol

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