Modified Gravity versus Dark Matter and dark energy

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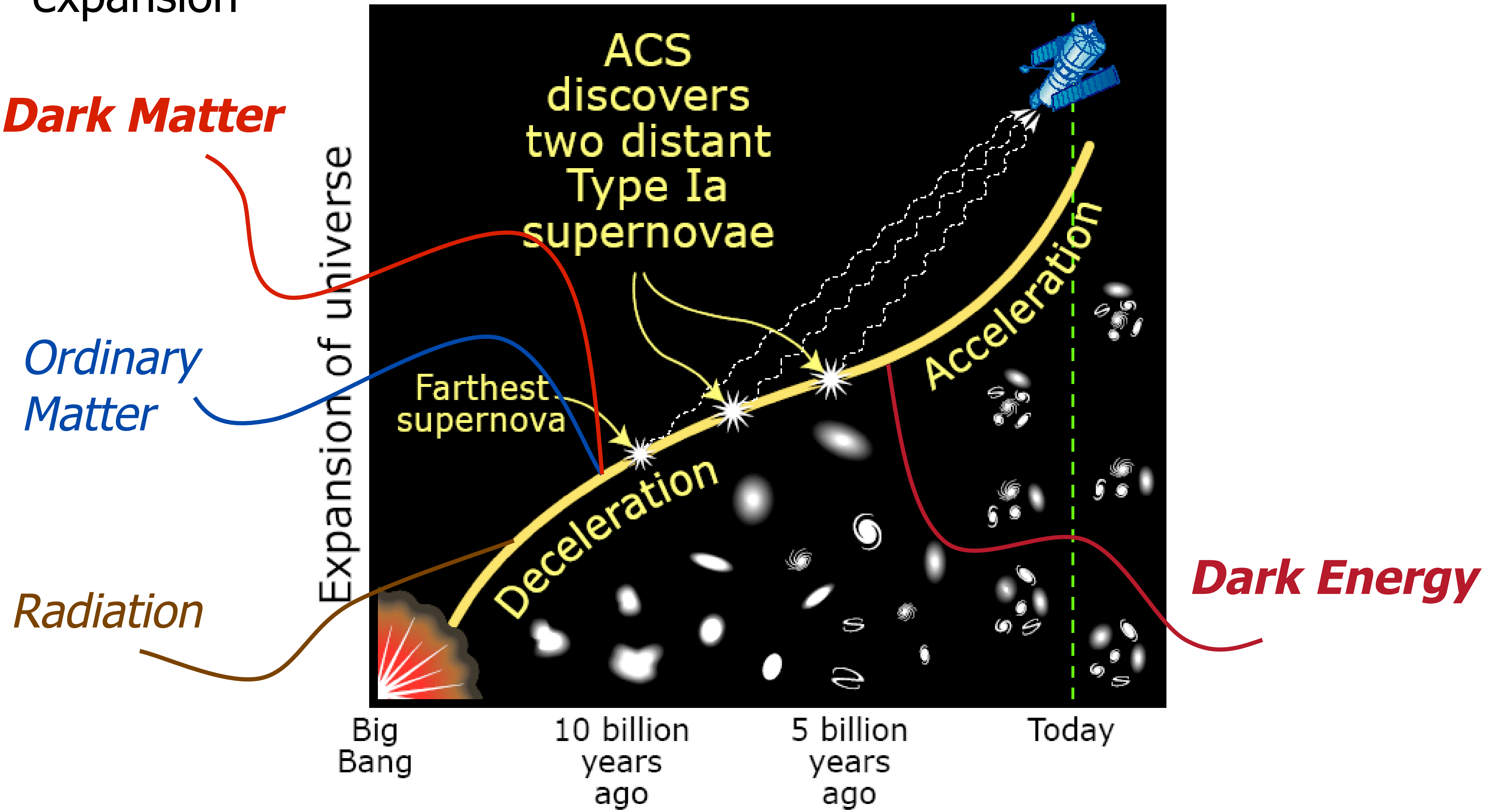
The Observed Universe

- Universe evolution is characterized by different phases of expansion



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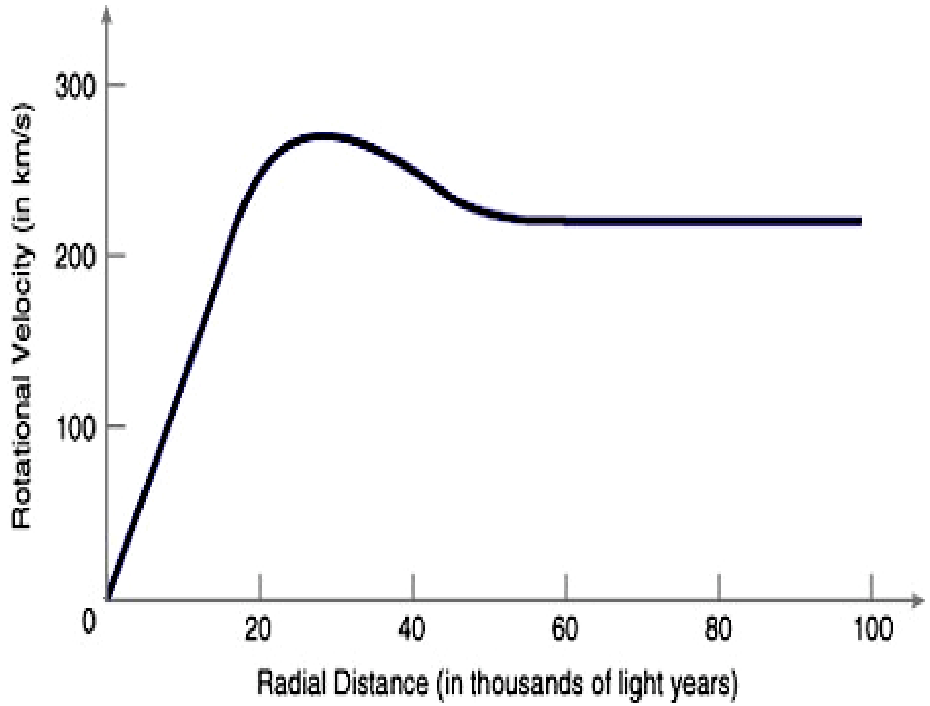
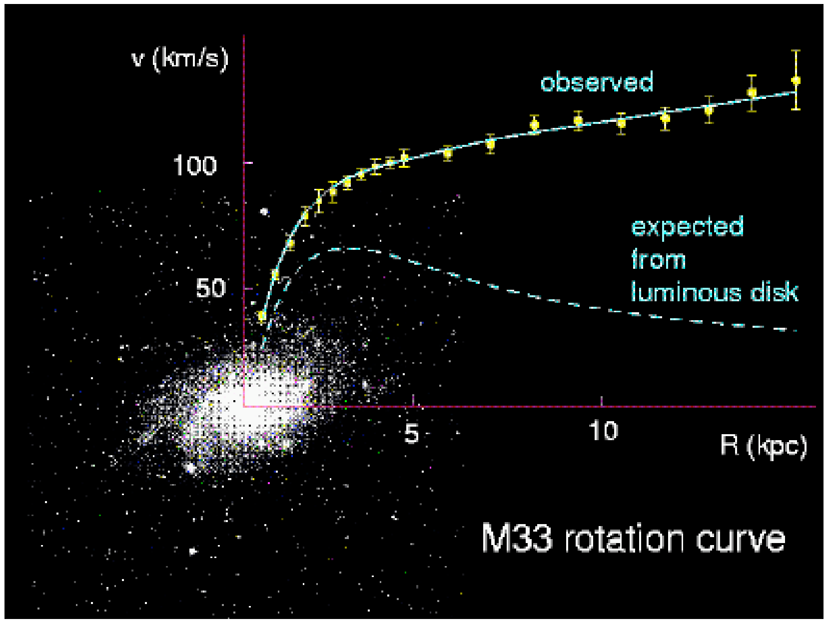
Dark Matter sector

- ✓ The presence of Dark Matter components has been revealed since 1933 by Zwicky as a lack in the mass content of galaxy clusters. The most peculiar effect of Dark Matter is the discover of a non-decaying velocity of rotation curves of galaxies



Dark matter differences in clusters and galaxies

An important difference between the distribution of dark matter in galaxies and clusters is that whereas dark matter seems to increase with distance in galaxies [M33_Roy D.P. physics/0007025], it is just the opposite situation in clusters. Thanks to gravitational lensing effects, it is possible to estimate that the most of dark matter in clusters is concentrated in the central regions (0.2-0.4 Mpc).



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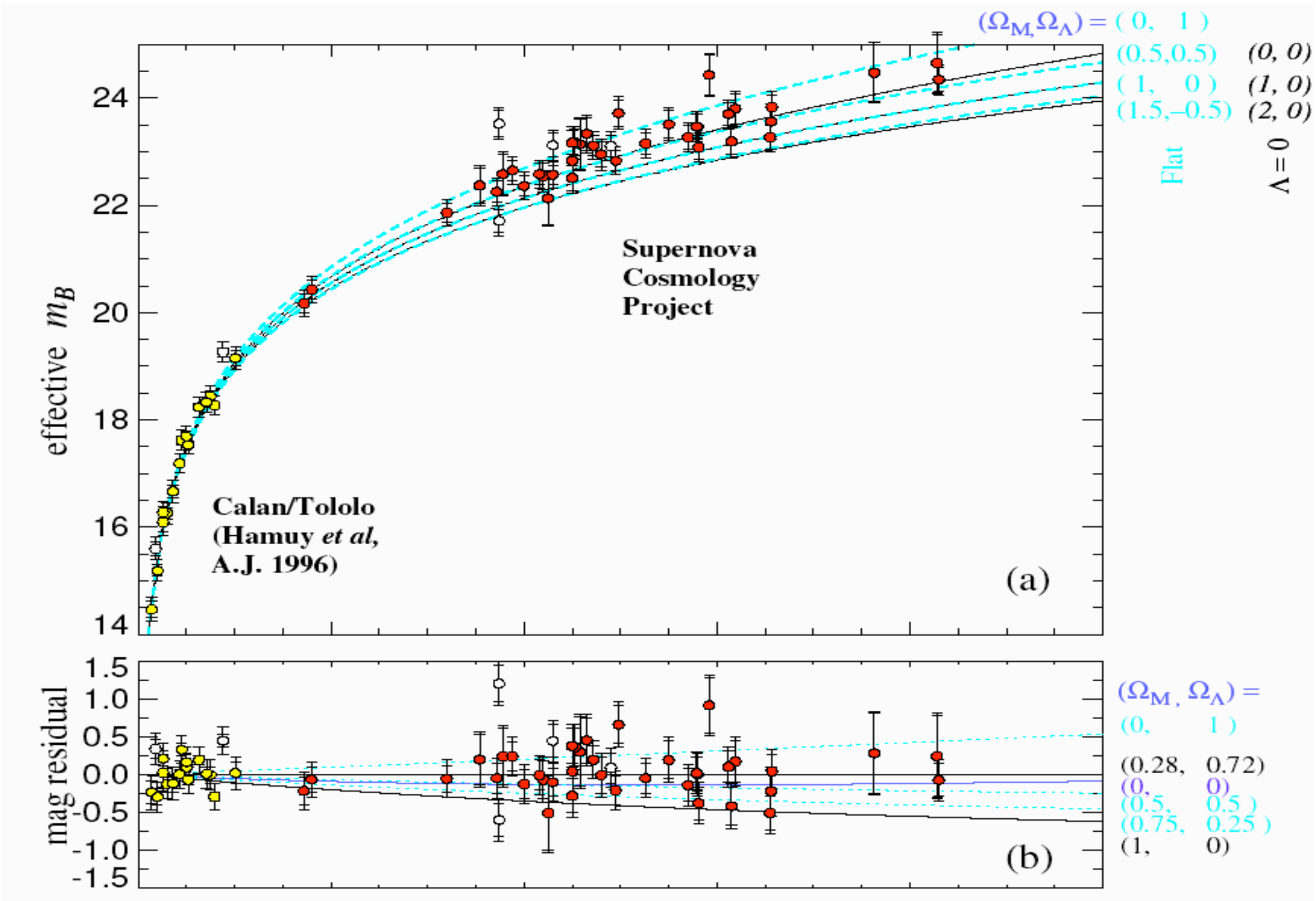
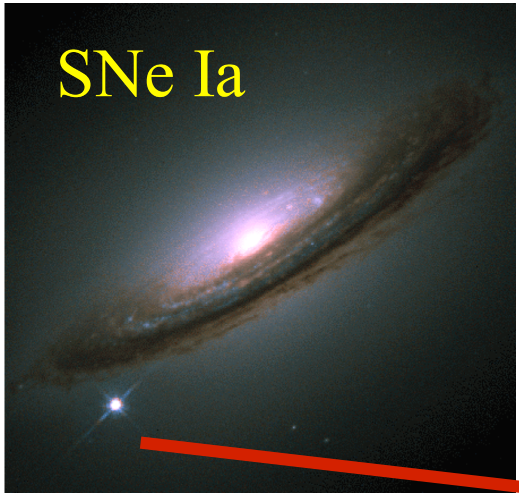
Dark Energy sector

- ✓ The presence of a Dark Energy component has been proposed after the results of SNeIa observations (HZT [Riess A.G. et al. Ap.J. 116, 1009 (1998)]- SCP [Perlmutter S. et al. Nature 391, 58 (1998)] collaborations).

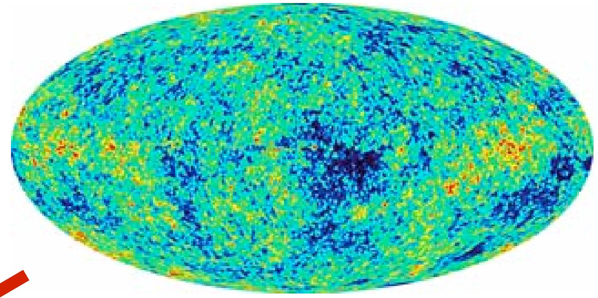


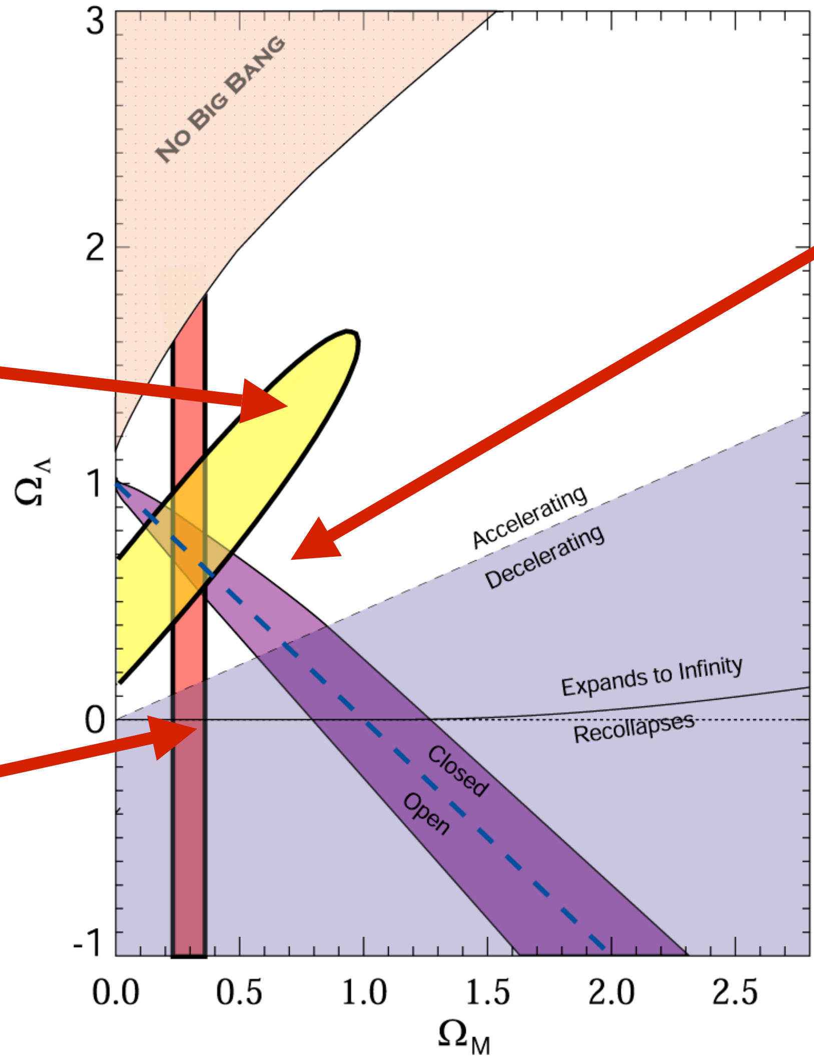
Main observational evidences for Dark Energy

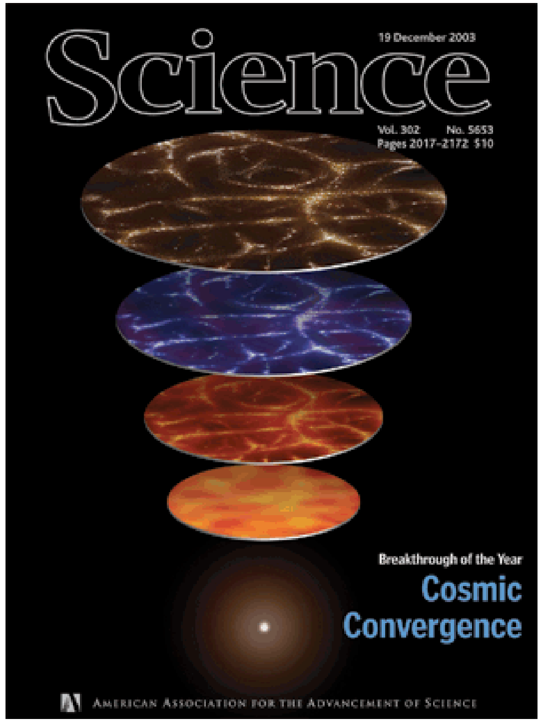
After 1998, more and more data have been obtained confirming this result.



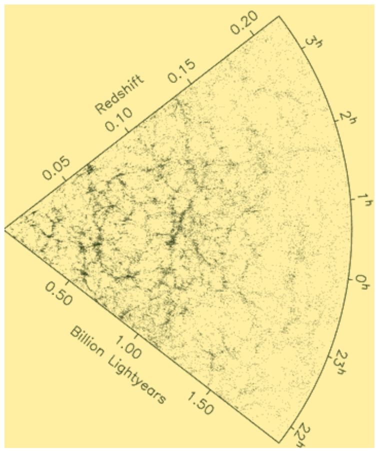
CMB(WMAP)







LSS



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- Combining SNeIa data with other observations and in particular with data coming from CMBR experiments (COBE, MAXIMA, BOOMERANG, WMAP) we have, up today, a ***"best fit"*** ***(The Concordance Model)*** universe which is filled with 30% of matter (dark and baryonic) and 70% of dark energy, a component, in principle, different from the standard dark matter. Dark Energy is always characterized by a negative pressure and does not give rise to clustered structures.

- The most important consequence of this result is that the universe is in a phase of ***accelerating*** expansion

$$q_0 = \Omega_R + \frac{1}{2}\Omega_M - \Omega_\Lambda + \frac{1}{2}\sum_x (1 + 3w_x)\,\Omega_x$$

$$\Omega = \frac{\rho}{\rho_C}$$

$$\rho_C = \frac{3H^2}{8\pi G}$$

Dark Matter

\+

Dark Energy

95%!

??

!

Dark Matter hints

- Neutrino _ Too low mass limit ($\sum_{\nu_i} m_{\nu_i} < 0.7\,\mathrm{eV}$ and relic neutrino density. Shortcomings in forming structures in a considerable time capable of explaining QSO and galaxies at high redshift. Mass requested ~ (10 eV) . However, neutrino mixing could work for DE giving a dynamical vacuum state [Capolupo, SC, Vitiello 2006].

- WIMP (Weakly Interacting Massive Particle) _ Neutralino (neutral supersymmetric fermionic partner of gauge and Higgs bosons [Roszkowsky 1999]). Since WIMPs cluster gravitationally, it should be possible to find some effects of these particles in the Solar System thanks to the nuclear recoil on nucleii targets. Answers from DAMA experiments? [Munoz 2004, Krauss 2004].

- MOND _ It is a modification of Newtonian physics which comes out at sufficiently low acceleration where $\mathbf{F} = ma\mu(a/a_0)$ $\mu(x) = x$ when $x \ll 1$ and $\mu(x) = 1$ when $x \gg 1$ Difficulties in explaining galaxy cluster dark matter and in embedding this approach in a general theory of gravity (*TeVeS*) [Sanders R.H., Mcgough S., Ann.Rev.Astron.Astrophys.40:263-317,2002 ; Bekenstein J.D., Phys. Rev D71:069901,2005]

Dark Energy hints *(Einstein Lore)*

- ✓ Cosmological constant _ Introduced by Einstein (1917) to get a static universe, it has been recovered in the last years to interpret the cosmic acceleration evidenced by SNeIa data through the Einstein equations

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik} + \Lambda g_{ik}$$

The force law is

$$\mathcal{F} = -\frac{GM}{R^2} + \frac{\Lambda}{3} R, \quad (R \equiv a)$$

which shows that the cosmological constant gives rise to a repulsive force which could be responsible for the acceleration of the universe. Since 60's, cosmological constant has been related to vacuum energy of fields. ⇒ Cosmological constant problem (126 orders of magnitude of difference between the theoretical estimate and the observational one) & Coincidence problem (the today observed equivale $\rho_\Lambda \simeq 10^{-47}\mathrm{GeV}^4$ f magnitude, of dark energy and dark matter densities).

✓ Dynamical dark energy (Quintessence) _ Allows to overshoot the coincidence problem considering a dynamical negative pressure component. The standard scheme is to consider a scalar field Lagrangian.

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi) \ ,$$

$$\rho \equiv T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi), \ p \equiv -T^\alpha_\alpha = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Potentials able to furnish interesting quintessential models:

Quintessence Potential	Reference
$V_0 \exp(-\lambda\phi)$	Ratra & Peebles (1988), Wetterich (1988), Ferreira & Joyce (1998)
$m^2\phi^2, \lambda\phi^4$	Frieman et al (1995)
$V_0/\phi^\alpha, \alpha > 0$	Ratra & Peebles (1988)
$V_0 \exp(\lambda\phi^2)/\phi^\alpha$	Brax & Martin (1999,2000)
$V_0(\cosh\lambda\phi - 1)^p$	Sahni & Wang (2000)
$V_0 \sinh^{-\alpha}(\lambda\phi)$	Sahni & Starobinsky (2000), Ureña-López & Matos (2000)
$V_0(e^{\alpha\kappa\phi} + e^{\beta\kappa\phi})$	Barreiro, Copeland & Nunes (2000)
$V_0(\exp M_p/\phi - 1)$	Zlatev, Wang & Steinhardt (1999)
$V_0[(\phi - B)^\alpha + A]e^{-\lambda\phi}$	Albrecht & Skordis (2000)

.....Resume....

The possible theoretical answers

DARK MATTER	DARK ENERGY

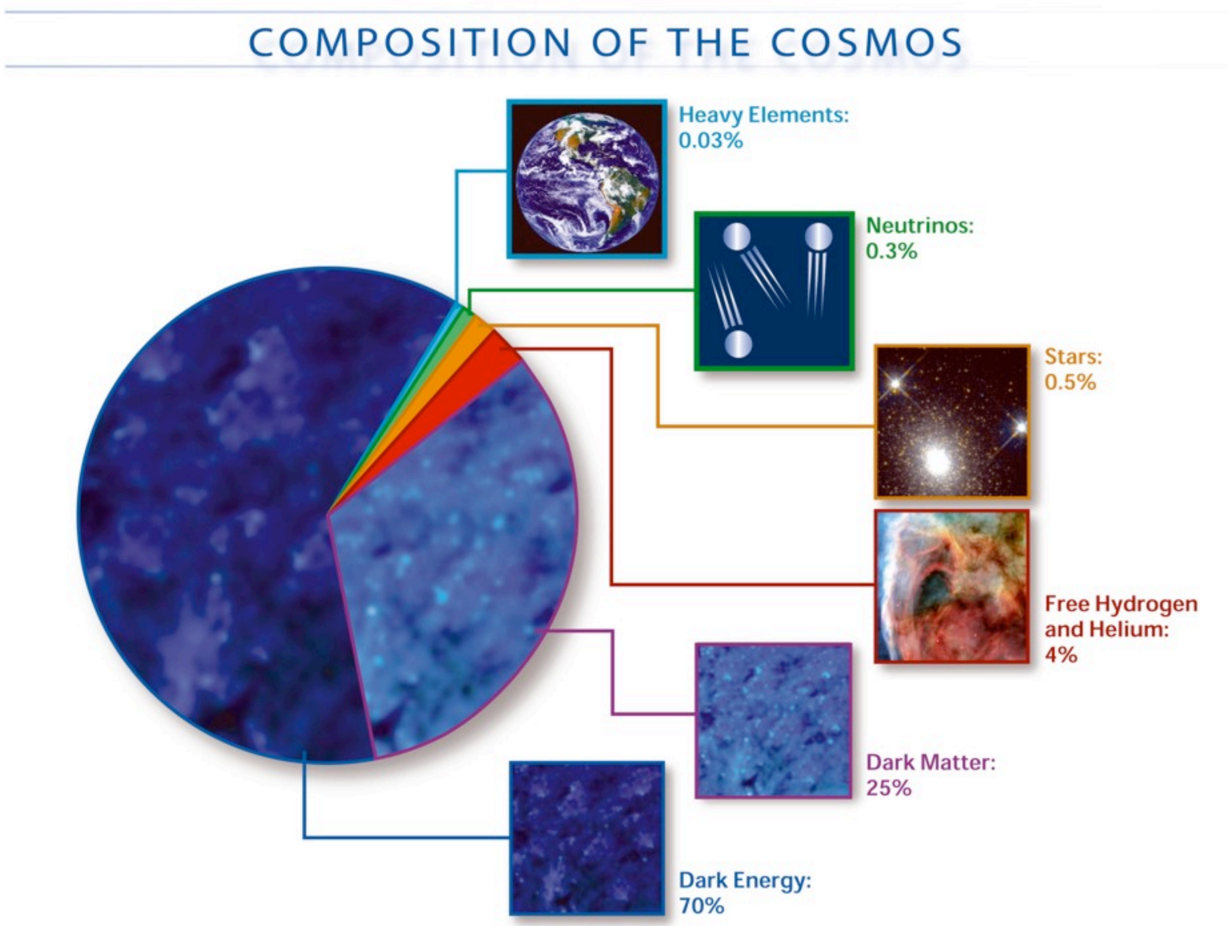
DARK MATTER

- ✓ Neutrinos
- ✓ WIMP
- ✓ Wimpzillas, Axions, the "particle forest".....
- ✓ MOND
- ✓ MACHOS
- ✓ Black Holes
- ✓ ......

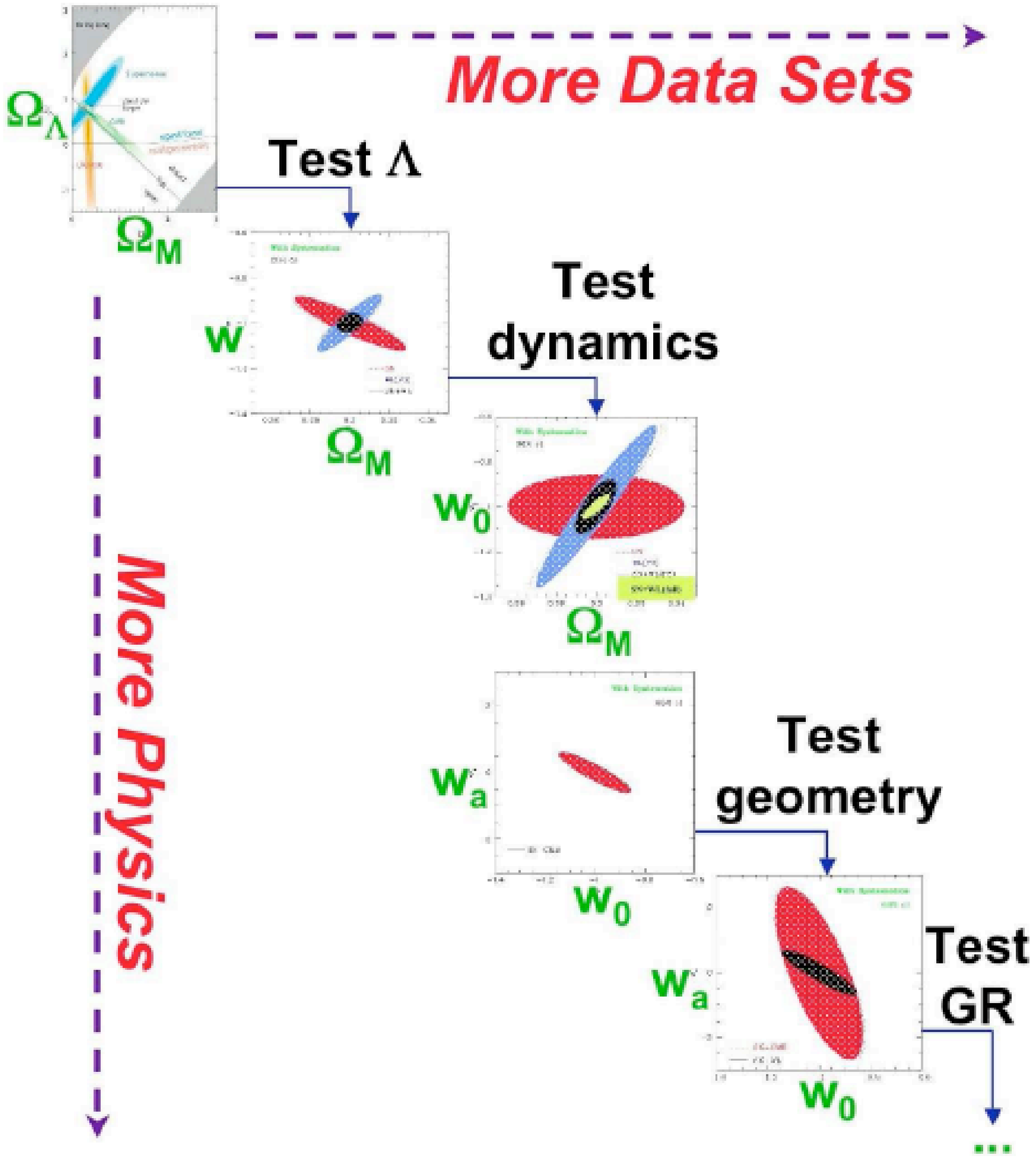
DARK ENERGY

- ✓ Cosmological constant
- ✓ Scalar field Quintessence
- ✓ Phantom fields
- ✓ String-Dilaton scalar field
- ✓ Braneworlds
- ✓ Unified theories
- ✓ ....

- In conclusion: The content of the universe is, up today, absolutely unknown for its largest part. The situation is very “DARK” while the observations are extremely good!



Incremental Exploration of the Unknown



Linder (astro-ph/0511197)

The problem could be reversed

We are able to observe and test only baryons and gravity

Dark Energy and Dark Matter as "shortcomings" of GR

The "correct" theory of gravity is derived by matching the largest number of observations

Accelerating behaviour (DE) and dynamical phenomena (DM)
as ***CURVATURE EFFECTS***

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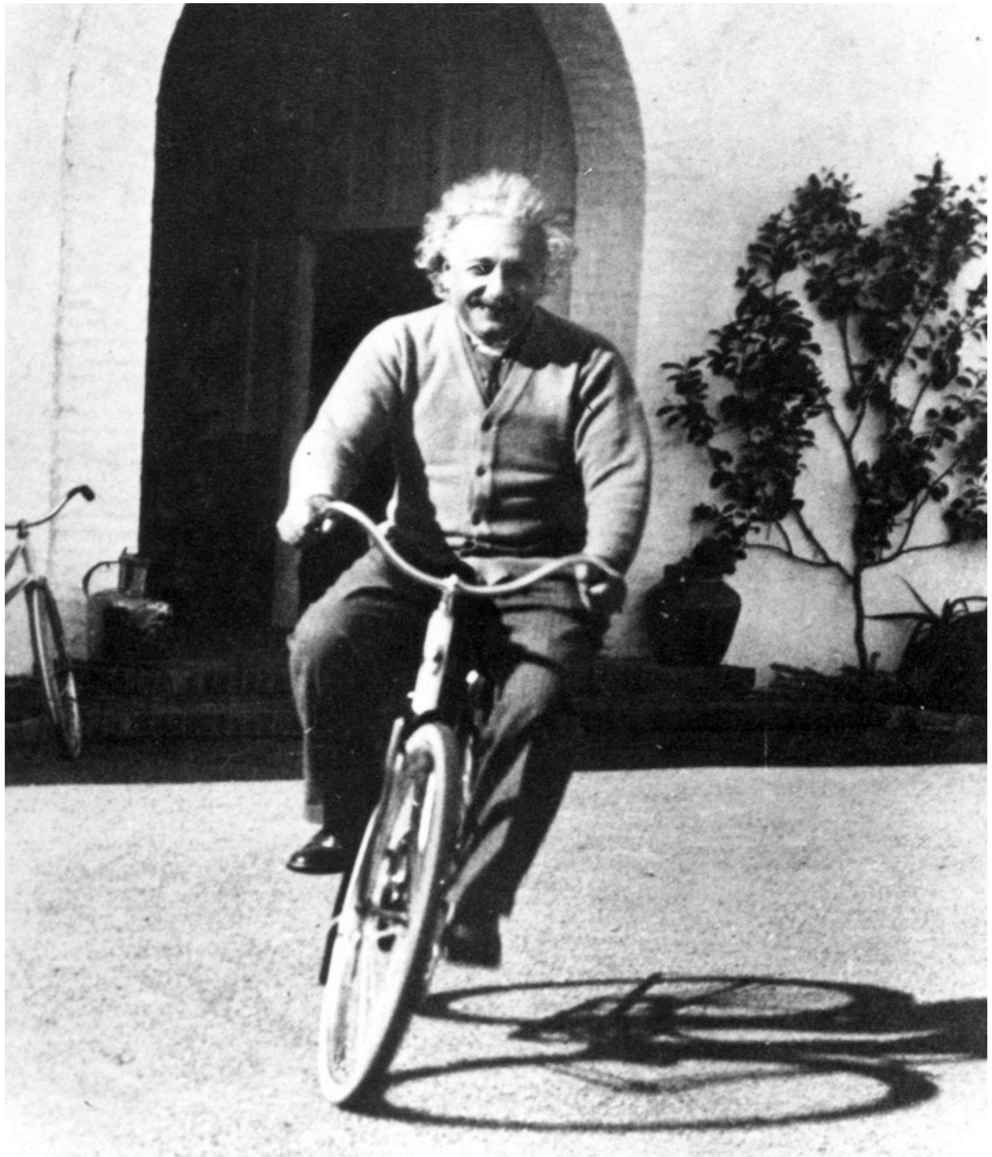
f(R) -proposal

✓ Generalization of the Hilbert-Einstein action to a generic (unknown) theory of gravity

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[f(R) + \mathcal{L}_{(matter)}\right]$$

$$f'(R)R_{\alpha\beta} - \frac{1}{2}f(R)g_{\alpha\beta} = f'(R)^{;\mu\nu}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu}) + \tilde{T}^{(matter)}_{\alpha\beta}$$

[SC., 2002; SC., V. Cardone, A. Troisi , 2003, 2004, 2005, 2006]



Theoretical motivations and features:

- Quantization on curved space-time needs higher-order invariants corrections to the Hilbert-Einstein Action.
- These corrections are also predicted by several unification schemes as String/M-theory, Kaluza-Klein, etc.
- A generic action is $\mathcal{A} = \int d^4x\sqrt{-g}\left[F\left(R, \Box R, \Box^2 R, \ldots \Box^k R\right) + \mathcal{L}_m\right]$
- We can consider only fourth order terms which give the main contributions.
- This scheme allows to obtain an "Einstein" two fluid model in which one component has a geometrical origin

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = T^{(curv)}_{\alpha\beta} + T^{(matter)}_{\alpha\beta}$$

$$T^{(curv)}_{\alpha\beta} = \frac{1}{f'(R)}\left\{\frac{1}{2}g_{\alpha\beta}\left[f(R) - Rf'(R)\right] + f'(R)^{;\mu\nu}\left(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu}\right)\right\}$$

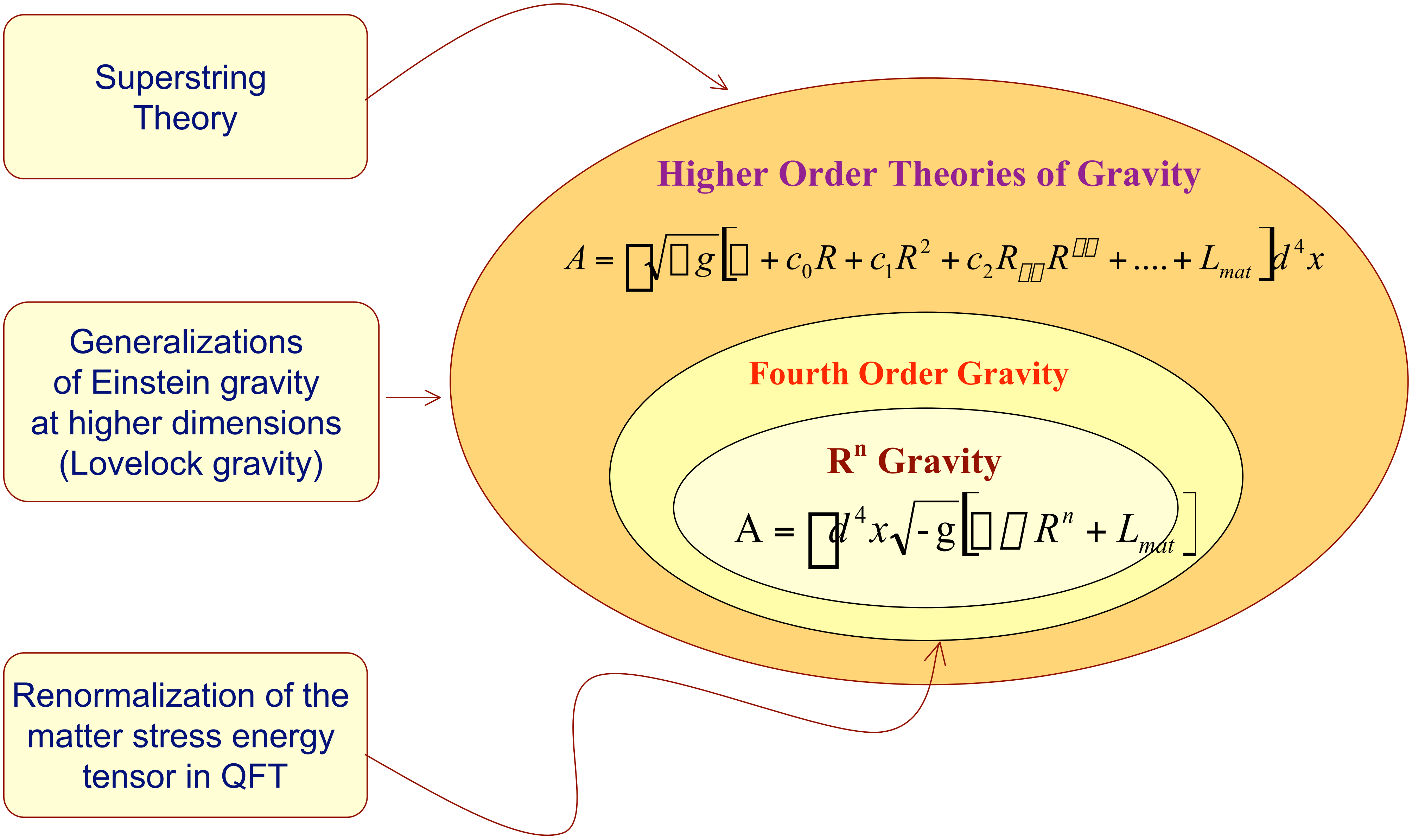
Conservation Properties of Higher Order Theories

The Hamiltonian derivative of any fundamental invariant is divergence free!!

- A key point in the above system is related to the **conservation equations**. Eddington proved in his book that every higher order correction to the Hilbert-Einstein Lagrangian produces terms that are divergence free: **the curvature fluid is conserved on his own** and **the matter follows the standard conservation equation**

$$\dot{\rho}_{curv} + 3H(\rho_{curv} + p_{curv}) = 0$$

In particular, the R^n Gravity:



Dark Energy as a curvature effect

It is possible to write a curvature pressure and a curvature energy density in the FRW metric

$$p_{(curv)} = \frac{1}{f'(R)}\left\{2\left(\frac{\dot{a}}{a}\right)\dot{R}f''(R) + \ddot{R}f''(R) + \dot{R}^2 f'''(R) - \frac{1}{2}\left[f(R) - Rf'(R)\right]\right\}$$

$$\rho_{(curv)} = \frac{1}{f'(R)}\left\{\frac{1}{2}\left[f(R) - Rf'(R)\right] - 3\left(\frac{\dot{a}}{a}\right)\dot{R}f''(R)\right\}$$

$$w_{curv} = -1 + \frac{\ddot{R}f''(R) + \dot{R}\left[\dot{R}f'''(R) - Hf''(R)\right]}{\frac{1}{2}\left[f(R) - Rf'(R)\right] - 3H\dot{R}f''(R)}$$

As a simple choice, we assume a power law function for *f(R)* and for the scale factor *a(t)*

$$f(R) = f_0 R^n\,, \qquad a(t) = a_0\left(\frac{t}{t_0}\right)^{\alpha}$$

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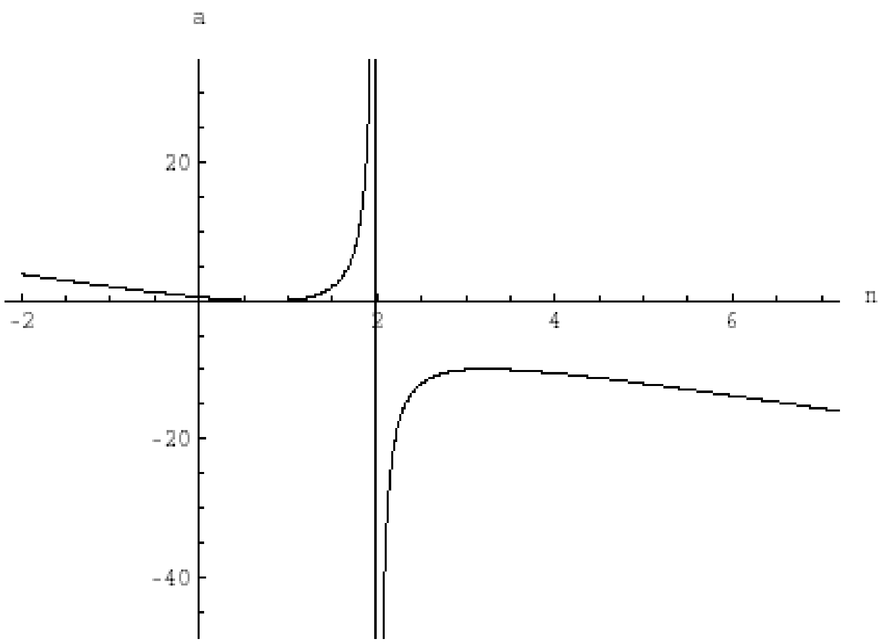
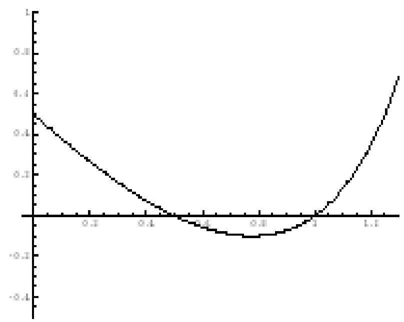
- The power law *f(R)* function is interesting since each analytical Lagrangian in R can be locally approximated by a Taylor polynomial expansion. The power law of the scale factor is coherent with the cases of dominant matter and radiation in universe dynamics (_ > 1 implies accelerating expansion).

(Vacuum case)   Solutions for

$$\alpha = \frac{2n^2 - 3n + 1}{2 - n}$$





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Matching with data

SNeIa data [SC, Carloni, Cardone, Troisi 2003]

The test with the SNeIa data has been done considering the so called "distance modulus" $\mu(z) = 5 \log \frac{c}{H_0} d_L(z) + 25$

comparing its theoretical estimate with the observed one.

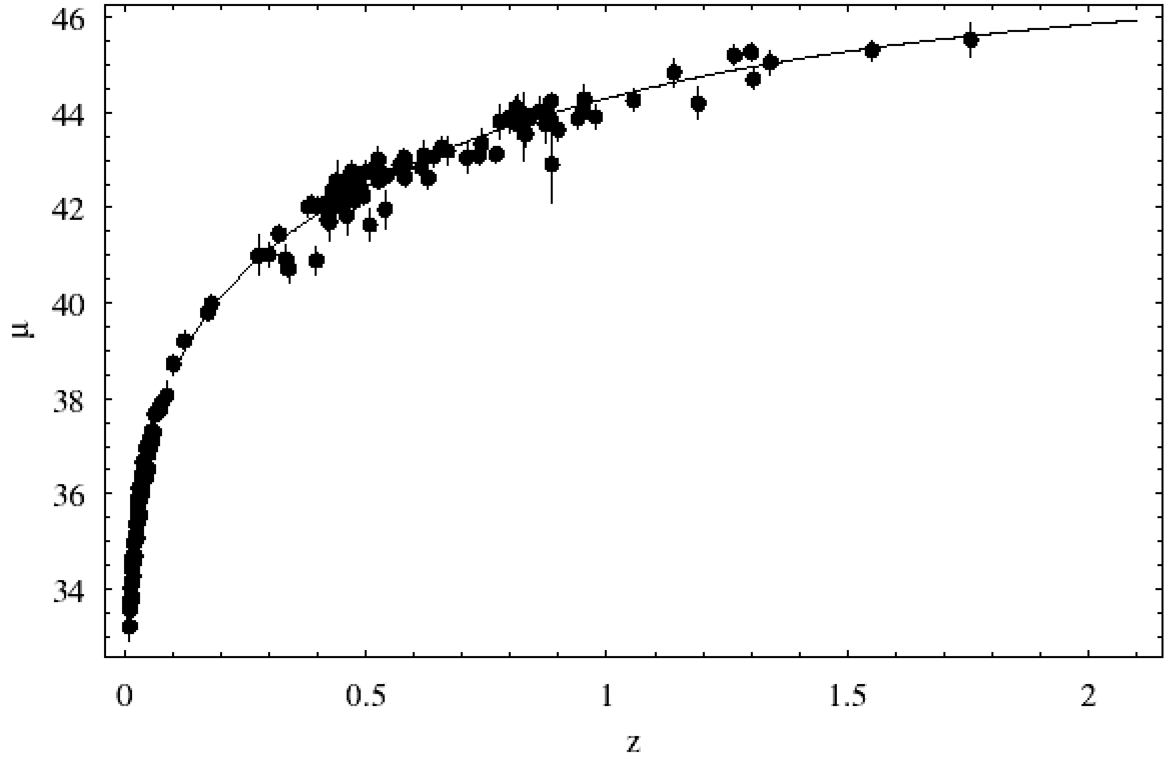
The luminosity distance $d_L(z)$ is defined in relation to the considered cosmological model. In our case, it is

$$d_L(z, H_0, n) = \frac{c}{H_0} \frac{\alpha}{\alpha - 1}(1+z)\left[(1+z)^{\frac{\alpha}{\alpha-1}} - 1\right]$$

The analysis is performed minimizing the $\chi^2(H_0, n) = \sum_i \frac{[\mu_i^{theor}(z_i|H_0, n) - \mu_i^{obs}]^2}{\sigma^2_{\mu 0,i} + \sigma^2_{mz,i}}$

Range	$H_0^{best}(km\,s^{-1}Mpc^{-1})$	n^{best}	χ^2
$-100 < n < 1/2(1-\sqrt{3})$	65	-0.73	1.003
$1/2(1-\sqrt{3}) < n < 1/2$	63	-0.36	1.160
$1/2 < n < 1$	100	0.78	348.97
$1 < n < 1/2(1+\sqrt{3})$	62	1.36	1.182
$1/2(1+\sqrt{3}) < n < 3$	65	1.45	1.003
$3 < n < 100$	70	100	1.418

f(R) solutions fitted against SNeIa



Very promising results! We have used recent GOODS survey
Giavalisco et al. ApJ, 600, L93 (2004)

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The Age test

The age of the universe can be theoretically calculated if one is capable of furnishing the Hubble parameter. We have:

$$t = \left(\frac{-2n^2 + 3n - 1}{n - 2} \right) H^{-1}$$

For the experimental value, we have considered both data coming from globular cluster observations and WMAP measurements of CMBR (this last estimate is very precise 13.7 ± 0.02 *Gyr*) [SC, Cardone, Troisi 2005]

Result → A fourth order theory $f(R) = f_0 R^n$ is able to fit SNeIa data and WMAP age prediction with

$Range$	$\Delta H (km\,s^{-1} Mpc^{-1})$	Δn	q_0
$-100 < n < 1/2(1-\sqrt{3})$	$50 - 80$	$-0.450 \leq n < -0.370$	< 0
$1/2(1-\sqrt{3}) < n < 1/2$	$57 - 69$	$-0.345 < n \leq -0.225$	> 0
$1 < n < 1/2(1+\sqrt{3})$	$56 - 70$	$1.330 \leq n < 1.360$	> 0
$1/2(1+\sqrt{3}) < n < 2$	$54 - 78$	$1.366 < n \leq 1.376$	< 0

f (R) theories, with small corrections to Einstein Gravity, seem in good agreement with DARK ENERGY, in particular for 1.366 < n < 1.376

Alternatively, observational H(z) gives f(R)

[SC, Cardone, Troisi 2005]

From field equations

$$\rho_m = \Omega_{M,0}\, \rho_{crit}\, a^{-3}$$

$$\frac{dH}{dt} = -\frac{1}{2f'(R)}\left\{3H_0^2\Omega_{M,0}(1+z)^3 + f''(R)\ddot{R} + \left[f'''(R)\dot{R} - Hf''(R)\right]\dot{R}\right\}$$

Fourth order equation for a(t)

Considering the relation between ***R*** and ***H*** (FRW-metric) and changing time variabile *t* with redshift *z*, we get a ***third order diff. eq. for f(R(z))***

$$R = -6\left[2H^2 - (1+z)H\frac{dH}{dz}\right]$$

$$\mathcal{H}_3(z)\frac{d^3f}{dz^3} + \mathcal{H}_2(z)\frac{d^2f}{dz^2} + \mathcal{H}_1(z)\frac{df}{dz} = -3H_0^2\Omega_{M,0}(1+z)^3$$

with H_i functions defined in term of *R(H(z),z)* and its *z* derivatives

✓ H(z) can be inferred from observations or by phenomenological approaches. The goal is numerically finding *f(z)* and then back tranforming this thanks to *z=z(R)* in *f(R)*

Q-essence, ***f(z)*** → ***f(R)*** $(l_1, l_2, l_3, l_4) = (2.6693, 0.5950, 0.0719, -3.0099)$

$$\ln(-f) = l_1\left[\ln(-R)\right]^{l_2}\left[1 + \ln(-R)\right]^{l_3} + l_4$$

…..L-CDM,

Quintessence...

Chaplygin, $(l_1, l_2, l_3, l_4) = (1.9814, 0.5558, 0.2665, -2.5337)$

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Can f(R)-theories reproduce also Dark Matter dynamics?

Main research interests:

1) Galactic dynamics (rotation curves of spiral galaxies)
2) DM in the Ellipticals
3) Galaxy cluster dynamics

The problem: we search for f (R)-solutions capable of fitting consistently the data. A nice feature is that the same f(R) – theory could work for Dark Energy (very large scales) and Dark Matter (small and clustered scales).

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DM in the galaxies as a curvature effect

✓ ***Theoretical issues***

Spiral galaxies are ideal candidates to test DM models. In particular, LSB galaxies are DM-dominated so that fitting their rotation curves WITHOUT DM is a good test for any alternative gravity theory

A further test is trying to fit Milky Way rotation curve WITHOUT DM. In this case, the approach could be problematic

In general, is it possible to define an *effective dark matter halo* induced by cuvature effects?

What about baryonic Tully-Fischer relation without DM?

DM as a Curvature effect:

- Let us consider again: $f(R) = f_0 R^n$

✓In the low energy limit, the spacetime metric can be written as

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2 d\Omega^2$$

✓A physically motivated hypothesis is

$$A(r) = \frac{1}{B(r)} = 1 + \frac{2\Phi(r)}{c^2}$$

✓An exact solution is

$$\Phi(r) = -\frac{Gm}{2r}\left[1 + \left(\frac{r}{r_c}\right)^{\beta}\right]$$

with

$$\beta = \frac{12n^2 - 7n - 1 - \sqrt{36n^4 + 12n^3 - 83n^2 + 50n + 1}}{6n^2 + 4n - 2}$$

✓***For n=1, $\beta = 0$. The approach is consistent with GR!***

✓The pointlike rotation curve is $$v_c^2(r) = \frac{Gm}{r}\left[1 + (1-\beta)\left(\frac{r}{r_c}\right)^{\beta}\right]$$

✓The galaxy can be modeled considering a thin disk and a bulge component.

✓The potential can be splitted in a Newtonian and a correction part

$$\Phi(r) = \Phi_N(r) + \Phi_c(r) = \frac{GM(r)}{r} + \Phi_c(r)$$

✓Integration can be performed considering a spherical symmetry for the bulge and a cylindrical symmetry for the disk.

$$\Phi_i(\tilde{r}, z) = \int_0^{\tilde{r}} \tilde{r}'\, d\tilde{r}' \int_0^{2\pi} d\theta \int_{-z}^{+z} dz'\, \rho(\tilde{r}', z')\, r^{\beta_i} \qquad \beta_i = \begin{cases} -1 & \text{i = N} \\ \beta(n) & \text{i = c} \end{cases}$$

✓The rotation curve is given by $$v_{circ}{}^2(\tilde{r}) = \tilde{r}\left.\frac{\partial \Phi_i}{\partial \tilde{r}}\right|_{z=0}$$

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DM in LSB galaxies as a curvature effect: results of fits on a sample of 15 galaxies

Id	β	$\log r_c$	f_g	$\Upsilon_\star$	χ^2/dof	σ_{rms}
UGC 1230	0.608	-0.24	0.26	7.78	3.24/8	0.54
UGC 1281	0.485	-2.46	0.57	0.88	3.98/21	0.41
UGC 3137	0.572	-1.97	0.77	5.54	49.4/26	1.31
UGC 3371	0.588	-1.74	0.49	2.44	0.97/15	0.23
UGC 4173	0.532	-0.17	0.49	5.01	0.07/10	0.07
UGC 4325	0.588	-3.04	0.75	0.37	0.20/13	0.11
NGC 2366	0.532	0.99	0.32	6.67	30.6/25	1.04
IC 2233	0.807	-1.68	0.62	1.38	16.29/22	0.81
NGC 3274	0.519	-2.65	0.72	1.12	19.62/20	0.92
NGC 4395	0.578	0.35	0.17	6.17	34.81/52	0.80
NGC 4455	0.775	-2.04	0.88	0.29	3.71/17	0.43
NGC 5023	0.714	-2.34	0.61	0.72	13.06/30	0.63
DDO 185	0.674	-2.37	0.90	0.21	6.04/5	0.87
DDO 189	0.526	-1.87	0.69	3.14	0.47/8	0.21
UGC 10310	0.608	-1.61	0.65	1.04	3.93/13	0.50

Best values of the model parameters from maximizing the joint likelihood function *L(β, log r_c, f_g)* . We report the value

$$\Upsilon_\star = \frac{[(1-f_g)\, M_g]}{f_g L_d}$$

with the gas fraction,

$$M_g = 1,4 M_{HI}$$

and the disk total mass and luminosity

$$M_d = \Upsilon_\star L_d$$

$$L_d = 2\pi I_0 R_d^2$$

dof=N-3, with N the number of datapoints. In the last column there is the root mean square of the fit residuals.

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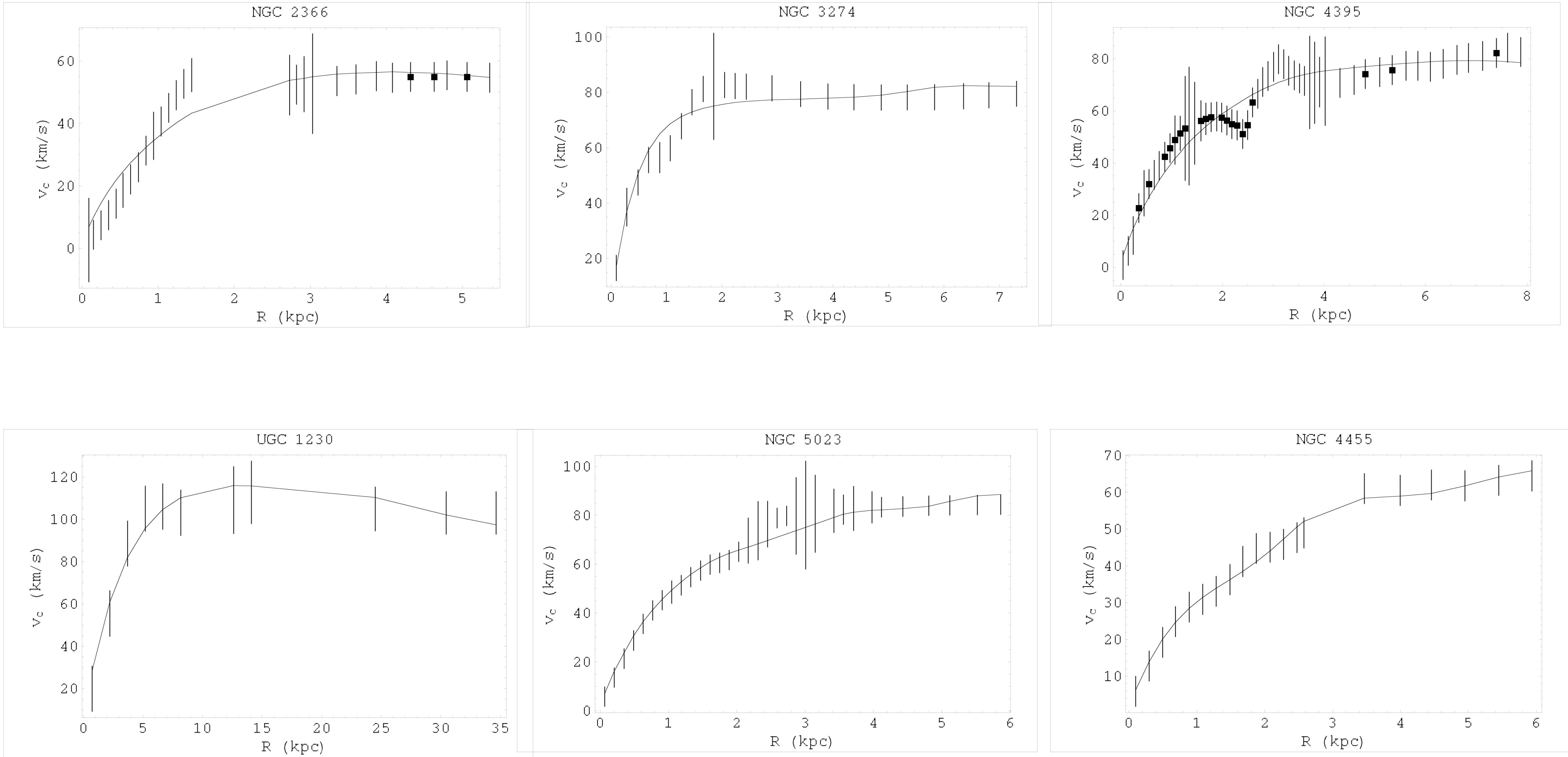
Experimental points vs. Fourth order gravity induced theoretical curves



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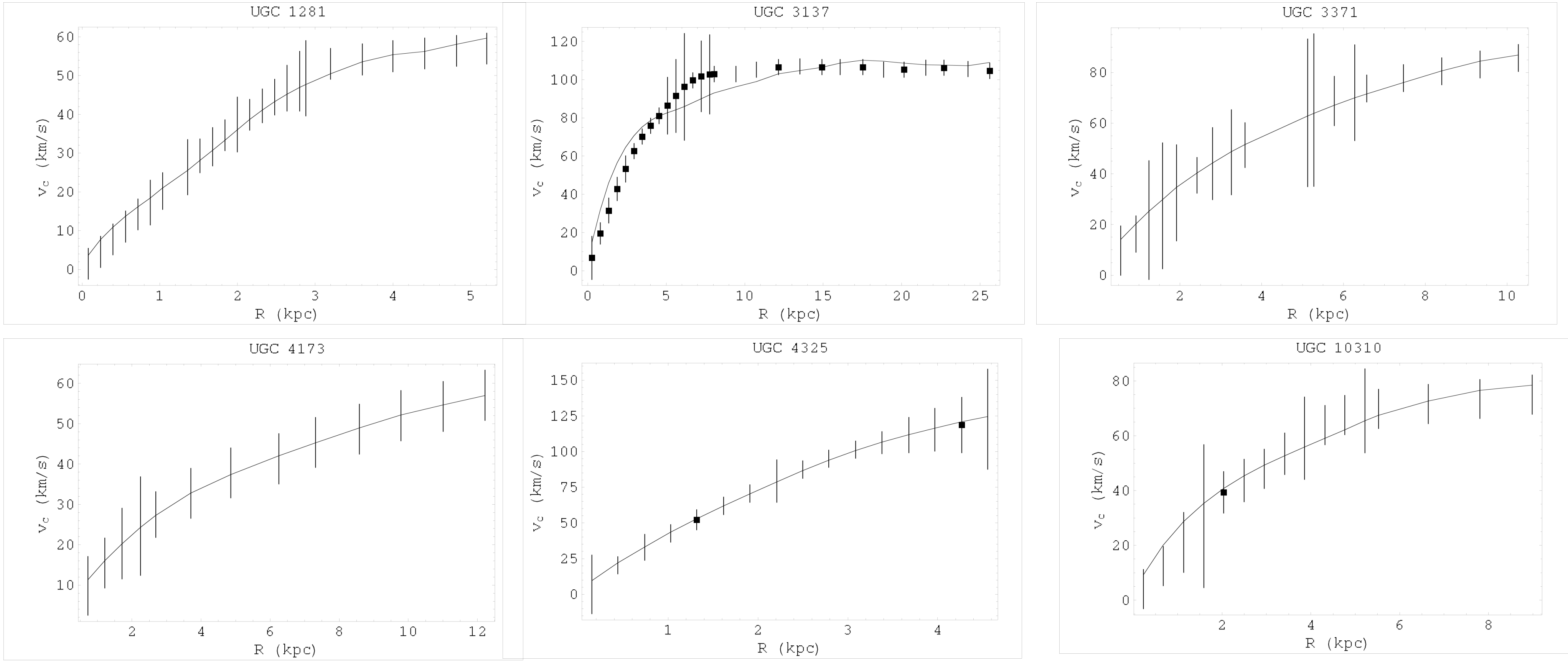

Data:

- Rotation curves
- Mass surface density
- Gas component
- Disk photometry

fit →

- M/L ratio $\Upsilon_\star$
- ß =universal parameter
- r_c = characteristic parameter related to the total mass of every galaxy

Results → Best Fit

$$\beta = 0.58 \pm 0.15$$

↓

$$1.34 < n < 2.41$$

No DARK component has been used!

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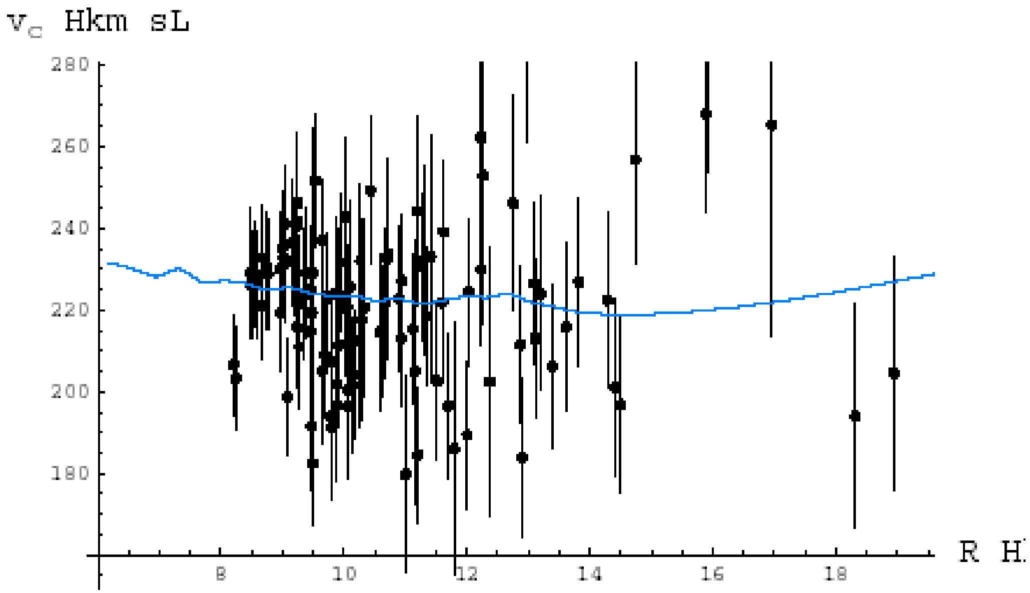
Milky Way data fit



Result

The upper points are affected by systematic errors as discussed in Pont et al. 1997

Also in this case, NO Dark Matter is needed

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At this point, it is worth wondering whether a link can be found between fourth order gravity and the standard approach, based on *effective dark matter haloes,* since both approaches fit equally well the data

Considering $v_c^2(r) = v_{c,N}^2(r) + v_{c,corr}^2(r)$ **due to modified gravity**

and $v_c^2(r) = v_{c,disk}^2(r) + v_{c,DM}^2(r)$ **due to effective DM halo**

being $v_{c,DM}^2(r) = GM_{DM}(r)/r$

Equating the two expressions, we get

$$M_{DM}(\eta) = 2^{\beta-5}\eta_c^{-\beta}\pi(1-\beta)\Sigma_0 R_d^2 \eta^{\frac{\beta+1}{2}} \mathcal{I}_0(\eta,\beta)$$

where $\eta = r/R_d,\ \Sigma_0 = \Upsilon_\star I_0$ and

$$\mathcal{I}_0(\eta,\beta) = \int_0^\infty \mathcal{F}_0(\eta,\eta',\beta) k^{3-\beta} \eta'^{\frac{\beta-1}{2}} e^{-\eta'} d\eta'$$

this means that the mass profile of an effective spherically symmetric DM halo, which provides corrected disk rotation curves, can be reproduced.

For $\beta = 0.58 \pm 0.15$ ***Burkert models are exactly reproduced!*** [Burkert 1995]

Vulcano 2006

Baryonic Tully-Fisher relation

From Virial theorem: $V_{vir} = GM_{vir}/R_{vir}$

$$M_d = \frac{(3/4\pi\delta_{th}\Omega_m\rho_{crit})^{\frac{1-\beta}{4}} R_d^{\frac{1+\beta}{2}} \eta_c^{\beta}}{2^{\beta-6}(1-\beta)G^{\frac{5-\beta}{4}}} \frac{V_{vir}^{\frac{5-\beta}{2}}}{\mathcal{I}_0(V_{vir}, \beta)}$$

Where $M_d \propto V_{vir}^a$ is the disk mass. For $\beta = 0.58 \pm 0.15$

empirical baryonic Tully-Fisher relation is reproduced!

[SC, Cardone, Troisi 2006]

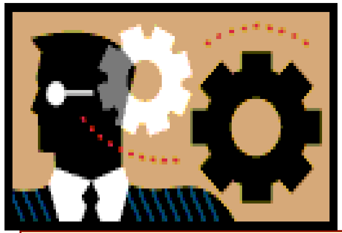
In summary: Dark Matter dynamics can be reproduced by f(R)-gravity

Results:

1) Galactic dynamics (rotation curves of LSB spiral galaxies)
2) Milky Way rotation curves
3) Burkert haloes
4) Tully-Fisher relation

Further issues: HSB galaxies, Elliptical galaxies, Clusters.
A nice feature could be that the same f(R) – theory works for Dark Energy (very large scales, unclustered) and Dark Matter (small and medium scales, clustered structures)

Vulcano 2006



Summarizing the results

- **Higher Order Curvature Theories gives solutions consistent with Dark Energy dynamics** [SC (2002), SC, Cardone, Carloni, Troisi (2003), SC, Cardone, Troisi (2005)]

- **Solutions agree with SNeIa e WMAP data (distance measurements).** [SC, Cardone, Carloni, Troisi 2003,2004]

- **Good agreement also with lookback time methods (time measurements)** [SC, Cardone, Funaro, Andreon (2004)]

- **Luminosity distance in f(R)** [SC, Cardone, Troisi (2004)]

- **f(R)- dynamical systems and stability analysis** [Carloni, SC, Dunsby, Troisi (2005)]

- **Rotation curves of LSB galaxies** [SC, Cardone, Troisi (2006)]

- **Baryonic Tully-Fisher, Burkert haloes** [SC, Cardone, Troisi (2006)]

Perspectives: → ***DE & DM as curvature effects***

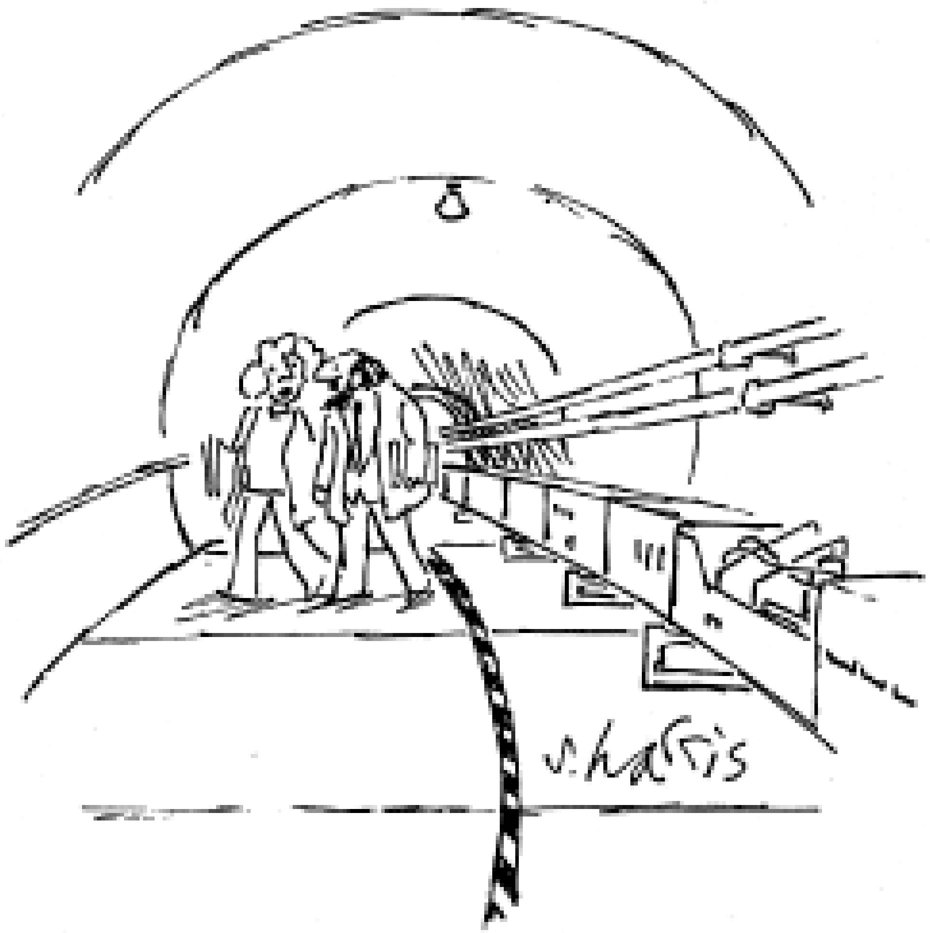
- Matching other DE models
- Jordan Frame and Einstein Frame
- Systematic studies of rotation curves of other galaxies
- Galaxy cluster dynamics (virial theorem, SZE, etc.)
- Luminosity profiles of galaxies in f(R).
- Faber-Jackson & Tully-Fisher

Weak Fields, GW and Static Fields →

- Systematic studies of PPN formalism
- Relativistic Experimental Tests in f(R)
- Gravitational waves and lensing
- Birkhoff ‘s Theorem in f (R)-gravity

WORK in PROGRESS! (suggestions are welcome!)

Ending with a joke: Dark Matter in the Lab!



….and if, after spending all this money,

are there no further particles to discover?