Transversity - a theoretical overview

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Outline

- Transversity - properties
- Accessing transversity: DSA & SSA
- The role of $k_T$ and $S_T$ dependent functions (Sivers, Collins, ...)
- Actual extraction of $h_1$ via SSA
- Open problems: process dependence, factorization, evolution, ...
- Interference fragmentation functions
- Role of $e^+e^-$ annihilation data
- Transversity inside unpolarized hadrons
Transversity

$h_1(x)$: distribution of transversely polarized quarks inside a transversely polarized proton

\[ \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S_T | \bar{\psi}(0) U(0, \lambda) i\sigma^i + \gamma_5 \psi(\lambda) | P, S_T \rangle = S_T^i \ h_1(x) \]

This is a chiral-odd/helicity flip quantity:

 Observables involving transversity should be (helicity flip)^2

$h_1(x)$ cannot be measured in inclusive DIS ($ep \rightarrow e'X$), suppressed by $m_a/Q$

In charged current exchange processes chiral-odd functions like $h_1(x)$ cannot be accessed

Hard to probe, one needs at least two hadrons in the process
Importance of transversity

If it is so hard to measure, why do the effort?

$h_1(x)$ encodes completely new information on the proton spin structure

The integral over $x$ is a fundamental charge, like the electric and the axial charge

$$\delta q \equiv \int dx \, h_1(x) = \text{tensor charge}$$

$$\langle P, S | \bar{\psi} q \sigma^{\mu\nu} \gamma_5 \psi_q(0) | P, S \rangle \sim \delta q \left[ P^\mu S^\nu - P^\nu S^\mu \right]$$

Unlike the electric or axial charge, $\delta q$ is not measurable in elastic scattering

Transverse polarization parton distributions are needed for the future step:

polarized hadron colliders $\longrightarrow$ polarized quark colliders

Perhaps learn something about chiral symmetry breaking
What is known about the size of $h_1$?

Information on the tensor charge from the lattice and from models

First lattice determination by Aoki et al. (PRD 56 (’97) 433) (at $\mu^2 = 2$ GeV$^2$)

$$
\delta u = +0.839(60) \\
\delta d = -0.321(55) \\
\delta s = -0.046(34)
$$

Other determinations find similar magnitudes [→ Philipp Hägler’s talk]

Most models find results roughly in the range:

$$
\delta u = +1.0 \pm 0.2 \\
\delta d = -0.2 \pm 0.2
$$

There is no reason to believe that $\delta q$ is small
Soffer bound

Soffer (PRL 74 (‘95) 1292) derived the following bounds:

\[ |\delta u| \leq \frac{3}{2} \]
\[ |\delta d| \leq \frac{1}{3} \]

Based on his inequality

\[ |h_1(x)| \leq \frac{1}{2} [f_1(x) + g_1(x)] \]

Remarks:

• At NLO one can devise factorization schemes that violate the bound
• Saturation of the bound is not preserved under evolution
• Satisfying the inequality is preserved under evolution to larger scales

Martin, Schäfer, Stratmann & Vogelsang, PRD 60 (‘99) 117502
Energy scale dependence

The evolution of $h_1(x, Q^2)$ is very different from that of $g_1(x, Q^2)$
In part, because there is no gluon transversity distribution

$\delta u$ grows with increasing $Q^2$ towards smaller $x$, but as $Q^2 \to \infty$: $h_1(x, Q^2) \to 0$

Hayashigaki, Kanazawa, Koike, PRD 56 ('97) 7350; Vogelsang, PRD 57 ('98) 1886
Unlike the electric or axial charge, the tensor charge is mildly energy scale dependent.

At $Q^2 \sim 10^{111}$, the tensor charge is still only reduced by factor 1/2 w.r.t. $Q^2 = 1$

The r.h.s. plot compares $\int dx \, x \, h_1(x)$ and $\int dx \, x \, f_1(x)$

Hayashigaki, Kanazawa, Koike, PRD 56 ('97) 7350
Accessing transversity

Transversity can be probed by using other hadrons

- $p^{\uparrow} p^{\uparrow} \rightarrow \ell \ell' X$ (Ralston-Soper '79; ...)
- $e p^{\uparrow} \rightarrow e' \pi X$ (Collins '93)
- $p p^{\uparrow} \rightarrow \Lambda^{\uparrow} X$ (De Florian, Soffer, Stratmann, Vogelsang '98)
- $e p^{\uparrow} \rightarrow \Lambda^{\uparrow} X$
- $e p^{\uparrow} \rightarrow e' (\pi^+ \pi^-) X$ (Ji '94; Collins, Heppelmann, Ladinsky '94; Jaffe, Jin, Tang '98; ...)
- ...

These suggestions can be categorized as follows:

- **Double transverse spin asymmetries (DSA)**
- **Single transverse spin asymmetries (SSA)**, using:
  - $k_T$-dependent functions
  - Interference fragmentation functions
  - Higher spin functions, e.g. $\rho$ fragmentation functions
  - Higher twist functions
Double spin asymmetry: Drell-Yan

The Drell-Yan Process

\[ H_1 + H_2 \rightarrow \ell + \bar{\ell} + X \]

\[
A_{TT} = \frac{\sigma(p^\uparrow p^\uparrow \rightarrow \ell \ell' X) - \sigma(p^\uparrow p^\downarrow \rightarrow \ell \ell' X)}{\sigma(p^\uparrow p^\uparrow \rightarrow \ell \ell' X) + \sigma(p^\uparrow p^\downarrow \rightarrow \ell \ell' X)} = \frac{\sin^2 \theta \cos 2\phi_S}{1 + \cos^2 \theta} \sum_{a,\bar{a}} e_a^2 h_1^a(x_1) \bar{h}_1^a(x_2)
\]

Problem: \( h_1 \) for antiquarks presumably much smaller than for quarks

Using Soffer’s inequality, \( A_{TT} \) shown to be too small at RHIC (percent level)

Martin, Schäfer, Stratmann & Vogelsang, PRD 60 (’99) 117502
Next to the (leading twist) distribution functions

\[ \Phi(x) = \frac{1}{2} \left[ f_1(x) P + g_1(x) \lambda \gamma_5 P + h_1(x) \gamma_5 S_T P \right] \]

there are also fragmentation functions

\[ \Delta(z) = \frac{1}{2} \left[ D_1(z) P + G_1(z) \lambda \gamma_5 P + H_1(z) \gamma_5 S_T P \right] \]

\( H_1 \) is relevant for transversely polarized hyperon production and DSA like:

- \( e p^\uparrow \rightarrow \Lambda^\uparrow X \) [HERMES, COMPASS]: \( D_{NN} \propto h_1 H_1 \)
- \( p p^\uparrow \rightarrow \Lambda^\uparrow X \) [E704, RHIC]
**Single spin asymmetries**

Double transverse spin asymmetries do not seem promising yet to extract the transversity distribution.

Remaining option: single spin asymmetries

Unpolarized final state [pions are easier than hyperons]

- \( \Delta(z) \rightarrow \Delta(z, k_T) \) (measure transverse momentum of the final state hadron compared to the jet direction): the Collins effect

- Hadrons with higher spin, e.g. \( \rho \), or related to it \( \pi^+\pi^- \) Interference fragmentation functions

- Higher twist SSA (\( \propto h_1 \times \text{twist-3 function} \))
Collins effect asymmetry in SIDIS

Collins, NPB 396 (’93) 161

\[
\frac{d\sigma(e p^\uparrow \rightarrow e' \pi X)}{d\phi_\pi^e d|P_{\pi}^\perp|^2} \propto \{1 + |S_T| \sin(\phi_\pi^e + \phi_S^e) A_T\}, \quad A_T \sim h_1 H_1^\perp
\]

ECT* Workshop on Transversity, Trento, June 14, 2004
Collins effect asymmetries

Experimental indications that the Collins effect is nonzero: SMC, HERMES, COMPASS

Possibly also related to the SSA in $p p^\uparrow \rightarrow \pi X$ [E704, AGS, STAR]

A left-right asymmetry
Pion distribution is asymmetric depending on transverse spin direction and on pion charge

Explanation at the quark-gluon level?

It could be the Collins effect (Anselmino, Boglione, D’Alesio, Murgia)

$$A_T \sim h_1(x_1) \otimes (f_1(x_2) \text{ or } g(x_2)) \otimes H_1^\perp(z, k_T)$$

Bourrely & Soffer, hep-ph/0311110
**Sivers effect**

First proposal of a $k_T$ & $S_T$ dependent distribution function by Sivers (PRD 41 ('90) 83)

$$f_{1T}^\perp = P_{x,k} - S_{x,k}$$

Inspired by data on $p p \uparrow \rightarrow \pi^0 X$ (Antille, PLB 94 ('80) 523) and the advent of E704

Intended as a test of perturbative QCD for large $p_T$ hadron production in $p p$ scattering
Sivers asymmetry in SIDIS

Sivers effect in semi-inclusive DIS \( e p^\uparrow \rightarrow e' \pi X \)

\[
\frac{d\sigma(e p^\uparrow \rightarrow e' \pi X)}{d\phi^e_{\pi} d|P^\pi_\perp|^2} \propto \{1 + |S_T| \sin(\phi^e_\pi - \phi^e_S) A_T\}, \quad A_T \sim f_{1T}^\perp D_1
\]

\( f_{1T}^\perp \) and \( H_1^\perp \) proposed because of SSA and \( h_1 \), but are of interest in their own right
**$k_T$-dependent transverse spin functions**

Besides $f_{1T}^{\perp}$ and $H_{1}^{\perp}$, there are two other, equally interesting $k_T$-odd functions

\[ D_{1T}^{\perp} = \begin{array}{c} \Lambda \\ \downarrow \end{array} k_T - \begin{array}{c} \Lambda \\ \downarrow \end{array} \]

\[ h_{1}^{\perp} = \begin{array}{c} q \\ \downarrow \end{array} k_T - \begin{array}{c} q \\ \downarrow \end{array} \]

Mulders & Tangeman, NPB 461 ('96) 197; D.B. & Mulders, PRD 57 ('98) 5780

These functions have unexpected properties, not yet fully studied

Recent developments spurred on by a model calculation of

Brodsky, Hwang & Schmidt (PLB 530 ('02) 99)
One problem is that the $k_T$-dependent functions appear in convolution integrals. For example, Sivers effect in SIDIS:

$$\frac{d\sigma(e p^\uparrow \rightarrow e' \pi X)}{d^2 q_T} \propto |S_T| \frac{\sin(\phi^e_\pi - \phi^e_S)}{Q_T} F \left[ \frac{q_T \cdot p_T}{M} f_{1T}^+ D_1 \right]$$

$$F[w f D] \equiv \int d^2 p_T \, d^2 k_T \, \delta^2(p_T + q_T - k_T) w(p_T, q_T, k_T) f(x, p_T^2) D(z, z^2 k_T^2)$$

One solution would be to measure “jet SIDIS”: $e p^\uparrow \rightarrow e' \text{ jet } X$

$$\frac{d\sigma(e p^\uparrow \rightarrow e' \text{ jet } X)}{d^2 q_T} \propto |S_T| \sin(\phi^e_\pi - \phi^e_S) \frac{Q_T}{M} f_{1T}^+(x, Q_T^2), \quad Q_T^2 = |P_{\perp}^{\text{jet}}|^2$$

One can probe the $k_T$-dependence of the Sivers function directly in this way!

For asymmetries involving chiral-odd quantities this cannot be done.
A more general solution is to consider weighted asymmetries.
Jet SIDIS $e p^\uparrow \rightarrow e'\text{jet} X$

Cross sections integrated, but weighted with function of *observed* transverse momentum

$$\langle W \rangle_{UT} \equiv \int dz \, d^2 P_{\perp}^{\text{jet}} \, W \frac{d\sigma^{[e p^\uparrow \rightarrow e'\text{jet} X]}}{dx \, dy \, dz \, d\phi_{\text{jet}}^e \, d|P_{\perp}^{\text{jet}}|^2}$$

Weighted asymmetries become expressions in terms of transverse moments

$$\langle \cos \phi_{\text{jet}}^e \frac{|P_{\perp}^{\text{jet}}|}{M} \rangle_{UT} \propto -\sin \phi_{S}^e \sum_{a,\bar{a}} e_a^2 x \, f_{1T}^{\perp(1) a}(x)$$

D.B. & Mulders, PRD 57 ('98) 5780

This asymmetry contains a *weighted* Sivers function:

$$f_{1T}^{\perp(1) a}(x) = \int d^2 k_T \frac{k_T^2}{2M^2} f_{1T}^{\perp a}(x, k_T^2)$$

Such transverse moments appear in different asymmetries in exactly the same way
Actual extraction

Even if one deconvolutes asymmetries by weighting, in case of chiral-odd quantities, one is always dealing with products of functions (summed over flavors)

\[ h_1 \bar{h}_1, \quad h_1 H_1, \quad h_1 H_1^{(1)} , \quad ... \]

Almost no experiment aiming to extract \( h_1 \) will be self-sufficient.

This would only apply to \( (h_1)^2 \) observables:

\[ \bar{p} \uparrow p \uparrow \rightarrow \ell \bar{\ell} X \quad [\text{GSI?}] \]

\[ p \uparrow p \uparrow \rightarrow \text{high } p_T \text{ jet } + X \quad [\text{RHIC?}] \]

Anselmino et al., hep-ph/0403114
Efremov, Goeke, Schweitzer, hep-ph/0403124
Jaffe & Saito, PLB 382 ('96) 165
Vogelsang ('00)

A global transversity analysis is needed, therefore:

Process dependence and evolution of \( k_T \)-dependent functions are very relevant issues.
Process dependence

Gauge invariant definition of Sivers function in DIS contains a future pointing Wilson line, whereas in Drell-Yan (DY) it is past pointing:

\[ f_{1T}^\perp \propto \langle P, S_T | \overline{\psi}(0) U(0, \xi) \gamma_\perp \psi(\xi) | P, S_T \rangle \]

Belitsky, Ji & Yuan, NPB 656 ('03) 165

As a consequence (Collins, PLB 536 ('02) 43):

\[ (f_{1T}^\perp)_{\text{DIS}} = -(f_{1T}^\perp)_{\text{DY}} \]

What about more complicated processes? (Bomhof, Mulders & Pijlman, hep-ph/0406099)

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Factorization and related issues

• The process dependence or universality of fragmentation functions

  On the basis of symmetry restrictions alone one finds schematically

  \[
  (H_1^\perp)_{\text{SIDIS}} \equiv A + B \\
  (H_1^\perp)_{e^+e^-} = A - B
  \]

  But model calculation by Metz (PLB 549 ('02) 139) shows

  \[
  (H_1^\perp)_{\text{SIDIS}} = (H_1^\perp)_{e^+e^-}
  \]

• All open issues related to: establishing the proper factorization theorems

  Collins & Soper, NPB 193 ('81) 381; Ji, Ma & Yuan, hep-ph/0404183; hep-ph/0405085

• Should answer the question “What exactly is a parton density?”, posed by Collins
  (hep-ph/0304122)

• Evolution or energy scale dependence of functions and asymmetries, e.g.,
  Sudakov suppression of azimuthal spin asymmetries

  D.B., NPB 603 ('01) 195; D.B. & Vogelsang, PRD 69 ('04) 094025
Illustration: Collins effect asymmetry in SIDIS

Consider the Collins effect asymmetry in SIDIS:

\[
\frac{d\sigma(e p^\uparrow \to e'\pi X)}{dx dz dy d\phi_e d^2 q_T} \propto \{1 + |S_T| \sin(\phi^e_\pi + \phi^e_S) A(q_T)\}
\]

Assume Gaussian transverse momentum dependence for \(H_1^\perp\)

\[
H_1^\perp(z, k_T^2) = H_1^\perp(z) \frac{R^2}{z^2 \pi} \exp(-R^2 k_T^2)
\]

The asymmetry analyzing power is then given by

\[
A(q_T) = \frac{(1 - y)}{(1 - y + \frac{1}{2} y^2)} \sum_a e_a^2 h_a^a(x) H_1^{(1)a}(z) \sum_b e_b^2 f_b^1(x) D_b^1(z) A(Q_T)
\]

At tree level:

\[
A(Q_T) = M_\pi Q_T R^4 \frac{R^4}{R_u^2} e^{- (R^2 - R_u^2) Q_T^2 / 2}
\]
Beyond tree level (in CS-81 spirit)

\[ A(Q_T) = M_\pi \frac{\int db \ b^2 J_1(bQ_T) \exp \left( -S(b_*) - S_{NP}(b) \right)}{\int db \ b J_0(bQ_T) \exp \left( -S(b_*) - S_{NP}(b) \right)} \]

\[ b_* = \frac{b}{\sqrt{1 + b^2/b_{\text{max}}^2}} \]

\( S(b_*) \) can be calculated perturbatively and \( S_{NP} \) has to be fitted to experiment.

Generic example (leading log and \( S_{NP} \) from Ladinsky-Yuan '94):

\[ A(Q_T, Q) \sim Q^{-0.5} - Q^{-0.6} \text{ (partial power suppression)} \]

D.B., NPB 603 ('01) 195
Beyond tree level

Consider instead the weighted observable

$$
\mathcal{O} \equiv \frac{\langle \sin(\phi^e_\pi + \phi^e_S) | P_{\perp}^\pi | M_{\pi} \rangle}{[4\pi \alpha^2 s/Q^4]} = |S_T| (1-y) \sum_{a, \bar{a}} e^2_a x h^a_1(x) z H_{1(1)}^{\perp}(z)
$$

$$
H_{1}^{\perp(1)}(z) \equiv \int d^2 k_T \frac{k_T^2}{2z^2 M_{\pi}^2} H_{1}^{\perp}(z, k_T^2)
$$

\(\mathcal{O}\) is not sensitive to Sudakov suppression

Hence, if one measures \(\mathcal{O}(Q_{0}^2)\) and \(\mathcal{O}(Q_{1}^2)\), one can relate them via LO evolution

NLO evolution equation is known for \(h_1\)

LO evolution equation for \(H_{1}^{\perp(1)}\) still to be established

Conclusion: from the theoretical point of view Collins effect asymmetries are not like ordinary leading twist asymmetries
SSA: Interference Fragmentation Functions

Consider the final state \(|\pi^+ \pi^- X\rangle\), with \(\pi^+\) and \(\pi^-\) belonging to same jet

Ji ('94); Collins, Heppelmann, Ladinsky ('94); Jaffe, Jin, Tang ('98); ...

\[
\Delta(z) \propto \left[ D_1(z) P + i H_1^\downarrow \frac{R_T P}{2M_\pi} \right]
\]

\[
z = z^+ + z^-
\]

\[
P = P_{\pi^+} + P_{\pi^-}
\]

\[
R_T = (z^+ P_{\pi^-} - z^- P_{\pi^+})/z
\]

Radici, Jakob, Bianconi, PRD 65 ('02) 074031

Nonzero \(H_1^\downarrow\) due to interference between different partial waves of the \((\pi^+ \pi^-)\) system

This leads to single spin asymmetries:

\[
e p^\uparrow \rightarrow e' (\pi^+ \pi^-) X
\]

\[
p p^\uparrow \rightarrow (\pi^+ \pi^-) X
\]

\[
h_1 \otimes H_1^\downarrow
\]

\[
f_1/g \otimes h_1 \otimes H_1^\downarrow
\]

Ji, PRD 49 ('94) 114; ...
SSA: Interference Fragmentation Functions

The SSA in $e p^\uparrow \rightarrow e' (\pi^+ \pi^-) X$ according to Jaffe, Jin, Tang (PRL 80 ('98) 1166)

$$\langle \sin(\phi_{ST}^e + \phi_{RT}^e) \rangle \propto |S_T||R_T| F(m^2) \sum_{a,\bar{a}} e_a^2 x h_1^a(x) \delta q_I(z)$$

$F(m^2) = \sin \delta_0 \sin \delta_1 \sin(\delta_0 - \delta_1)$, where $\delta_0, \delta_1$ are the strong $\pi^+\pi^-$ phase shifts and $m^2$ the invariant mass of $\pi^+\pi^-$

Conclusion: best done around the $\rho$ mass region (rapid change around $F(m^2_\rho) = 0$)

But is based on assumption of factorization of $z$ and $m^2$ dependence
More general dependences are possible
Bianconi et al., PRD 62 ('00) 034008 & 034009; Radici et al., PRD 65 ('02) 074031

In addition, the T-odd $\rho$ fragmentation function ($H_{1LT}(z)$) consists of more than $s$-$p$ wave interference; there is also a piece from the $p$-wave only
Bacchetta, Mulders, PRD 62 ('00) 114004; Bacchetta, Radici, PRD D67 ('03) 094002
Advantages

Collinear factorization (no Sudakov suppression, no process dependence, etc) Only functions of lightcone momentum fractions

Evolution of $H_1^Q(z)$ same as for $H_1(z)$ (known to NLO)
Stratmann & Vogelsang, PRD 65 ('02) 057502

Reminder

Ordinary fragmentation function $D_1^{\pi^+\pi^-}$ needs to be determined separately
Role of electron-positron annihilation data

Almost all proposed transversity measurements contain unknown fragmentation functions

- \(e^+ e^- \rightarrow \Lambda^\uparrow \bar{\Lambda}^\uparrow X: (H_1)^2\)  
  Contogouris et al., PLB 344 ('95) 370

- \(e^+ e^- \rightarrow \pi^+ \pi^- X: \langle \cos(2\phi_1) \rangle \propto (H_1^\perp)^2\)  
  D.B., Jakob & Mulders, NPB 504 ('97) 345

- \(e^+ e^- \rightarrow (\pi^+ \pi^-)_{\text{jet} 1} (\pi^+ \pi^-)_{\text{jet} 2} X:\)  
  \(\langle \cos(\phi_{R1T}^e + \phi_{R2T}^e) \rangle \propto (H_1^\perp)^2\)  
  Artru & Collins, ZPC 69 ('96) 277

One can use off-resonance data of \(B\)-factories (BELLE, BABAR)

Grosse Perdekamp et al., NPA 711 ('02) 69c

Using the two-hadron FF is easiest, since there is no asymmetric background
Transversity inside unpolarized hadrons

What does nonzero $h_{1\perp}$ mean?

The transverse polarization of a noncollinear quark inside an unpolarized hadron in principle can have a preferred direction

Implies an intrinsic handedness

$$h_{1\perp} = P - P$$
The unpolarized Drell-Yan process

NA10 Collab. ('86/'88) & E615 Collab. ('89) measured the angular distribution of lepton pairs

\[ \frac{1}{\sigma} \frac{d\sigma}{d\Omega} \propto \left( 1 + \lambda \cos^2 \theta + \mu \sin^2 \theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right) \]

Perturbative QCD relation \( (\mathcal{O}(\alpha_s)) \): \( 1 - \lambda - 2\nu = 0 \)

Data: considerable deviation from this Lam-Tung relation

NLO doesn’t fix it

Data for \( \pi N \rightarrow \mu^+ \mu^- X \), with \( N = D, W \); \( \pi \)-beams of 140-286 GeV; \( Q \sim 4 - 12 \) GeV
Explaining the unpolarized DY data

\[ h_1^\perp \neq 0 \implies \text{deviation from Lam-Tung relation} \]

Offers a tree level (\( \lambda = 1, \mu = 0 \)) explanation of NA10 data:

\[ \nu \propto h_1^\perp(\pi) h_1^\perp(N) \]

Fit \( h_1^\perp \) to data

D.B., PRD 60 (’99) 014012
The polarized Drell-Yan process

In the case of one polarized hadron (choosing $\lambda = 1$ and $\mu = 0$):

$$\frac{d\sigma}{d\Omega \, d\phi_S} \propto 1 + \cos^2 \theta + \sin^2 \theta \left[ \frac{\nu}{2} \cos 2\phi - \rho |S_T| \sin(\phi + \phi_S) \right] + \ldots$$

Assuming $u$-quark dominance and Gaussian $k_T$ dependence for $h_1^\perp$:

$$\rho = \frac{1}{2} \sqrt{\frac{\nu}{\nu_{\text{max}}} \frac{h_1^u}{f_1^u}}$$

It offers another probe of transversity.
Data to test $h_1$ hypothesis

Possible future DY data

RHIC: can measure $\nu$ and $\rho \implies$ information on $h_1$

Fermilab: $\nu$ in $p\bar{p} \rightarrow \mu^+\mu^- X$ (probably yields larger results)

GSI: future PANDA experiment $p\bar{p} \rightarrow \mu^+\mu^- X$; $Q \lesssim 6$ GeV

Semi-inclusive DIS

The $\langle \cos 2\phi \rangle$ in SIDIS would be $\propto h_1 H_1$

ZEUS data seems to follow pQCD, hence this could be a sign that the magnitude of $H_1$ is smaller than that of $h_1$

But perhaps $Q^2$ is too high to probe this contribution: due to Sudakov suppression
Sudakov suppression

Assuming Gaussian $k_T$ dependence for $h_1^{1/2}$, the $\cos(2\phi)$ asymmetry is proportional to

$$\mathcal{A}(Q_T) \equiv M^2 \frac{\int_0^\infty db \, b^3 \, J_2(bQ_T) \exp \left( -S(b_*) - S_{NP}(b) \right)}{\int_0^\infty db \, b \, J_0(bQ_T) \exp \left( -S(b_*) - S_{NP}(b) \right)}$$

Considerable Sudakov suppression with increasing $Q$: $\sim 1/Q$ (effectively twist-3)
Conclusions

- Transversity is a worthwhile quantity to try to measure
- Difficult to measure, no info from elastic scattering or inclusive DIS
- The lattice and models indicate that the tensor charge is not small
- Double transverse spin asymmetries not promising yet
- $k_T$-dependent functions $(H^+_1, h^+_1)$ can be used as tools to extract transversity
- $k_T$-dependence of Sivers function can be directly accessed in jet SIDIS
- Weighting with observed transverse momentum important
- Process dependence, scale dependence, factorization requires further study
- $e^+e^-$ annihilation data needed, no experiment self-sufficient
- Two-hadron fragmentation functions most promising