# Transversity - a theoretical overview

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## Outline

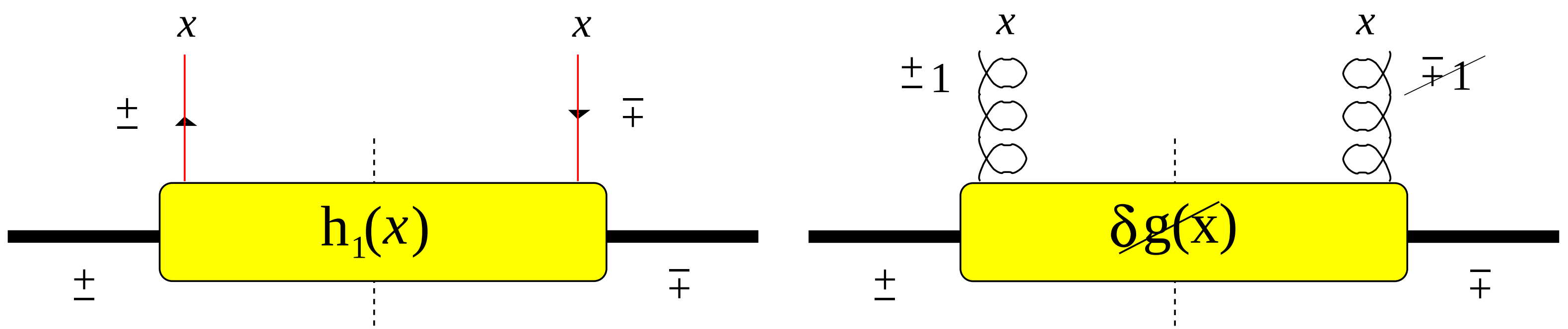
- Transversity - properties
- Accessing transversity: DSA & SSA
- The role of $k_T$ and $S_T$ dependent functions (Sivers, Collins, ...)
- Actual extraction of $h_1$ via SSA
- Open problems: process dependence, factorization, evolution, ...
- Interference fragmentation functions
- Role of $e^+e^-$ annihilation data
- Transversity inside unpolarized hadrons

# Transversity

$h_1(x)$: distribution of transversely polarized quarks inside a transversely polarized proton

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S_T | \overline{\psi}(0) U(0,\lambda) i\sigma^{i+} \gamma_5 \psi(\lambda) | P, S_T \rangle = S_T^i \, h_1(x)$$

This is a chiral-odd/helicity flip quantity:




Observables involving transversity should be (helicity flip)$^2$

$h_1(x)$ cannot be measured in inclusive DIS ($ep \to e'X$), suppressed by $m_a/Q$

In charged current exchange processes chiral-odd functions like $h_1(x)$ cannot be accessed

Hard to probe, one needs at least two hadrons in the process

# Importance of transversity

If it is so hard to measure, why do the effort?

$h_1(x)$ encodes completely new information on the proton spin structure

The integral over $x$ is a fundamental charge, like the electric and the axial charge

$$\delta q \equiv \int dx\, h_1(x) = \text{ tensor charge}$$

$$\langle P, S | \overline{\psi}_q \sigma^{\mu\nu} \gamma_5 \psi_q(0) | P, S \rangle \sim \delta q \left[ P^\mu S^\nu - P^\nu S^\mu \right]$$

Unlike the electric or axial charge, $\delta q$ is not measurable in elastic scattering

Transverse polarization parton distributions are needed for the future step:

polarized hadron colliders $\longrightarrow$ polarized quark colliders

Perhaps learn something about chiral symmetry breaking

# What is known about the size of $h_1$?

Information on the tensor charge from the lattice and from models

First lattice determination by Aoki *et al.* (PRD 56 ('97) 433) (at $\mu^2 = 2$ GeV$^2$)

$$\begin{aligned} \delta u &= +0.839(60) \\ \delta d &= -0.321(55) \\ \delta s &= -0.046(34) \end{aligned}$$

Other determinations find similar magnitudes [$\rightarrow$ Philipp Hägler's talk]

Most models find results roughly in the range:

$$\begin{aligned} \delta u &= +1.0 \pm 0.2 \\ \delta d &= -0.2 \pm 0.2 \end{aligned}$$

There is no reason to believe that $\delta q$ is small

# Soffer bound

Soffer (PRL 74 ('95) 1292) derived the following bounds:

$$
\begin{aligned}
|\delta u| &\leq 3/2 \\
|\delta d| &\leq 1/3
\end{aligned}
$$

Based on his inequality

$$|h_1(x)| \leq \frac{1}{2}\left[f_1(x) + g_1(x)\right]$$

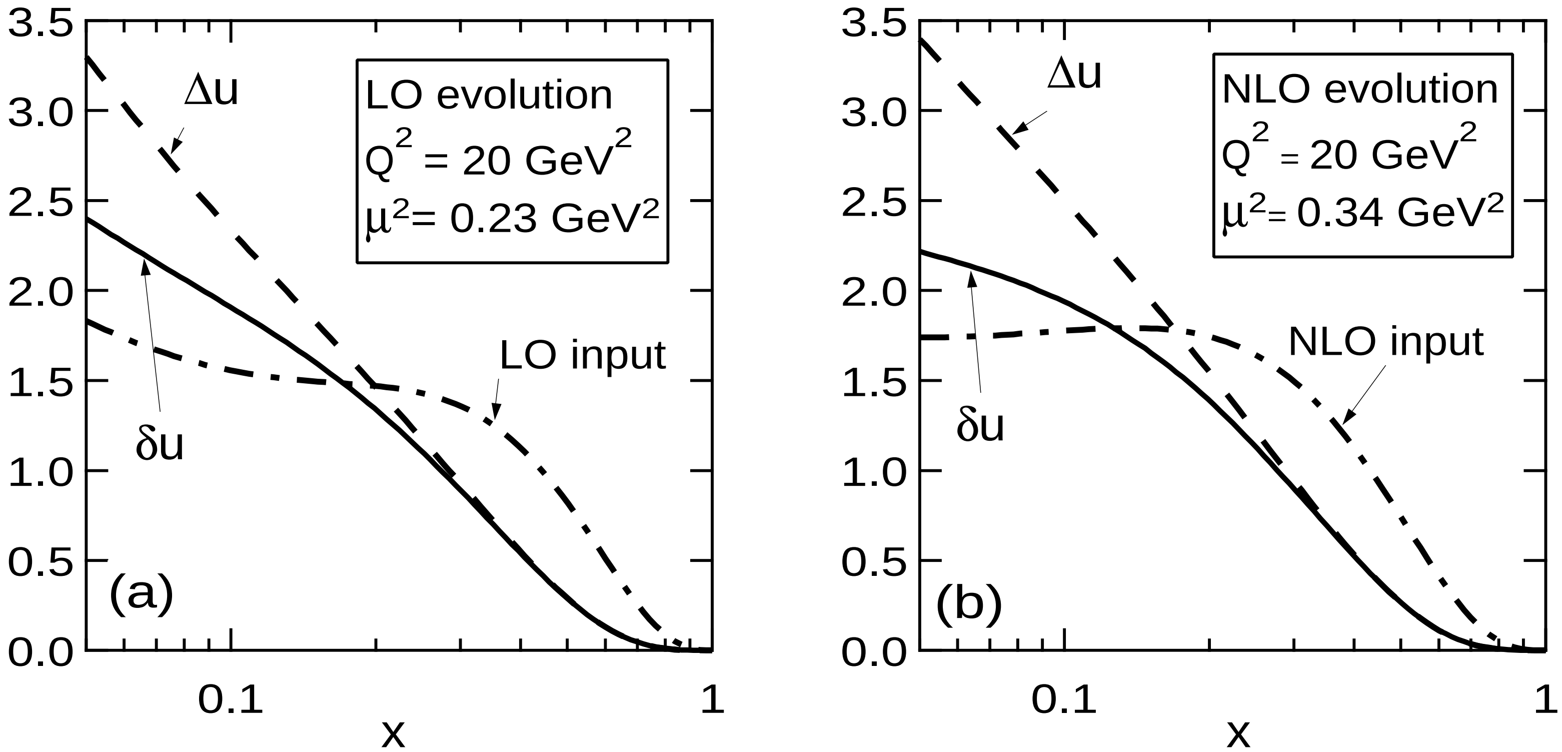
Remarks:

- At NLO one can devise factorization schemes that violate the bound
- Saturation of the bound is not preserved under evolution
- Satisfying the inequality is preserved under evolution to larger scales

Martin, Schäfer, Stratmann & Vogelsang, PRD 60 ('99) 117502

# Energy scale dependence

The evolution of $h_1(x, Q^2)$ is very different from that of $g_1(x, Q^2)$
In part, because there is no gluon transversity distribution

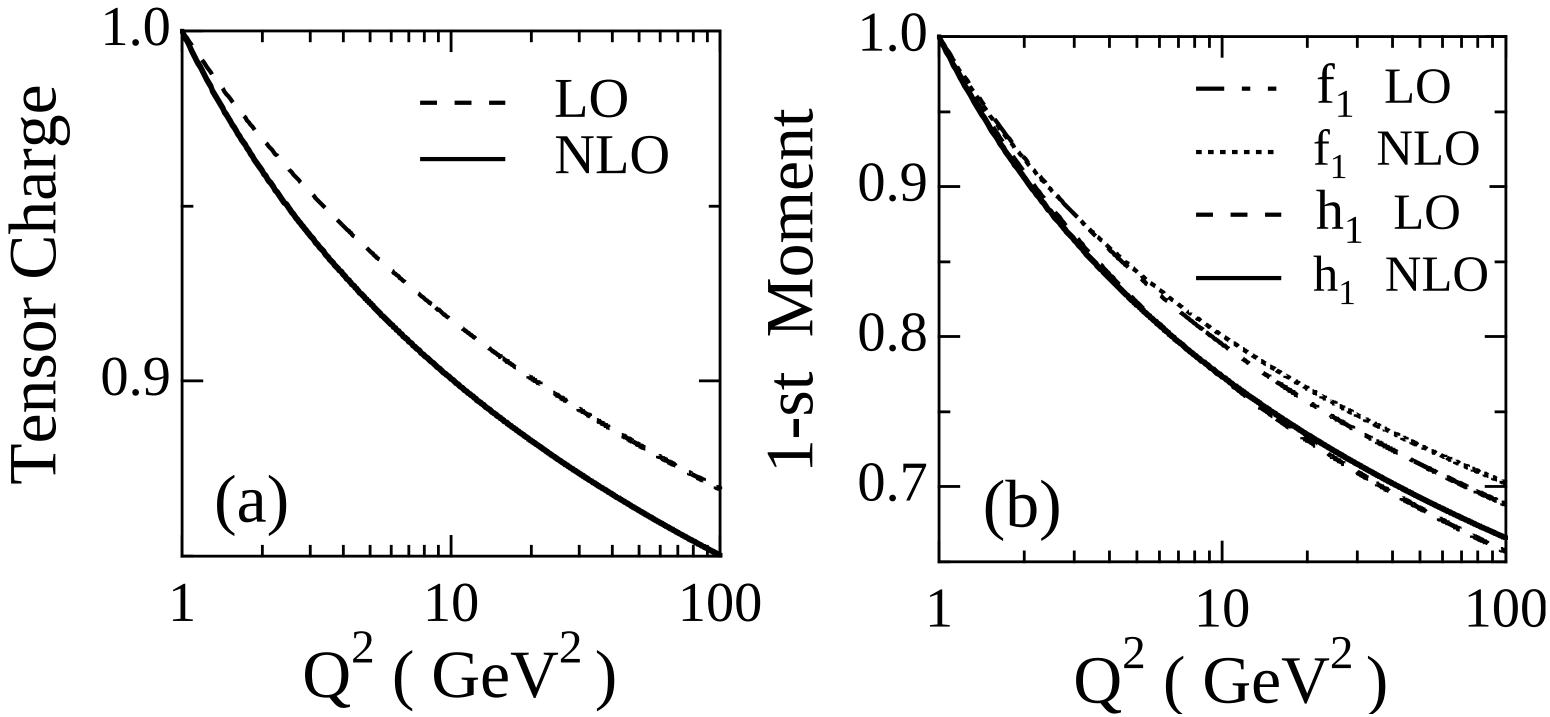



Hayashigaki, Kanazawa, Koike, PRD 56 ('97) 7350; Vogelsang, PRD 57 ('98) 1886

$h_1$ grows with increasing $Q^2$ towards smaller $x$, but as $Q^2 \to \infty$: $h_1(x, Q^2) \to 0$

# Energy scale dependence

Unlike the electric or axial charge, the tensor charge is mildly energy scale dependent




Hayashigaki, Kanazawa, Koike, PRD 56 ('97) 7350

At $Q^2 \sim 10^{111}$, the tensor charge is still only reduced by factor $1/2$ w.r.t. $Q^2 = 1$

The r.h.s. plot compares $\int dx\, x\, h_1(x)$ and $\int dx\, x\, f_1(x)$

# Accessing transversity

Transversity can be probed by using other hadrons

- $p^\uparrow \, p^\uparrow \longrightarrow \ell \, \ell' \, X$ (Ralston-Soper '79; ...)
- $e \, p^\uparrow \longrightarrow e' \, \pi \, X$ (Collins '93)
- $p \, p^\uparrow \longrightarrow \Lambda^\uparrow \, X$ (De Florian, Soffer, Stratmann, Vogelsang '98)
- $e \, p^\uparrow \longrightarrow \Lambda^\uparrow \, X$
- $e \, p^\uparrow \longrightarrow e' \, (\pi^+ \, \pi^-) \, X$ (Ji '94; Collins, Heppelmann, Ladinsky '94; Jaffe, Jin, Tang '98; ...)
- ...

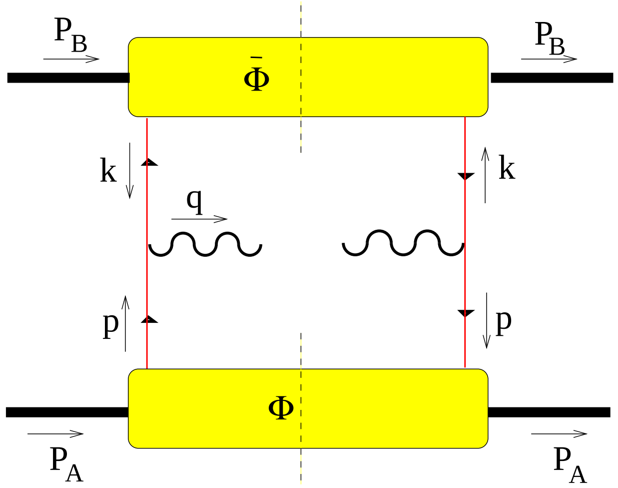
These suggestions can be categorized as follows:

- Double transverse spin asymmetries (DSA)
- Single transverse spin asymmetries (SSA), using:
  - $k_T$-dependent functions
  - Interference fragmentation functions
  - Higher spin functions, e.g. $\rho$ fragmentation functions
  - Higher twist functions

# Double spin asymmetry: Drell-Yan

The Drell-Yan Process

$H_1 + H_2 \to \ell + \bar{\ell} + X$



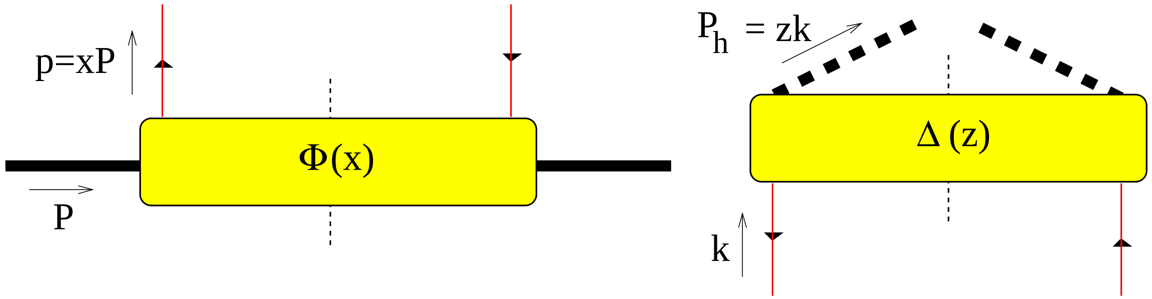

$$A_{TT} = \frac{\sigma(p^\uparrow\, p^\uparrow \to \ell\, \ell'\, X) - \sigma(p^\uparrow\, p^\downarrow \to \ell\, \ell'\, X)}{\sigma(p^\uparrow\, p^\uparrow \to \ell\, \ell'\, X) + \sigma(p^\uparrow\, p^\downarrow \to \ell\, \ell'\, X)} = \frac{\sin^2\theta \cos 2\phi_S^\ell}{1+\cos^2\theta} \frac{\sum_{a,\bar{a}} e_a^2\, h_1^a(x_1)\, \overline{h}_1^a(x_2)}{\sum_{a,\bar{a}} e_a^2\, f_1^a\, \overline{f}_1^a}$$

Problem: $h_1$ for antiquarks presumably much smaller than for quarks

Using Soffer's inequality, $A_{TT}$ shown to be too small at RHIC (percent level)

Martin, Schäfer, Stratmann & Vogelsang, PRD 60 ('99) 117502

# DSA: transversity fragmentation function




Next to the (leading twist) distribution functions

$$\Phi(x) = \frac{1}{2}\,[f_1(x)\,\not{P} + g_1(x)\,\lambda\gamma_5\,\not{P} + h_1(x)\,\gamma_5\,\not{S}_T\,\not{P}\,]$$

there are also fragmentation functions

$$\Delta(z) = \frac{1}{2}\,[D_1(z)\,\not{P} + G_1(z)\,\lambda\gamma_5\,\not{P} + H_1(z)\,\gamma_5\,\not{S}_T\,\not{P}\,]$$

$H_1$ is relevant for transversely polarized hyperon production and DSA like:

- $e\,p^\uparrow \to \Lambda^\uparrow\,X$ [HERMES, COMPASS]: $D_{NN} \propto h_1 H_1$
- $p\,p^\uparrow \to \Lambda^\uparrow\,X$ [E704, RHIC]

# Single spin asymmetries

Double transverse spin asymmetries do not seem promising yet to extract the transversity distribution

Remaining option: single spin asymmetries

Unpolarized final state [pions are easier than hyperons]

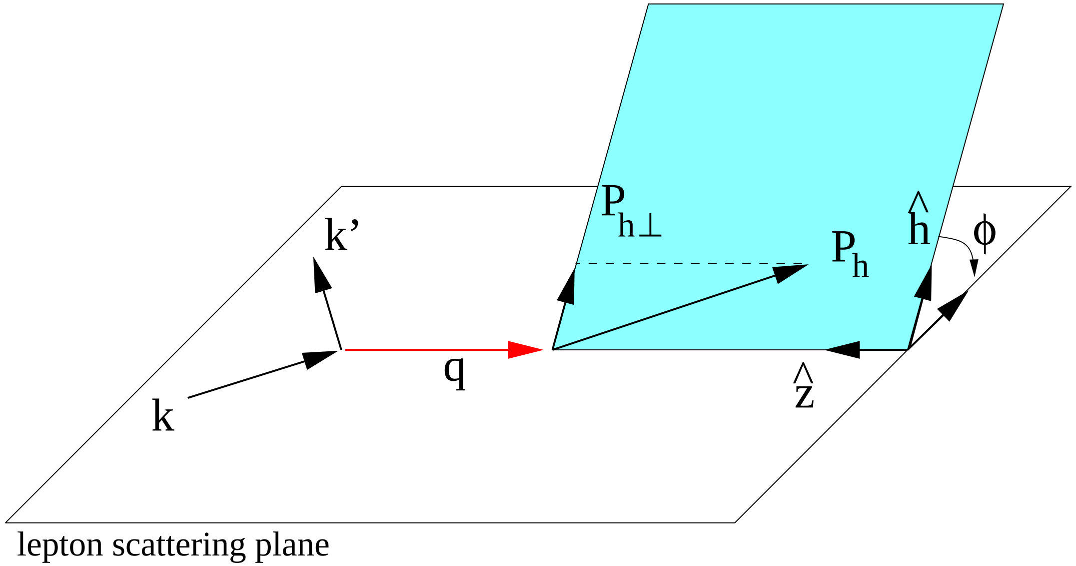
- $\Delta(z) \to \Delta(z, \boldsymbol{k}_T)$ (measure transverse momentum of the final state hadron compared to the jet direction): the Collins effect

- Hadrons with higher spin, e.g. $\rho$, or related to it $\pi^+\pi^-$
  Interference fragmentation functions

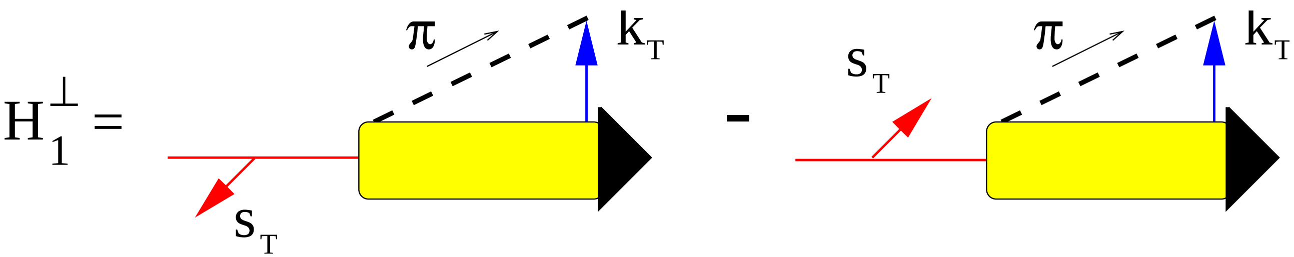
- Higher twist SSA ($\propto h_1 \times$ twist-3 function)

# Collins effect asymmetry in SIDIS

Collins, NPB 396 ('93) 161



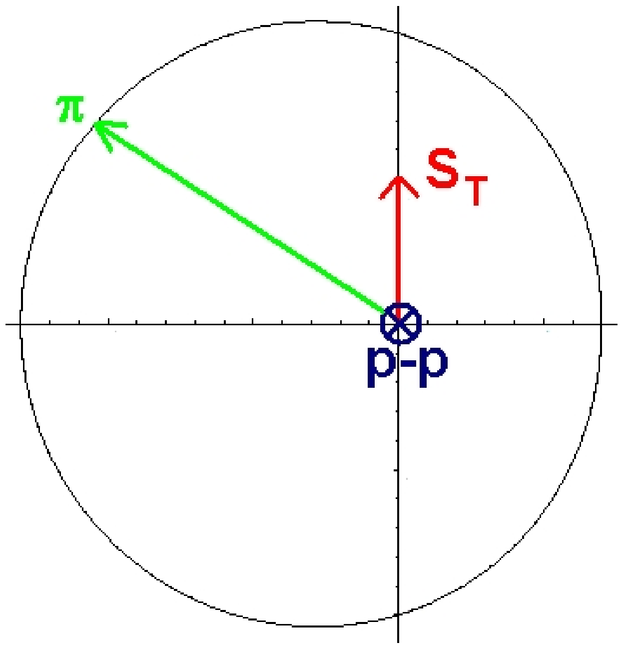

$$\frac{d\sigma(e\,p^{\uparrow} \to e'\pi X)}{d\phi_\pi^e d|\boldsymbol{P}_\perp^\pi|^2} \propto \{1 + |\boldsymbol{S}_T|\ \sin(\phi_\pi^e + \phi_S^e)\ A_T\}\,, \qquad A_T \sim h_1 H_1^\perp$$



# Collins effect asymmetries

Experimental indications that the Collins effect is nonzero: SMC, HERMES, COMPASS

Possibly also related to the SSA in $p\,p^{\uparrow} \to \pi\,X$ [E704, AGS, STAR]




A left-right asymmetry

Pion distribution is asymmetric depending on transverse spin direction and on pion charge

Explanation at the quark-gluon level?

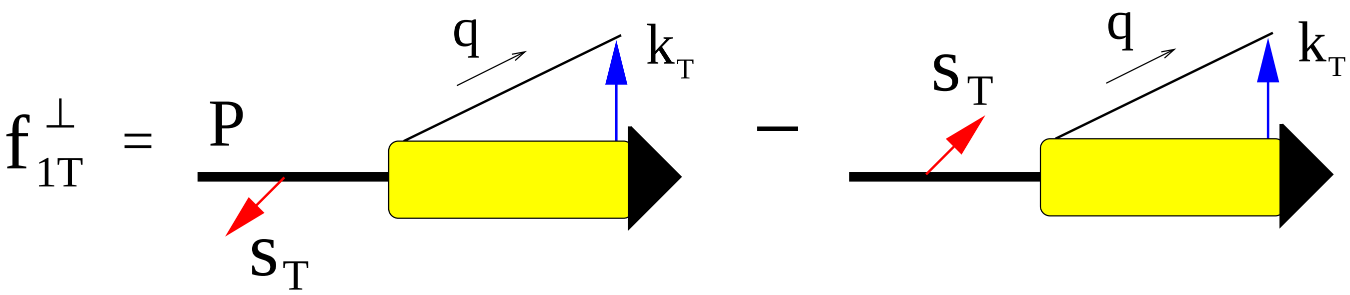
Bourrely & Soffer, hep-ph/0311110

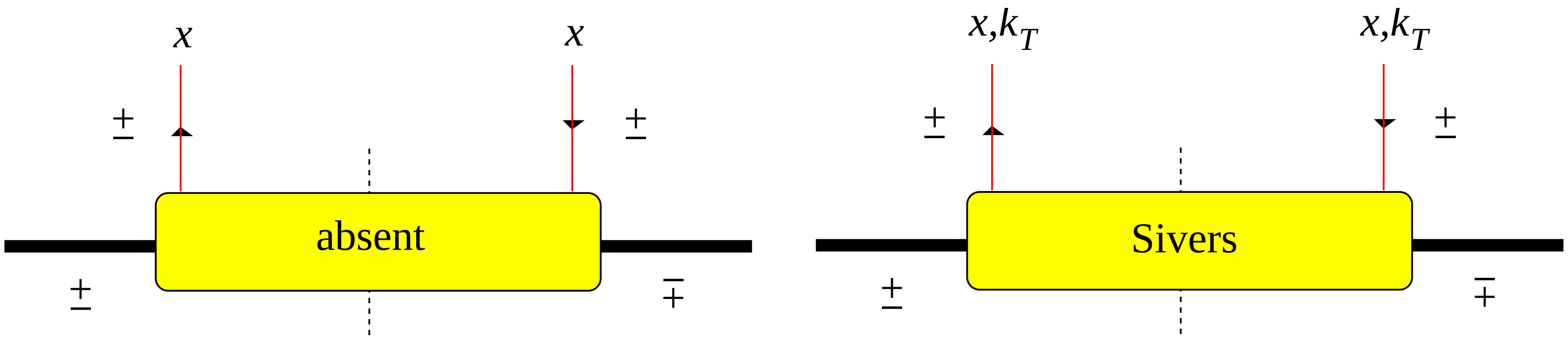
It could be the Collins effect (Anselmino, Boglione, D'Alesio, Murgia)

$$A_T \sim h_1(x_1) \otimes (f_1(x_2) \text{ or } g(x_2)) \otimes H_1^{\perp}(z, \boldsymbol{k}_T)$$

# Sivers effect

First proposal of a $\boldsymbol{k_T}$ & $\boldsymbol{S_T}$ dependent distribution function by Sivers (PRD 41 ('90) 83)



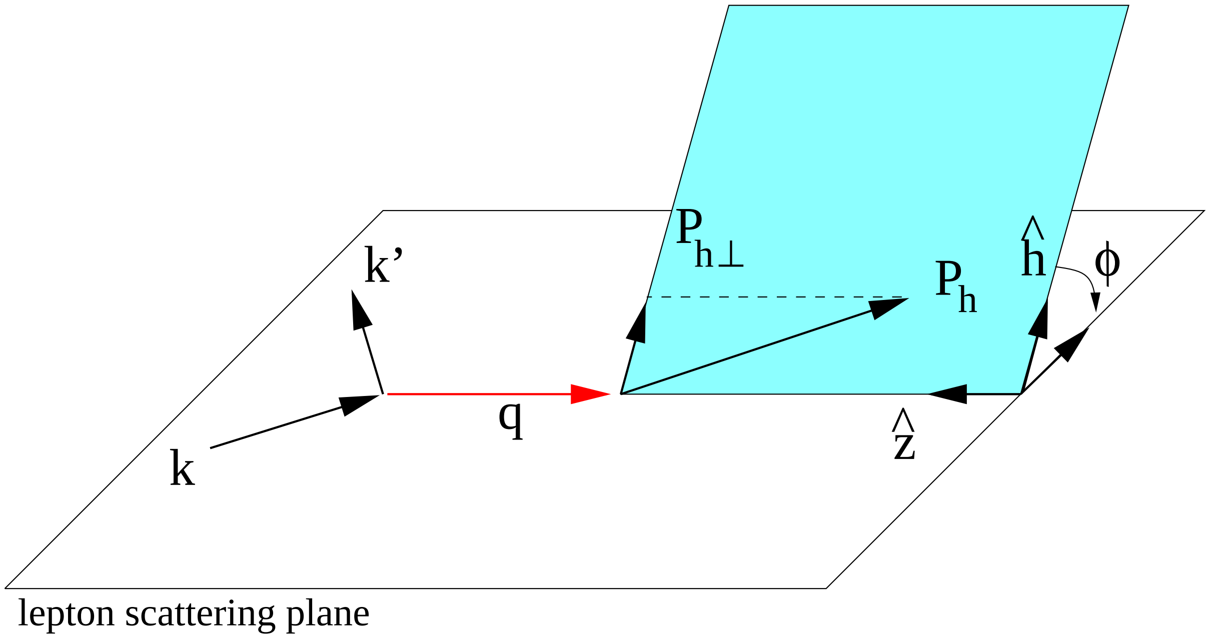




Inspired by data on $p\,p^{\uparrow} \to \pi^0\,X$ (Antille, PLB 94 ('80) 523) and the advent of E704

Intended as a test of perturbative QCD for large $\boldsymbol{p_T}$ hadron production in $p\,p$ scattering

# Sivers asymmetry in SIDIS

Sivers effect in semi-inclusive DIS $e\,p^{\uparrow} \to e'\,\pi\,X$

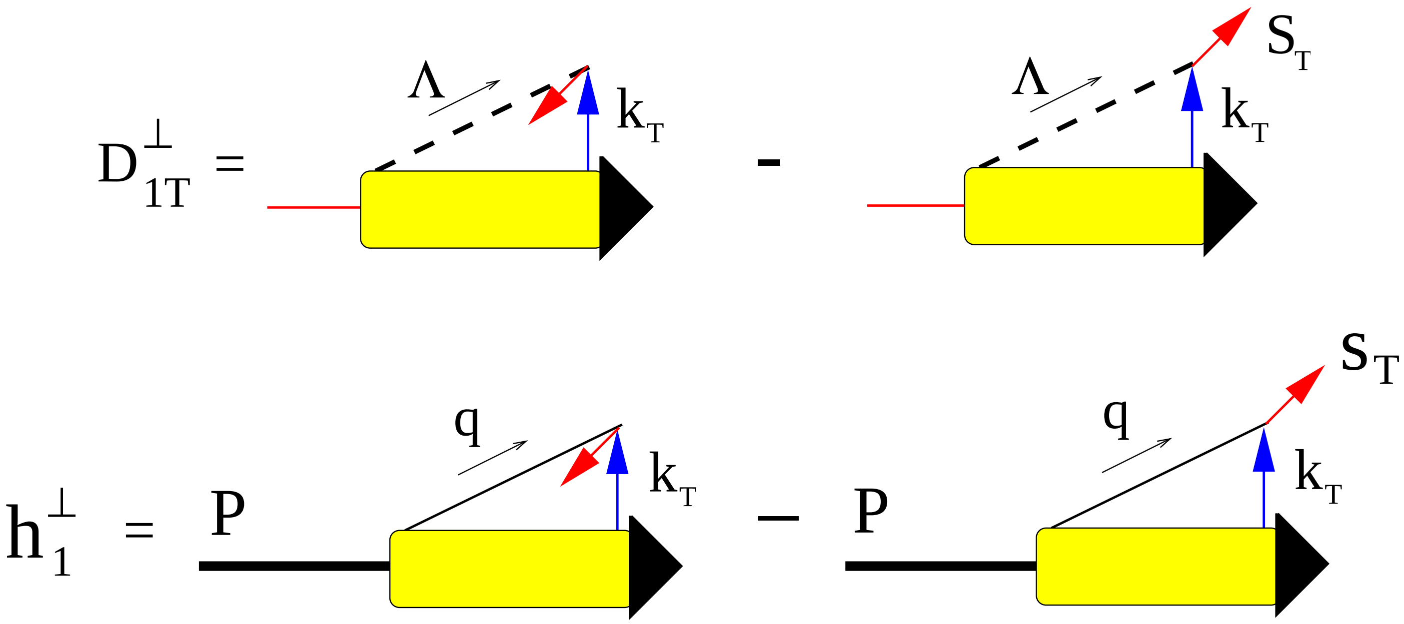



$$\frac{d\sigma(e\,p^{\uparrow} \to e'\pi X)}{d\phi_\pi^e d|\boldsymbol{P}_\perp^\pi|^2} \propto \{1 + |\boldsymbol{S}_T|\ \sin(\phi_\pi^e - \phi_S^e)\ A_T\}\,, \qquad A_T \sim f_{1T}^{\perp} D_1$$

$f_{1T}^{\perp}$ and $H_1^{\perp}$ proposed because of SSA and $h_1$, but are of interest in their own right

# $k_T$-dependent transverse spin functions

Besides $f_{1T}^{\perp}$ and $H_1^{\perp}$, there are two other, equally interesting $k_T$-odd functions




Mulders & Tangerman, NPB 461 ('96) 197; D.B. & Mulders, PRD 57 ('98) 5780

These functions have unexpected properties, not yet fully studied

Recent developments spurred on by a model calculation of

Brodsky, Hwang & Schmidt (PLB 530 ('02) 99)

# Actual extraction

One problem is that the $\boldsymbol{k}_T$-dependent functions appear in convolution integrals

For example, Sivers effect in SIDIS:

$$\frac{d\sigma(e\,p^{\uparrow} \to e'\,\pi\,X)}{d^2\boldsymbol{q}_T} \propto \frac{|\boldsymbol{S}_T|}{Q_T}\,\sin(\phi_\pi^e - \phi_S^e)\;\mathcal{F}\left[\frac{\boldsymbol{q}_T\cdot\boldsymbol{p}_T}{M}\,f_{1T}^{\perp}D_1\right]$$

$$\mathcal{F}\left[w\,f\,D\right] \equiv \int d^2\boldsymbol{p}_T\;d^2\boldsymbol{k}_T\;\delta^2(\boldsymbol{p}_T+\boldsymbol{q}_T-\boldsymbol{k}_T)\,w(\boldsymbol{p}_T,\boldsymbol{q}_T,\boldsymbol{k}_T)\,f(x,\boldsymbol{p}_T^2)D(z,z^2\boldsymbol{k}_T^2)$$

One solution would be to measure "jet SIDIS": $e\,p^{\uparrow} \to e'\,\text{jet}\,X$

$$\frac{d\sigma(e\,p^{\uparrow} \to e'\,\text{jet}\,X)}{d^2\boldsymbol{q}_T} \propto |\boldsymbol{S}_T|\;\sin(\phi_\pi^e - \phi_S^e)\,\frac{Q_T}{M}f_{1T}^{\perp}(x,Q_T^2), \qquad Q_T^2 = |\boldsymbol{P}_\perp^{\text{jet}}|^2$$

One can probe the $k_T$-dependence of the Sivers function directly in this way!

For asymmetries involving chiral-odd quantities this cannot be done

A more general solution is to consider weighted asymmetries

# Jet SIDIS $e\,p^\uparrow \to e'\,\textbf{jet}\,X$

Cross sections integrated, but weighted with function of *observed* transverse momentum

$$\langle W\rangle_{UT} \equiv \int dz\; d^2\boldsymbol{P}_\perp^{\text{jet}}\; W\, \frac{d\sigma^{[e\,p^\uparrow\to e'\,\text{jet}\,X]}}{dx\,dy\,dz\,d\phi^e_{\text{jet}} d|\boldsymbol{P}_\perp^{\text{jet}}|^2}$$

Weighted asymmetries become expressions in terms of transverse moments

$$\Big\langle \cos\phi^e_{\text{jet}}\,\frac{|\boldsymbol{P}_\perp^{\text{jet}}|}{M}\Big\rangle_{UT} \propto -\sin\phi^e_S \sum_{a,\bar a} e_a^2\, x\, f_{1T}^{\perp(1)a}(x)$$

D.B. & Mulders, PRD 57 ('98) 5780

This asymmetry contains a *weighted* Sivers function:

$$f_{1T}^{\perp(1)a}(x) = \int d^2\boldsymbol{k}_T \frac{\boldsymbol{k}_T^2}{2M^2} f_{1T}^{\perp a}(x,\boldsymbol{k}_T^2)$$

Such transverse moments appear in different asymmetries in exactly the same way

# Actual extraction

Even if one deconvolutes asymmetries by weighting, in case of chiral-odd quantities, one is always dealing with products of functions (summed over flavors)

$$h_1 \bar{h}_1, \quad h_1 H_1, \quad h_1 H_1^{\perp(1)}, \quad \ldots$$

Almost no experiment aiming to extract $h_1$ will be self-sufficient

This would only apply to "$(h_1)^2$" observables:

| | | | |
|---|---|---|---|
| $\bar{p}^\uparrow p^\uparrow \quad \to \quad \ell \, \bar{\ell} \, X$ | [GSI?] | Anselmino *et al.*, hep-ph/0403114<br>Efremov, Goeke, Schweitzer, hep-ph/0403124 |
| $p^\uparrow p^\uparrow \quad \to \quad$ high $p_T$ jet $+ X$ | [RHIC?] | Jaffe & Saito, PLB 382 ('96) 165<br>Vogelsang ('00) |

A global transversity analysis is needed, therefore:

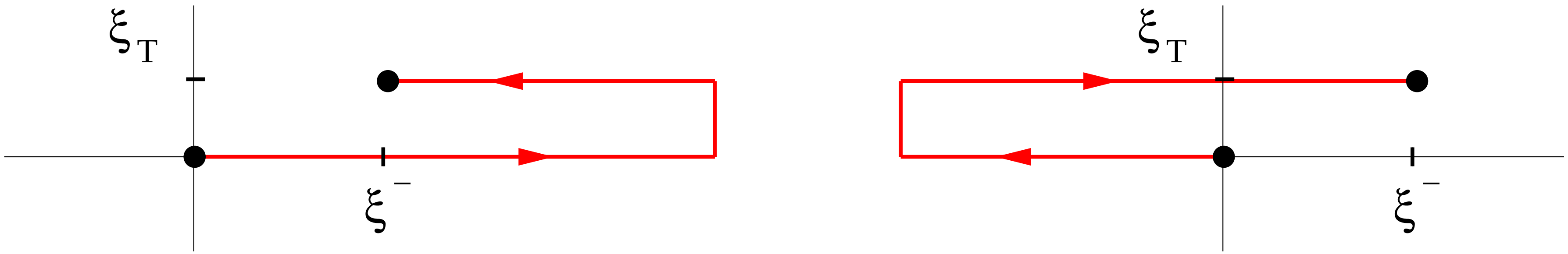
Process dependence and evolution of $\boldsymbol{k}_T$-dependent functions are very relevant issues

# Process dependence

Gauge invariant definition of Sivers function in DIS contains a future pointing Wilson line, whereas in Drell-Yan (DY) it is past pointing

$$f_{1T}^{\perp} \propto \langle P, S_T | \overline{\psi}(0)\, U(0,\xi)\, \not{n}_-\, \psi(\xi) | P, S_T \rangle$$

Belitsky, Ji & Yuan, NPB 656 ('03) 165




As a consequence (Collins, PLB 536 ('02) 43):

$$(f_{1T}^{\perp})_{\rm DIS} \quad = \quad -(f_{1T}^{\perp})_{\rm DY}$$

What about more complicated processes? (Bomhof, Mulders & Pijlman, hep-ph/0406099)

# Factorization and related issues

- The process dependence or universality of fragmentation functions

  On the basis of symmetry restrictions alone one finds schematically

$$
\begin{aligned}
(H_1^\perp)_{\text{SIDIS}} &\equiv A + B \\
(H_1^\perp)_{e^+e^-} &= A - B
\end{aligned}
$$

  D.B., Mulders, Pijlman, NPB 667 ('03) 201

  But model calculation by Metz (PLB 549 ('02) 139) shows $(H_1^\perp)_{\text{SIDIS}} = (H_1^\perp)_{e^+e^-}$

- All open issues related to: establishing the proper factorization theorems

  Collins & Soper, NPB 193 ('81) 381; Ji, Ma & Yuan, hep-ph/0404183; hep-ph/0405085

- Should answer the question "What exactly is a parton density?", posed by Collins (hep-ph/0304122)

- Evolution or energy scale dependence of functions and asymmetries, e.g., Sudakov suppression of azimuthal spin asymmetries

  D.B., NPB 603 ('01) 195; D.B. & Vogelsang, PRD 69 ('04) 094025

# Illustration: Collins effect asymmetry in SIDIS

Consider the Collins effect asymmetry in SIDIS:

$$\frac{d\sigma(e\,p^{\uparrow} \to e'\pi X)}{dx dz dy d\phi_e d^2\boldsymbol{q}_T} \propto \{1 + |\boldsymbol{S}_T|\,\sin(\phi_\pi^e + \phi_S^e)\; A(\boldsymbol{q}_T)\}$$

Assume Gaussian transverse momentum dependence for $H_1^\perp$

$$H_1^\perp(z, \boldsymbol{k}_T^2) = H_1^\perp(z)\,\frac{R^2}{z^2\pi}\;\exp\left(-R^2\boldsymbol{k}_T^2\right)$$

The asymmetry analyzing power is then given by

$$A(\boldsymbol{q}_T) = \frac{(1-y)}{(1-y+\frac{1}{2}y^2)}\,\frac{\sum_a\, e_a^2\; h_1^a(x) H_1^{\perp(1)a}(z)}{\sum_b\, e_b^2\; f_1^b(x) D_1^b(z)}\mathcal{A}(Q_T)$$

At tree level:

$$\mathcal{A}(Q_T) = M_\pi\, Q_T\,\frac{R^4}{R_u^2}\; e^{-(R^2-R_u^2)Q_T^2/2}$$

# Beyond tree level (in CS-81 spirit)

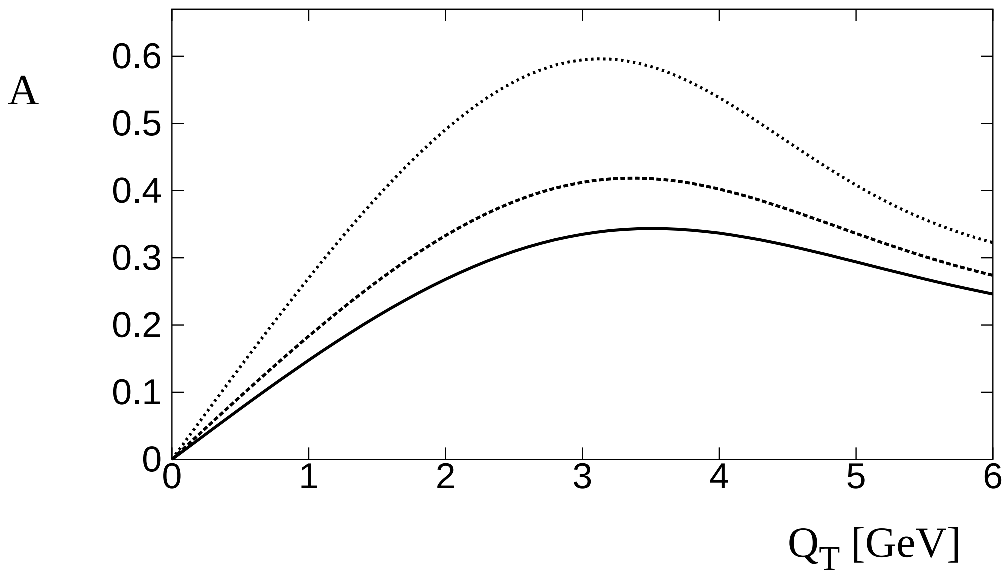
$$\mathcal{A}(Q_T) = M_\pi \frac{\int db\, b^2\, J_1(bQ_T)\, \exp\left(-S(b_*) - S_{NP}(b)\right)}{\int db\, b\, J_0(bQ_T)\, \exp\left(-S(b_*) - S_{NP}(b)\right)} \qquad b_* = \frac{b}{\sqrt{1 + b^2/b_{\max}^2}}$$

$S(b_*)$ can be calculated perturbatively and $S_{NP}$ has to be fitted to experiment

Generic example (leading log and $S_{NP}$ from Ladinsky-Yuan '94):

$\mathcal{A}(Q_T)$

$Q = 30$ GeV (upper)
$Q = 60$ GeV (middle)
$Q = 90$ GeV (lower)




$\mathcal{A}(Q_T, Q) \sim Q^{-0.5} - Q^{-0.6}$ (partial power suppression) D.B., NPB 603 ('01) 195

# Beyond tree level

Consider instead the weighted observable

$$\mathcal{O} \equiv \frac{\left\langle \sin(\phi_\pi^e + \phi_S^e)\, |\boldsymbol{P}_\perp^\pi|/M_\pi \right\rangle}{[4\pi\, \alpha^2\, s/Q^4]} = |\boldsymbol{S}_T|\, (1-y) \sum_{a,\bar{a}} e_a^2\, x\, h_1^a(x) z H_1^{\perp(1)}(z)$$

$$H_1^{\perp(1)}(z) \equiv \int d^2\boldsymbol{k}_T \frac{\boldsymbol{k}_T^2}{2z^2 M_\pi^2} H_1^\perp(z, \boldsymbol{k}_T^2)$$

$\mathcal{O}$ is not sensitive to Sudakov suppression

Hence, if one measures $\mathcal{O}(Q_0^2)$ and $\mathcal{O}(Q_1^2)$, one can relate them via LO evolution

NLO evolution equation is known for $h_1$

LO evolution equation for $H_1^{\perp(1)}$ still to be established

Conclusion: from the theoretical point of view Collins effect asymmetries are not like ordinary leading twist asymmetries

# SSA: Interference Fragmentation Functions

Consider the final state $|\pi^+ \pi^- X\rangle$, with $\pi^+$ and $\pi^-$ belonging to same jet

Ji ('94); Collins, Heppelmann, Ladinsky ('94); Jaffe, Jin, Tang ('98); ...

$$\Delta(z) \propto \left[ D_1(z) \not{P} + i H_1^{\sphericalangle} \frac{\not{R}_T \not{P}}{2M_\pi} \right] \qquad \begin{aligned} z &= z^+ + z^- \\ P &= P_{\pi^+} + P_{\pi^-} \\ R_T &= (z^+ P_{\pi^-} - z^- P_{\pi^+})/z \end{aligned}$$

Radici, Jakob, Bianconi, PRD 65 ('02) 074031

Nonzero $H_1^{\sphericalangle}$ due to interference between different partial waves of the $(\pi^+ \pi^-)$ system

This leads to single spin asymmetries:

$$e\, p^\uparrow \to e'\, (\pi^+ \pi^-)\, X \qquad h_1 \otimes H_1^{\sphericalangle} \qquad \text{Ji, PRD 49 ('94) 114; ...}$$

$$p\, p^\uparrow \to (\pi^+ \pi^-)\, X \qquad f_1/g \otimes h_1 \otimes H_1^{\sphericalangle}$$

# SSA: Interference Fragmentation Functions

The SSA in $e\,p^{\uparrow} \rightarrow e'\,(\pi^+\,\pi^-)\,X$ according to Jaffe, Jin, Tang (PRL 80 ('98) 1166)

$$\left\langle \sin(\phi^e_{S_T} + \phi^e_{R_T}) \right\rangle \propto |\boldsymbol{S}_T||\boldsymbol{R}_T|\,F(m^2)\,\sum_{a,\bar{a}} e_a^2\,x\,h_1^a(x)\delta\hat{q}_I(z)$$

$F(m^2) = \sin\delta_0 \sin\delta_1 \sin(\delta_0 - \delta_1)$, where $\delta_0, \delta_1$ are the strong $\pi^+\pi^-$ phase shifts and $m^2$ the invariant mass of $\pi^+\pi^-$

Conclusion: best done around the $\rho$ mass region (rapid change around $F(m_\rho^2) = 0$)

But is based on assumption of factorization of $z$ and $m^2$ dependence
More general dependences are possible
Bianconi et al., PRD 62 ('00) 034008 & 034009; Radici et al., PRD 65 ('02) 074031

In addition, the T-odd $\rho$ fragmentation function $(H_{1LT}(z))$ consists of more than $s$-$p$ wave interference; there is also a piece from the $p$-wave only
Bacchetta, Mulders, PRD 62 ('00) 114004; Bacchetta, Radici, PRD D67 ('03) 094002

# SSA: Interference Fragmentation Functions

## Advantages

Collinear factorization (no Sudakov suppression, no process dependence, etc)
Only functions of lightcone momentum fractions

Evolution of $H_1^{\sphericalangle}(z)$ same as for $H_1(z)$ (known to NLO)

Stratmann & Vogelsang, PRD 65 ('02) 057502

## Reminder

Ordinary fragmentation function $D_1^{\pi^+\pi^-}$ needs to be determined separately

# Role of electron-positron annihilation data

Almost all proposed transversity measurements contain unknown fragmentation functions

- $e^+ \, e^- \to \Lambda^\uparrow \, \overline{\Lambda}^\uparrow X$: $(H_1)^2$ Contogouris *et al.*, PLB 344 ('95) 370
- $e^+ \, e^- \to \pi^+ \, \pi^- \, X$: $\langle \cos(2\phi_1) \rangle \propto (H_1^\perp)^2$ D.B., Jakob & Mulders, NPB 504 ('97) 345
- $e^+ \, e^- \to (\pi^+ \, \pi^-)_{\text{jet}\,1} \, (\pi^+ \, \pi^-)_{\text{jet}\,2} \, X$:

$$\left\langle \cos(\phi^e_{R_{1T}} + \phi^e_{R_{2T}}) \right\rangle \propto (H_1^{\sphericalangle})^2$$

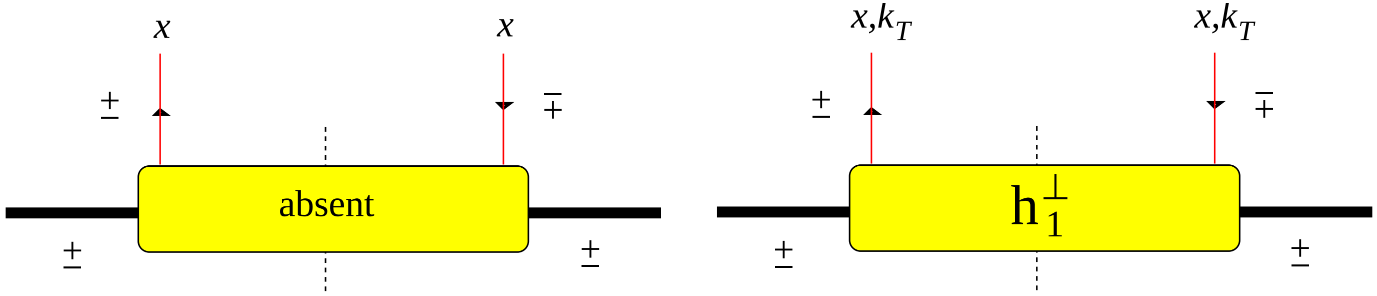
Artru & Collins, ZPC 69 ('96) 277

One can use off-resonance data of $B$-factories (BELLE, BABAR)
Grosse Perdekamp *et al.*, NPA 711 ('02) 69c

Using the two-hadron FF is easiest, since there is no asymmetric background

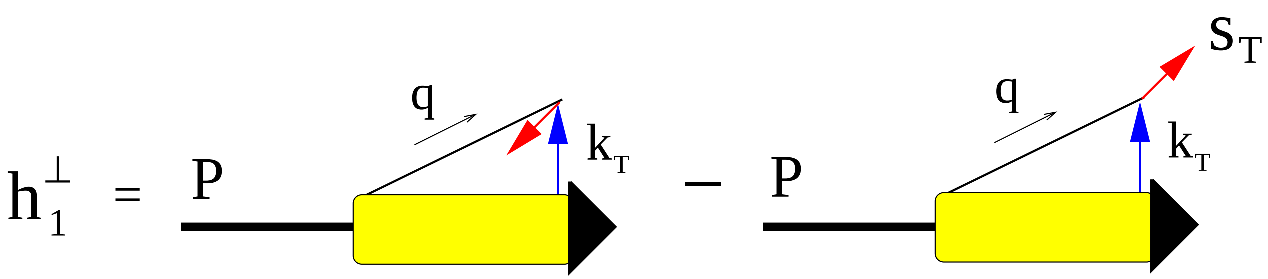
# Transversity inside unpolarized hadrons

What does nonzero $h_1^\perp$ mean?




The transverse polarization of a noncollinear quark inside an unpolarized hadron in principle can have a preferred direction

Implies an intrinsic handedness



# The unpolarized Drell-Yan process

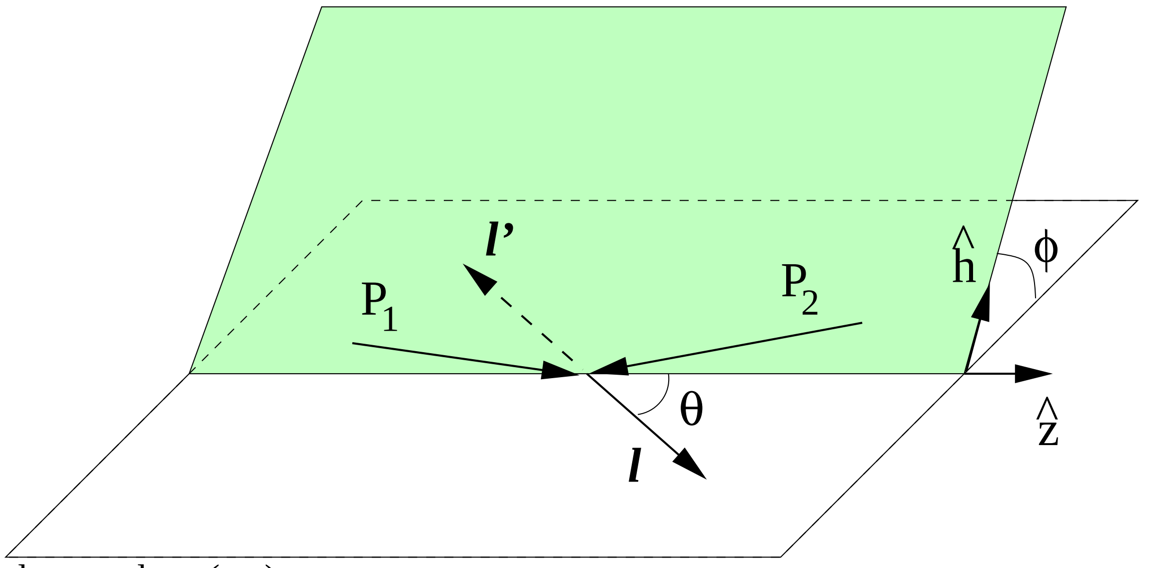
NA10 Collab. ('86/'88) & E615 Collab. ('89) measured the angular distribution of lepton pairs

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda \cos^2\theta + \mu \sin^2\theta \, \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi\right)$$

Perturbative QCD relation ($\mathcal{O}(\alpha_s)$): $1 - \lambda - 2\nu = 0$

Data: considerable deviation from this Lam-Tung relation

NLO doesn't fix it




Data for $\pi N \to \mu^+\mu^- X$, with $N = D, W$; $\pi$-beams of 140-286 GeV; $Q \sim 4 - 12$ GeV

# Explaining the unpolarized DY data

$h_1^\perp \neq 0 \quad \Longrightarrow \quad$ deviation from Lam-Tung relation

Offers a tree level ($\lambda = 1,\ \mu = 0$) explanation of NA10 data:

$$\nu \propto h_1^\perp(\pi)\, h_1^\perp(N)$$

Fit $h_1^\perp$ to data

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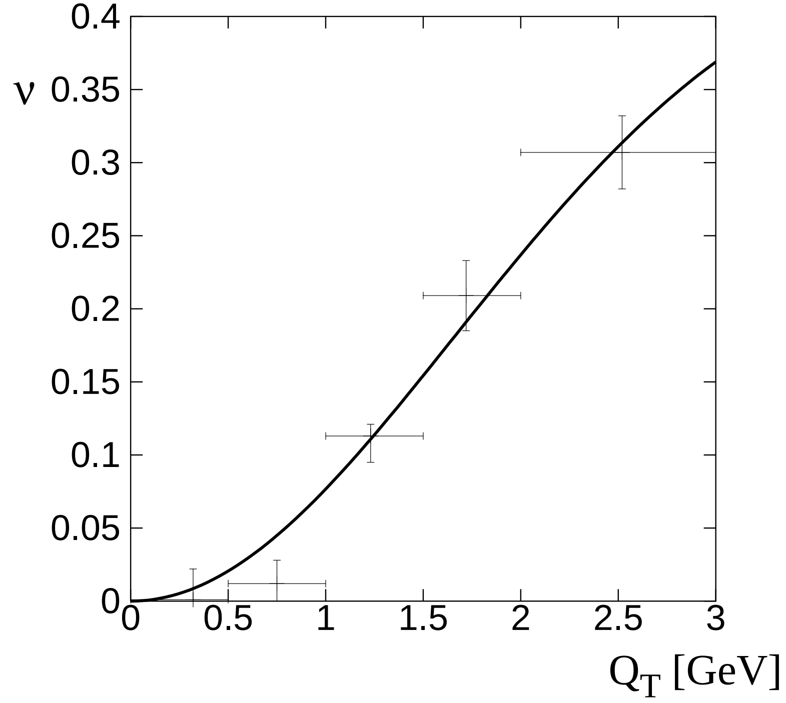


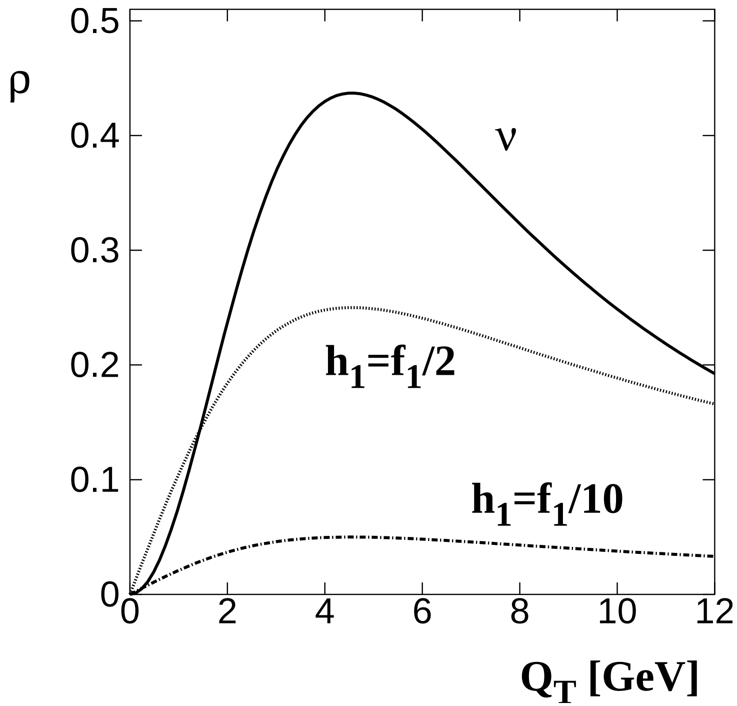
# The polarized Drell-Yan process

In the case of one polarized hadron (choosing $\lambda = 1$ and $\mu = 0$):

$$\frac{d\sigma}{d\Omega\, d\phi_S} \propto 1 + \cos^2\theta + \sin^2\theta \left[\frac{\nu}{2}\, \cos 2\phi - \rho\, |\boldsymbol{S}_T|\, \sin(\phi + \phi_S)\right] + \ldots$$

Assuming $u$-quark dominance and Gaussian $k_T$ dependence for $h_1^\perp$:

$$\rho = \frac{1}{2}\sqrt{\frac{\nu}{\nu_{\text{max}}}\,\frac{h_1^u}{f_1^u}}$$




It offers another probe of transversity

# Data to test $h_1^\perp$ hypothesis

Possible future DY data

RHIC: can measure $\nu$ and $\rho \Longrightarrow$ information on $h_1$

Fermilab: $\nu$ in $p\,\bar{p} \to \mu^+\mu^- X$ (probably yields larger results)

GSI: future PANDA experiment $p\,\bar{p} \to \mu^+\mu^- X$; $Q \lesssim 6$ GeV

Semi-inclusive DIS

The $\langle \cos 2\phi \rangle$ in SIDIS would be $\propto h_1^\perp H_1^\perp$

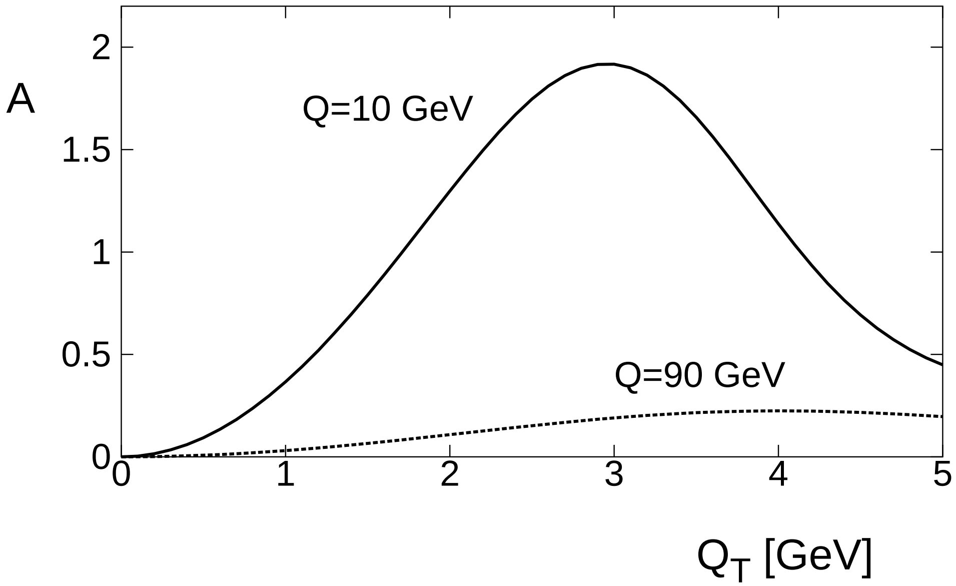
ZEUS data seems to follow pQCD, hence this could be a sign that the magnitude of $H_1^\perp$ is smaller than that of $h_1^\perp$

But perhaps $Q^2$ is too high to probe this contribution: due to Sudakov suppression

# Sudakov suppression

Assuming Gaussian $k_T$ dependence for $h_1^{\perp}$, the $\cos(2\phi)$ asymmetry is proportional to

$$\mathcal{A}(Q_T) \equiv M^2 \frac{\int_0^\infty db\, b^3\, J_2(bQ_T)\, \exp\left(-S(b_*) - S_{NP}(b)\right)}{\int_0^\infty db\, b\, J_0(bQ_T)\, \exp\left(-S(b_*) - S_{NP}(b)\right)}$$

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Considerable Sudakov suppression with increasing $Q$: $\sim 1/Q$ (effectively twist-3)

# Conclusions

- Transversity is a worthwhile quantity to try to measure
- Difficult to measure, no info from elastic scattering or inclusive DIS
- The lattice and models indicate that the tensor charge is not small
- Double transverse spin asymmetries not promising yet
- $\boldsymbol{k}_T$-dependent functions $(H_1^\perp, h_1^\perp)$ can be used as tools to extract transversity
- $\boldsymbol{k}_T$-dependence of Sivers function can be directly accessed in jet SIDIS
- Weighting with observed transverse momentum important
- Process dependence, scale dependence, factorization requires further study
- $e^+e^-$ annihilation data needed, no experiment self-sufficient
- Two-hadron fragmentation functions most promising