Some consideration on multidimensional likelyhood fits

Marco Mirazita
INFN – Laboratori Nazionali di Frascati

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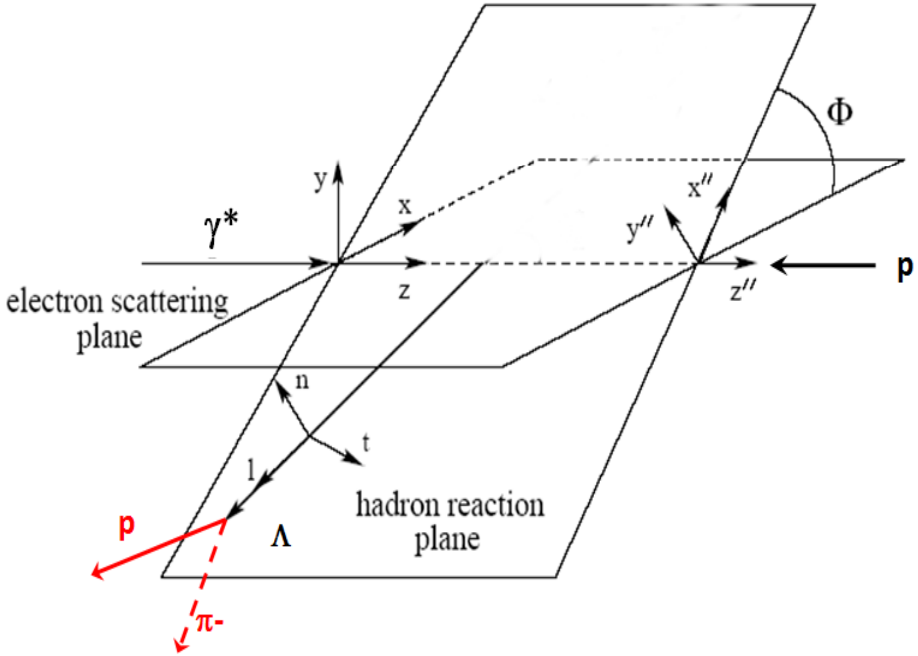
Physics environment

Extraction of the Λ polarization in SIDIS with CLAS data

$$\vec{e}p \rightarrow e'\Lambda X$$
$$\hookrightarrow p\pi^-$$

$$d\sigma = d\sigma_U \left(1 + S_\mu P_\Lambda^\mu\right)$$



decay proton angular distribution

$$\frac{dN}{d\cos\vartheta_p} \propto 1 + \alpha_w P_\Lambda \cos\vartheta_p$$

$$P_{\Lambda,z} = P_{\Lambda,z}^I + \lambda P_{\Lambda,z}^T$$

$$P_{\Lambda,z}^I = C_z^s \sin\varphi + C_z^{2s} \sin 2\varphi$$

$$P_{\Lambda,z}^T = D_z^0 + D_z^c \cos\varphi$$

The ϕ and θ_p dependencies are defined
Other dependencies are encoded in the C and D parameters

Unbinned likelyhood fit

$$L = \prod_{i=1}^{n} f^{norm}(X_i,\alpha) = \prod_{i=1}^{n} \frac{f(X_i,\alpha)}{N} = \frac{\prod_{i=1}^{n} f(X_i,\alpha)}{N^n}$$

n=number of events

X = set of kinematic variables

α = set of parameters to be determined

f(X, α) = probability distribution function

N = normalization factor of the PDF

↑↓ = polarization states

Probability distribution function:

$$f^{\uparrow,\downarrow} = 1 + \alpha_w P_\Lambda^{\pm} \cos\vartheta_p$$

$$P_\Lambda^{\pm} = P_\Lambda^I \pm P_\Lambda^T$$

- all kinematics dependencies can be fitted at the same time (if known)
- higher statistic precision compared to binned analysis
- no needs to study binning effects

$$LogL = \sum_{i=i}^{n^\uparrow} Logf^{\uparrow}(X_i,\alpha) + \sum_{i=i}^{n^\downarrow} Logf^{\downarrow}(X_i,\alpha) - n^\uparrow LogN^\uparrow - n^\downarrow LogN^\downarrow$$

Likelyhood normalization

$$N = \int dX_i \, f(X_i, \alpha)\varepsilon(X_i)$$

- ensures that total probability is 1
- takes into account holes in the acceptance where the probability is zero

In some cases this term can be dropped, for example

$$\begin{cases} f(X_i, \alpha) \propto 1 + \alpha_0 \cos\varphi \\ \varepsilon(X_i) \text{ doesn't depend on } \varphi \end{cases}$$

The integral is a constant term

Normalization using MC data

$$N \to \sum_{j=1}^{n_{MC}} f(X_i^{\,j}, \alpha)$$

- need good Monte Carlo, including polarizations
- variations of the fitted parameters through the fits should modify the angular distributions in the MC data

Would the fit never converging at all?

Normalization using exp data

Prescription:

- sum over experimental events

- include ALL events in both $N^{\uparrow}$ and $N^{\downarrow}$

$$N^{\uparrow,\downarrow} = \sum_{i=1}^{n^{\uparrow}+n^{\downarrow}} \left(1 + \alpha_w P_{\Lambda}^{\pm} \cos\vartheta_p\right)$$

Advantages:

- **all the correct kinematics dependencies are there**
- **no MC simulation is necessary**

Drawbacks:

- **limited kinematics**
- **what are the limitations of the prescription?**

Background subtraction

Good events selected within some kinematic cut contain background

$$L=\prod_{i=1}^{n}\frac{f(X_i,\alpha)}{N}=\underbrace{\left[\prod_{i=1}^{n_S}\frac{f(X_i,\alpha)}{N}\right]}_{\text{SIGNAL}}\underbrace{\left[\prod_{i=1}^{n_B}\frac{f(X_i,\alpha)}{N}\right]}_{\text{BACKGROUND}}$$

To subtract the BKG contribution in the selected events, add n_B background events taken from outside the signal region with weight -1

$$L=\prod_{i=1}^{n}\frac{f(X_i,\alpha)}{N}=\left[\prod_{i=1}^{n_S}\frac{f(X_i,\alpha)}{N}\right]\left[\prod_{i=1}^{n_B}\frac{f(X_i,\alpha)}{N}\right]\left[\prod_{i=1}^{n_B}\frac{f(X_i,\alpha)}{N}\right]^{-1}$$

Events from the side-band region

The total number of events for the fit is now $n+n_B$

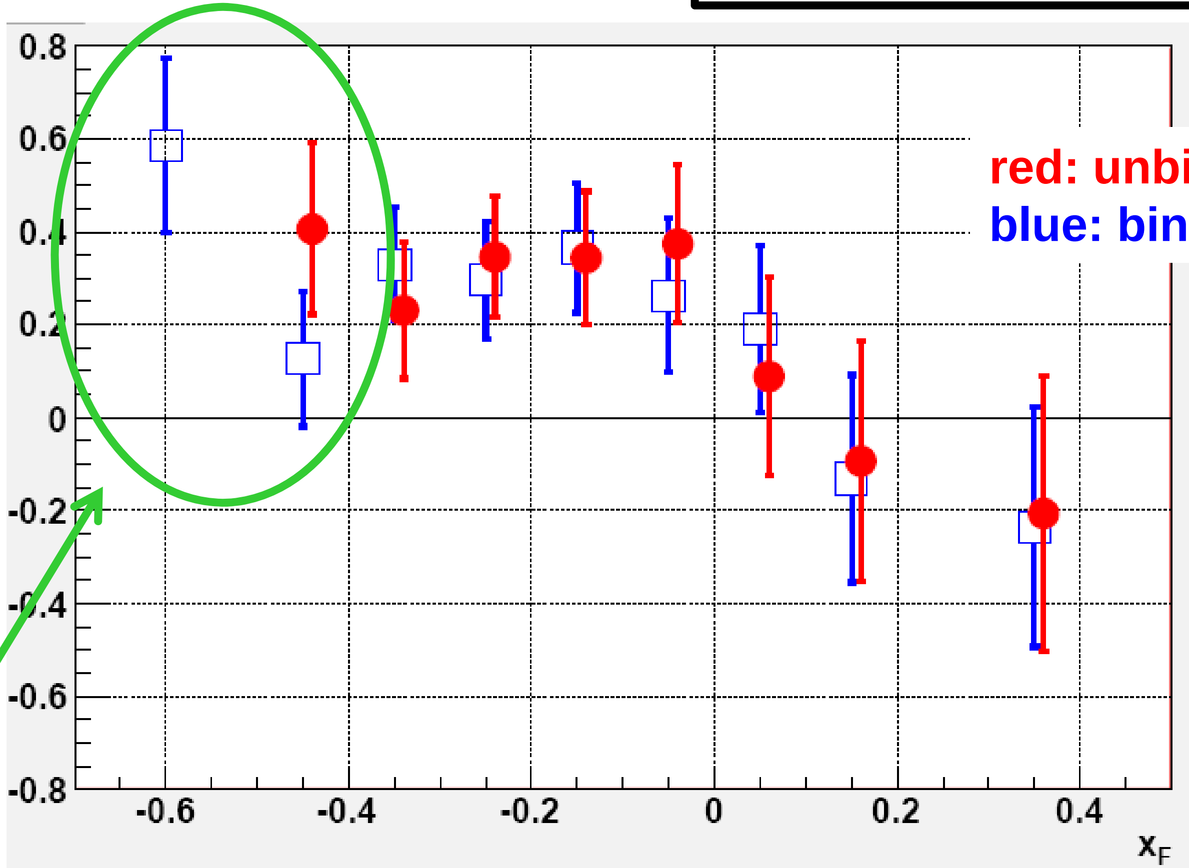
peak events: $w_i=+1$

side band events: $w_i=-1$

Extracting transferred polarization

Fit of the constant term $P^{T}_{\Lambda,z} = D^{0}_{z}$

results verified with binned analysis and χ^2 fit



where the statistics in too low, the LL fit gives better result doesn't converge

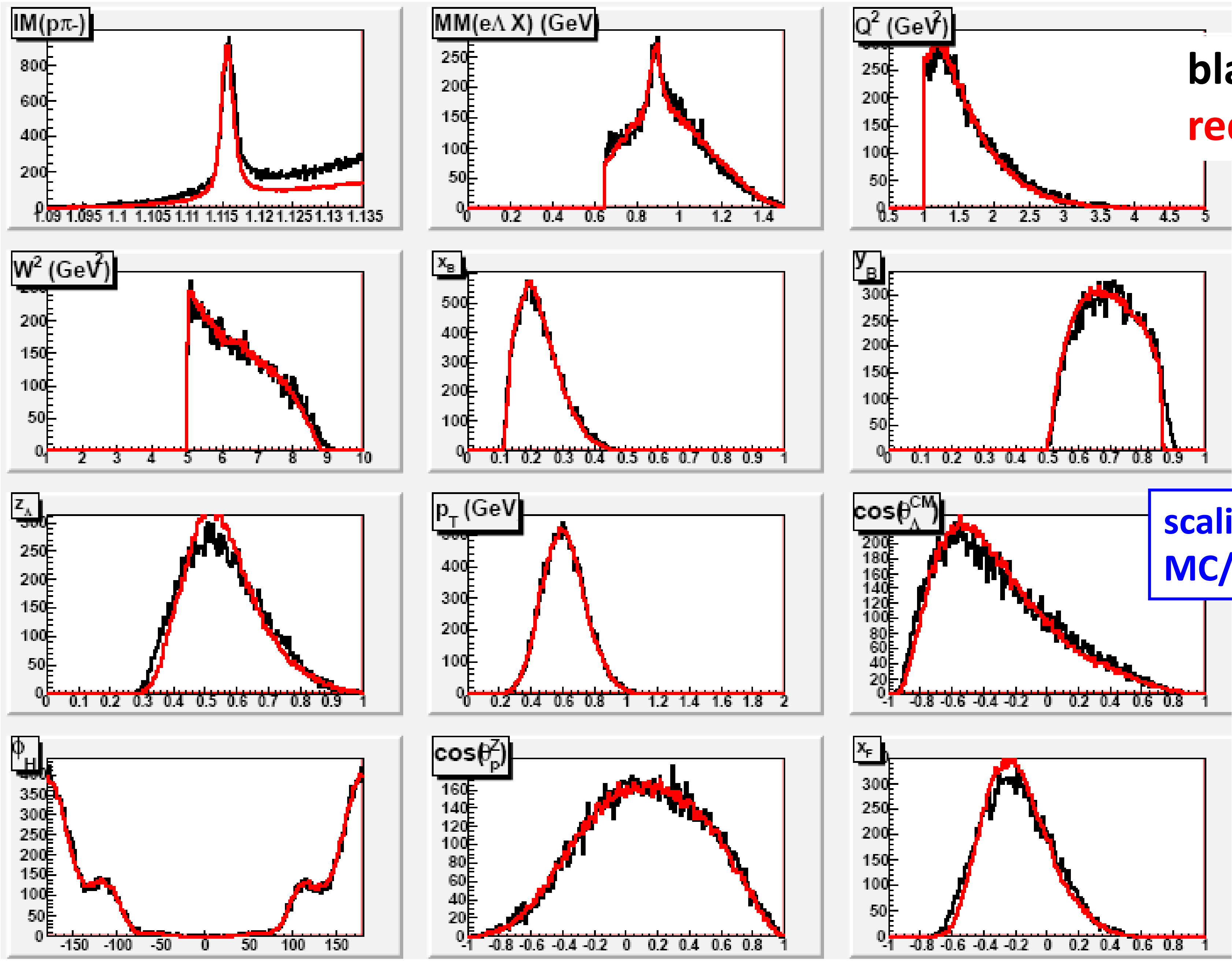
Monte Carlo simulation

clasDIS event generator based on pythia-JETSET

- adjustment needed for JETSET parameters to simulate Λ production in the target fragmentation
- standard parameters for exclusive ΛK^* events
- the two event samples are combined by fitting the experimental data

- full simulation of the CLAS detector
- analysis as for the experimental raw data

Comparison MC/EXP data



black: EXP

red: MC

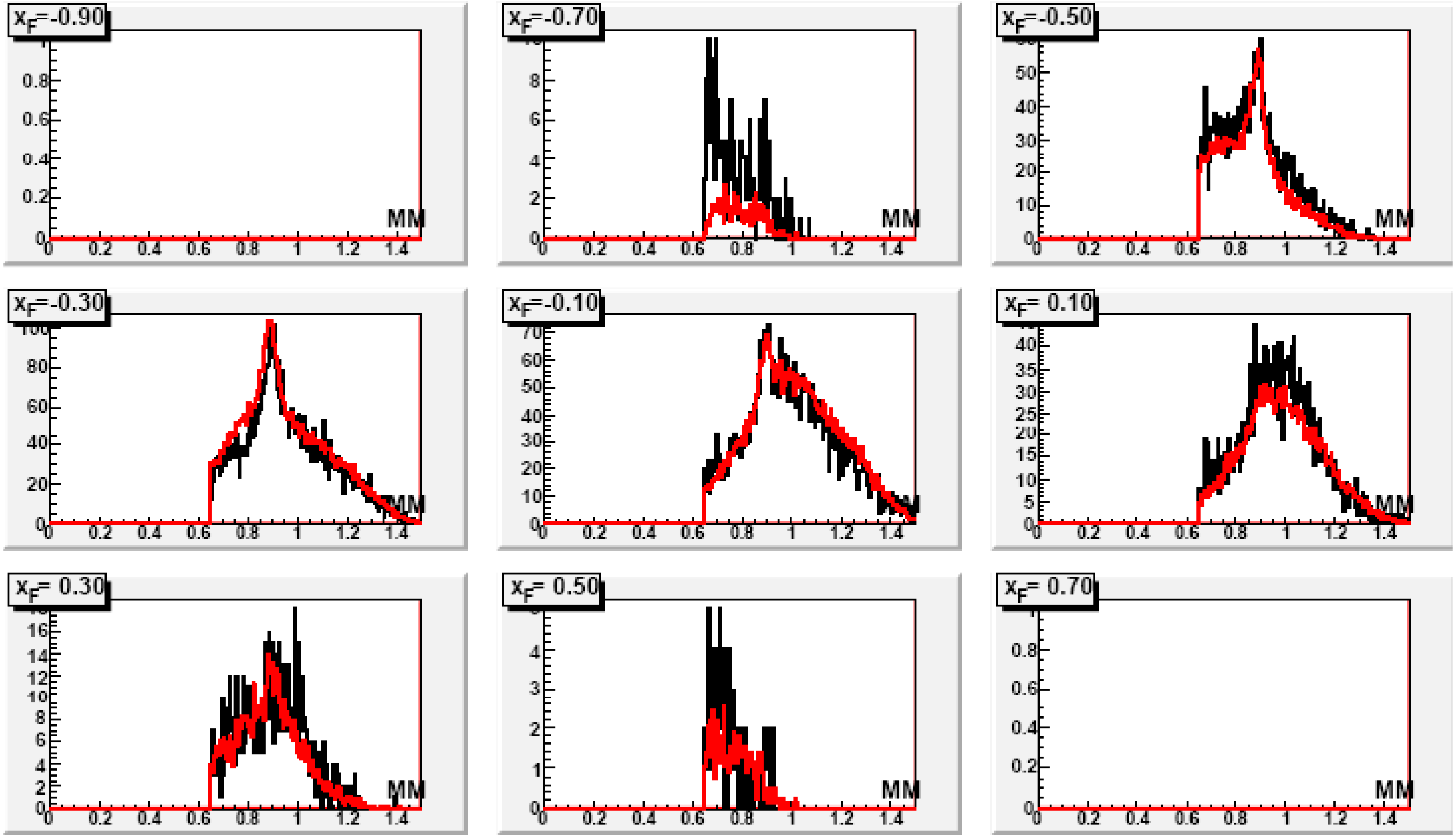
scaling factor MC/EXP ~8

Comparison MC/EXP data

MM(e Λ X) distributions in x_F bins

black: EXP

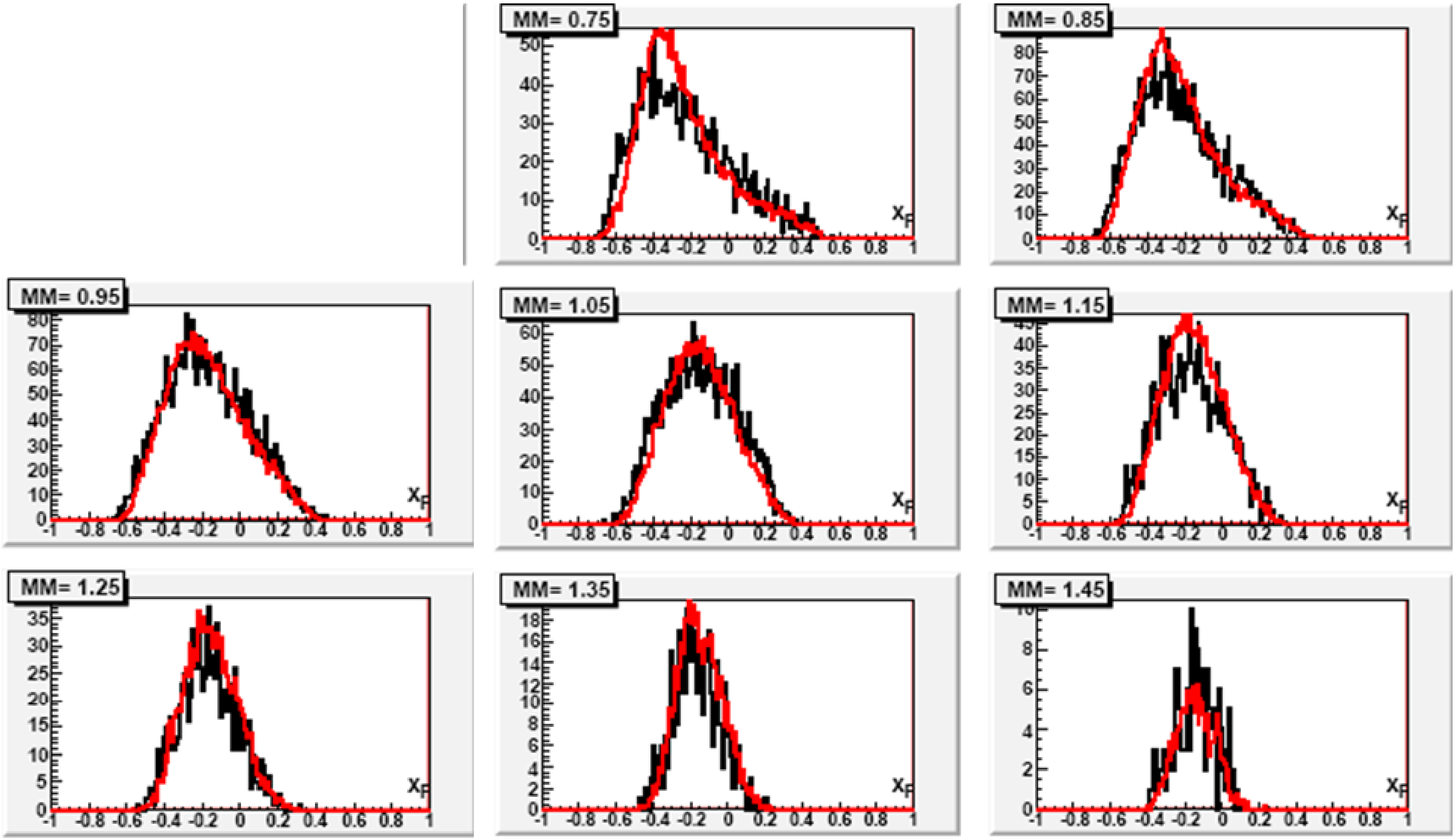
red: MC



Comparison MC/EXP data

x_F distributions in MM(e Λ X) bins

black: EXP
red: MC



Polarization in the MC data

MC is unpolarized ⇒

- θ_p generated distributions are flat (P^I is zero)
- no asymmetry between positive and negative beam helicity (P^T is zero)

For a given model of the polarization, i.e. a given (ϕ-dependent) value of P^I and P^T:

- probability of having an event at θ_p

$$\mathcal{P}\left(\cos\vartheta_p\right) = \frac{1+\alpha_w P^I \cos\vartheta_p}{\left(1+\alpha_w P^I\right)}$$

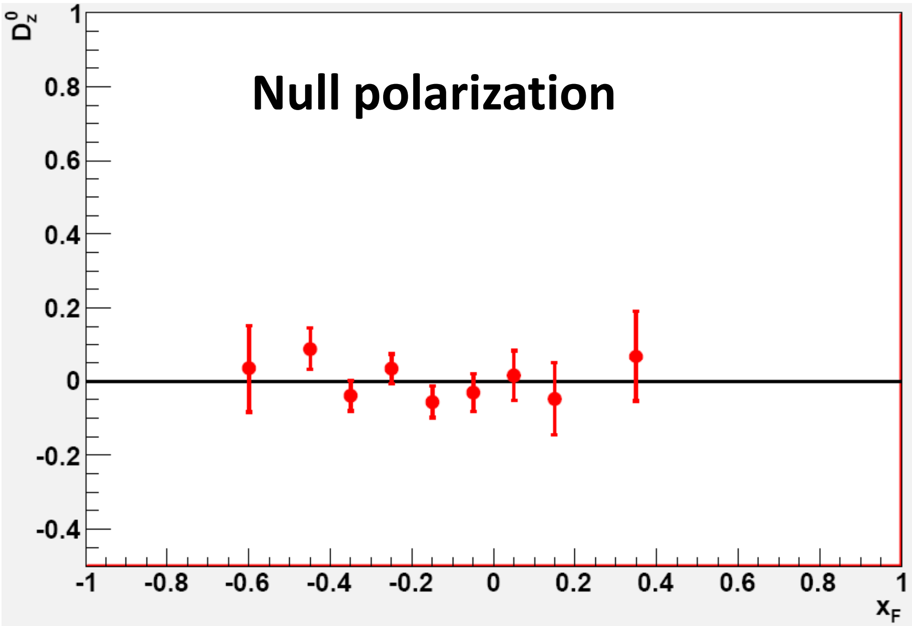
- probability of event produced by an electron with helicity λ

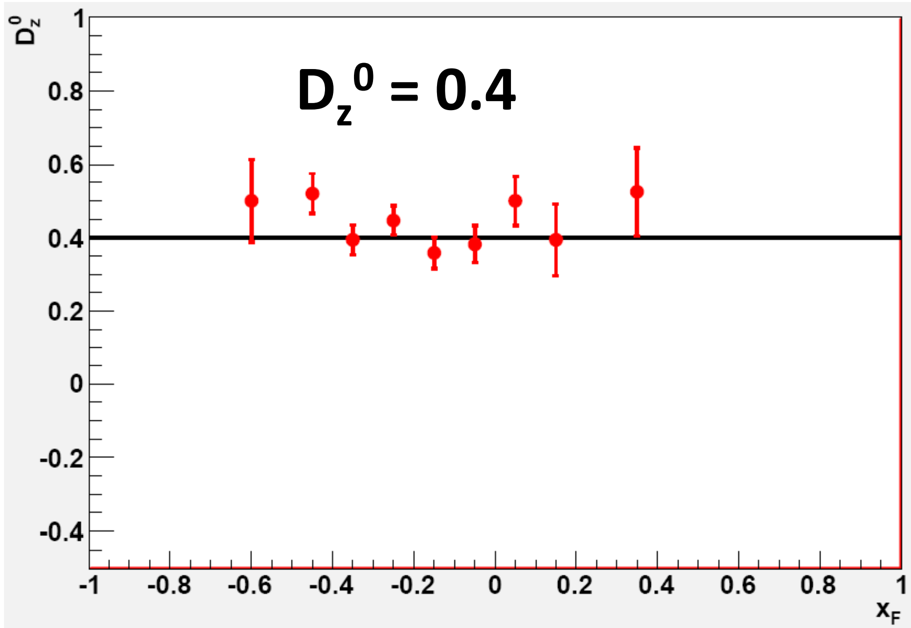
$$\mathcal{P}(\lambda) = \frac{1+\alpha_w\left(P^I + \lambda P^T\right)\cos\vartheta_p}{\left(1+\alpha_w P^I \cos\vartheta_p\right)}$$

MLE of polarization in MC data

Check of the procedure using Monte Carlo data
- no false polarization from the acceptance
- put some polarization in the unpolarized MC
- verify that the likelyhood fit is able to recover the input polarization

Fit of the constant term $P^{T}_{\Lambda,z} = D^{0}_{z}$



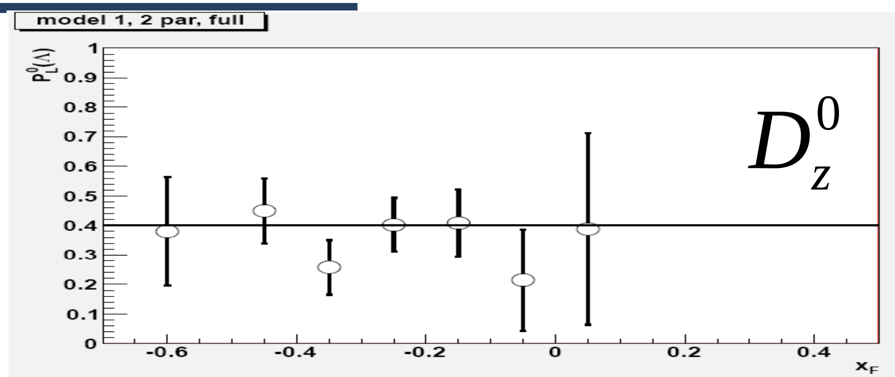


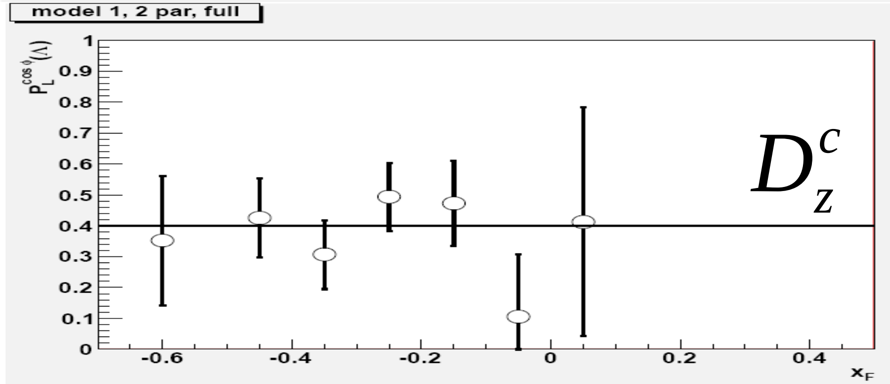
Acceptance effects

Monte Carlo data with ϕ dependent polarization

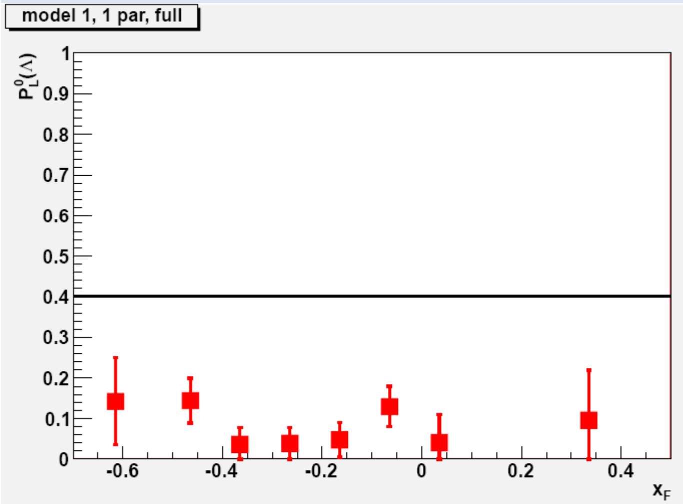
$$P^T_{\Lambda,z} = D^0_z + D^c_z \cos\varphi$$

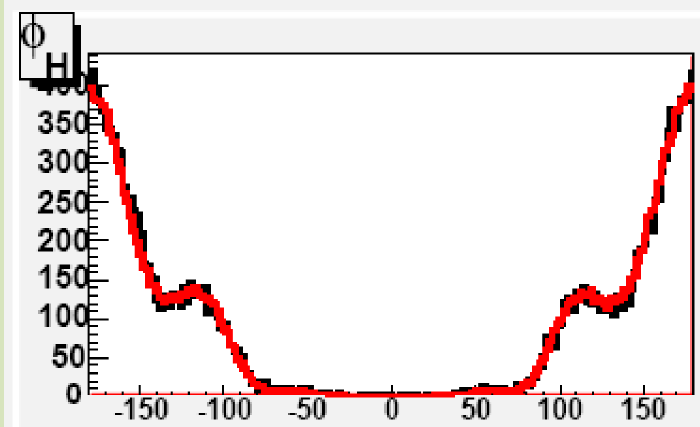
$$D^0_z = D^c_z = 0.4$$





Likelyhood fit of the constant term only:





black: EXP
red: MC

$$P^T_{\Lambda,z} = P^0_{\Lambda,z} + P^c_{\Lambda,z}\left\langle \cos(\phi)\right\rangle \approx P^0_{\Lambda,z} - 0.85 P^c_{\Lambda,z}$$

Extracting the induced polarization

- consider the induced polarization only

$$P_{\Lambda,z} = P^{I}_{\Lambda,z} = C_z^{s} \sin\varphi + C_z^{2s} \sin 2\varphi$$

- pdf function

$$f^{\uparrow,\downarrow} = 1 + \alpha_w P_{\Lambda,z}{}^{I} \cos\vartheta_p$$

No dependence on the beam helicity

$$f^{\uparrow} = f^{\downarrow} \qquad N^{\uparrow} = N^{\downarrow}$$

- likelyhood function with normalization prescription

$$LogL = \sum_{i=i}^{n} Logf\left(X_i, \alpha\right) - nLog\sum_{i=i}^{n} f\left(X_i, \alpha\right)$$

- maximization of the likelyhood

$$\frac{\partial LogL}{\partial P^{I}_{\Lambda,z}} = 0 \quad \Rightarrow \quad P^{I}_{\Lambda,z} = 0$$

verified with MC data with different values of the input polarization

Conclusion

- Likelyhood unbinned fits are a powerful tool for multidimensional analysis
- Monte Carlo studies are important to check the reliability of the analysis procedures

- No easy estimate of goodness-of-fit
- Normalization of the likelyhood function is a crucial point and must be studied carefully
- Can MC data be used to normalize the likelyhood function if the MC is not perfectly reproducing the experimental data?