Monte Carlo generators for TMD extractions at HERMES

(and some applications)

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Part 1: MC generators for 1h SIDIS at HERMES

MC generators at HERMES

GMC (Generator Monte Carlo) is a general framework for MC generators that simulate different physics processes. The most important MC generators for Hermes are:

Inclusive/Semi-Inclusive

- **disNG**
- **Pythia**
- **gmc_trans**
- Lepto with Cahn
- gmc_decay

Exclusive

- gmcDVCS
- gmc_Dual
- gmc_exclpion
- gmc_autpion
- rhoMC

Radiative corrections are calculate using the **RADGEN** program

- takes as input the observed kinematics of an event and (potentially) **generates a radiative photon** according to the probabilities for these kinematics
- **returns the true kinematics at the interaction vertex** (which are different from the observed kinematics in case a photon has been generated)

Fragmentation of partons into final state hadrons is performed by **JETSET**

- is an implementation of the **Lund string fragmentation model**

HMC and HSG

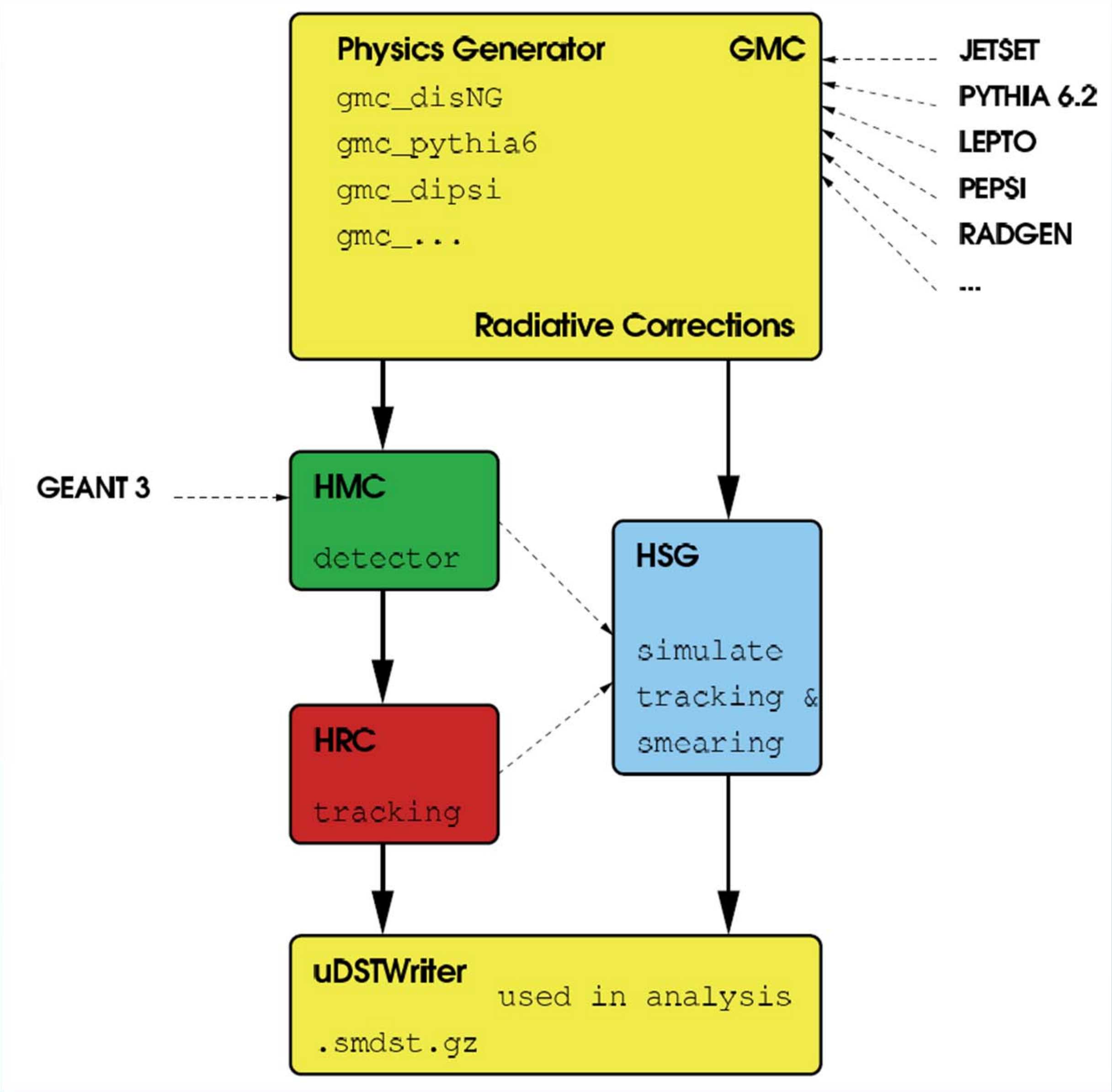
The HERMES MC (**HMC**) takes generated events and runs them through a **simulation of the HERMES detector**, based on GEANT3

- The HMC code has options to run with particular detector elements "off“.
- beam/spectrometer misalignment (specified in the geometry file) can be taken into account by using **misaligned MC**
- PYTHIA uses HMC

The **HERMES smearing generator HSG** is used to simulate detectors smearing on generated MC events without actually running a full detector simulation via HMC.

- Track **momenta** and **angles** are **modified on a statistical basis** using MC generated **lookup tables** which takes into account the **track momentum** and the **particle type (lepton or hadron)**
- **Advantage:** is fast! (~10% of the time needed for fully tracked MC)
- **But:** it's a simplified statistical model, there are no specific detector responses, no PID values, ...
- GMC_trans uses HSG

The structure of HERMES MC chain



disNG

- Based on **LEPTO** (unpolarized) or **PEPSI** (polarized) generator:
  - **LEPTO** is a Monte Carlo generator which simulates **unpolarized lepton-nucleon DIS scattering**.
  - **PEPSI** (*Polarized Electron Proton Scattering Interactions*) is an extension of LEPTO for **polarized beams and targets**
- **Accurate description of inclusive cross section** (but not ideal for SIDIS)
- **Processes:** DIS +1st order QCD processes: QCD Compton, Photon Gluon Fusion
- **Uses event weights** to simulate kinematic dependence of the cross section
- **Working range**: $Q^2 > 0.5\ GeV^2$ and $W^2 > 4\ GeV^2$
- Fragmentation process is performed by **JETSET**
- QED Radiative processes are provided by **RADGEN**

PYTHIA

- General purpose MC generator for particle physics
- The **PYTHIA** version used at HERMES is based on PYTHIA6 tuned to HERMES multiplicities
- provides a **much more complete** description of **semi-inclusive cross section** (compared to disNG)
- **Main processes:** DIS, QCD Compton, Photon-Gluon Fusion,...
- **Working range:** $Q^2 \approx 0 \ GeV^2$ and $W^2 > 4 \ GeV^2$
  $\Rightarrow$ suitable also for photo-production
- **Targets:** proton, neutron
- Fragmentation process is performed by **JETSET**
- QED Radiative processes are evalueted by **RADGEN**

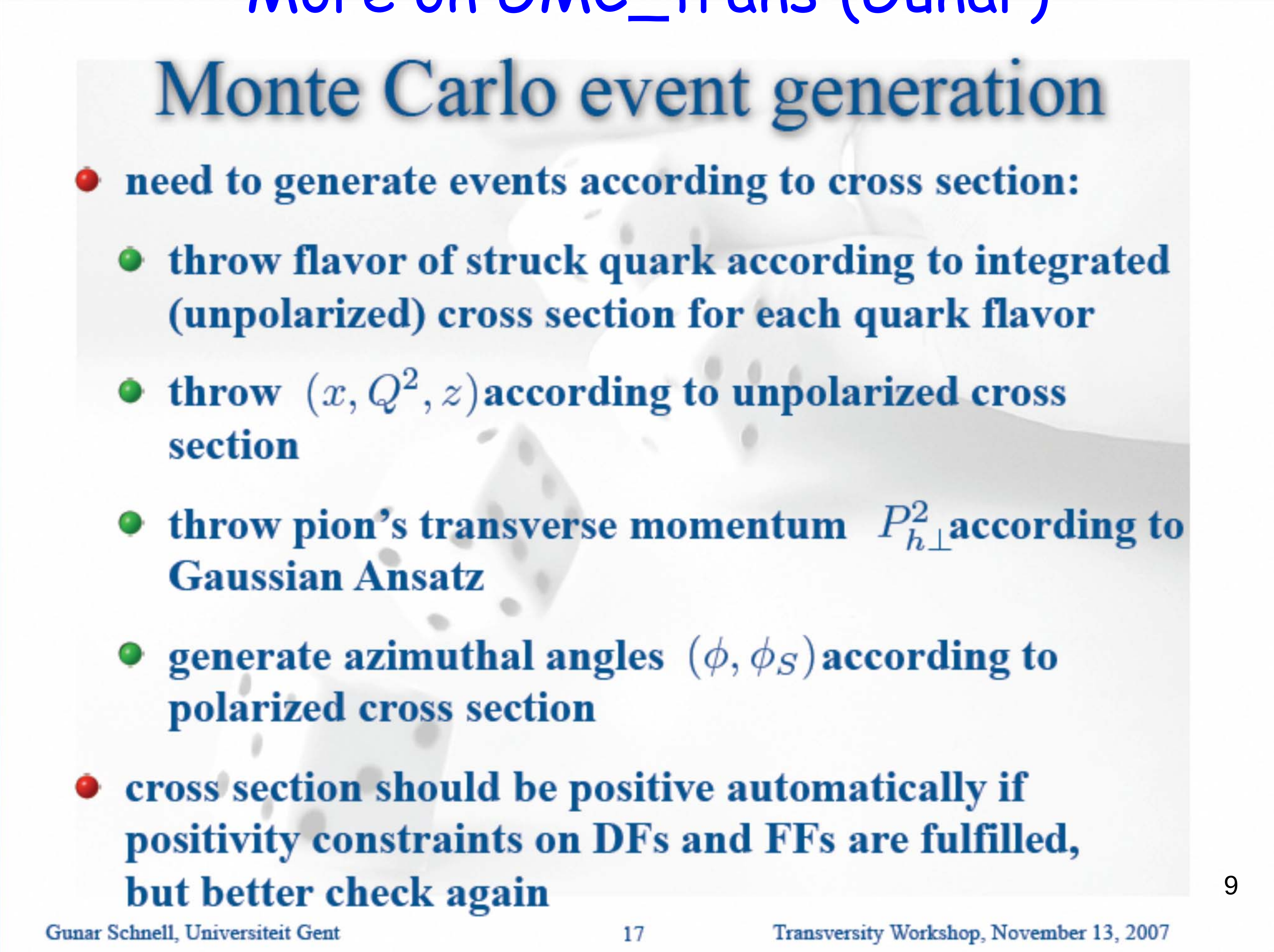
GMC_trans (1-h)

- **simulates semi-inclusive production on transversely polarized protons**
- azimuthal ampl. implemented: **Sivers, Collins, Boer-Mulders, Cahn, sinφ_s**
- throws hadron kinematics according to 6D SIDIS cross section

  (Mulders & Tangerman, Nucl. Phys. B461 1996)
- uses standard PDFs and FFs from fits/parametrizations (e.g. Kretzer, DSS,etc)
- uses **Gaussian ansatz** for **transverse-momentum** distributions
- presently available are **pions, charged kaons and protons**
- allows comparison between input model and extracted amplitudes
- is fast (no full track reconstruction → HSG)
- **more details in the next slides (from Gunar)**

The official gmc_trans version is being replaced with a **new C++ GMC_Trans**.

More on GMC_trans (Gunar)

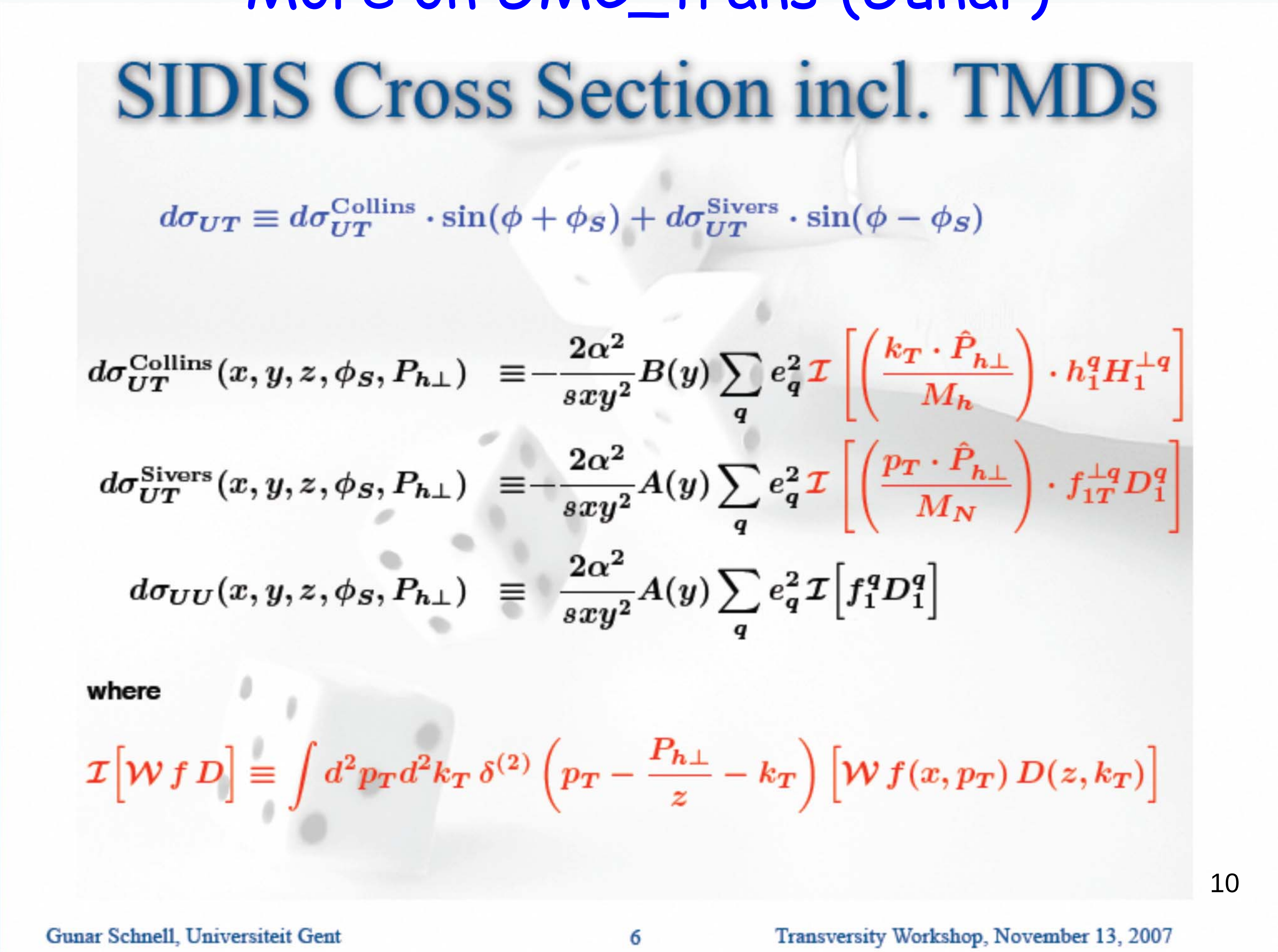


Monte Carlo event generation

- need to generate events according to cross section:
  - throw flavor of struck quark according to integrated (unpolarized) cross section for each quark flavor
  - throw (x, Q^2, z) according to unpolarized cross section
  - throw pion's transverse momentum $P_{h\perp}^2$ according to Gaussian Ansatz
  - generate azimuthal angles (ϕ, ϕ_S) according to polarized cross section
- cross section should be positive automatically if positivity constraints on DFs and FFs are fulfilled, but better check again

Gunar Schnell, Universiteit Gent 17 Transversity Workshop, November 13, 2007

More on GMC_trans (Gunar)



SIDIS Cross Section incl. TMDs

$$d\sigma_{UT} \equiv d\sigma_{UT}^{\text{Collins}} \cdot \sin(\phi + \phi_S) + d\sigma_{UT}^{\text{Sivers}} \cdot \sin(\phi - \phi_S)$$

$$d\sigma_{UT}^{\text{Collins}}(x, y, z, \phi_S, P_{h\perp}) \equiv -\frac{2\alpha^2}{sxy^2} B(y) \sum_q e_q^2 \mathcal{I}\left[\left(\frac{k_T \cdot \hat{P}_{h\perp}}{M_h}\right) \cdot h_1^q H_1^{\perp q}\right]$$

$$d\sigma_{UT}^{\text{Sivers}}(x, y, z, \phi_S, P_{h\perp}) \equiv -\frac{2\alpha^2}{sxy^2} A(y) \sum_q e_q^2 \mathcal{I}\left[\left(\frac{p_T \cdot \hat{P}_{h\perp}}{M_N}\right) \cdot f_{1T}^{\perp q} D_1^q\right]$$

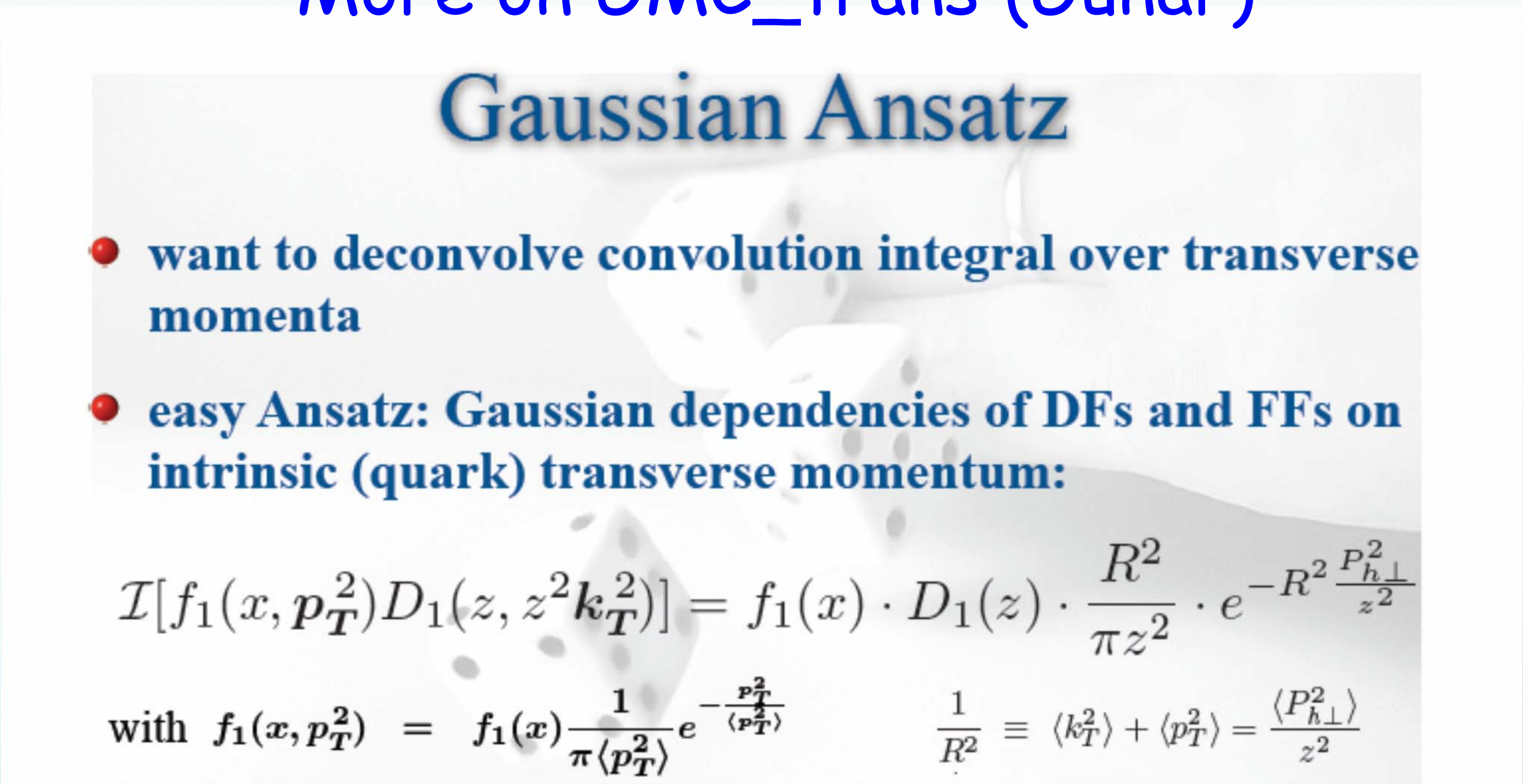
$$d\sigma_{UU}(x, y, z, \phi_S, P_{h\perp}) \equiv \frac{2\alpha^2}{sxy^2} A(y) \sum_q e_q^2 \mathcal{I}\left[f_1^q D_1^q\right]$$

where

$$\mathcal{I}\left[\mathcal{W} f D\right] \equiv \int d^2 p_T d^2 k_T \, \delta^{(2)}\left(p_T - \frac{P_{h\perp}}{z} - k_T\right) \left[\mathcal{W} f(x, p_T) D(z, k_T)\right]$$

Gunar Schnell, Universiteit Gent 6 Transversity Workshop, November 13, 2007

More on GMC_trans (Gunar)

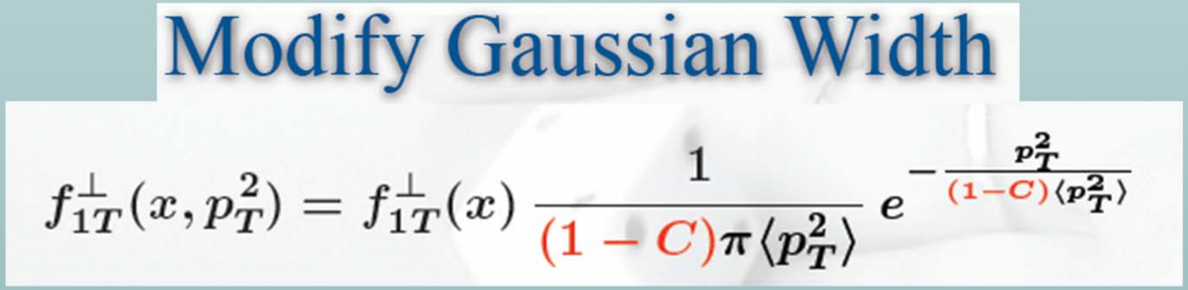


Gaussian Ansatz

- want to deconvolve convolution integral over transverse momenta
- easy Ansatz: Gaussian dependencies of DFs and FFs on intrinsic (quark) transverse momentum:

$$\mathcal{I}[f_1(x,p_T^2)D_1(z,z^2k_T^2)] = f_1(x)\cdot D_1(z)\cdot \frac{R^2}{\pi z^2}\cdot e^{-R^2\frac{P_{h\perp}^2}{z^2}}$$

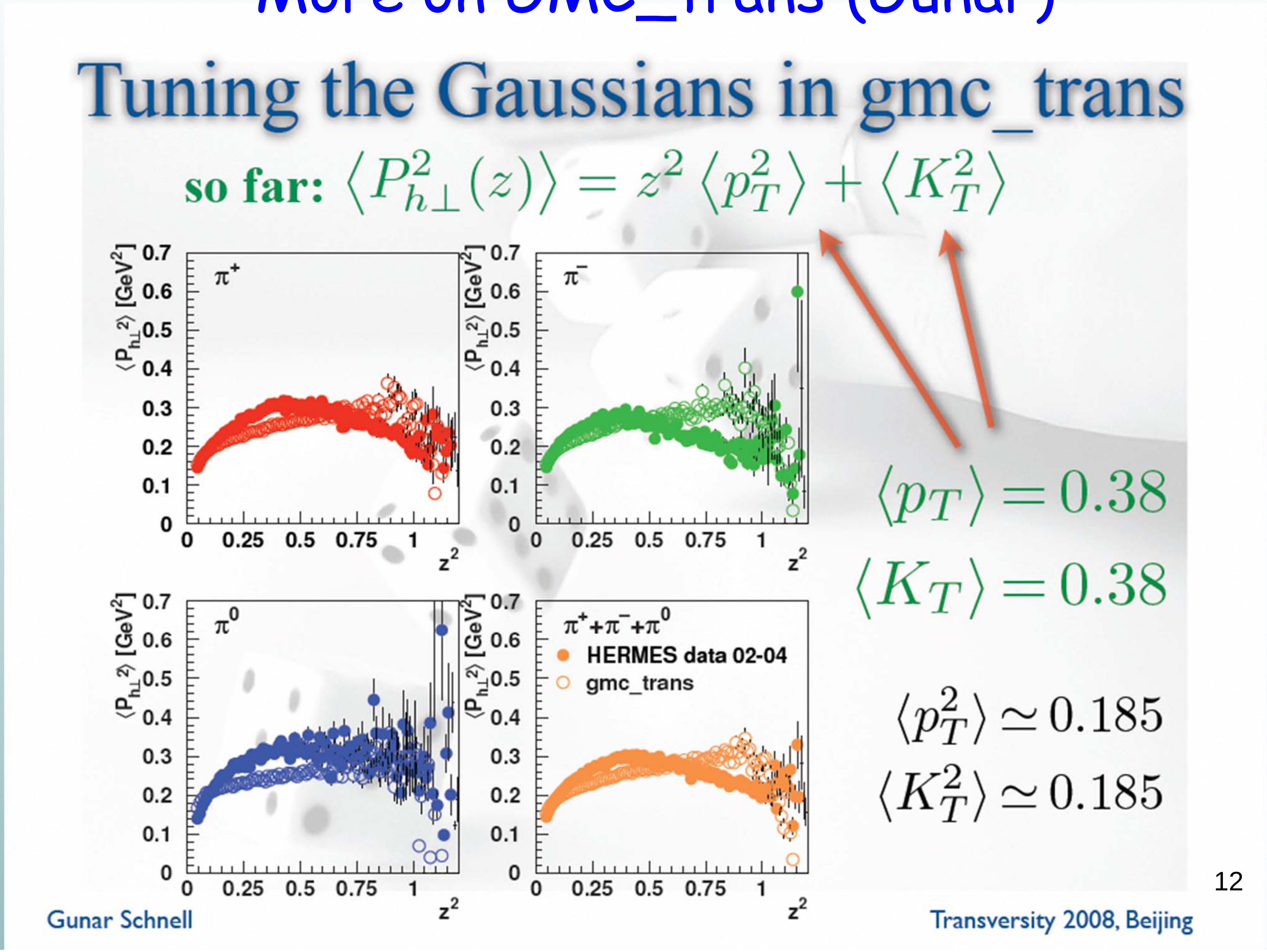
with $f_1(x,p_T^2) = f_1(x)\frac{1}{\pi\langle p_T^2\rangle}e^{-\frac{p_T^2}{\langle p_T^2\rangle}}$ $\quad \frac{1}{R^2} \equiv \langle k_T^2\rangle + \langle p_T^2\rangle = \frac{\langle P_{h\perp}^2\rangle}{z^2}$



Modify Gaussian Width

$$f_{1T}^{\perp}(x,p_T^2) = f_{1T}^{\perp}(x)\,\frac{1}{(1-C)\pi\langle p_T^2\rangle}\,e^{-\frac{p_T^2}{(1-C)\langle p_T^2\rangle}}$$

More on GMC_trans (Gunar)



Tuning the Gaussians in gmc_trans

so far: $\langle P_{h\perp}^2(z)\rangle = z^2\,\langle p_T^2\rangle + \langle K_T^2\rangle$

$\langle p_T\rangle = 0.38$

$\langle K_T\rangle = 0.38$

$\langle p_T^2\rangle \simeq 0.185$

$\langle K_T^2\rangle \simeq 0.185$

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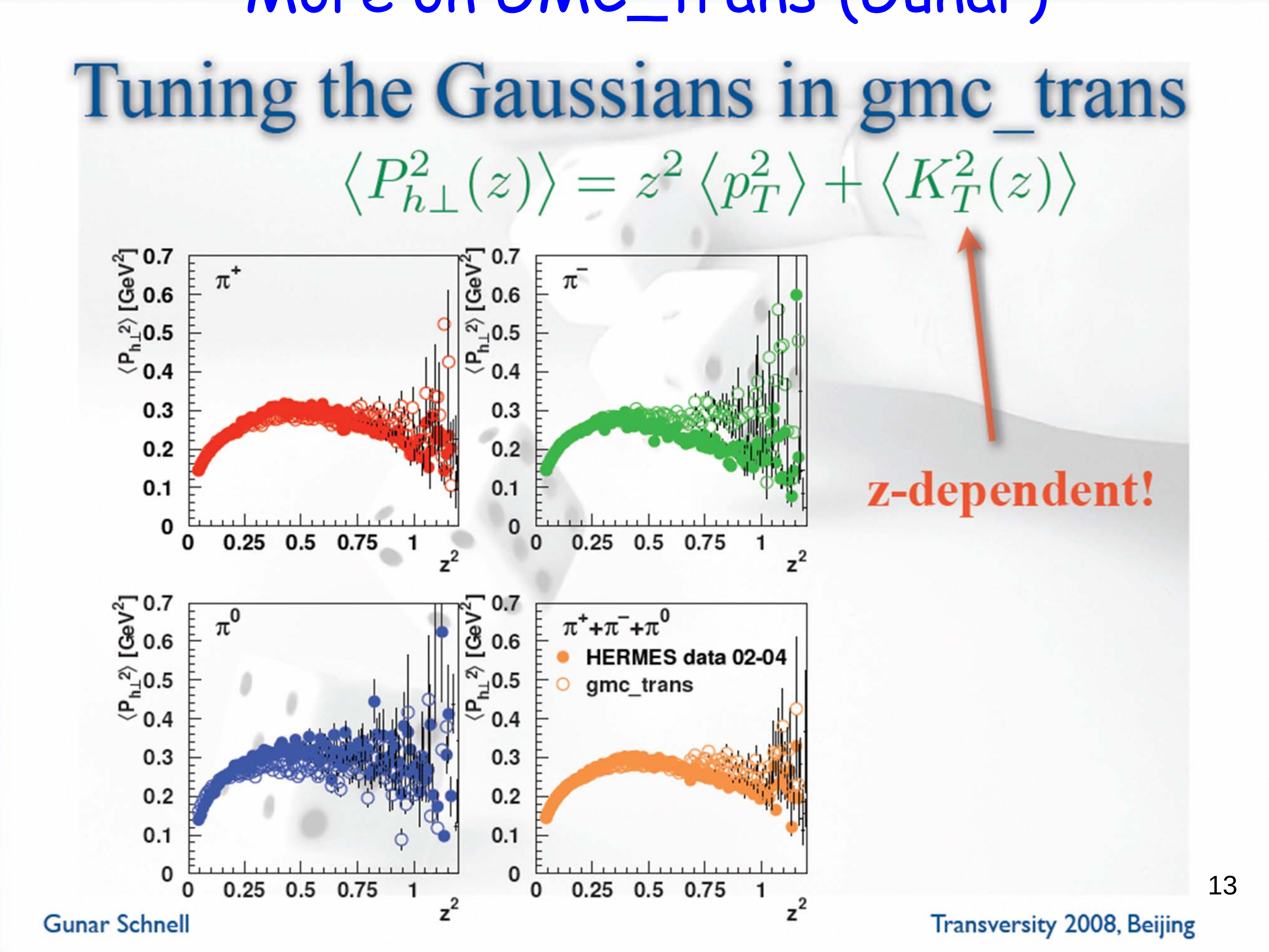
Transversity 2008, Beijing

More on GMC_trans (Gunar)



Tuning the Gaussians in gmc_trans

$$\langle P_{h\perp}^2(z)\rangle = z^2\,\langle p_T^2\rangle + \langle K_T^2(z)\rangle$$

z-dependent!

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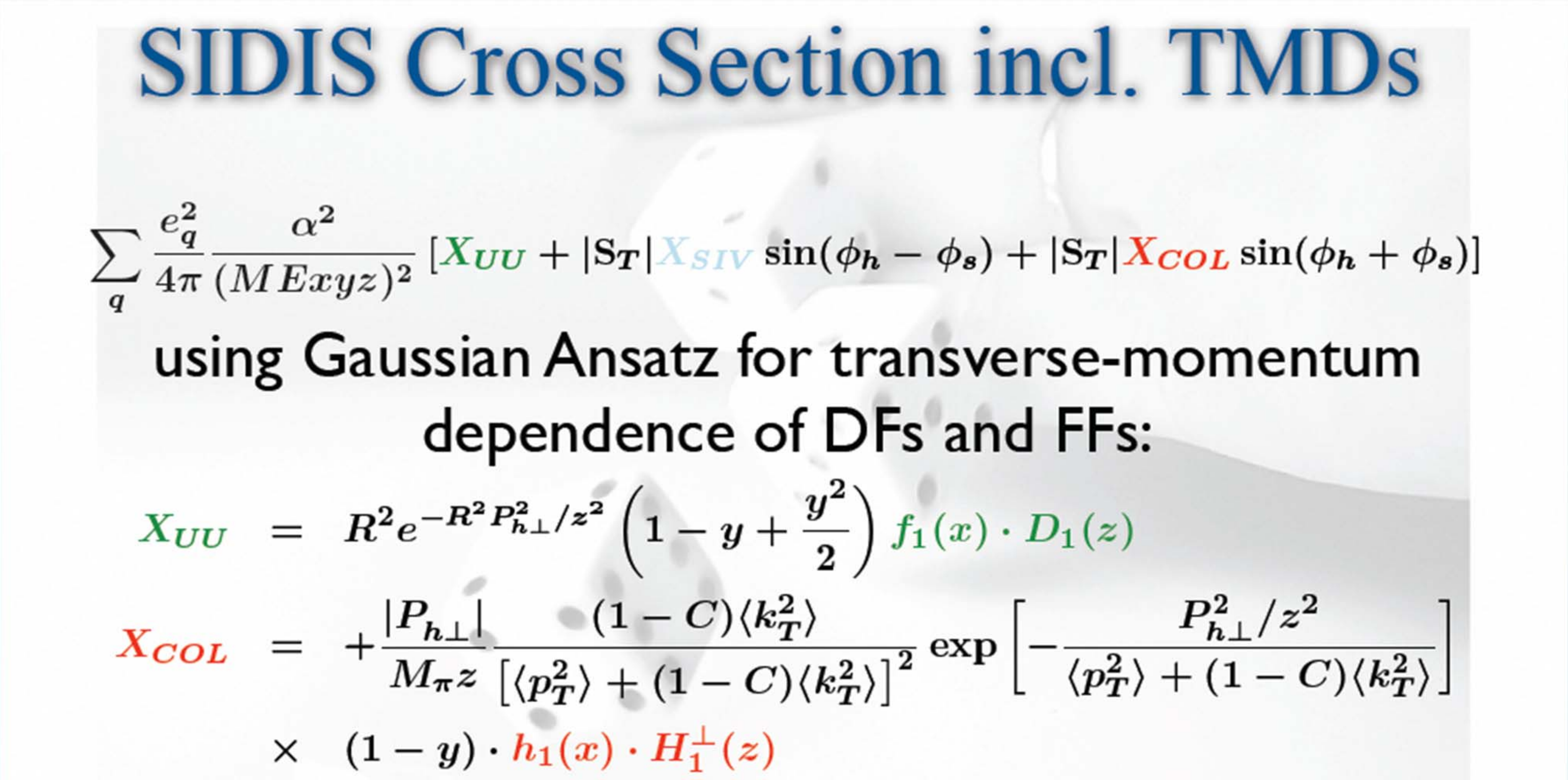
Transversity 2008, Beijing

More on GMC_trans (Gunar)

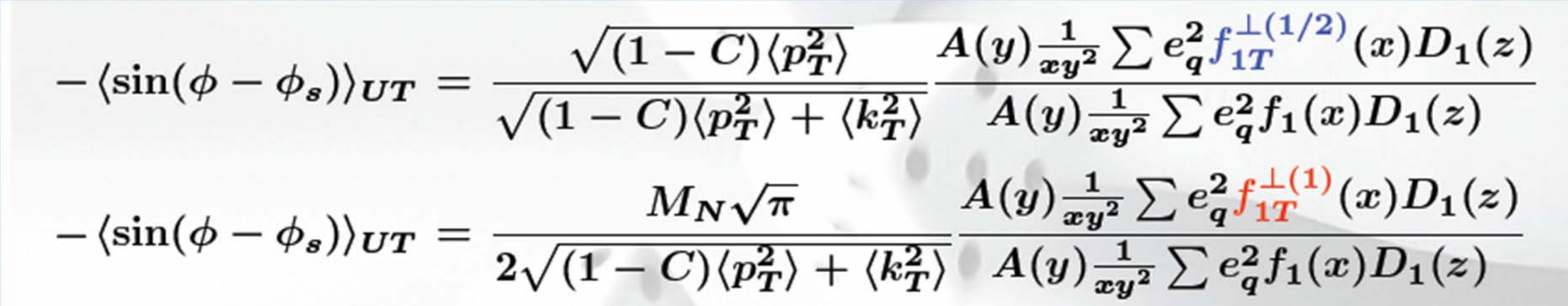


SIDIS Cross Section incl. TMDs

$$\sum_q \frac{e_q^2}{4\pi}\frac{\alpha^2}{(MExyz)^2}\left[X_{UU} + |\mathbf{S}_T| X_{SIV}\sin(\phi_h-\phi_s) + |\mathbf{S}_T| X_{COL}\sin(\phi_h+\phi_s)\right]$$

using Gaussian Ansatz for transverse-momentum dependence of DFs and FFs:

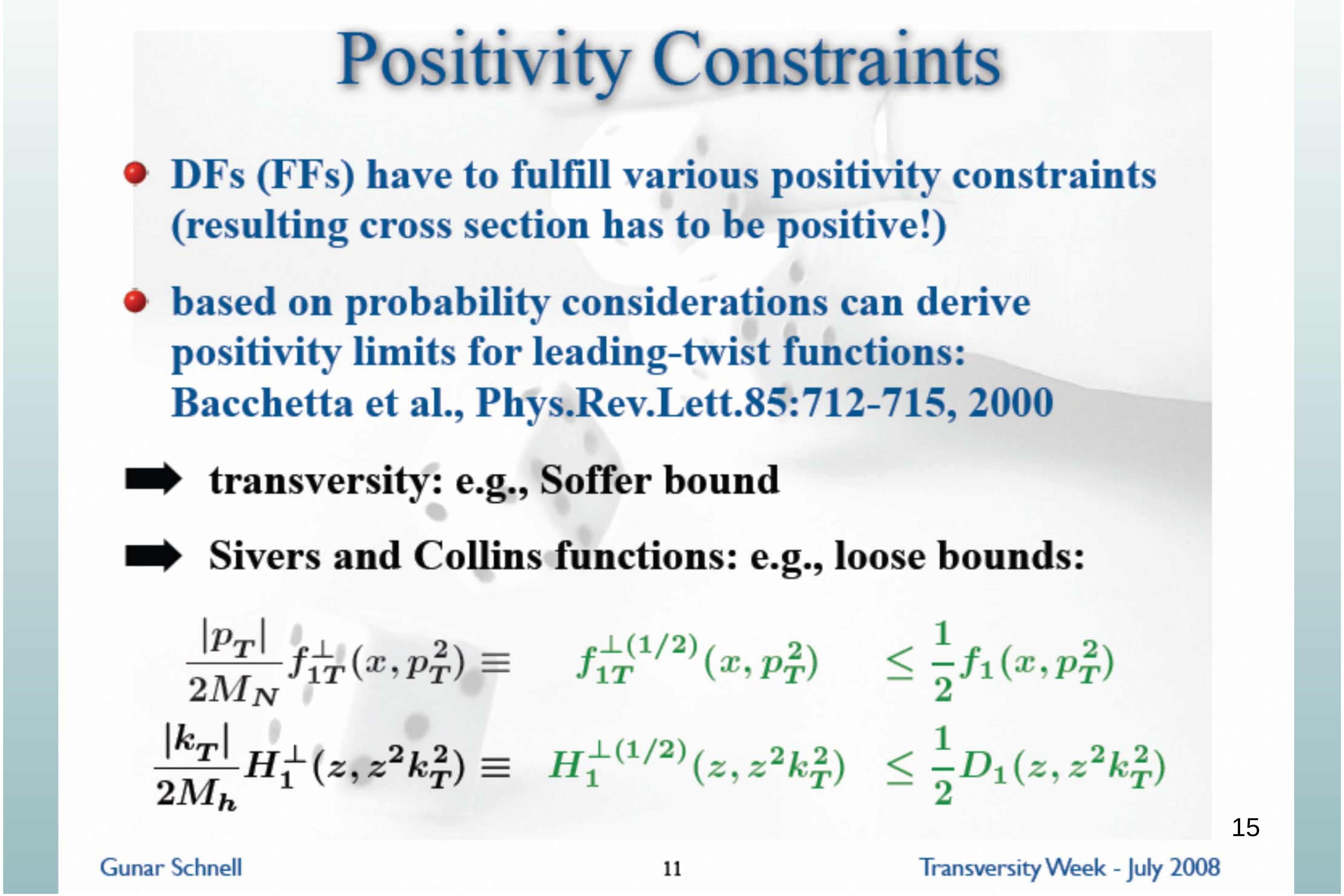
$$\begin{aligned} X_{UU} &= R^2 e^{-R^2 P_{h\perp}^2/z^2}\left(1-y+\frac{y^2}{2}\right) f_1(x)\cdot D_1(z) \\ X_{COL} &= +\frac{|P_{h\perp}|}{M_\pi z}\frac{(1-C)\langle k_T^2\rangle}{\left[\langle p_T^2\rangle+(1-C)\langle k_T^2\rangle\right]^2}\exp\left[-\frac{P_{h\perp}^2/z^2}{\langle p_T^2\rangle+(1-C)\langle k_T^2\rangle}\right] \\ &\times \quad (1-y)\cdot h_1(x)\cdot H_1^\perp(z) \end{aligned}$$



$$-\langle\sin(\phi-\phi_s)\rangle_{UT} = \frac{\sqrt{(1-C)\langle p_T^2\rangle}}{\sqrt{(1-C)\langle p_T^2\rangle+\langle k_T^2\rangle}}\frac{A(y)\frac{1}{xy^2}\sum e_q^2 f_{1T}^{\perp(1/2)}(x)D_1(z)}{A(y)\frac{1}{xy^2}\sum e_q^2 f_1(x)D_1(z)}$$

$$-\langle\sin(\phi-\phi_s)\rangle_{UT} = \frac{M_N\sqrt{\pi}}{2\sqrt{(1-C)\langle p_T^2\rangle+\langle k_T^2\rangle}}\frac{A(y)\frac{1}{xy^2}\sum e_q^2 f_{1T}^{\perp(1)}(x)D_1(z)}{A(y)\frac{1}{xy^2}\sum e_q^2 f_1(x)D_1(z)}$$

More on GMC_trans (Gunar)



Positivity Constraints

- DFs (FFs) have to fulfill various positivity constraints (resulting cross section has to be positive!)
- based on probability considerations can derive positivity limits for leading-twist functions: Bacchetta et al., Phys.Rev.Lett.85:712-715, 2000

➡ transversity: e.g., Soffer bound

➡ Sivers and Collins functions: e.g., loose bounds:

$$\frac{|p_T|}{2M_N} f_{1T}^{\perp}(x,p_T^2) \equiv f_{1T}^{\perp(1/2)}(x,p_T^2) \leq \frac{1}{2} f_1(x,p_T^2)$$

$$\frac{|k_T|}{2M_h} H_1^{\perp}(z,z^2k_T^2) \equiv H_1^{\perp(1/2)}(z,z^2k_T^2) \leq \frac{1}{2} D_1(z,z^2k_T^2)$$

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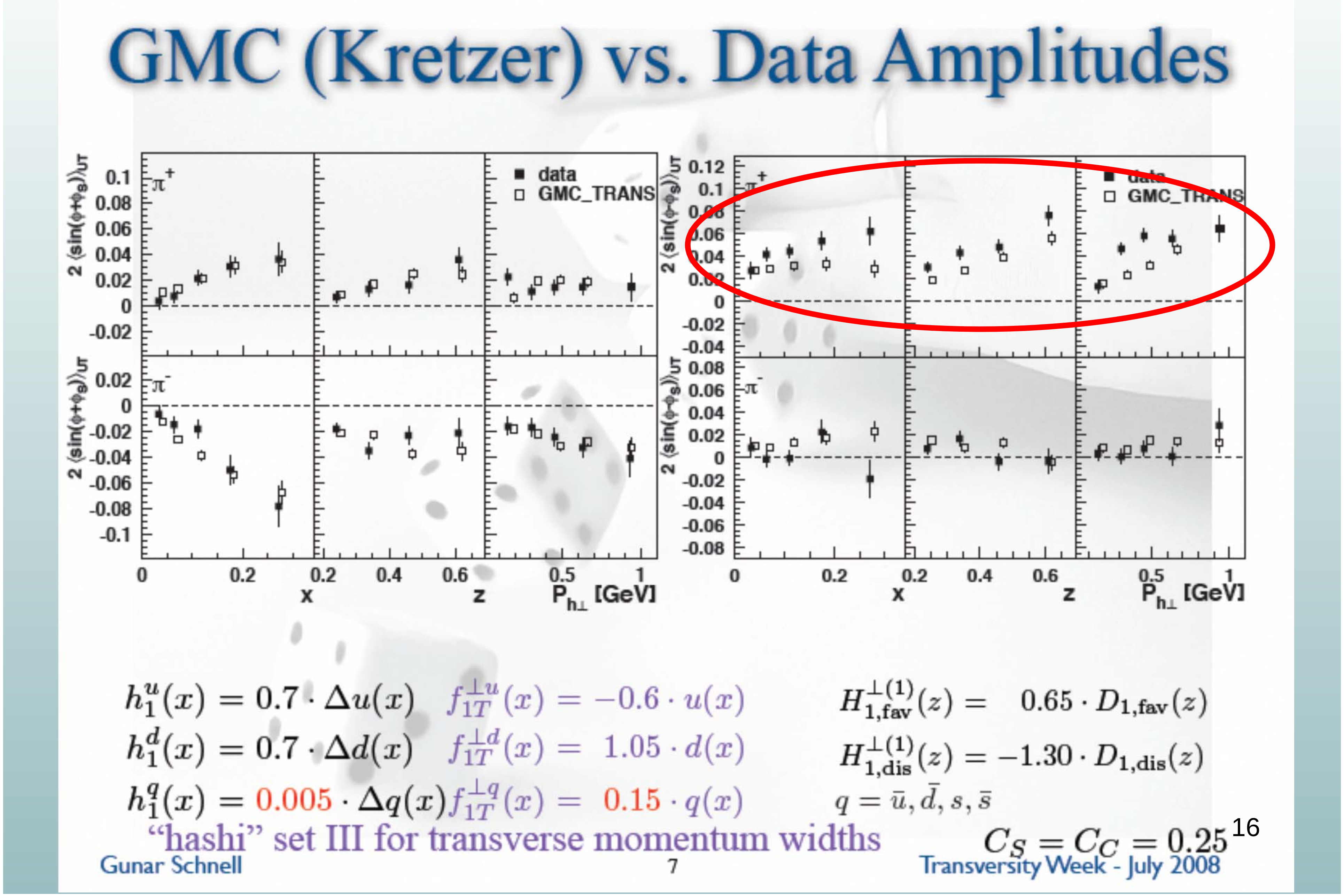
More on GMC_trans (Gunar)



GMC (Kretzer) vs. Data Amplitudes

$h_1^u(x) = 0.7 \cdot \Delta u(x)$ $f_{1T}^{\perp u}(x) = -0.6 \cdot u(x)$ $H_{1,\mathrm{fav}}^{\perp(1)}(z) = 0.65 \cdot D_{1,\mathrm{fav}}(z)$

$h_1^d(x) = 0.7 \cdot \Delta d(x)$ $f_{1T}^{\perp d}(x) = 1.05 \cdot d(x)$ $H_{1,\mathrm{dis}}^{\perp(1)}(z) = -1.30 \cdot D_{1,\mathrm{dis}}(z)$

$h_1^q(x) = 0.005 \cdot \Delta q(x) f_{1T}^{\perp q}(x) = 0.15 \cdot q(x)$ $q = \bar{u}, \bar{d}, s, \bar{s}$

“hashi” set III for transverse momentum widths $C_S = C_C = 0.25$

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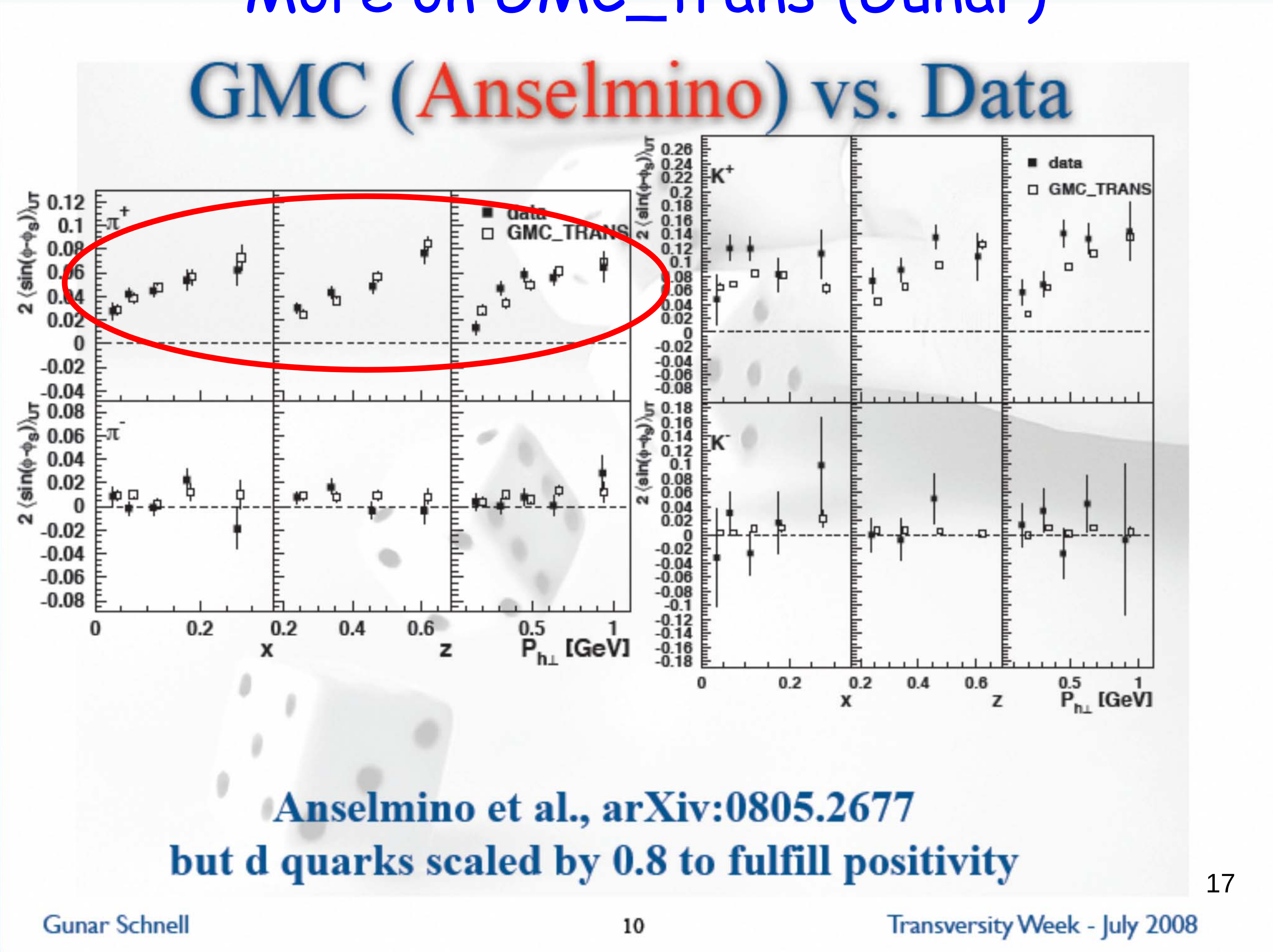
More on GMC_trans (Gunar)



GMC (Anselmino) vs. Data

Anselmino et al., arXiv:0805.2677
but d quarks scaled by 0.8 to fulfill positivity

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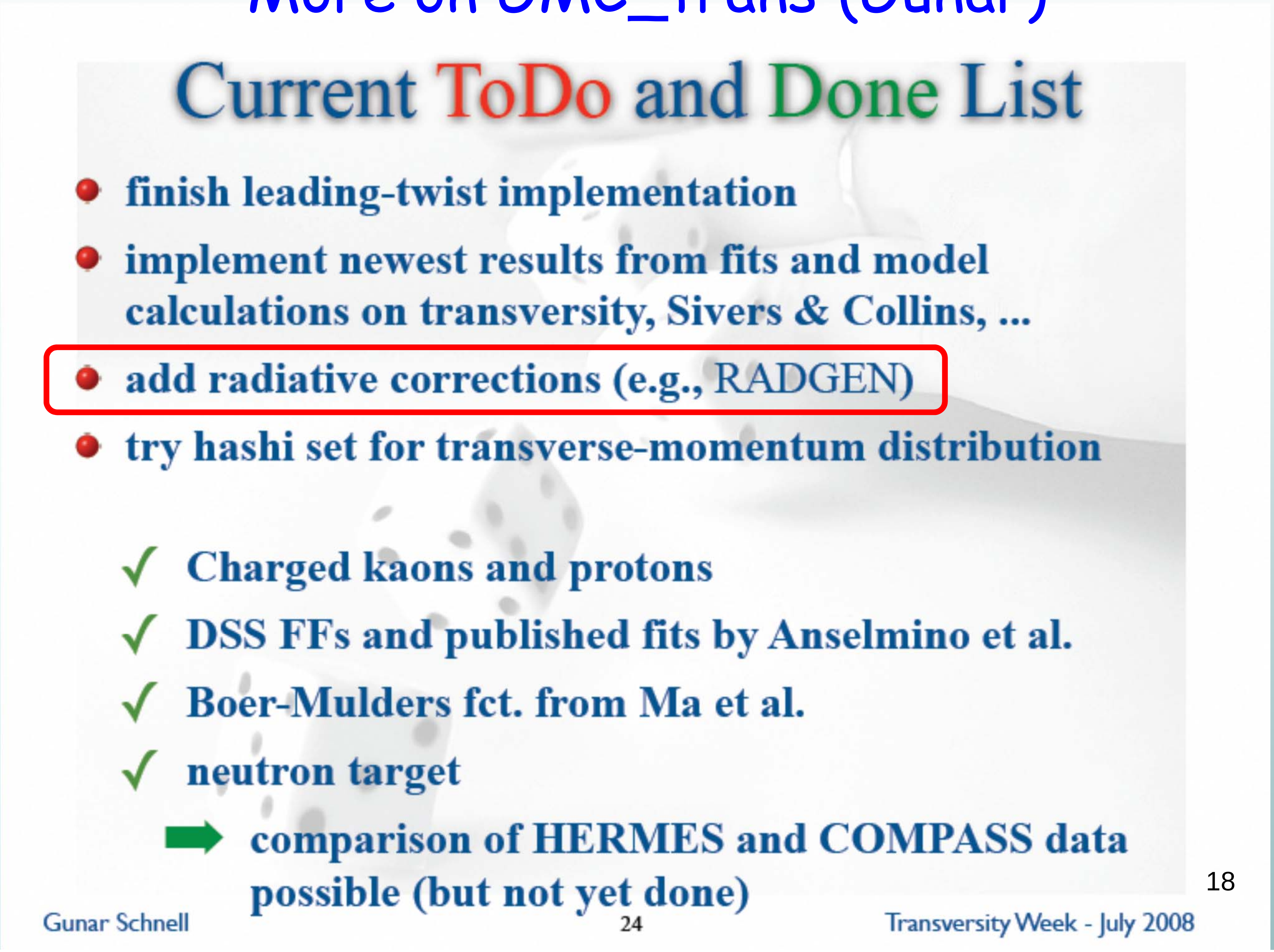
More on GMC_trans (Gunar)



Current ToDo and Done List

- finish leading-twist implementation
- implement newest results from fits and model calculations on transversity, Sivers & Collins, ...
- add radiative corrections (e.g., RADGEN)
- try hashi set for transverse-momentum distribution

✓ Charged kaons and protons

✓ DSS FFs and published fits by Anselmino et al.

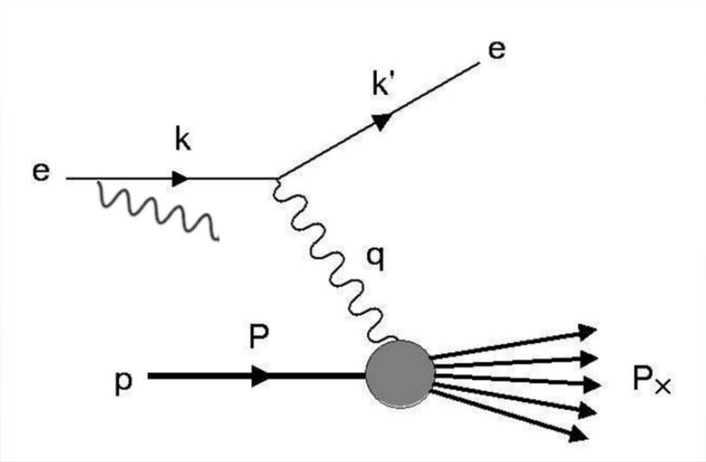
✓ Boer-Mulders fct. from Ma et al.

✓ neutron target

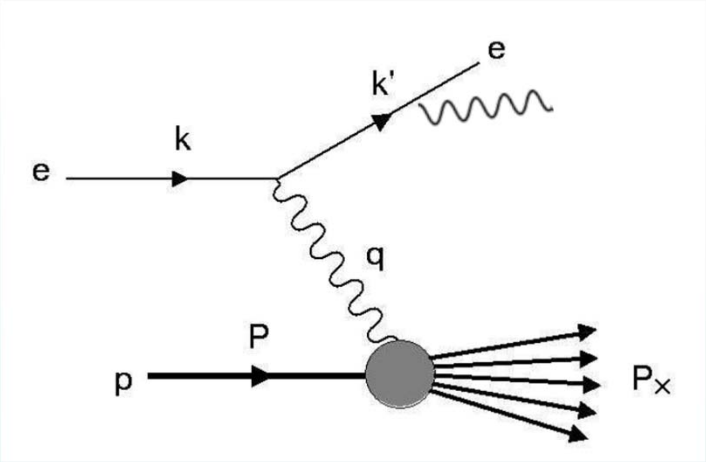
➡ comparison of HERMES and COMPASS data possible (but not yet done)

Part 2: applications of MC in 1h SIDIS analyses

QED radiative effects and "True" kinematics



I.S.R



F.S.R

Influence reaction kinematics:

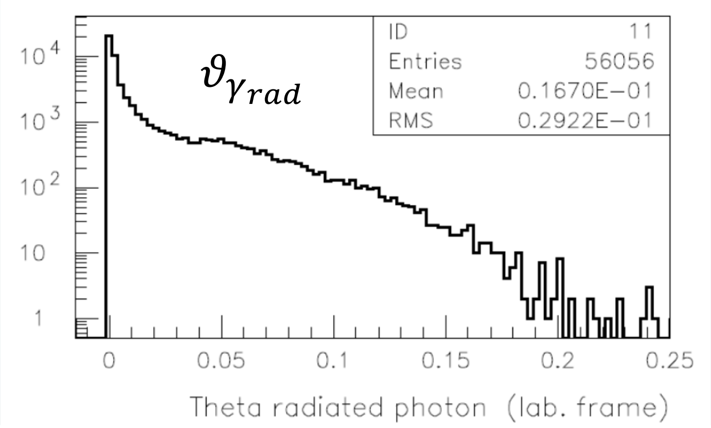
- virtual photon 4-momentum $q_{\gamma^*} = k - k' - q_{rad}$
- azimuthal angles ϕ and ϕ_S (defined w.r.t. the virtual photon direction)
- scattering plane differs between I.S.R and F.S.R.

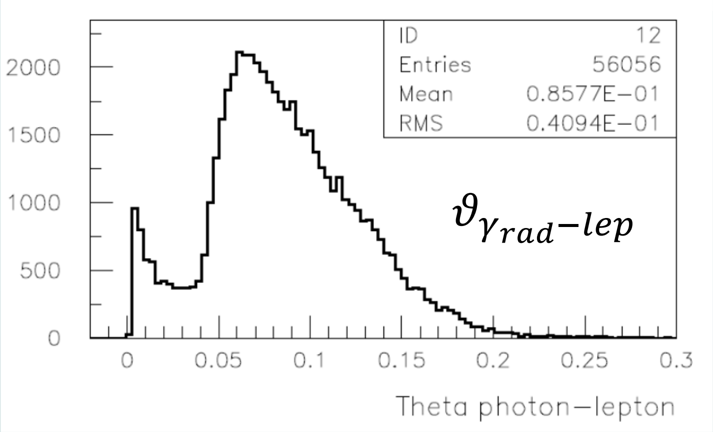
- **PYTHIA (RADGEN) provides "True" kinematics for the standard DIS variables, but:**
- **need to correct virtual photon 4-momentum and azimuthal angles!**
- **need to distinguish between I.S.R. and F.S.R. (not provided bt Pythia!)**

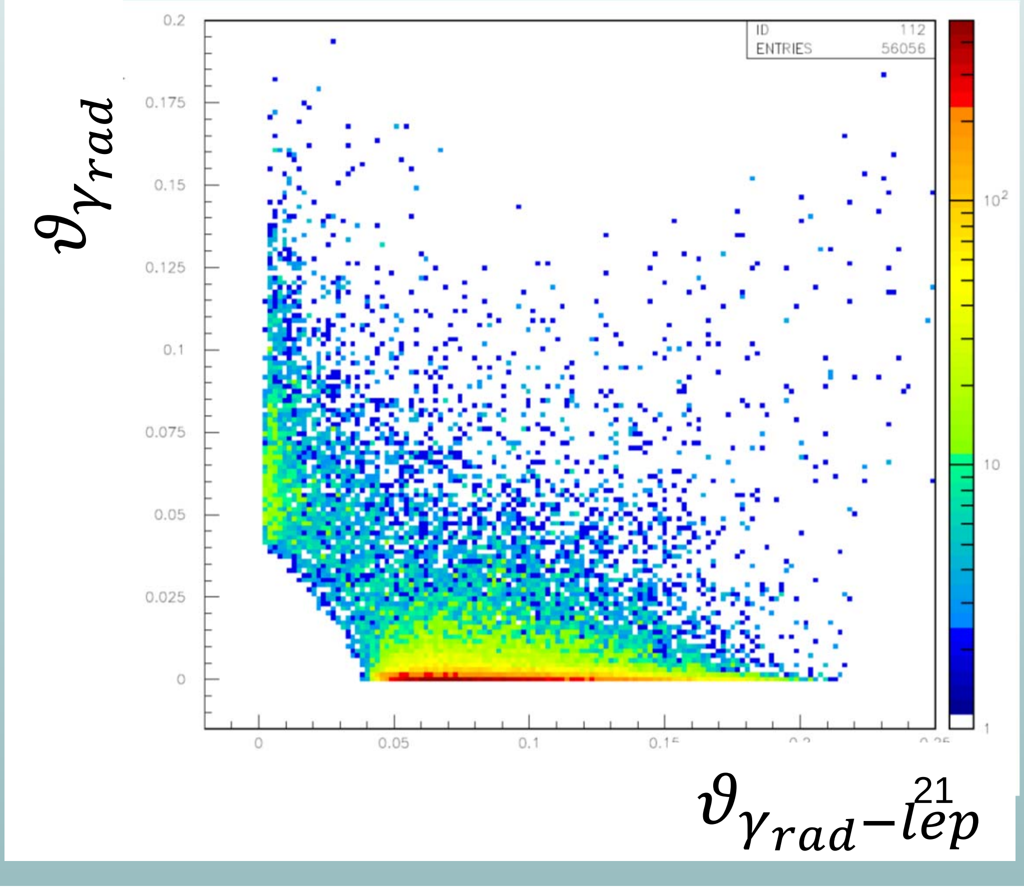
QED radiative effects and "True" kinematics

PYTHIA stores the polar angle ϑ_{rad} (and 4-momentum q_{rad}) of the real photon

one can distinguish between I.S.R. and F.S.R. by combining the information on the polar angle of the real photon ($\vartheta_{\gamma_{rad}}$) and the relative polar angle between real photon and scattered lepton ($\vartheta_{\gamma_{rad}-lep}$).







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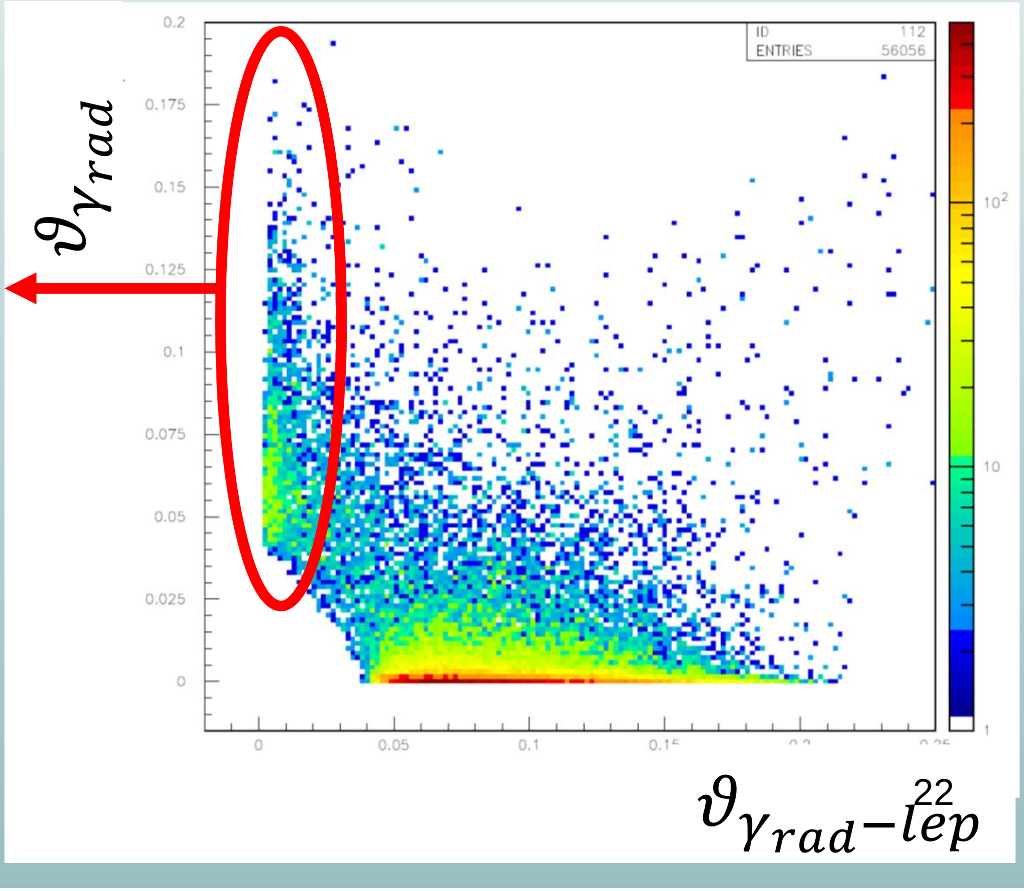
→ one can distinguish between I.S.R. and F.S.R. by combining the information on the polar angle of the real photon ($\vartheta_{\gamma_{rad}}$) and the relative polar angle between real photon and scattered lepton ($\vartheta_{\gamma_{rad}-lep}$).

real photon almost collinear with scattered lepton

(F.S.R)

← $\theta_{\gamma_{rad}-lep} \approx 0$



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real photon almost collinear with scattered lepton

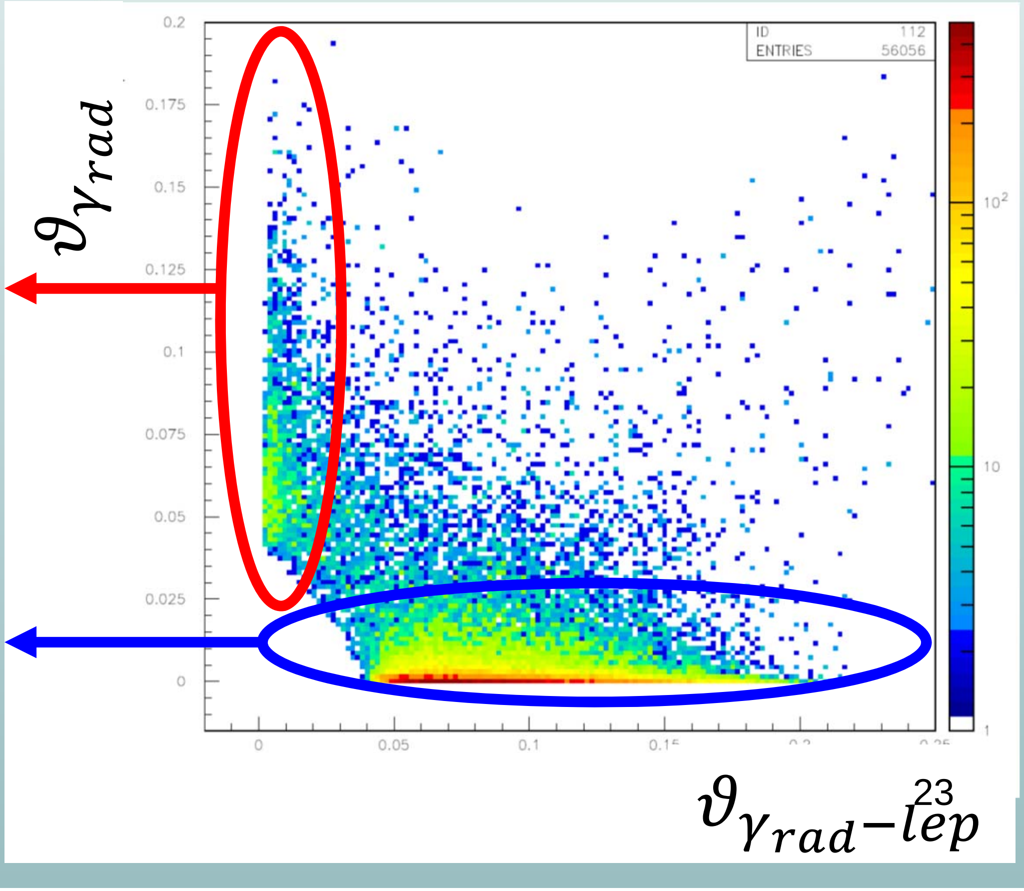
(F.S.R)

⇐ $\theta_{\gamma_{rad}-lep} \approx 0$

real photon almost collinear with incident (beam) lepton

(I.S.R)

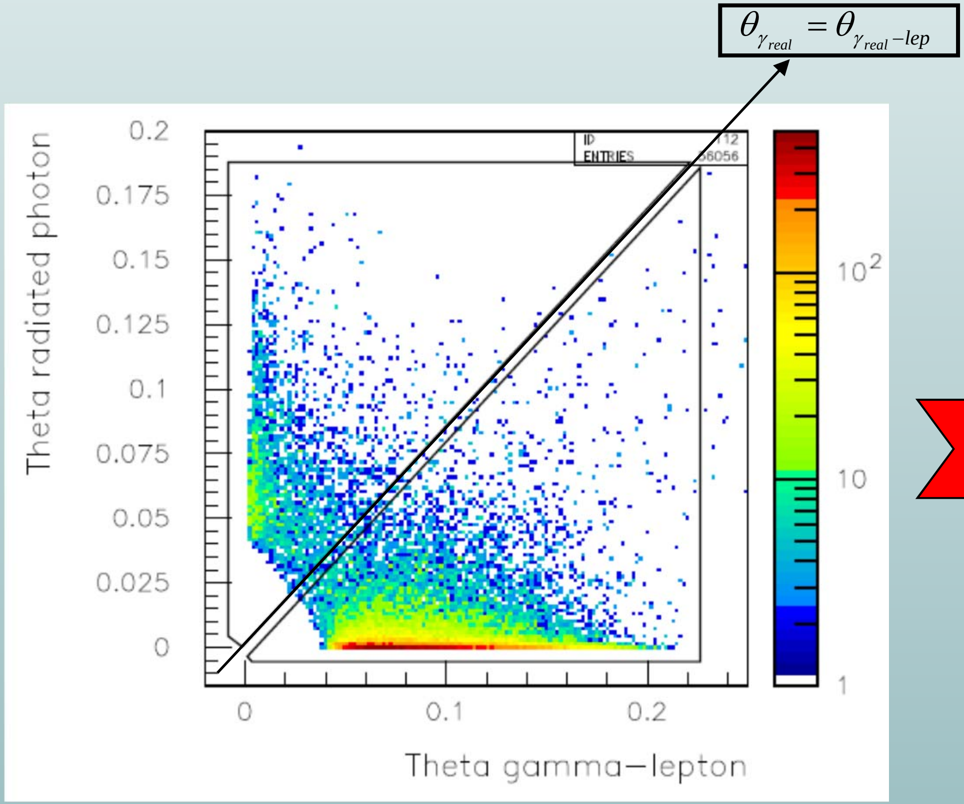
⇐ $\theta_{\gamma_{rad}} \approx 0$



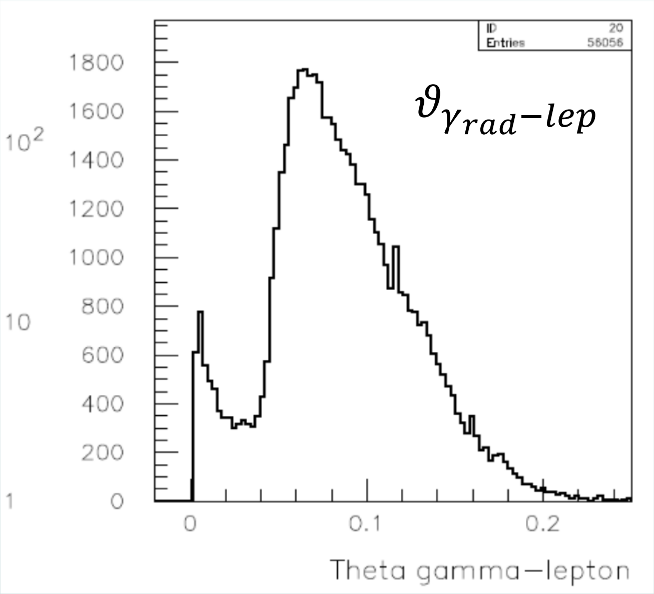
QED radiative effects and "True" kinematics

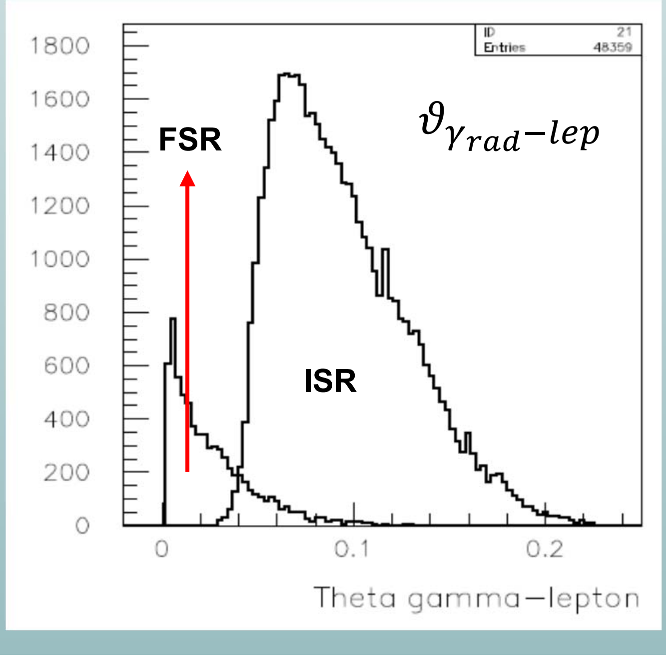
$$\theta_{\gamma_{real}} > \theta_{\gamma_{real}-lep} \Rightarrow FSR$$

$$\theta_{\gamma_{real}} < \theta_{\gamma_{real}-lep} \Rightarrow ISR$$

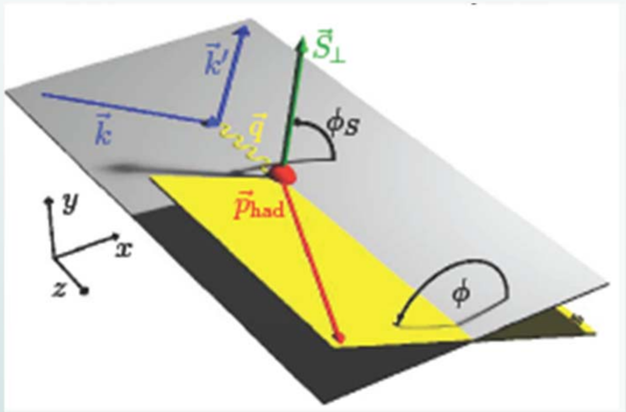


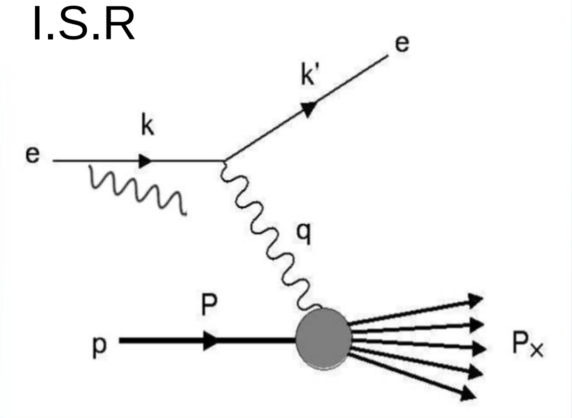
$\theta_{\gamma_{real}} = \theta_{\gamma_{real}-lep}$

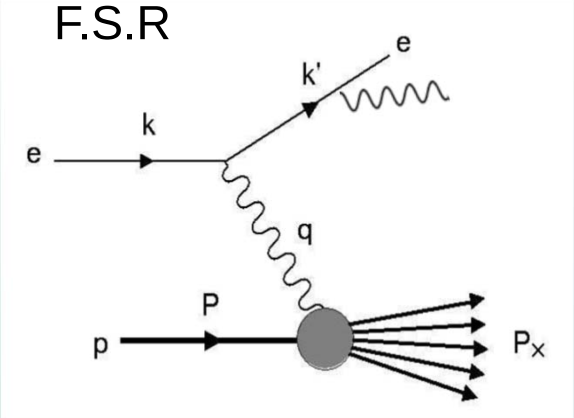




QED radiative effects and “True” kinematics







Calculate the “True” values of ϕ and ϕ_S :

$$\phi_S = \operatorname{sgn}\left(\mathbf{q}\times\mathbf{k}\cdot\hat{\mathbf{S}}_\perp\right)\arccos\left(\frac{\mathbf{q}\times\mathbf{k}\cdot\mathbf{q}\times\hat{\mathbf{S}}_\perp}{|\mathbf{q}\times\mathbf{k}|\cdot|\mathbf{q}\times\hat{\mathbf{S}}_\perp|}\right)$$

$$\phi = \operatorname{sgn}\left(\mathbf{q}\times\mathbf{k}\cdot\mathbf{P}_h\right)\arccos\left(\frac{\mathbf{q}\times\mathbf{k}\cdot\mathbf{q}\times\mathbf{P}_h}{|\mathbf{q}\times\mathbf{k}|\cdot|\mathbf{q}\times\mathbf{P}_h|}\right)$$

- Correct virtual photon 4-momentum: $q = k - k' - q_{rad}$
- use incident lepton momentum (k) for FSR $(\theta_{\gamma_{rad}} > \theta_{\gamma_{rad}-lep})$
- use scattered lepton momentum (k’) for ISR $(\theta_{\gamma_{rad}} < \theta_{\gamma_{rad}-lep})$

Acceptance, smearing and radiative effects

Need a MC generator with:

- Polarized cross section (including Collins, Sivers, etc)
- Full description of the apparatus
- Radiative effects (RADGEN)

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GMC_trans:

🙂 Polarized cross section (including Collins, Sivers, etc) and transverse momentum dependence implemented

😐 Has not full description of the apparatus (HSG)

☹ Radiative effects (RADGEN) not yet implemented (fortran version)

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🙂 Polarized cross section (including Collins, Sivers, etc) and transverse momentum dependence implemented

😐 Has not full description of the apparatus (HSG)

🙁 Radiative effects (RADGEN) not yet implemented (fortran version)

PYTHIA:

🙂 Full description of the apparatus (HMC)

🙂 Radiative effects (RADGEN)

🙁 generates events according to the spin-independent (Born) cross-section
➡ polarized cross section (including Collins, Sivers, etc) not implem.

"Polarizing" PYTHIA

The spin-dependent part of the cross section (e.g. Collins and Sivers effects) can be implemented in PYTHIA a-posteriori using a suitable parametrization (or model)

1. Extract the full kinematic dependence (e.g. Taylor expansion in $x, Q^2, z, P_{h\perp}$) of the various azimuthal moments (e.g. Collins and Sivers) through a fully-differential ML fit of real data (using full set of SIDIS events):

$$f(\overline{x}, P_t; c) = 1 + P_t \cdot [A_{Collins}(\overline{x}; c_i) \cdot \sin(\phi + \phi_S) + A_{Sivers}(\overline{x}; c_i) \cdot \sin(\phi - \phi_S) + ...]$$

e.g.: $A_{Collins}(\overline{x}, c) = c_0 + c_1 \cdot x + c_2 \cdot z + c_3 \cdot Q^2 + c_4 \cdot P_{h\perp} + c_5 \cdot x^2 + ... + c_{22} \cdot x^2 \cdot z \cdot P_{h\perp}$

"Polarizing" PYTHIA

The spin-dependent part of the cross section (e.g. Collins and Sivers effects) can be implemented in PYTHIA a-posteriori using a suitable parametrization (or model)

1. Extract the full kinematic dependence (e.g. Taylor expansion in $x, Q^2, z, P_{h\perp}$) of the various azimuthal moments (e.g. Collins and Sivers) through a fully-differential ML fit of real data (using full set of SIDIS events):

$$f(\overline{x},P_t;c)=1+P_t\cdot[A_{Collins}(\overline{x};c_i)\cdot\sin(\phi+\phi_S)+A_{Sivers}(\overline{x};c_i)\cdot\sin(\phi-\phi_S)+...]$$

e.g.: $A_{Collins}(\overline{x},c)=c_0+c_1\cdot x+c_2\cdot z+c_3\cdot Q^2+c_4\cdot P_{h\perp}+c_5\cdot x^2+...+c_{22}\cdot x^2\cdot z\cdot P_{h\perp}$

model assumptions minimized

acceptance effects vanish

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2. evaluate this parametrization (model) at the True kinematics (⇒ virtually free from accept., smearing and radiative effects!!) and construct the probability functions:

$$\begin{cases} \sigma_+ = \sigma_0 / 2 \, \left[1 + \left(A_{Coll} \sin(\phi + \phi_S) + ...\right)\right] \\ \sigma_- = \sigma_0 / 2 \, \left[1 - \left(A_{Coll} \sin(\phi + \phi_S) + ...\right)\right] \end{cases}$$

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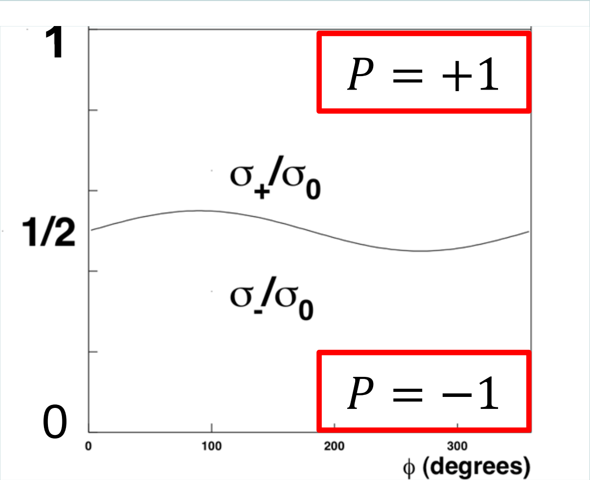
$$\Rightarrow \sigma_+ / \sigma_0 + \sigma_- / \sigma_0 = 1$$

3. randomly assign the sign of the target polarization (+1 or -1) event-by-event according to these probabilities:



4. ⇒ obtain a (transverse) **spin-dependent PYTHIA data sample!**

Evaluation of acceptance effects ("old" approach)

1. Extract a parametrization from a fully-differential ML fit of real data

$$f(\overline{x},P_t;c)=1+P_t\cdot[A_{Collins}(\overline{x};c_i)\cdot\sin(\phi+\phi_S)+A_{Sivers}(\overline{x};c_i)\cdot\sin(\phi-\phi_S)+\ldots]$$

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2. Fold the extracted parametriz. (models) with the spin-independent cross section (e.g. from PYTHIA or GMC_trans) in 4π ($\sigma_{UU}^{4\pi}$) and within the acceptance ($\sigma_{UU}^{acc.}$) :

$$\langle\sin(\phi\pm\phi_S)\rangle_{UT}^{acc,4\pi}(x)=\frac{\int\sigma_{UU}^{acc,4\pi}(\bar{x})\;A_{Collins,Sivers}(\bar{x};c_i)}{\int\sigma_{UU}^{acc,4\pi}(\bar{x})}\approx\frac{\sum_{j=1}^{N_{MC}}W_j^{MC}\cdot A_{Collins,Sivers}(\bar{x};c_i)}{\sum_{j=1}^{N_{MC}}W_j^{MC}}$$

"**MODEL_ACC**" = model folded with Born Xsection in acceptance

"**MODEL_4π**" = model folded models with Born Xsection in 4π

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➡

"MODEL_ACC" = model folded with Born Xsection in acceptance

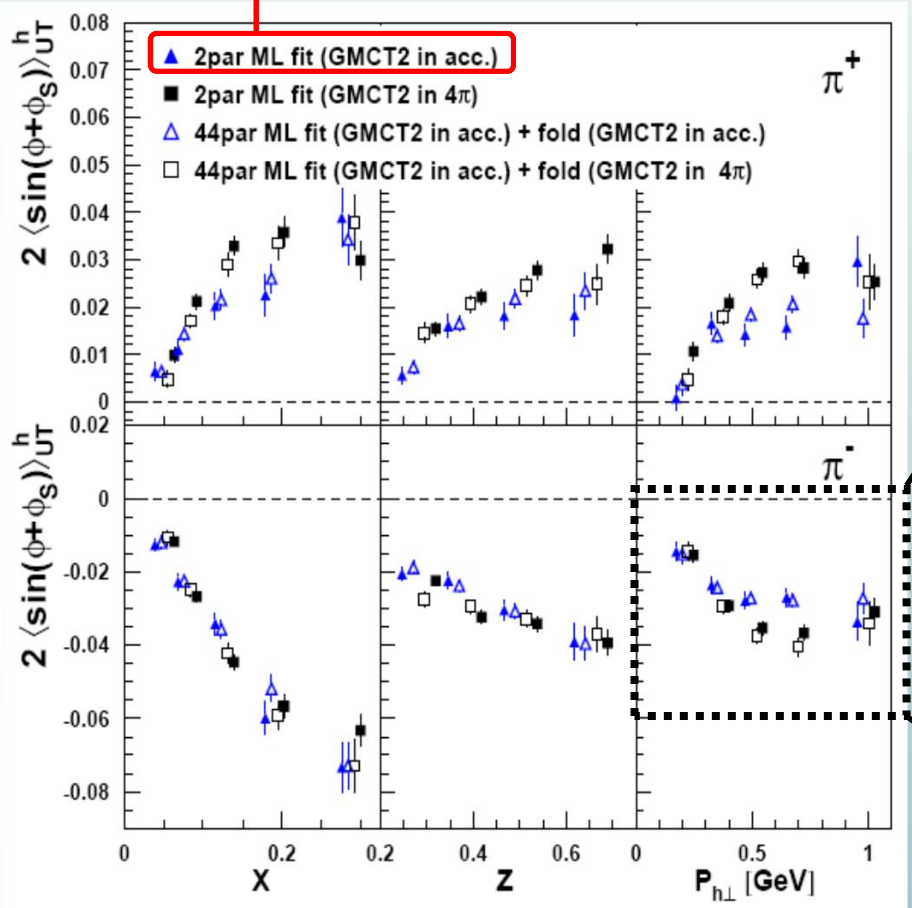
"MODEL_4π" = model folded models with Born Xsection in 4π

3. **Acceptance effects** = "**MODEL_4π**" – "**MODEL_ACC**"

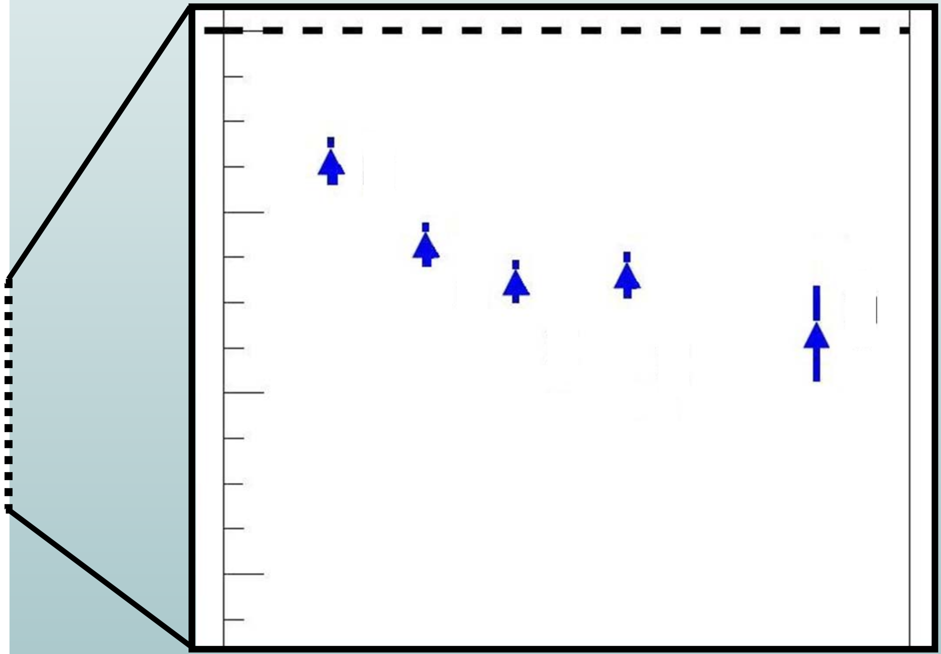
Evaluation of acceptance effects ("old" approach)

applying the method on MC (GMC_trans) data

Standard extraction method



▲ Simulate real data (in acc.)



Evaluation of acceptance effects ("old" approach)

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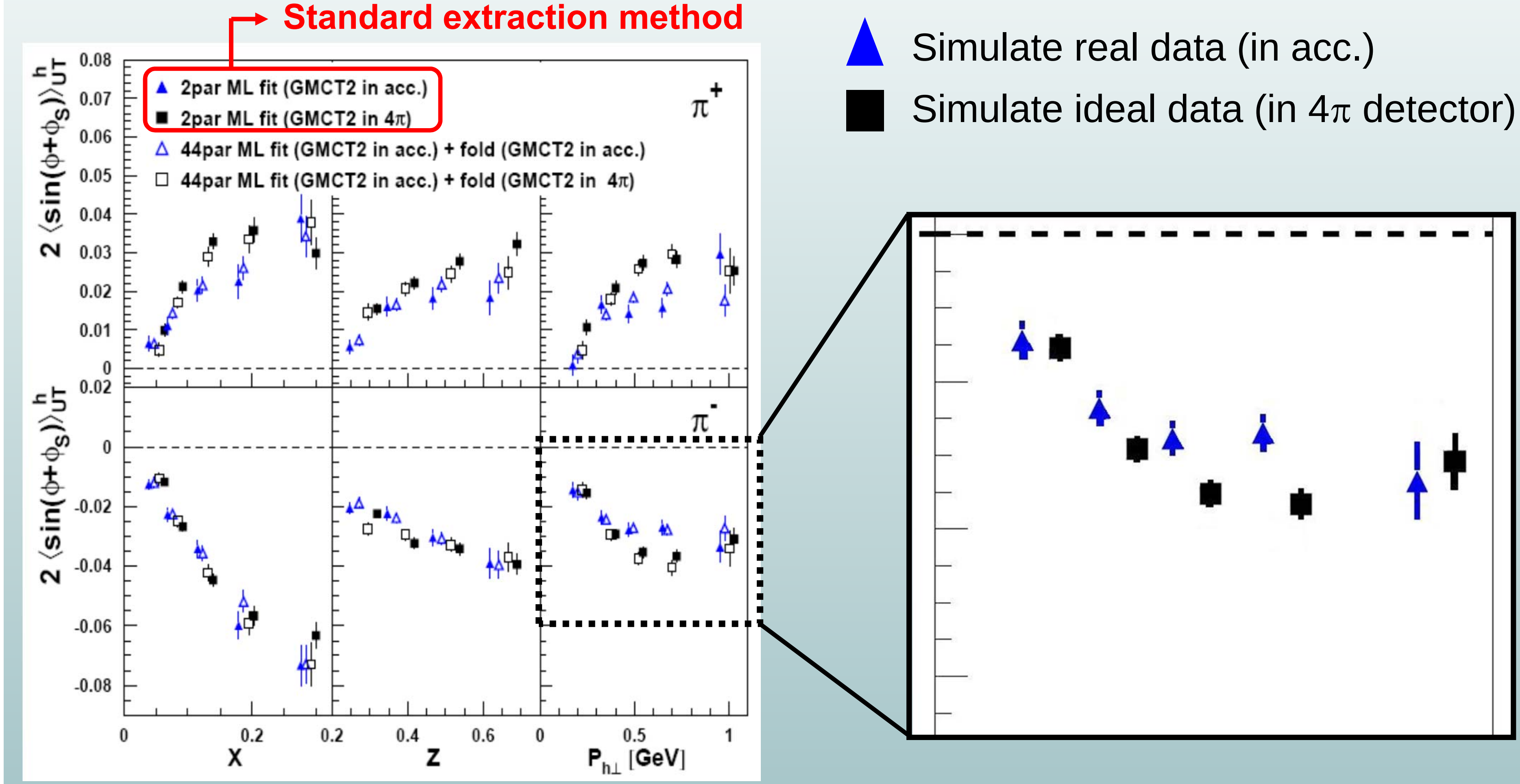

▲ Simulate real data (in acc.)

■ Simulate ideal data (in 4π detector)

- Large acceptance effects in MC data

Evaluation of acceptance effects ("old" approach)

applying the method on MC (GMC_trans) data

Standard extraction method

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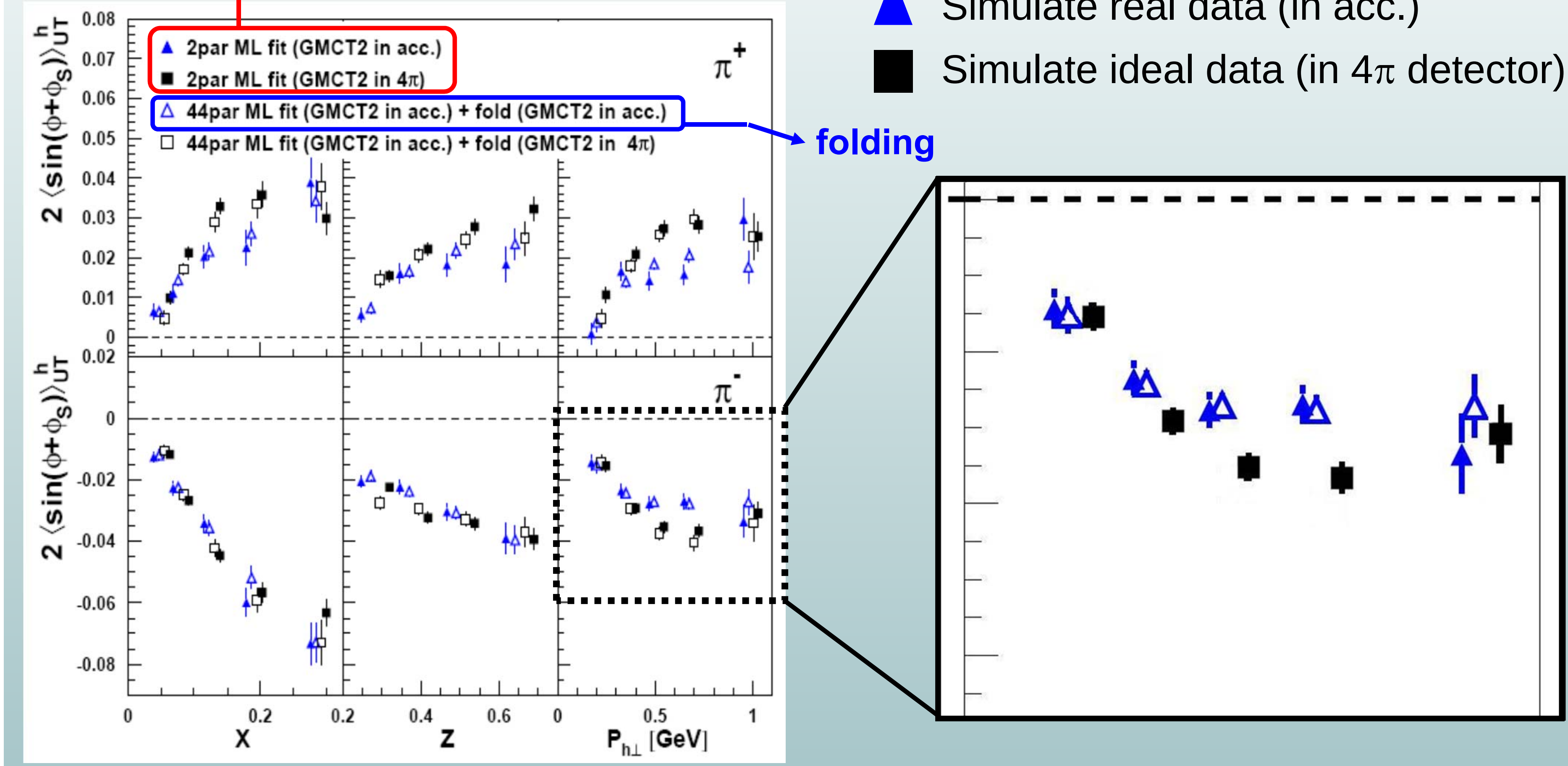


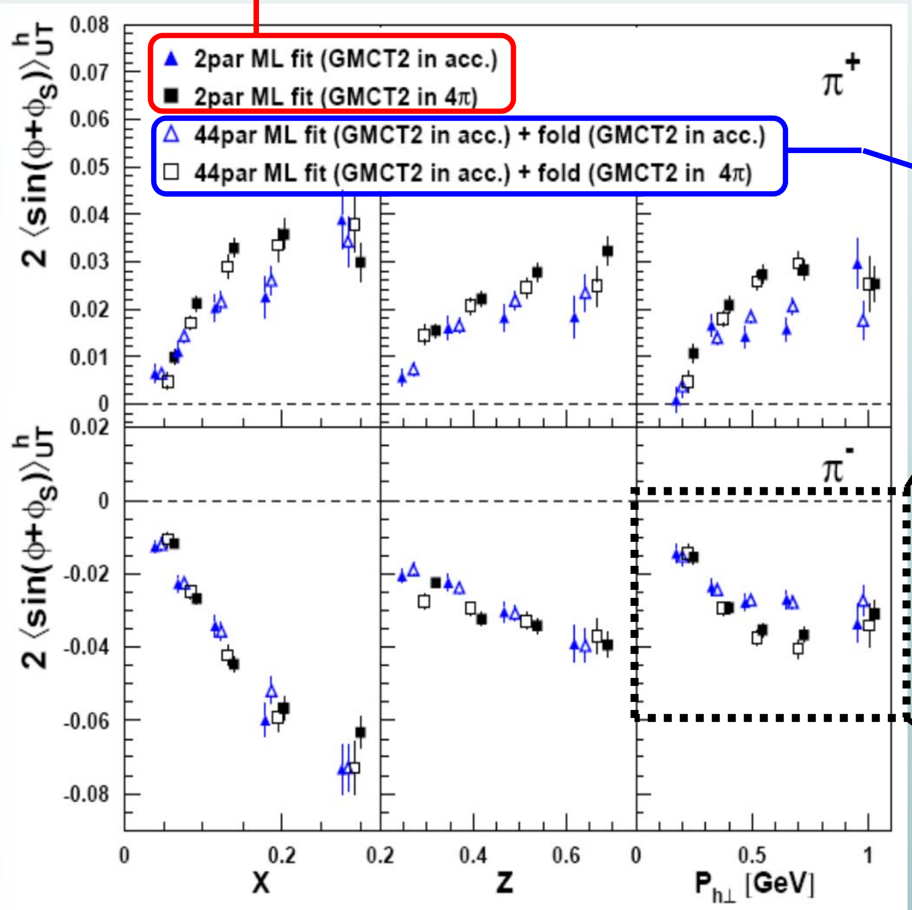
folding

- Large acceptance effects in MC data

Evaluation of acceptance effects ("old" approach)

applying the method on MC (GMC_trans) data

Standard extraction method



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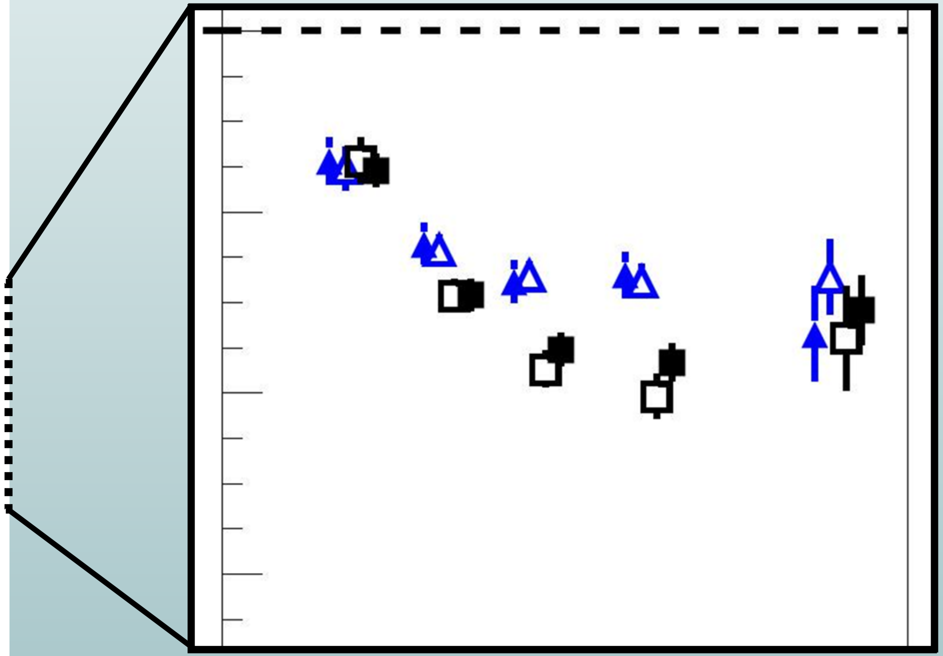
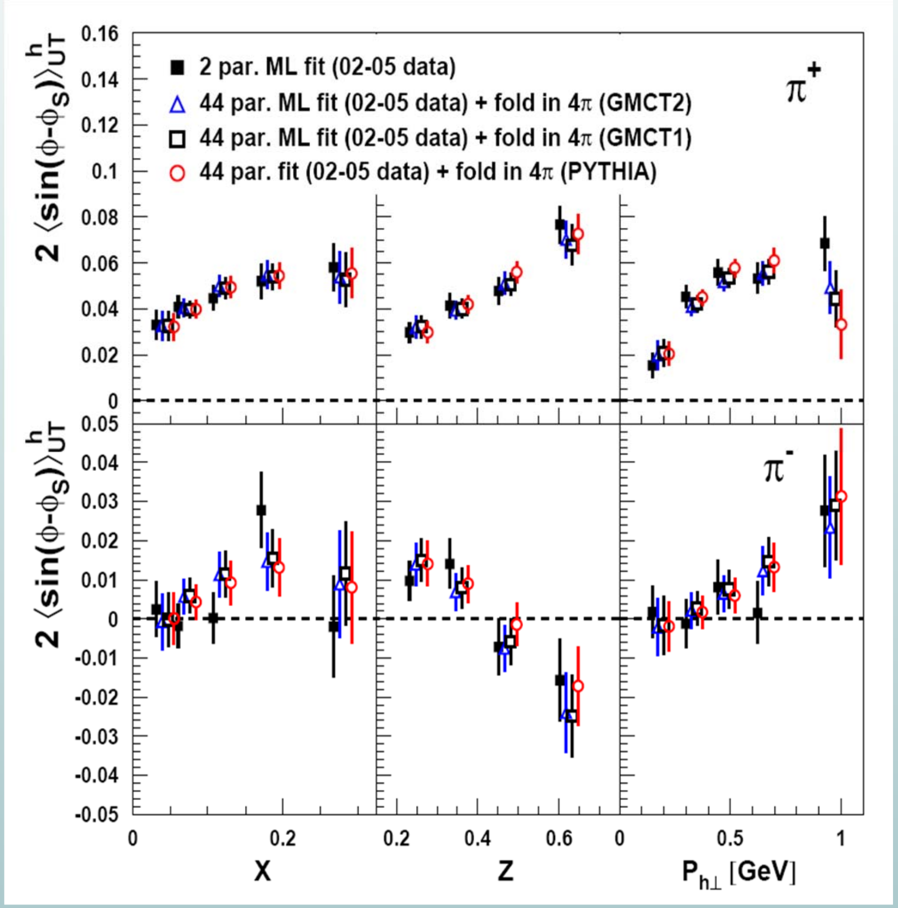


- Large acceptance effects in MC data
- The method is tested successfully using MC data!

Evaluation of acceptance effects ("old" approach)

The choice of MC for the σ_{UU} (folding)

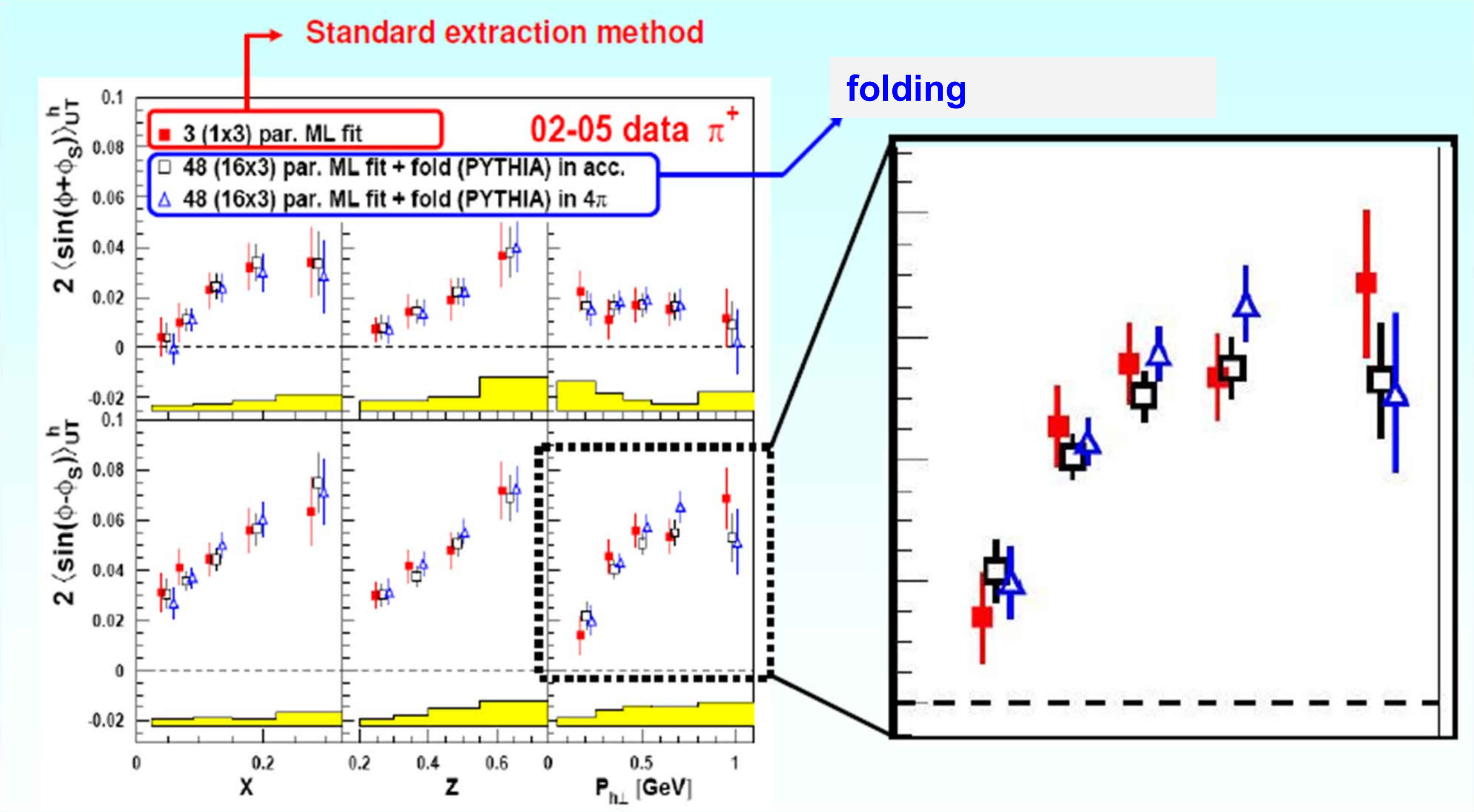


"2nd order" systematic uncertainties:

- arising from choice of the model for the unpolarized cross-section
- arising from the choice of the truncation of the Taylor expansion

Evaluation of acceptance effects ("old" approach)

applying the method on real data


Standard extraction method

folding

- acceptance effects smaller in data than in MC

Smearing and radiative effects ("old" approach)

Procedure:

1. Run over a "fully reconstructed" PYTHIA production to generate an output list containing, for each event:

- Reconstructed variables (includes radiative + smearing + acceptance)
- Generated variables (includes radiative + acceptance)
- True variables (including ϕ and ϕs) (includes acceptance effects only)

2. Evaluate the model at the True kinematics and use to "polarize" PYTHIA

3. Extract the azimuthal moments (Collins, Sivers,…) by fitting "polarized"PYTHIA events 3 times:

- Reconstructed kinematics → (**"PYTHIA REC"**)
- Generated kinematics → (**"PYTHIA GEN"**)
- True kinematics → (**"PYTHIA TRUE"**)

Smearing and radiative effects ("old" approach)

1. Azimuthal moments at Reconstructed kinematics **("PYTHIA REC")**
2. Azimuthal moments at Generated kinematics **("PYTHIA GEN")**
3. Azimuthal moments at True kinematics **("PYTHIA TRUE")**

Combining all the information above one can evaluate detector smearing and radiative effects:

"PYTHIA REC" – "PYTHIA GEN" ⇒ **smearing**

"PYTHIA GEN" – "PYTHIA TRUE" ⇒ **radiative**

"PYTHIA REC" – "PYTHIA TRUE" ⇒ **radiative + smearing**

Acceptance, smearing and radiative effects

("old" approach)

1. Azimuthal moments at Reconstructed kinematics **("PYTHIA REC")**
2. Azimuthal moments at Generated kinematics **("PYTHIA GEN")**
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"PYTHIA REC" – "PYTHIA GEN" ⟹ **smearing**

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"PYTHIA REC" – "PYTHIA TRUE" ⟹ **radiative + smearing**

…and combining with

4. Model folded with Born Xsection in acceptance **("MODEL_ACC")**
5. Model folded with Born Xsection in 4π **("MODEL_4π")**

"MODEL_4π" – MODEL_ACC ⟹ **acceptance**

"MODEL_4π" – "PYTHIA REC" ⟹ **rad. + smear. + acc.**

Acceptance, smearing and radiative effects

("old" approach)

Main advantages

1. The kinematic dependence of e.g. Collins and Sivers is extracted from the data → no need to rely on a model (e.g. gmc_trans model) for Collins, Sivers,…
2. The model is extracted in a fully differential fit of data → free from accept. effects
3. Acceptance, smearing and radiative effects can be evaluated individually or at once

Main limits

1. truncation of Taylor expansion is reasonable but arbitrary → can test different truncations and estimate a systematic error
2. Folding: need anyway a model for the spin-independent cross section, e.g. PYTHIA model → can use different models to test stability and estimate a systematic error
3. Folding: kinematic dependence of azimuthal moments outside the acceptance is assumed to be the same as inside
4. differences between amplitudes in 4π and in acceptance are biased by the different mean kinematics, resulting in an overestimate of the real acceptance effects

Acceptance, smearing and radiative effects
("old" approach)

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can be avoided → **"new" approach (all-in-one)**

All-in-one systematics: the “new” approach

Acceptance
Smearing
Radiative effects
Hadron (mis)identification
Detector misalignement

estimated simultaneously (“all-in-one” approach) in a MC simulation using a model constrained from data

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Acceptance
Smearing
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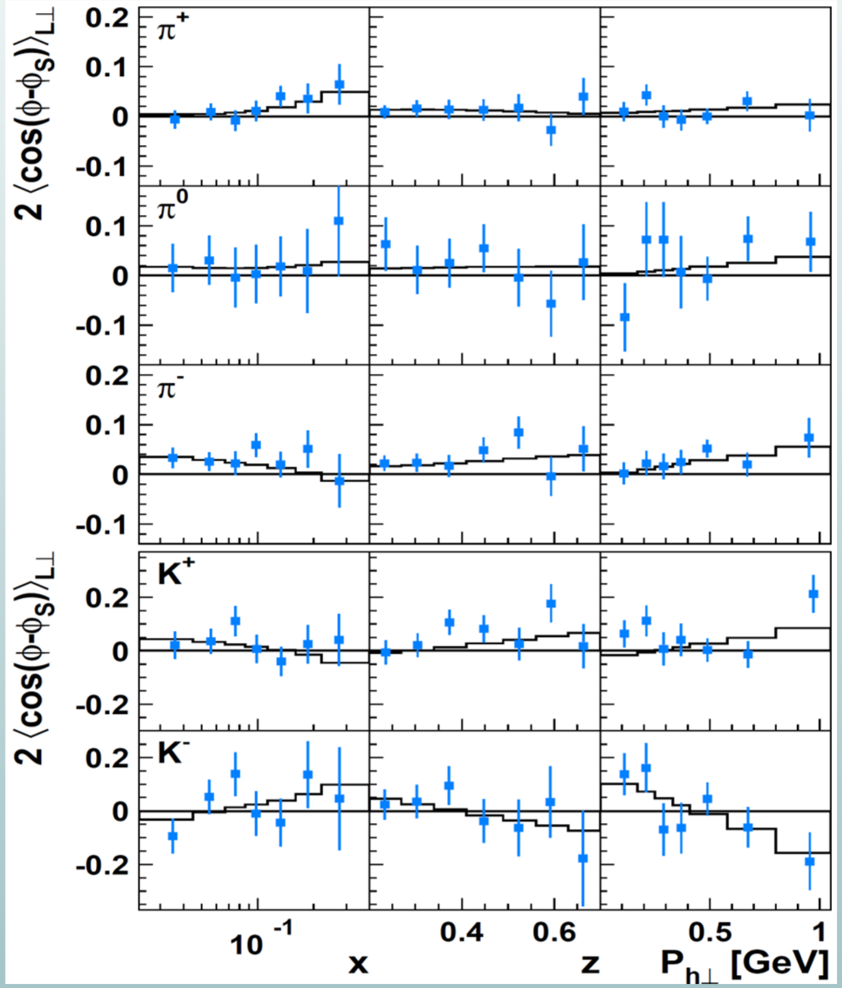
Detector misalignement

} estimated simultaneously (“all-in-one” approach) in a MC simulation using a model constrained from data

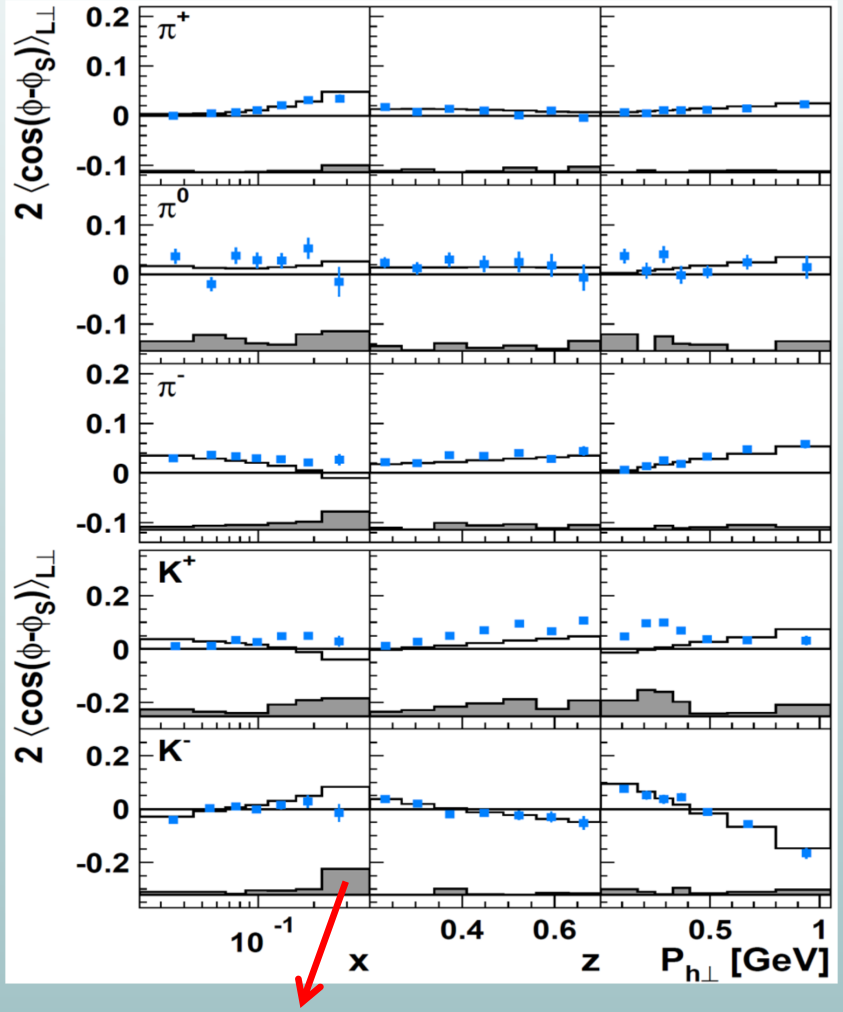
1. Generate a MC data set using a PYTHIA simulation including:
   - full track reconstruction (HMC)
   - radiative effects (RADGEN)
   - simulation of the RICH PID
   - estimate of beam and spectrometer misalignment
2. Extract a **model** from a fully-differential ML fit of real data (full SIDIS events sample) (the model is virtually free from acceptance effects)
3. **‘’polarize’’ PYTHIA events**: assigning target spin states to reconstructed PYTHIA (unpol.) events according to the model (calculated at the *True* kinematics)
4. Extract the azimuthal amplitudes from the (polarized) PYTHIA reconstructed events using the “standard” ML fit (same extraction method used for real data) → “**PYTHIA REC**”
5. calculate the model at the mean kinematics of the reconstr. PYTHIA events → “**MODEL REC**”
6. **all-in-one syst. uncertainty** = “PYTHIA REC ” – “MODEL_REC”
7. Reduce statistical effects by smoothing the resulting systematic “bands” with a slope

All-in-one systematics: the “new”approach

Model vs. data



Model vs. “modeled” PYTHIA



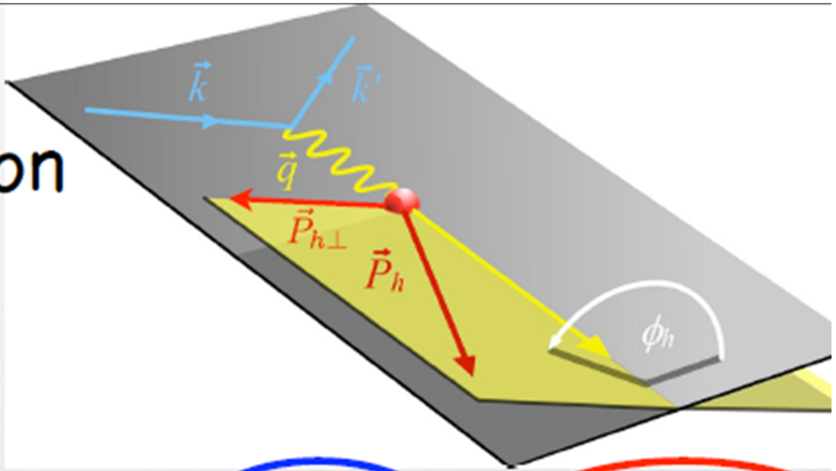
Syst. errors before smoothing

Conclusions

- Several MC generators for SIDIS analysis are available at HERMES
- **GMC_trans** is the only one with polarized SIDIS X-section (including models for several TMDs) …but radiative effects non yet implemented
- events modulated by polarized SIDIS X-section can also be extracted from **PYTHIA** using a fully-differential parametrization constrained by data
- Both GMC_trans and PYTHIA have been heavily used at HERMES in 1h SIDIS analyses for the evaluation of the main systematic uncertainties
- Another important application of MC at HERMES is the multi-dimensional unfolding used to correct for acceptance, smearing and radiative effects.
  (see application to $\cos(n\phi)_{UU}$ analysis in back-up slides)

Back-up

The LO, subleading twist unpolarized SIDIS cross section



$$\frac{d\sigma}{dx\;dy\;dz\;d\phi\;dP_{h\perp}^2} =$$

$$2\pi\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big[F_{UU,T}+\epsilon F_{UU,L}+\sqrt{2\epsilon(1+\epsilon)}\cos\phi F_{UU}^{\cos\phi}+\epsilon\cos(2\phi)F_{UU}^{\cos(2\phi)}$$

subleading twist

leading twist

Cahn effect

$$F_{UU,T} = \mathcal{C}[f_1 D_1]$$

$$F_{UU,L} = 0$$

$$F_{UU}^{\cos\phi} = \frac{2M}{Q}\mathcal{C}\Big[-\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_T}{M_h}\left(x\tilde{h}H_1^{\perp}+\frac{p_T^2}{M^2}h_1^{\perp}H_1^{\perp}+\frac{M_h}{M}f_1\frac{\tilde{D}^{\perp}}{z}\right)-\frac{\hat{\mathbf{h}}\cdot\mathbf{p}_T}{M}\left(x\tilde{f}^{\perp}D_1+f_1D_1+\frac{M_h}{M}h_1^{\perp}\frac{\tilde{H}}{z}\right)$$

$$F_{UU}^{\cos(2\phi)} = \mathcal{C}\left[-\frac{2(\hat{\mathbf{h}}\cdot\mathbf{k}_T)(\hat{\mathbf{h}}\cdot\mathbf{p}_T)-\mathbf{k}_T\cdot\mathbf{p}_T}{MM_h}h_1^{\perp}H_1^{\perp}\right]$$

$h_1^{\perp}$ =  –  Boer-Mulders DF

Collins FF $H_1^{\perp}$ =  – 

The LO, subleading twist unpolarized SIDIS cross section

$$\frac{d\sigma}{dx\ dy\ dz\ d\phi\ dP_{h\perp}^2} =$$

$$2\pi \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \Big[F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} \cos\phi F_{UU}^{\cos\phi} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)}$$

$$= A + B\cos(\phi) + C\cos(2\phi)$$

$$\int d^5\sigma = \int dx\ dy\ dz\ dP_{h\perp}^2\ d\phi \frac{d^5\sigma}{dx\ dy\ dz\ dP_{h\perp}^2\ d\phi}$$

$$\left.\begin{aligned} \langle\cos(\phi)\rangle &= \frac{\int \cos(\phi) d^5\sigma}{\int d^5\sigma} = \frac{1}{2}\frac{B}{A} \\ \langle\cos(2\phi)\rangle &= \frac{\int \cos(2\phi) d^5\sigma}{\int d^5\sigma} = \frac{1}{2}\frac{C}{A} \end{aligned}\right\}$$

Definition of azimuthal moments

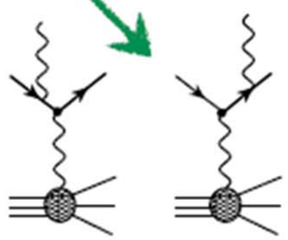
$$w = (x, y, z, P_{h\perp})$$

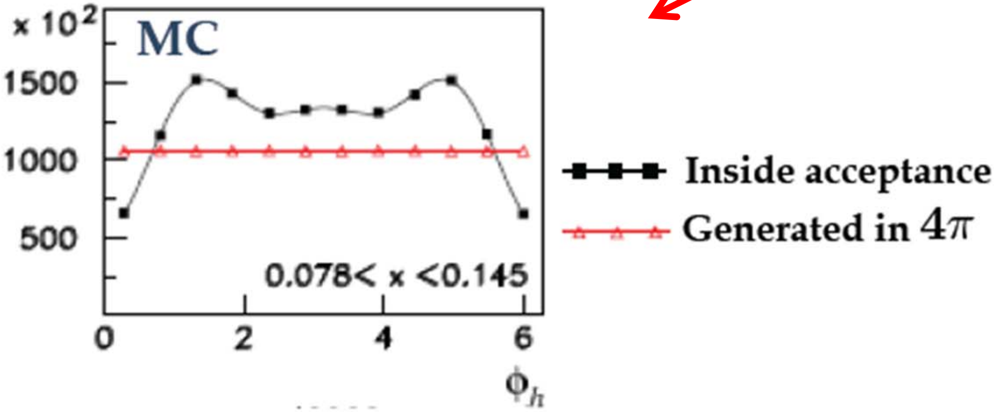
$$n = \int L\sigma_w^0[1 + 2\langle\cos\phi_h\rangle_w + 2\langle\cos 2\phi_h\rangle_w]\epsilon_{w,\phi_h}^{acc}\epsilon_{w,\phi_h}^{rad}dw$$

$$w = (x, y, z, P_{h\perp})$$

$$n = \int L\sigma_w^0[1 + 2\langle\cos\phi_h\rangle_w + 2\langle\cos 2\phi_h\rangle_w]\,\epsilon_{w,\phi_h}^{acc}\,\epsilon_{w,\phi_h}^{rad}\,dw$$

MC simulation of spectrometers to correct for acceptance/QED radiation





$$n_A = \int \sigma_w^0|_{mc}\epsilon_{w,\phi_h}^{acc}\,\epsilon_{w,\phi_h}^{rad}\,dw$$

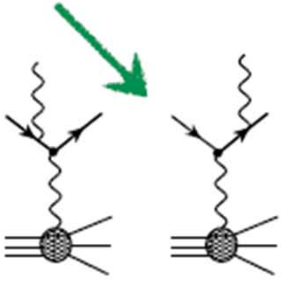
$$n_{4\pi} = \int \sigma_w^0|_{mc}dw$$

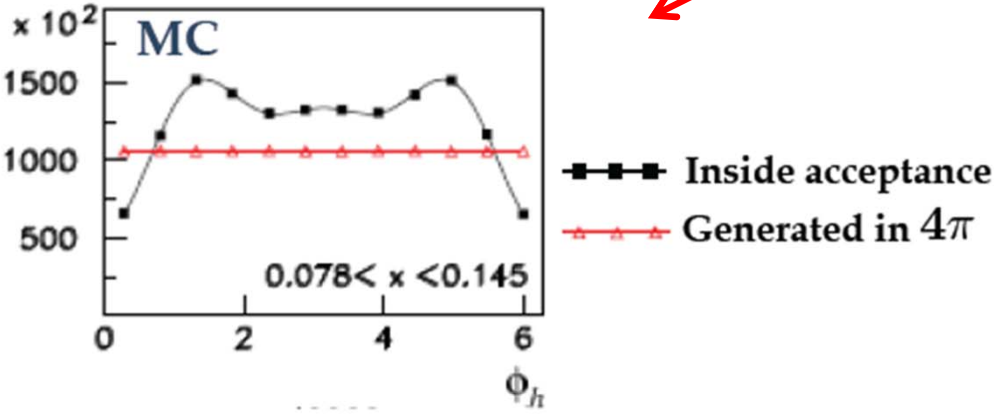
Acceptance Correction

$$w = (x, y, z, P_{h\perp})$$

$$n = \int L\sigma^0_w[1 + 2\langle\cos\phi_h\rangle_w + 2\langle\cos 2\phi_h\rangle_w]\epsilon^{acc}_{w,\phi_h}\epsilon^{rad}_{w,\phi_h}dw$$

MC simulation of spectrometers to correct for acceptance/QED radiation





$$\frac{n_A = \int \cancel{\sigma^0_w|_{mc}}\boxed{\epsilon^{acc}_{w,\phi_h}\epsilon^{rad}_{w,\phi_h}}dw}{n_{4\pi} = \int \cancel{\sigma^0_w|_{mc}}dw}$$

Not allowed!

only if fully differential ratio (4D binning)
and only in the limit of infinitely small bins

Model independent correction

Multi-dimensional unfolding

Acceptance Correction

Francesca Giordano

Standard Unfolding

$$\sigma_{rec}^{data}(i)=\sum_{j=0}^{N} S(i,j)\sigma_{born}^{data}(j)$$

Standard Unfolding

i = index of
"measured" bins
1-6000

$$\sigma^{data}_{rec}(i) = \sum_{j=0}^{N} S(i,j)\,\sigma^{data}_{born}(j)$$

$$\sigma^{data}_{rec}(i) = N^{data}_{rec}(i)\,\frac{\sigma_{DIS}}{N^{data}_{DIS}}$$

→ from tracked Pythia

What we actually measure

Standard Unfolding

i = index of
"measured" bins
1-6000

$$\sigma^{data}_{rec}(i) = \sum_{j=0}^{N} S(i,j)\,\sigma^{data}_{born}(j)$$

What we'd like to know!

j = index
of "born" bins
0-6000

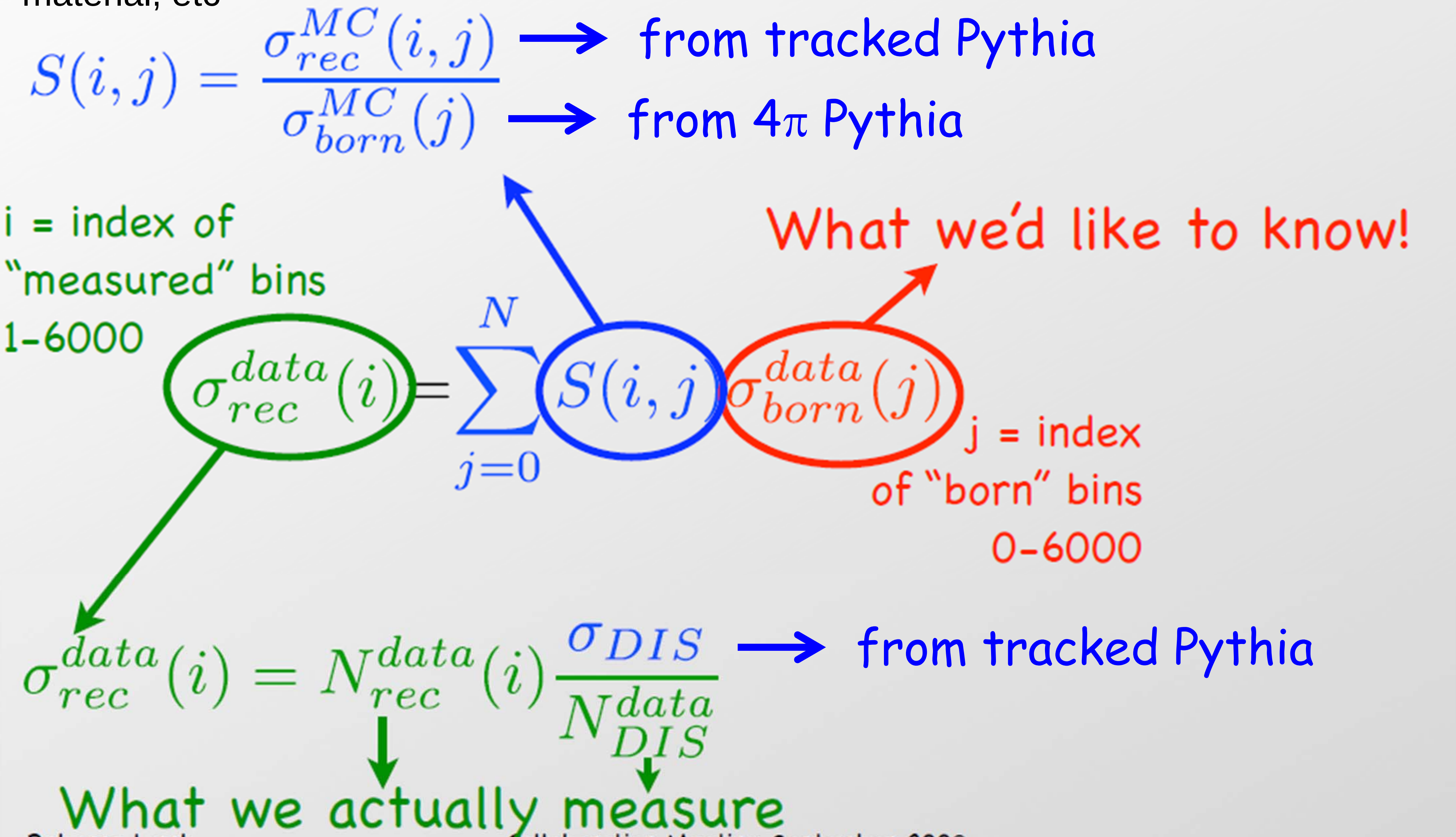
$$\sigma^{data}_{rec}(i) = N^{data}_{rec}(i)\,\frac{\sigma_{DIS}}{N^{data}_{DIS}} \longrightarrow$$

from tracked Pythia

What we actually measure

Standard Unfolding

Smearing matrix: describes migration of events from True to reconstr. kinematics due to: mis-reconstructed track kinematics, radiative effects,multiple scattering in detector material, etc



$$S(i,j) = \frac{\sigma_{rec}^{MC}(i,j)}{\sigma_{born}^{MC}(j)}$$

$\sigma_{rec}^{MC}(i,j)$ → from tracked Pythia

$\sigma_{born}^{MC}(j)$ → from 4π Pythia

i = index of "measured" bins 1-6000

$$\sigma_{rec}^{data}(i) = \sum_{j=0}^{N} S(i,j)\,\sigma_{born}^{data}(j)$$

What we'd like to know!

j = index of "born" bins 0-6000

$$\sigma_{rec}^{data}(i) = N_{rec}^{data}(i)\,\frac{\sigma_{DIS}}{N_{DIS}^{data}}$$

σ_{DIS} → from tracked Pythia

What we actually measure

Standard Unfolding

$$\sigma^{data}_{rec}(i) = \sum_{j=0}^{N} S(i,j)\,\sigma^{data}_{born}(j)$$

$$S(i,j) = \frac{\sigma^{MC}_{rec}(i,j)}{\sigma^{MC}_{born}(j)}$$

$$= \sum_{\boxed{j=1}}^{N} S'(i,j)\,\sigma^{data}_{born}(j) + S(i,0)\,\sigma^{MC}_{born}(0)$$

"zerobin" of events that smear into acceptance

$$= \sum_{j=1}^{N} S'(i,j)\,\sigma^{data}_{born}(j) + \sigma^{MC}_{rec}(i,0)$$

$$\boxed{\sigma^{data}_{born}(j) = \sum_{j=1}^{N} S'(i,j)^{-1}\left[\sigma^{data}_{rec}(i) - \sigma^{MC}_{rec}(i,0)\right]}$$

Standard Fitting

$$A + B\cos(\phi) + C\cos(2\phi)$$

$$\chi^2 = (\sigma^{data}_{born} - X\beta)^T {C^{data}_{born}}^{-1} (\sigma^{data}_{born} - X\beta)$$

$$X = \left(\begin{array}{cccccccccccc}
1 & \cos(\phi_1) & \cos(2\phi_1) & 0 & 0 & 0 & \ldots & \top & \ldots & 0 & 0 & 0 \\
1 & \cos(\phi_2) & \cos(2\phi_2) & 0 & 0 & 0 & \ldots & | & \ldots & 0 & 0 & 0 \\
 & \ldots & & & & & & & & & & \\
1 & \cos(\phi_{12}) & \cos(2\phi_{12}) & 0 & 0 & 0 & \ldots & | & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & \cos(\phi_1) & \cos(2\phi_1) & \ldots & | & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & \cos(\phi_2) & \cos(2\phi_2) & \ldots & | & \ldots & 0 & 0 & 0 \\
 & & & & \ldots & & & & & & & \\
0 & 0 & 0 & 1 & \cos(\phi_{12}) & \cos(2\phi_{12}) & \ldots & | & \ldots & 0 & 0 & 0 \\
 & & & & \ldots & & & | & & & & \\
 & & & & & & & 500\text{ kin bins} & & & & \\
\vdash -- & 500 & \text{kin bins} & \text{x} & 3\text{param} & - & - & * & - & - & - & -\dashv \\
 & & & & & & & 12\phi\text{ bins} & & & & \\
 & & & & \ldots & & & | & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots & \bot & \ldots & 1 & \cos(\phi_{12}) & \cos(2\phi_{12})
\end{array}\right)$$

Standard Fitting

$$A + B\cos(\phi) + C\cos(2\phi)$$

$$\chi^2 = (\sigma_{born}^{data} - X\beta)^T {C_{born}^{data}}^{-1} (\sigma_{born}^{data} - X\beta)$$

$$\beta = \begin{pmatrix} A_1 \\ B_1 \\ C_1 \\ A_2 \\ B_2 \\ C_2 \\ . \\ . \\ . \\ A_{500} \\ B_{500} \\ C_{500} \end{pmatrix}$$

Standard Fitting

$$A + B\cos(\phi) + C\cos(2\phi)$$

$$\chi^2 = (\sigma_{born}^{data} - X\beta)^T {C_{born}^{data}}^{-1} (\sigma_{born}^{data} - X\beta)$$

$${C_{born}^{data}}^{-1} = \left(\begin{array}{ccccccc} \frac{1}{\delta{\sigma_{rec}^{data}}^2(1)} & 0 & 0 & . \; . \; . & 0 & 0 & 0 \\ 0 & \frac{1}{\delta{\sigma_{rec}^{data}}^2(2)} & 0 & . \; . \; . & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\delta{\sigma_{rec}^{data}}^2(3)} & . \; . \; . & 0 & 0 & 0 \\ & & & . & & & \\ & & & . & & & \\ & & & . & & & \\ 0 & 0 & 0 & . \; . \; . & 0 & 0 & \frac{1}{\delta{\sigma_{rec}^{data}}^2(6000)} \end{array} \right) S\imath^{-}$$

Standard Fitting

$$A + B\cos(\phi) + C\cos(2\phi)$$

$$\chi^2 = (\sigma_{born}^{data} - X\beta)^T {C_{born}^{data}}^{-1} (\sigma_{born}^{data} - X\beta)$$

$$ {C_{born}^{data}}^{-1} = \begin{pmatrix} \frac{1}{\delta\sigma_{rec}^{data\,2}(1)} & 0 & 0 & . & . & . & 0 & 0 & 0 \\ 0 & \frac{1}{\delta\sigma_{rec}^{data\,2}(2)} & 0 & . & . & . & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\delta\sigma_{rec}^{data\,2}(3)} & . & . & . & 0 & 0 & 0 \\ & & & & . & & & & \\ & & & & . & & & & \\ & & & & . & & & & \\ 0 & 0 & 0 & . & . & . & 0 & 0 & \frac{1}{\delta\sigma_{rec}^{data\,2}(6000)} \end{pmatrix} $$

$S\imath^{-}$

Need to invert a 6000X6000 matrix!

Standard Fitting

$$A + B\cos(\phi) + C\cos(2\phi)$$

$$\chi^2 = (\sigma_{born}^{data} - X\beta)^T {C_{born}^{data}}^{-1} (\sigma_{born}^{data} - X\beta)$$

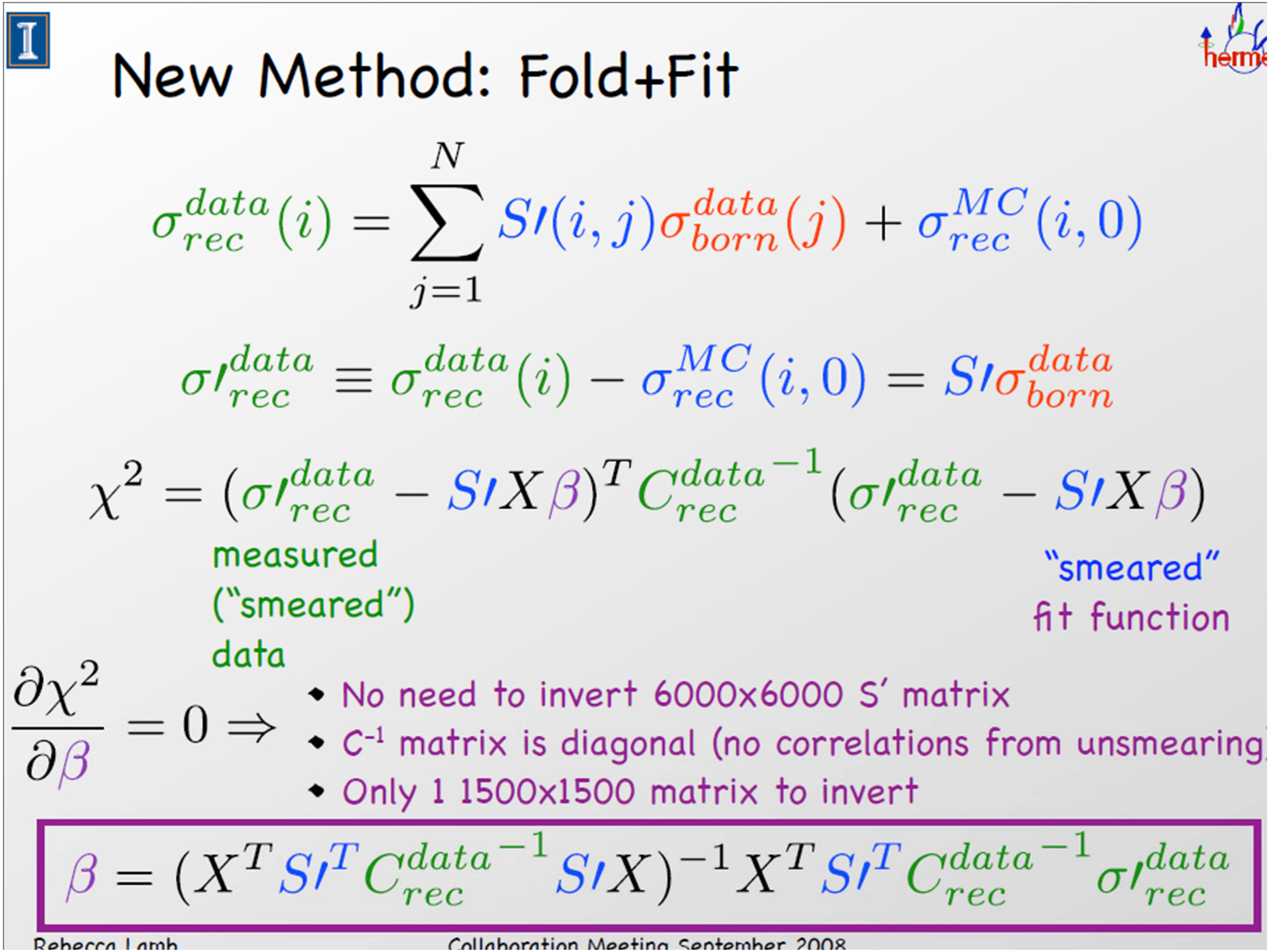
$$\frac{\partial\chi^2}{\partial\beta} = 0 \Rightarrow$$

$$\beta = (X^T {C_{born}^{data}}^{-1} X)^{-1} X^T {C_{born}^{data}}^{-1} \sigma_{born}^{data}$$

New Method: Fold+Fit

$$\sigma_{rec}^{data}(i) = \sum_{j=1}^{N} S\prime(i,j)\sigma_{born}^{data}(j) + \sigma_{rec}^{MC}(i,0)$$

$$\sigma\prime_{rec}^{data} \equiv \sigma_{rec}^{data}(i) - \sigma_{rec}^{MC}(i,0) = S\prime\sigma_{born}^{data}$$



New Method: Fold+Fit

$$\sigma_{rec}^{data}(i) = \sum_{j=1}^{N} S'(i,j)\sigma_{born}^{data}(j) + \sigma_{rec}^{MC}(i,0)$$

$$\sigma'^{data}_{rec} \equiv \sigma_{rec}^{data}(i) - \sigma_{rec}^{MC}(i,0) = S'\sigma_{born}^{data}$$

$$\chi^2 = (\sigma'^{data}_{rec} - S'X\beta)^T {C_{rec}^{data}}^{-1} (\sigma'^{data}_{rec} - S'X\beta)$$

measured
("smeared")
data

"smeared"
fit function

$$\frac{\partial\chi^2}{\partial\beta} = 0 \Rightarrow$$

- No need to invert 6000x6000 S′ matrix
- C^{-1} matrix is diagonal (no correlations from unsmearing
- Only 1 1500x1500 matrix to invert

$$\boxed{\beta = (X^T S'^T {C_{rec}^{data}}^{-1} S'X)^{-1} X^T S'^T {C_{rec}^{data}}^{-1} \sigma'^{data}_{rec}}$$

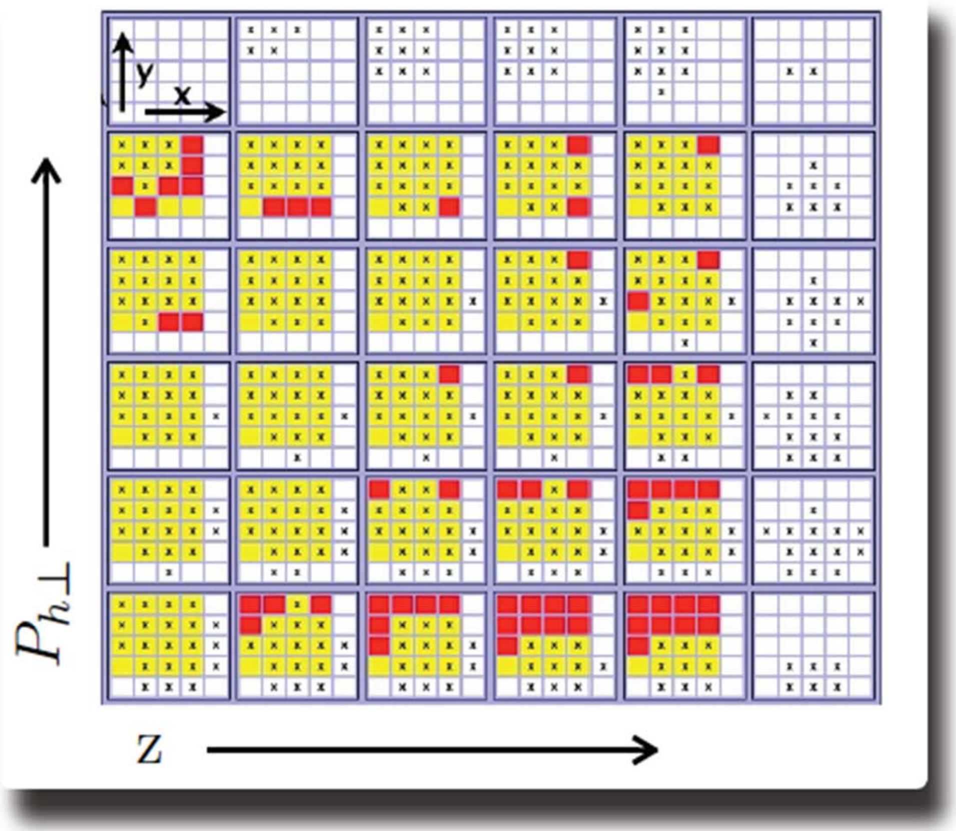
β ⇨ Moments ⇨ Projections

- Moments are calculated by ratios of the β parameters
  - $2\cos(\phi) = B/A$
  - $2\cos(2\phi) = C/A$

- 1-dimensional projection are calculated as the weighted average of the moments
  - weights are the born cross section (from Pythia)
  - highest z-bin not used in the projection vs x, y, p_T
  - Bad kinematic bins have a weight of 0
    - (yes this could cause a bias - see born based bad bin study!)

$$w = (x, y, z, P_{h\perp})$$

$$n = \int L\sigma_w^0[1 + 2\langle\cos\phi_h\rangle_w + 2\langle\cos 2\phi_h\rangle_w]\epsilon_{w,\phi_h}^{acc}\epsilon_{w,\phi_h}^{rad}dw$$



4-dimensional (w) unfolding

Binning
900 kinematic bins x 12ϕ_h -bins

Variable	Bin limits							#
x	0.023	0.042	0.078	0.145	0.27	0.6		5
y	0.2	0.3	0.45	0.6	0.7	0.85		5
z	0.2	0.3	0.4	0.5	0.6	0.75	1	6
$P_{h\perp}$	0.05	0.2	0.35	0.5	0.7	1	1.3	6

$$A(1 + B\cos\phi_h + C\cos 2\phi_h)$$

Projection Versus The Single Variable

☑ Model dependence of unfolding procedure:

$$n_{born}= S^{-1}[n - B_0]$$

Model independent only if fully differential ratio (4D binning)

only in the limit of infinitely small bins

are the bins small enough?

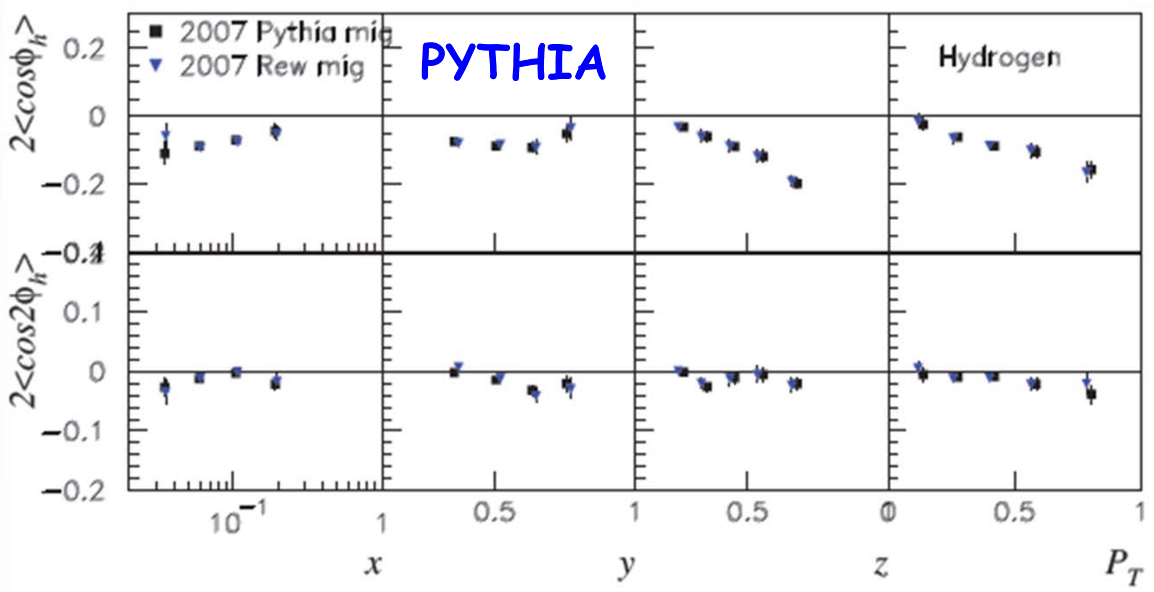
Different cross section models used for corrections

1. $\sigma^0_w|_{mc}$ **Pythia**

2. $\sigma^0_w|_{mc} M(\cos\phi_h, \cos 2\phi_h)$ **Pythia + azimuthal modulations**

$$
\begin{aligned}
P =\Big[\big(&A_1 + A_2x + A_3y + A_4z + A_5P_{h\perp} + A_6x^2 + A_7y^2 + A_8z^2 + A_9P_{h\perp}^2 + \\
&A_{10}x\,y + A_{11}x\,z + A_{12}x\,P_{h\perp} + A_{13}y\,z + A_{14}y\,P_{h\perp} + A_{15}z\,P_{h\perp} + \\
&+ A_{16}x^3 + A_{17}y^3 + A_{18}z^3 + A_{19}P_{h\perp}^3\big)\Big]cos\phi_h + \\
&\Big[\big(A_{20} + A_{21}x + A_{22}y + A_{23}z + A_{24}P_{h\perp} + A_{25}x^2 + A_{26}y^2 + A_{27}z^2 + A_{28}P_{h\perp}^2 + \\
&A_{29}x\,y + A_{30}x\,z + A_{31}x\,P_{h\perp} + A_{32}y\,z + A_{33}y\,P_{h\perp} + A_{34}z\,P_{h\perp} + \\
&+ A_{35}x^3 + A_{36}y^3 + A_{37}z^3 + A_{38}P_{h\perp}^3\big)\Big]cos2\phi_h
\end{aligned}
$$

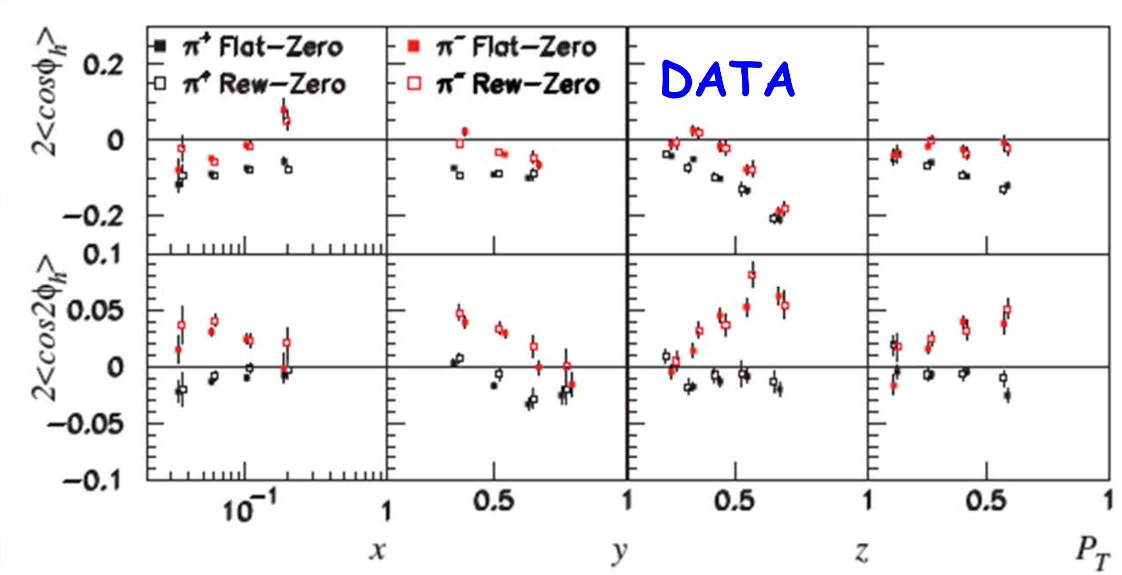
Used to generate a MC production whose cross section resemble the cosine modulation extracted from data



■ Subsample corrected with flat MC
▼ Subsample corrected with MC that resemble data modulations

$$
\begin{aligned}
P = &\Big[\big(A_1 + A_2 x + A_3 y + A_4 z + A_5 P_{h\perp} + A_6 x^2 + A_7 y^2 + A_8 z^2 + A_9 P_{h\perp}^2 + \\
&A_{10} x\, y + A_{11} x\, z + A_{12} x\, P_{h\perp} + A_{13} y\, z + A_{14} y\, P_{h\perp} + A_{15} z\, P_{h\perp} + \\
&+ A_{16} x^3 + A_{17} y^3 + A_{18} z^3 + A_{19} P_{h\perp}^3\big)\Big] cos\phi_h + \\
&\Big[\big(A_{20} + A_{21} x + A_{22} y + A_{23} z + A_{24} P_{h\perp} + A_{25} x^2 + A_{26} y^2 + A_{27} z^2 + A_{28} P_{h\perp}^2 + \\
&A_{29} x\, y + A_{30} x\, z + A_{31} x\, P_{h\perp} + A_{32} y\, z + A_{33} y\, P_{h\perp} + A_{34} z\, P_{h\perp} + \\
&+ A_{35} x^3 + A_{36} y^3 + A_{37} z^3 + A_{38} P_{h\perp}^3\big)\Big] cos2\phi_h
\end{aligned}
$$

Used to generate a MC production whose cross section resemble the cosine modulation extracted from data



π^+

■ Subsample corrected with flat background

□ Subsample corrected with background that resemble data modulations

π^-

🟥 Subsample corrected with flat background

□ Subsample corrected with background that resemble data modulations