

# MC-generators and flavor decomposition in SIDIS

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Introduction

MC studies of Bessel weighting

Extraction of  $k_T$ -dependent TMDs

    Kinematic dependences of  $A_1$

    Study effect of spin-orbit correlations on  $x$ -dependence

    Extraction of  $b_T$ -dependence of TMDs

Summary

# Flavor Decomposition for PDF (LO)

$$\frac{d\sigma_{\lambda,\lambda_e}}{dx dQ^2 dz} = \frac{\overset{\text{lepton helicity}}{\lambda_e} \overset{\text{proton helicity}}{\lambda} y (1 - \frac{1}{2} y) \sum_{a,\bar{a}} e_a^2 x_B g_1^a(x_B) D_1^a(z_h)}{(1 - y + \frac{1}{2} y^2) \sum_{a,\bar{a}} e_a^2 x_B f_1^a(x_B) D_1^a(z_h)}$$

$$d\sigma^{\pi^+} \propto e_u^2 u(x) D_u^{\pi^+}(z) + e_d^2 d(x) D_d^{\pi^+}(z)$$

$$d\sigma^{\pi^-} \propto e_d^2 d(x) D_d^{\pi^-}(z) + e_u^2 u(x) D_u^{\pi^-}(z)$$

$D_u^{\pi^+} = D_d^{\pi^-}$ , favored;  
 $D_u^{\pi^-} = D_d^{\pi^+}$ , unfavored

$$A_p^{\pi^+ - \pi^-}(x) = \frac{4\Delta u_V(x) - \Delta d_V(x)}{4u_V(x) - d_V(x)}$$

$$A_d^{\pi^+ - \pi^-}(x) = \frac{\Delta u_V(x) + \Delta d_V(x)}{u_V(x) + d_V(x)}$$

$$\Delta u = \frac{4}{15} [A_{1,p}^{\pi}(4u + d) - A_{1,n}^{\pi}(d + u/4)]$$

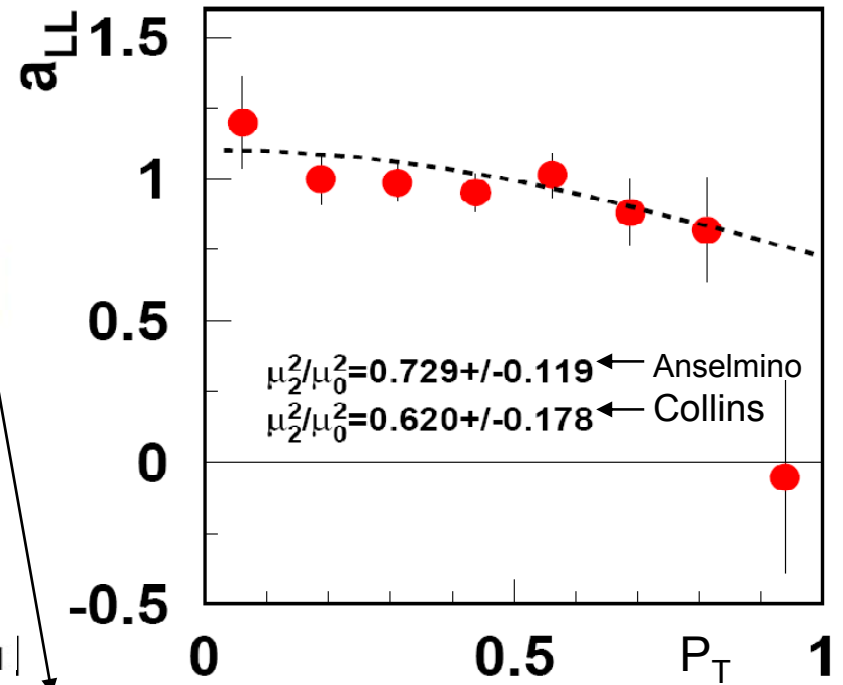
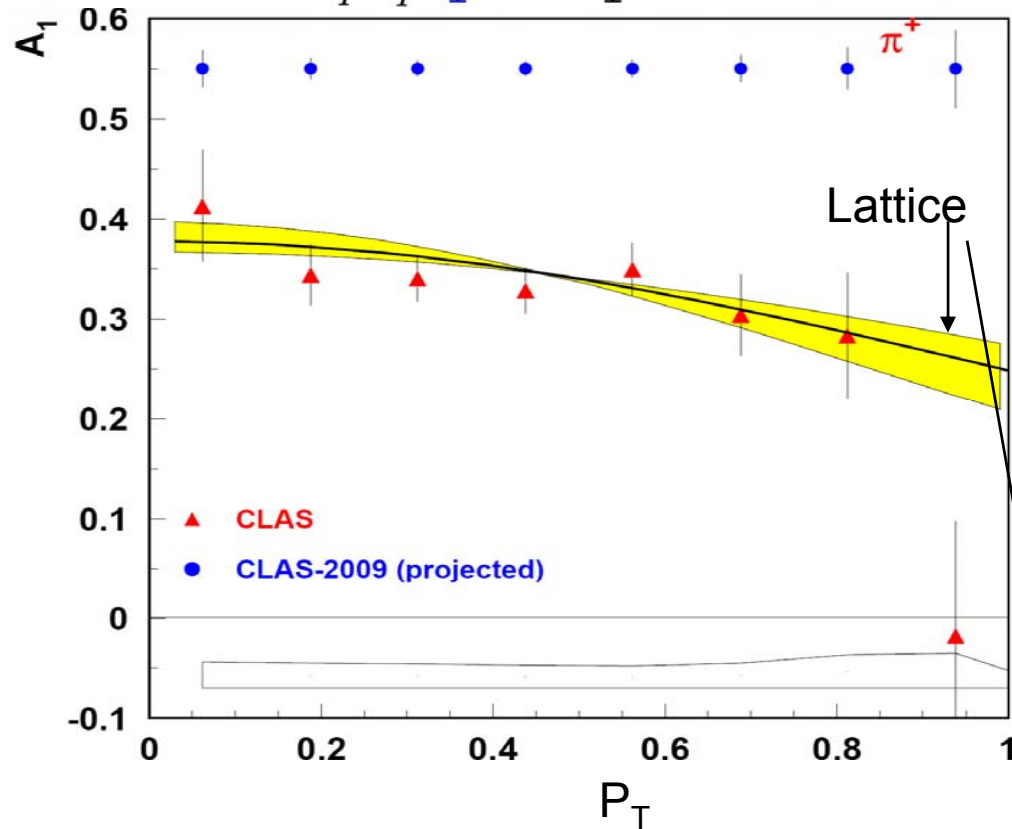
$$\Delta d = \frac{4}{15} [A_{1,n}^{\pi}(4d + u) - A_{1,p}^{\pi}(u + d/4)]$$

# $A_1$ $P_T$ -dependence

arXiv:1003.4549

$$A_1(\pi) \propto \frac{\sum_q e_q^2 g_1^q(x) D_1^{q \rightarrow \pi}(z)}{\sum_q e_q^2 f_1^q(x) D_1^{q \rightarrow \pi}(z)}$$

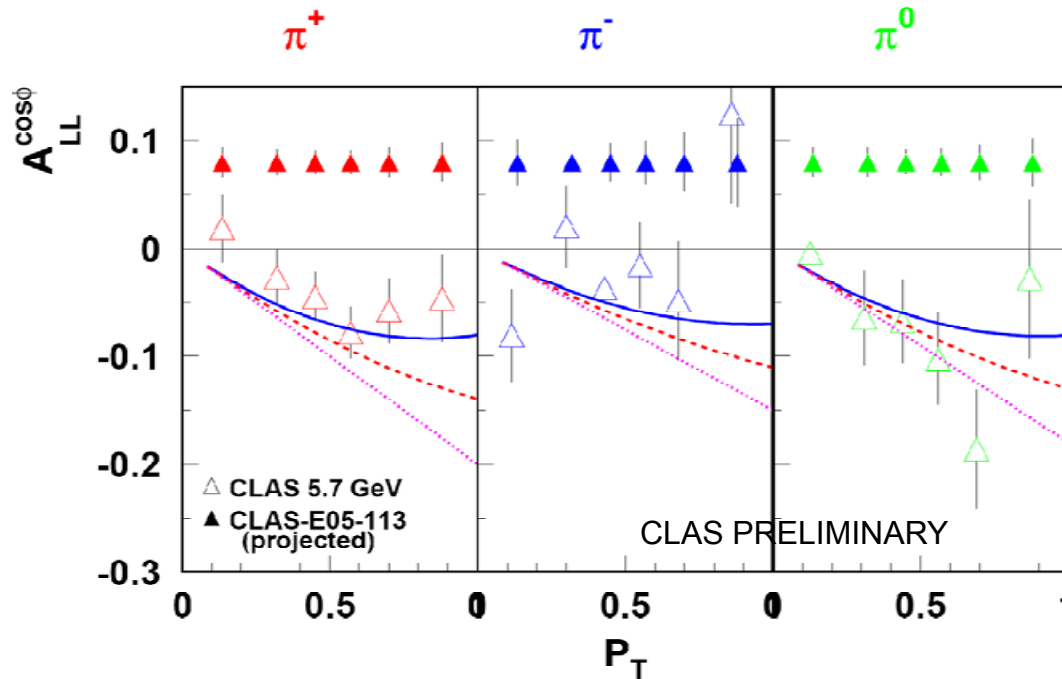
$$A_1(x, z, P_T) = A_1(x, z) \underbrace{\frac{\langle P_T^{2,unp} \rangle}{\langle P_T^{2,pol} \rangle} \exp(-P_T^2 / \langle P_T^{2,pol} \rangle - P_T^2 / \langle P_T^{2,unp} \rangle)}_{a_{LL}}$$



$$\mu_2^2 / \mu_0^2 = 0.692 \pm 0.039 \pm 0.045$$

CLAS data suggests that width of  $g_1$  is less than the width of  $f_1$

# cos $\phi$ moment in $A_{LL}$ - $P_T$ -dependence



hep-ph/0608048

$\mu_0^2 = 0.25 \text{ GeV}^2$

$\mu_D^2 = 0.2 \text{ GeV}^2$

—  $\mu_2^2 = 0.10$   
 - - -  $\mu_2^2 = 0.17$   
 ···  $\mu_2^2 = 0.25$

$\triangle$  CLAS 5.7 GeV  
 $\blacktriangle$  CLAS-E05-113  
 (projected)

CLAS PRELIMINARY

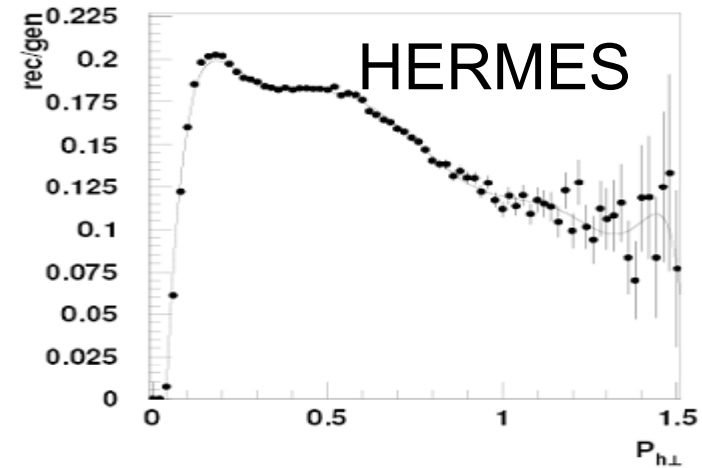
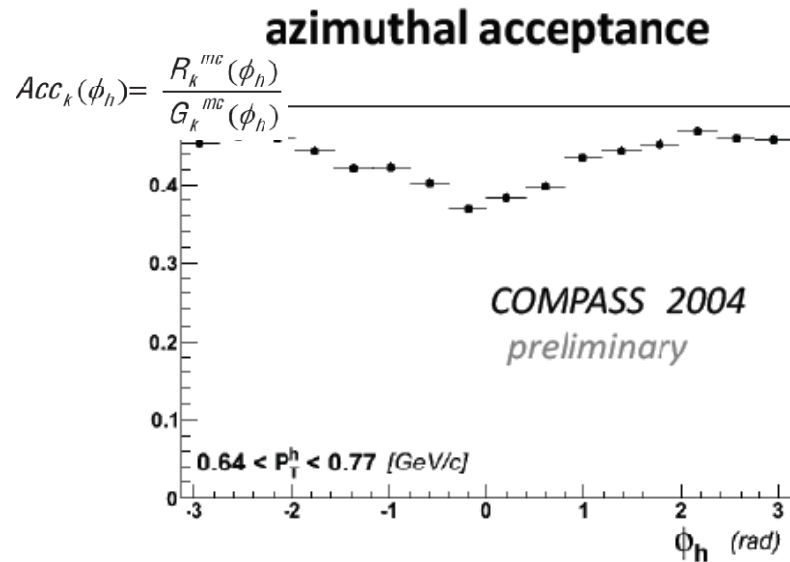
$$\sigma_0 = \frac{1 + (1 - y)^2}{xy^2} \frac{1}{\mu_D^2 + z^2 \mu_0^2} \exp\left(-\frac{P_{hT}^2}{\mu_D^2 + z^2 \mu_0^2}\right) \sum_q e_q^2 f_1^q(x) D_q^h(z)$$

$$A_{LL}^{\text{COS } \phi} \sim e_L H_1^\perp$$

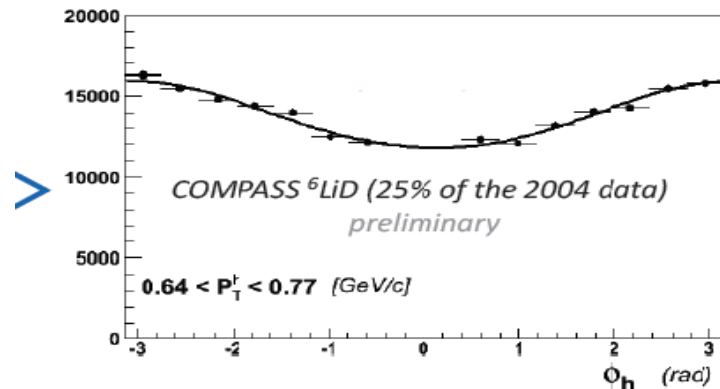
$$A_{LL}^{\text{COS } \phi} \sim g_L^\perp D_1$$

$P_T$ -dependence of  $\cos\phi$  moment of double spin asymmetry is most sensitive to  $k_T$ -distributions of quarks with spin orientations along and opposite to the proton spin.

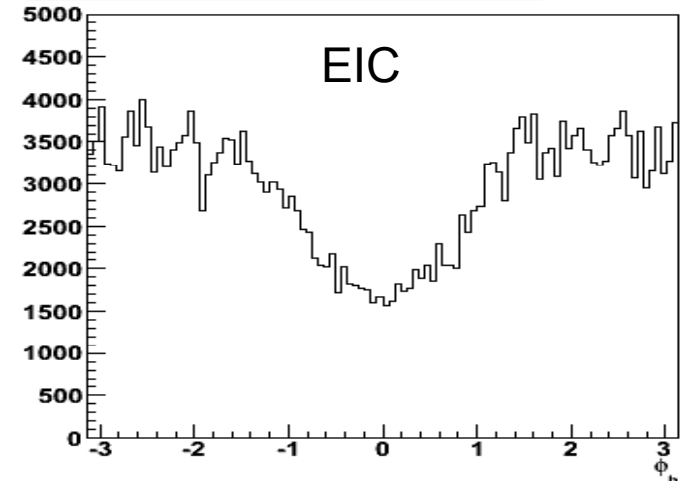
# Acceptances and efficiencies



**measured azimuthal distributions corrected by the acceptance**



$0.5 > z > 0.4 \ \&\& \ 10 > Q^2 > 1 \ \&\& \ 1 > P_T > 0.5 \text{ GeV}$

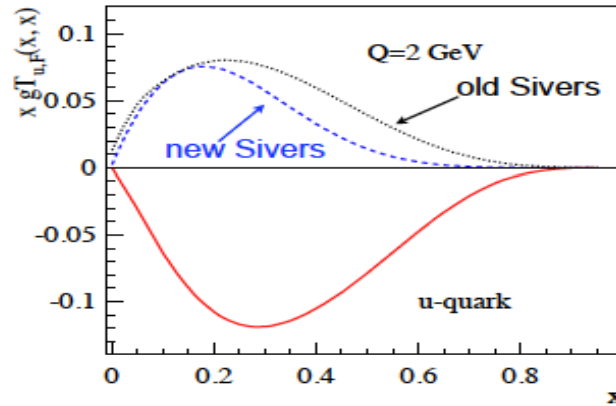
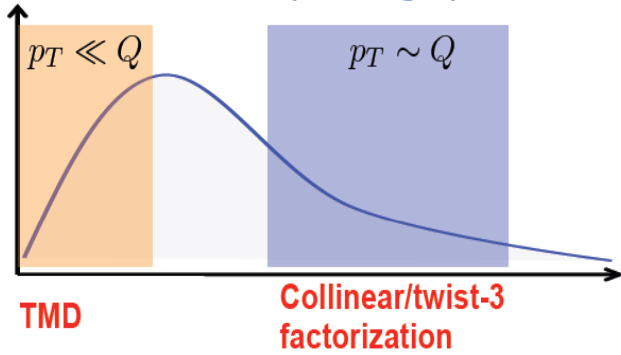


How acceptance in  $\phi$  and  $P_T$  affect the  $A_1$  and  $\Delta_s$  extractions in SIDIS?

# $k_T$ -dependence of TMDs

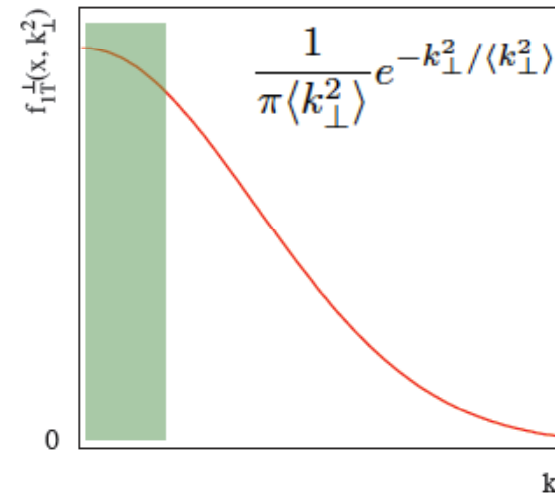
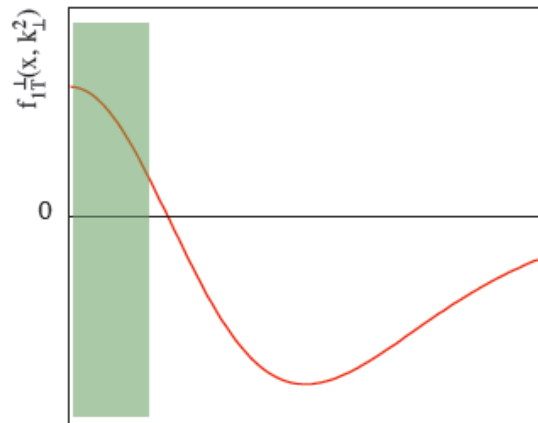
|       |          |          |                |
|-------|----------|----------|----------------|
| N \ q | U        | L        | T              |
| U     | $f_1$    |          | $h_{1T}^+$     |
| L     |          | $g_1$    | $h_{1L}$       |
| T     | $f_{1T}$ | $g_{1T}$ | $h_1$ $h_{1T}$ |

Transition from low  $p_T$  to high  $p_T$



Directly obtained ETQS functions are opposite in sign to those from  $k_T$  moments "sign mismatch"

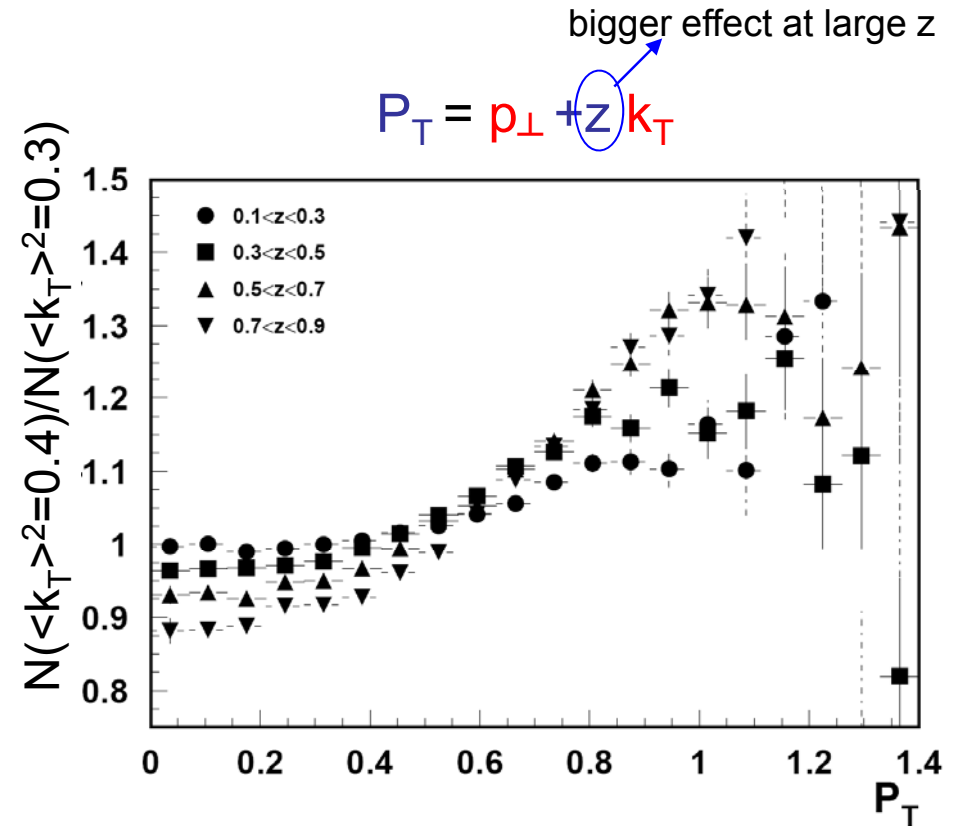
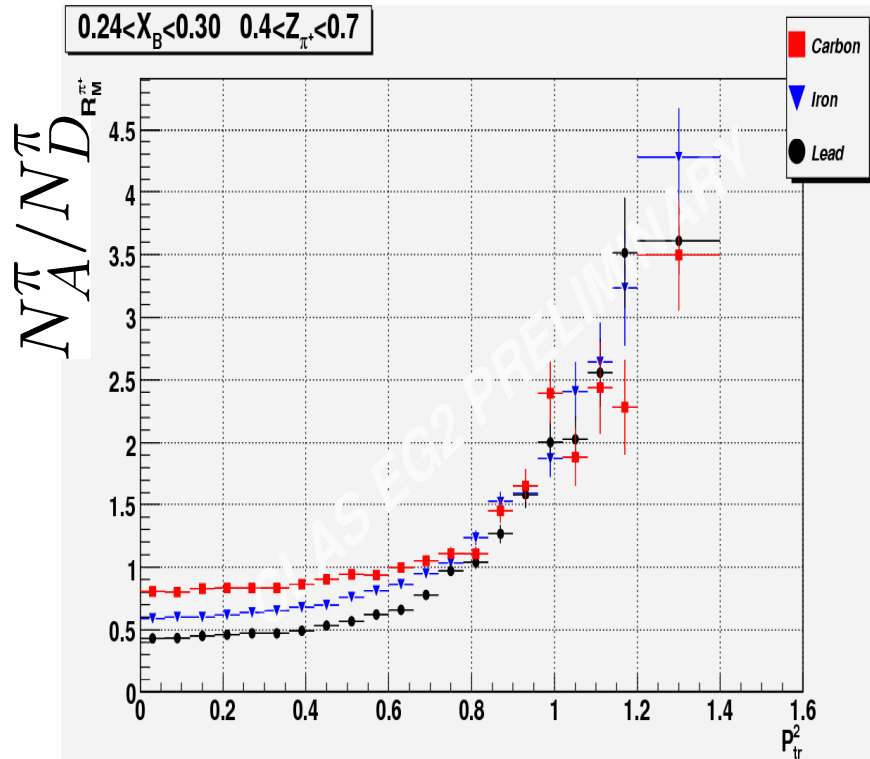
Siverts function extracted assuming  $k_T$  distribution is gaussian



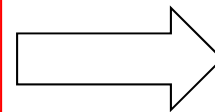
- With orbital angular momentum TMD can't be gaussian
- How to measure  $k_T$ -dependences of TMDs

(Z. Kang et al, 2011)

# $k_T$ -distributions & dilution factor



Higher probability to find a hadron at large  $P_T$  in nuclei



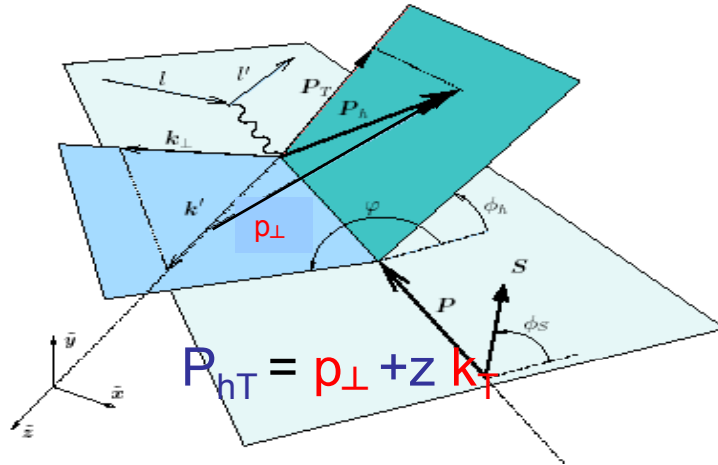
$k_T$ -distributions may be wider in nuclei?

Understanding of nuclear modifications of hadronic distributions is crucial for polarized target data analysis

# SIDIS x-section

$$\text{SIDIS } \ell(l) + N(P, S) \rightarrow \ell(l) + h(P_h) + X$$

$$\begin{aligned} \frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = & \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} \right. \\ & + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} \\ & + S_{\parallel} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\ & + S_{\parallel} \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\ & + |S_{\perp}| \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\ & + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\ & + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \left. \right] \\ & + |S_{\perp}| \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} \right. \\ & \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \left. \right\}, \end{aligned}$$



$$F_{UU,T} = x \sum_a e_a^2 f_1^a(x) D_1^a(z)$$

$$F_{UU,T} = x \sum_a e_a^2 \int d^2 p_T d^2 k_T \delta^{(2)}(p_T - k_T - P_{h\perp}/z) w(p_T, k_T) f^a(x, p_T^2) D^a(z, k_T^2),$$

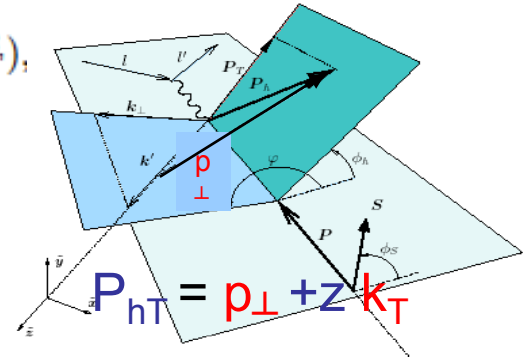


# SIDIS with Bessel weighting

$$F_{UU,T} = x \sum_a e_a^2 \int d^2 p_T d^2 k_T \delta^{(2)}(\mathbf{p}_T - \mathbf{k}_T - \mathbf{P}_{h\perp}/z) w(\mathbf{p}_T, \mathbf{k}_T) f^a(x, p_T^2) D^a(z, k_T^2),$$

$$\delta^{(2)}(z\mathbf{p}_T + \mathbf{K}_T - \mathbf{P}_{h\perp}) = \int \frac{d^2 \mathbf{b}_T}{(2\pi)^2} e^{i\mathbf{b}_T(z\mathbf{p}_T + \mathbf{K}_T - \mathbf{P}_{h\perp})}$$

$$F_{UU,T} = x_B \sum_a e_a^2 \int \frac{d|\mathbf{b}_T|}{(2\pi)} |\mathbf{b}_T| J_0(|\mathbf{b}_T| |\mathbf{P}_{h\perp}|) \tilde{f}_1(x, z^2 \mathbf{b}_T^2) \tilde{D}_1(z, \mathbf{b}_T^2)$$

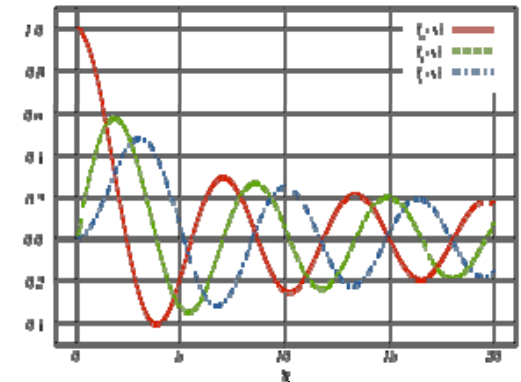


$$\int_0^\infty d|\mathbf{P}_{h\perp}| |\mathbf{P}_{h\perp}| J_n(|\mathbf{P}_{h\perp}| |\mathbf{b}_T|) J_n(|\mathbf{P}_{h\perp}| \mathcal{B}_T) = \frac{1}{\mathcal{B}_T} \delta(|\mathbf{b}_T| - \mathcal{B}_T)$$

$$\tilde{f}_1^q(x, z^2 \mathbf{b}_T^2) \tilde{D}_1^{q \rightarrow \pi}(z, \mathbf{b}_T^2)$$

$$\tilde{f}(x, \mathbf{b}_T^2) \equiv \int d^2 p_T e^{i\mathbf{b}_T \cdot \mathbf{p}_T} f(x, p_T^2) = 2\pi \int d|\mathbf{p}_T| |\mathbf{p}_T| J_0(|\mathbf{b}_T| |\mathbf{p}_T|) f(x, p_T^2)$$

$$F_{LL} = x_B \sum_a e_a^2 \int \frac{d|\mathbf{b}_T|}{(2\pi)} |\mathbf{b}_T| J_0(|\mathbf{b}_T| |\mathbf{P}_{h\perp}|) \tilde{g}_{1L}(x, z^2 \mathbf{b}_T^2) \tilde{D}_1(z, \mathbf{b}_T^2)$$



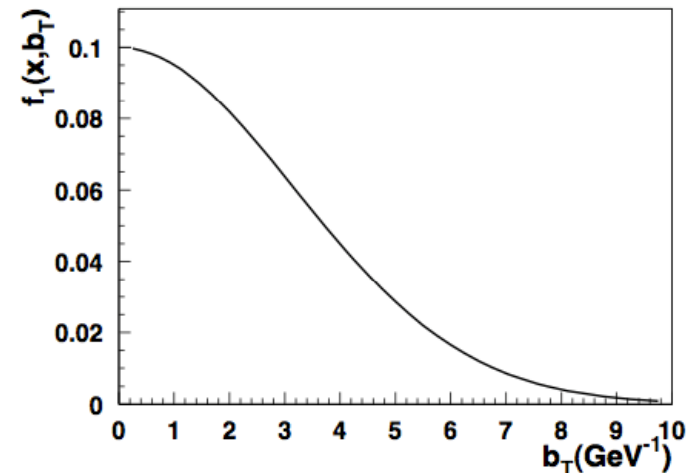
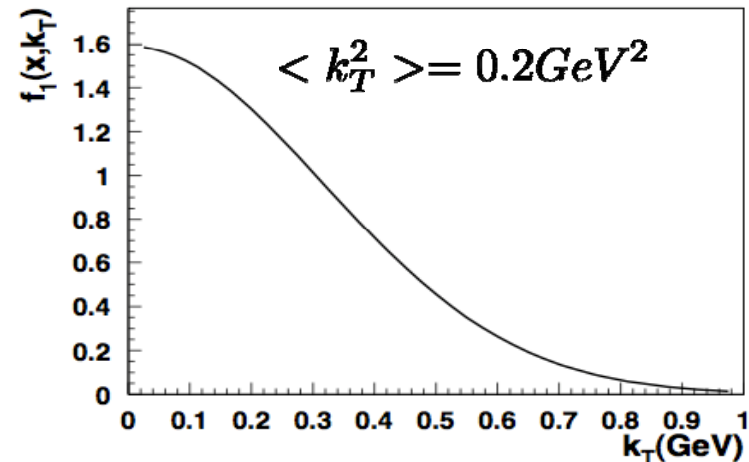
- the formalism in  $\mathbf{b}_T$ -space avoids convolutions
- provides a model independent way to study kinematical dependences of TMD

# SIDIS with Bessel weighting

$$\tilde{f}(x, b_T^2) \equiv \int d^2 p_T e^{i\mathbf{b}_T \cdot \mathbf{p}_T} f(x, p_T^2) = 2\pi \int d|p_T| |p_T| J_0(|b_T| |p_T|) f(x, p_T^2)$$

$$f_1(x, k_T) = \frac{N}{\pi \langle k_T^2 \rangle} e^{-\frac{k_T^2}{\langle k_T^2 \rangle}}$$

$$\tilde{f}_1(x, b_T^2) = \frac{1}{2} \langle k_T^2 \rangle N e^{-\frac{\langle k_T^2 \rangle b_T^2}{4}}$$

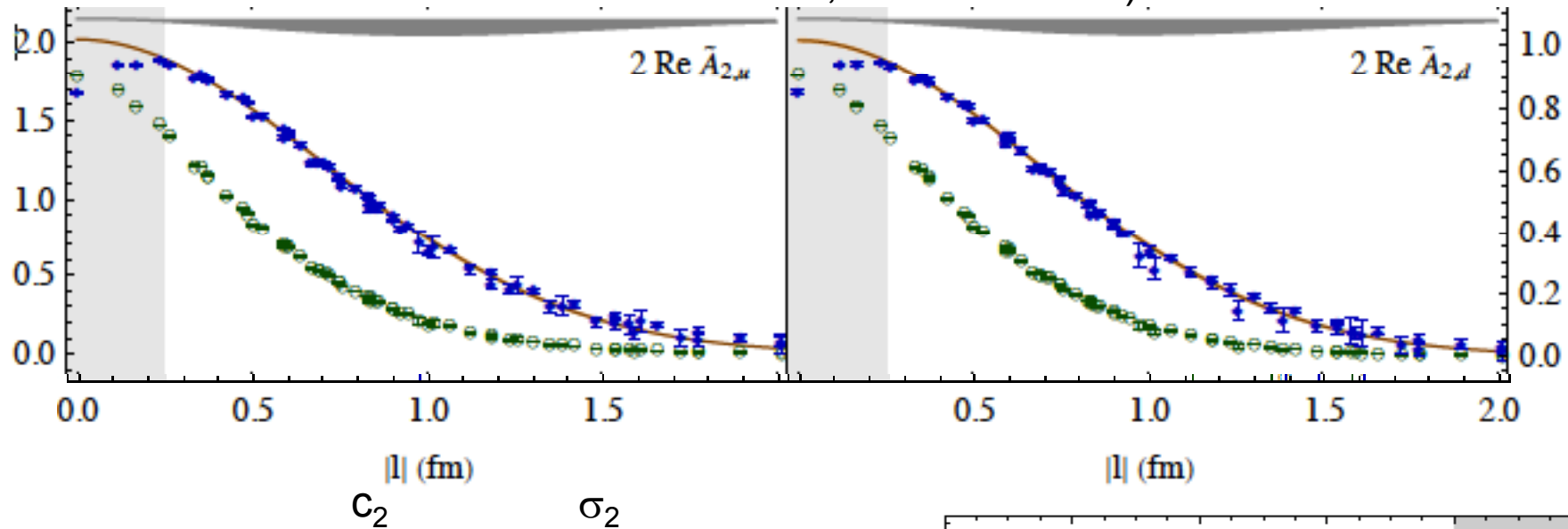


•the data analysis can be performed in the  $\mathbf{b}_T$ -space.

# Lattice calculations and $b_T$ -space

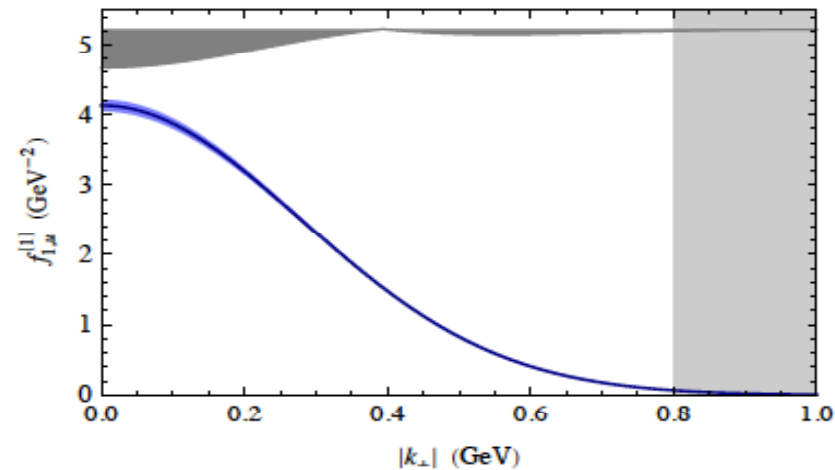
(PDFs in terms of Lorenz invariant amplitudes  
Musch et al, arXiv:1011.1213)

$\tilde{A}_i$

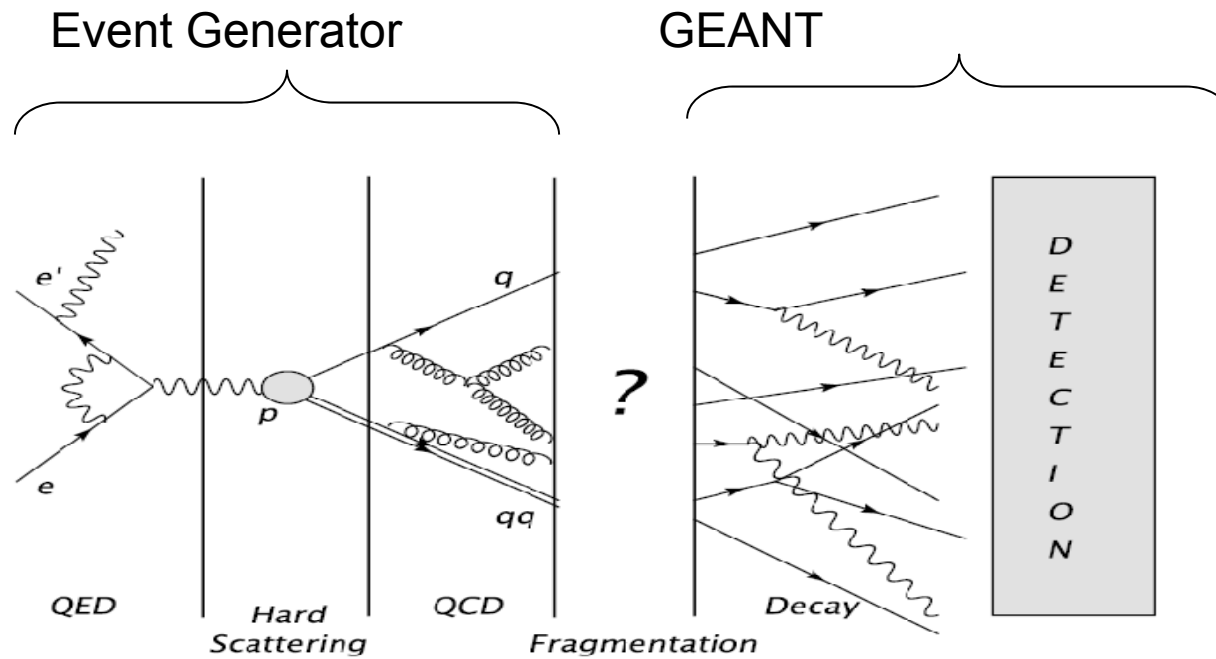


|                   |                                |                             |
|-------------------|--------------------------------|-----------------------------|
| $\tilde{A}_{2,u}$ | $2.0186 \pm 0.0063 \pm 0.0008$ | $1.001 \pm 0.010 \pm 0.068$ |
| $\tilde{A}_{2,d}$ | $1.0171 \pm 0.0064 \pm 0.0005$ | $0.975 \pm 0.012 \pm 0.063$ |

$$f_1^{[1]}(k_{\perp}^2) = \frac{c_2 \sigma_2^2}{4\pi} e^{-\frac{k_{\perp}^2}{(2/\sigma_2)^2}}$$

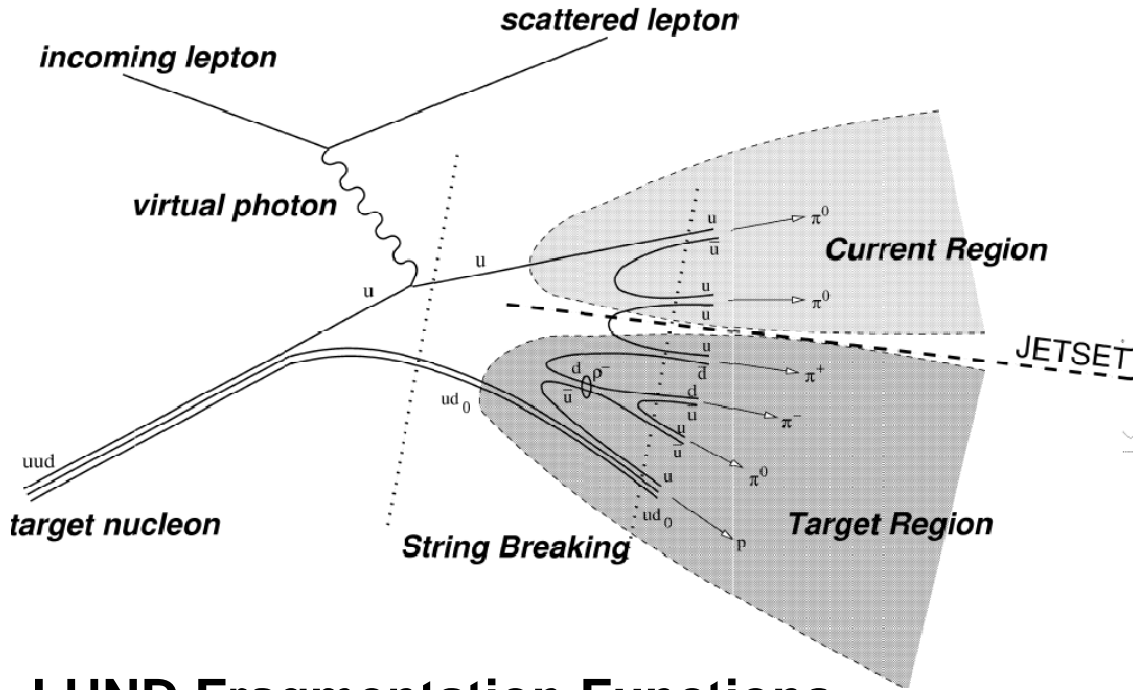


# Hadron production in hard scattering



LUND MC (LEPTO/PEPSI/JETSET)  
Model based (NJL,...)  
SIDIS x-section based dedicated MC

# JETSET: Single particle production in hard scattering



High probability to detect leading hadron in the forward detector

DIS:  
 $Q^2 > 1 \text{ GeV}^2$   
 $W^2 > 4 \text{ GeV}^2$  (10)  
 $y < 0.85$

## LUND Fragmentation Functions



The primary hadrons produced in string fragmentation come from the string as a whole, rather than from an individual parton.

# LUND Fragmentation Functions

Relative suppression of di-quark pairs compared to quark pairs ( $qq_{supp}$ ), strange quarks to u quarks ( $q_{supp}$ ), and extra suppression of strange diquarks ( $q^s q^s_{supp}$ )

$$f(P_T) \sim e^{-\frac{P_T^2}{\mu_D^2}}$$

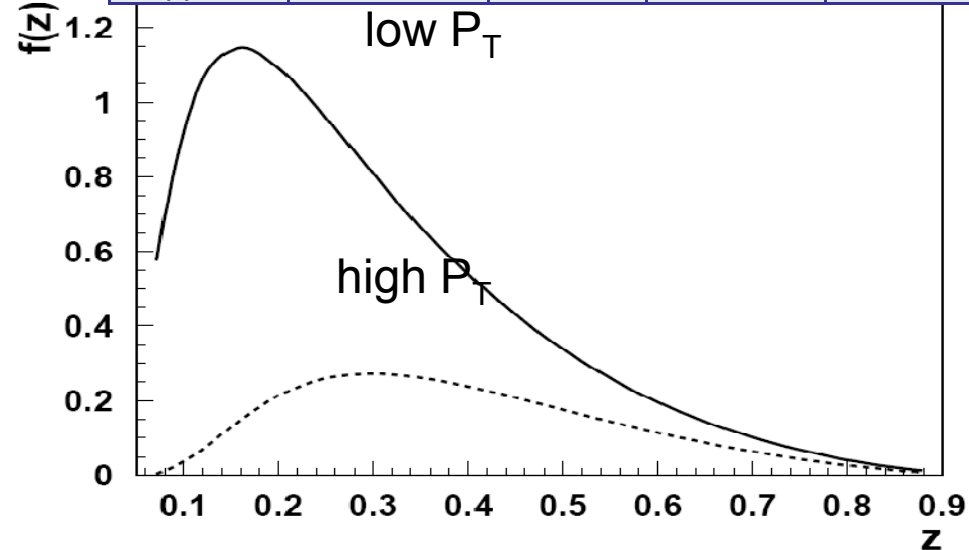
Distributions in  $P_T$  given by a gaussian with width  $\mu_D$  with additional exponential  $e^{-P_T/\mu_D}$  from fraction  $\eta$

$$f(z) \sim z^{-1}(1-z)^{a+a_{q\bar{q}}} e^{-\frac{b(m_h^2 + P_T^2)}{z}}$$

Suppression for diquarks at large  $z \rightarrow a_{q\bar{q}}$

PDF side: gaussian  $\langle k_T^2 \rangle$

| parameter        | JETSET   | default | HERMES | CLAS |
|------------------|----------|---------|--------|------|
| $q^s_{supp}$     | PARJ(2)  | 0.3     | 0.28   | 0.1  |
| $q^s q^s_{supp}$ | PARJ(3)  | 0.4     | 0.4    | 0.2  |
| S1               | PARJ(12) | 0.6     | 0.6    | 0.4  |
| $\mu_D$          | PARJ(21) | 0.36    | 0.38   | 0.3  |
| $\eta$           | PARJ(23) | 0.01    | 0.01   | 0.1  |
| $\Delta E$       | PARJ(33) | 0.8     | 0.8    | 0.3  |
| a                | PARJ(41) | 0.3     | 1.13   | 1.1  |
| $a_{qq}$         | PARJ(45) | 0.5     | 1.05   | 1.0  |



# FAST-MC for CLAS12

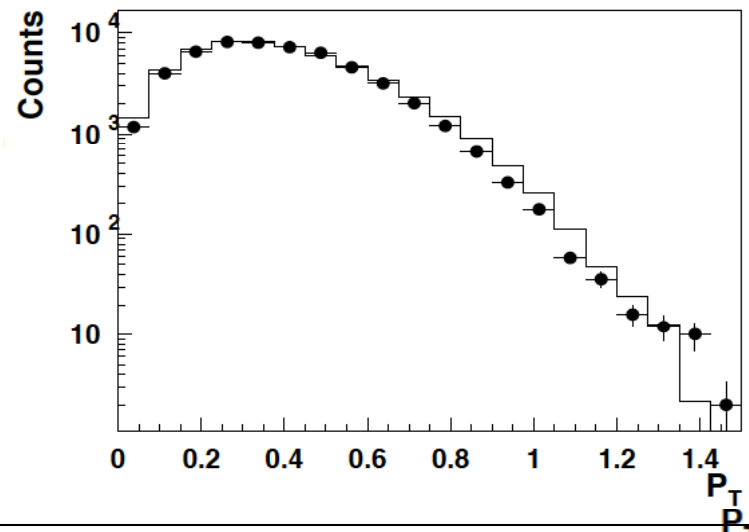
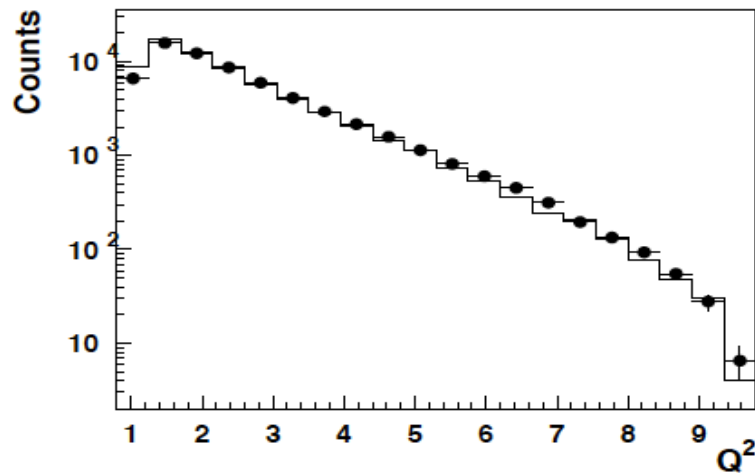
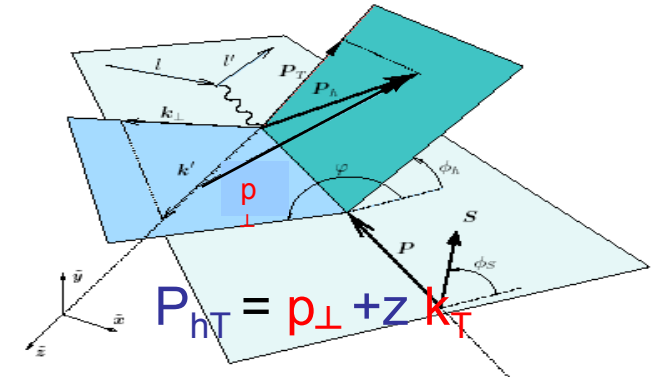
SIDIS MC in 7D ( $x, y, z, \phi, \phi_S, p_T, \lambda, \pi$ )

Simple model with 10% difference between  $f_1$  ( $0.2\text{GeV}^2$ ) and  $g_1$  widths with a fixed width for  $D_1$  ( $0.14\text{GeV}^2$ )

$$f_q(x, k_{\perp}) = f_q(x) \frac{1}{\pi \langle k_{\perp}^2 \rangle} e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}$$

CLAS12 acceptance & resolutions

Events in CLAS12



Reasonable agreement of kinematic distributions with realistic LUND simulation

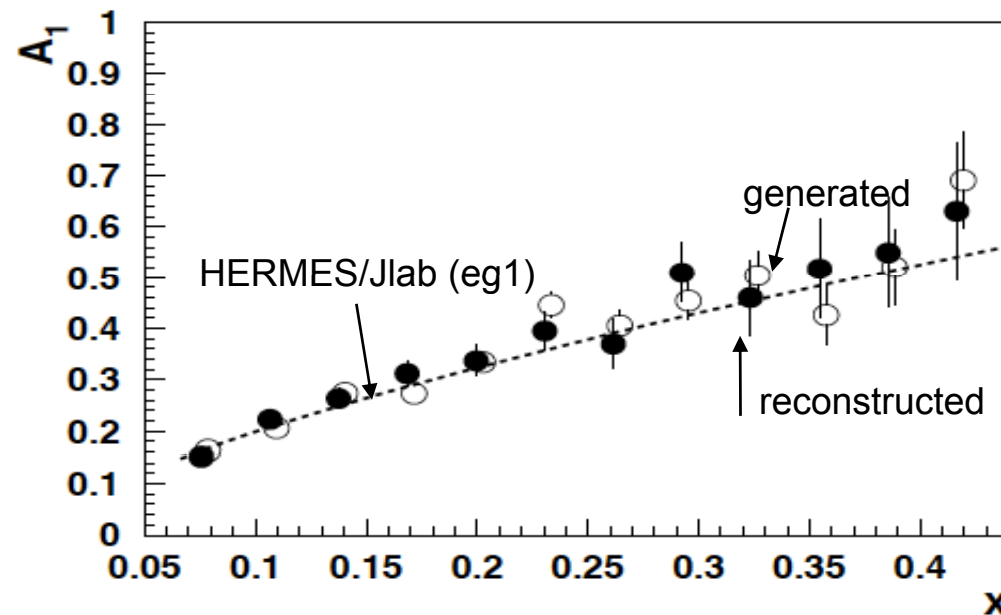
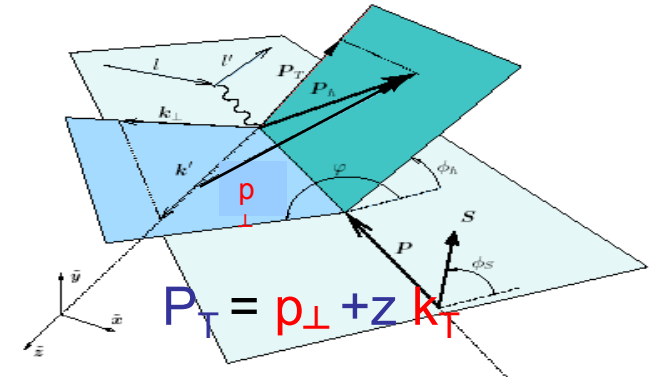
# FAST-MC for CLAS12

SIDIS MC in 7D ( $x, y, z, \phi, \phi_S, p_T, \lambda, \pi$ )

Simple model with 10% difference between  $f_1$  ( $0.2\text{GeV}^2$ ) and  $g_1$  widths with a fixed width for  $D_1$  ( $0.14\text{GeV}^2$ )

$$f_q(x, k_{\perp}) = f_q(x) \frac{1}{\pi \langle k_{\perp}^2 \rangle} e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}$$

CLAS12 acceptance & resolutions



Reasonable agreement of kinematic distributions with SIDIS data



# BGMP: extraction of $k_T$ -dependent PDFs

Need: project x-section onto Fourier mods in  $b_T$ -space to avoid convolution

Boer, Gamberg, Musch & Prokudin arXiv:1107.5294

$$\int_0^\infty d|P_{h\perp}| |P_{h\perp}| J_0(|P_{h\perp}||b_T|) \left[ \frac{d\sigma}{dx_B dy d\phi_S dz_h d\phi_h |P_{h\perp}| d|P_{h\perp}|} \right]$$

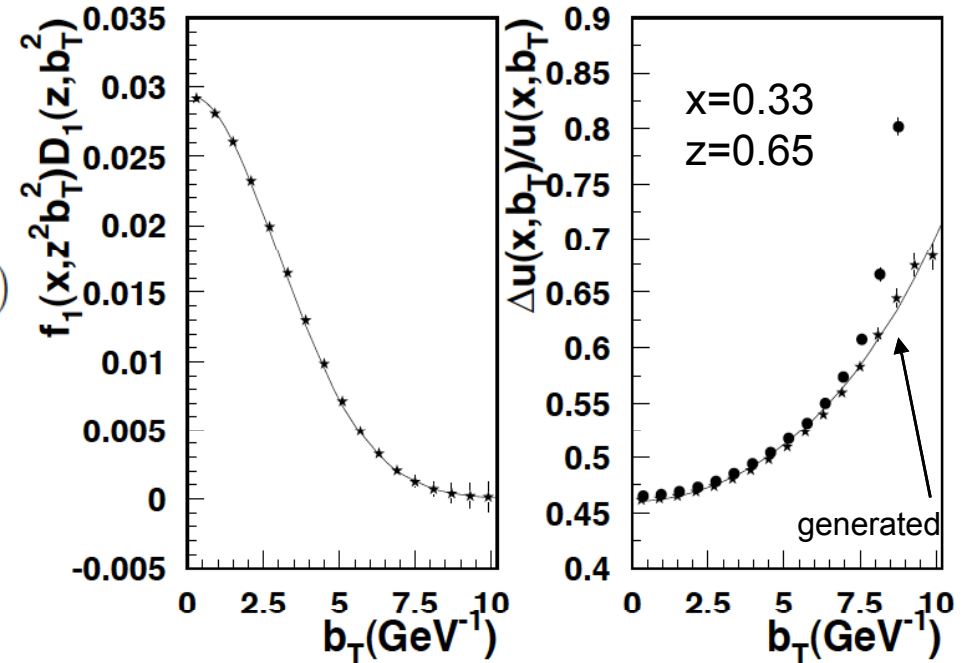
$$S_\pi^{unp\pm}(x_i, z_i, b_{Tj}) = \sum_{i=1}^{N_\pi^+ / N_\pi^-} J_0(b_{Tj} P_{Ti}) / \eta_i / A(x_i, y_i)$$

acceptance

$$A(x, y) = \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{(1-\varepsilon)} \left( 1 + \frac{\gamma^2}{2x_B} \right)$$

$$\tilde{f}_1^q(x, z^2 b_T^2) \tilde{D}_1^{q \rightarrow \pi}(z, b_T^2)$$

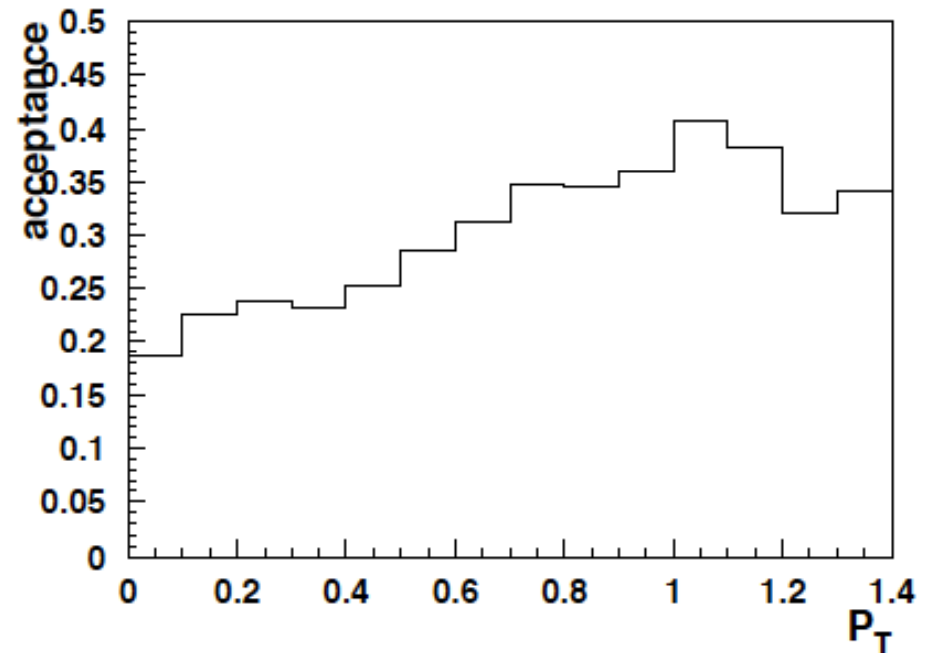
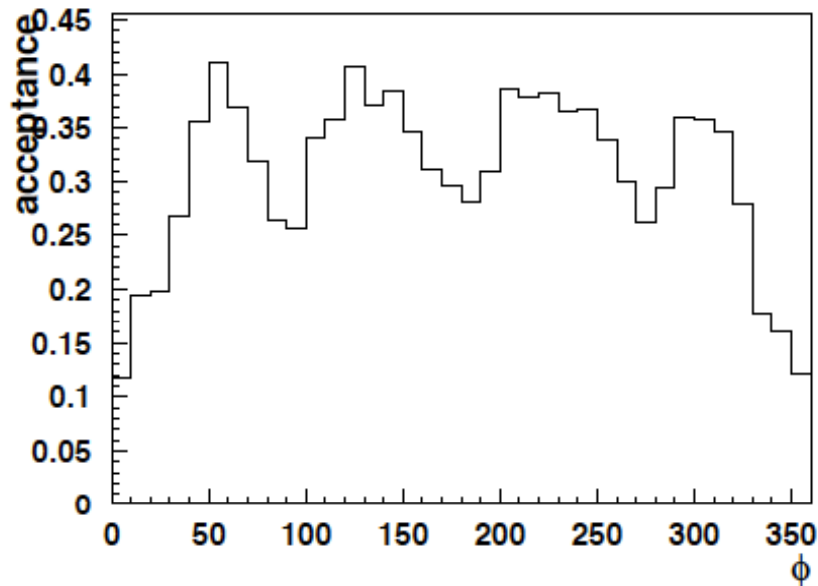
$$\Delta u(x, b_T) / u(x, b_T) = \frac{S_\pi^{pol+} - S_\pi^{pol-}}{S_\pi^{unp+} + S_\pi^{unp-}}$$



- BGMP provides a model independent way to extract  $k_T$ -dependences of helicity distributions
- requires wide range in hadron  $P_T$

# Acceptance effects

Acceptance for CLAS12 using CLAS-DIS generator using FAST-MC  
( $0.4 < z < 0.5, 0.1 < x < 0.2$ )



Acceptance corrections are important even for full kinematical coverage

# BGMP: extraction of $k_T$ -dependent TMDs

$$F_{UT,T}^{\sin(\phi_h - \phi_s)} = -x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^2 J_1(|b_T| |P_{h\perp}|) M z \tilde{f}_{1T}^{\perp(1)}(x, z^2 b_T^2) \tilde{D}_1(z, b_T^2)$$

$$F_{LL} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T| J_0(|b_T| |P_{h\perp}|) \tilde{g}_{1L}(x, z^2 b_T^2) \tilde{D}_1(z, b_T^2),$$

$$F_{LT}^{\cos(\phi_h - \phi_s)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^2 J_1(|b_T| |P_{h\perp}|) M z \tilde{g}_{1T}^{\perp(1)}(x, z^2 b_T^2) \tilde{D}_1(z, b_T^2),$$

$$F_{UT}^{\sin(\phi_h + \phi_s)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^2 J_1(|b_T| |P_{h\perp}|) M_h z \tilde{h}_1(x, z^2 b_T^2) \tilde{H}_1^{\perp(1)}(z, b_T^2),$$

$$F_{UU}^{\cos(2\phi_h)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^3 J_2(|b_T| |P_{h\perp}|) M M_h z^2 \tilde{h}_1^{\perp(1)}(x, z^2 b_T^2) \tilde{H}_1^{\perp(1)}(z, b_T^2),$$

$$F_{UL}^{\sin(2\phi_h)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^3 J_2(|b_T| |P_{h\perp}|) M M_h z^2 \tilde{h}_{1L}^{\perp(1)}(x, z^2 b_T^2) \tilde{H}_1^{\perp(1)}(z, b_T^2),$$

$$F_{UT}^{\sin(3\phi_h - \phi_s)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^4 J_3(|b_T| |P_{h\perp}|) \frac{M^2 M_h z^3}{4} \tilde{h}_{1T}^{\perp(2)}(x, z^2 b_T^2) \tilde{H}_1^{\perp(1)}(z, b_T^2).$$

| q \ N | U                | L        | T                      |
|-------|------------------|----------|------------------------|
| U     | $f_1$            |          | $h_1^{\perp}$          |
| L     |                  | $g_1$    | $h_{1L}^{\perp}$       |
| T     | $f_{1T}^{\perp}$ | $g_{1T}$ | $h_1$ $h_{1T}^{\perp}$ |

- BGMP provides a model independent way to extract  $k_T$ -dependences of TMD
- requires wide range in hadron  $P_T$

# Summary

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- Understanding of  $k_T$ -dependence is crucial for SIDIS data analysis
- New SIDIS MC developed allowing simulation in 8D  $(x, y, z, \phi, \phi_S, p_T, h, \lambda)$  to test the  $k_T$ -dependent TMD flavor decomposition
- Bessel weighting procedure tested with a simple model
- Ratio of TMDs recovered with good precision within CLAS12 accessible kinematics

## Plans

- Implement all structure functions
- Develop the procedure for  $k_T$ -dependent flavor decomposition for all TMDs (including  $f_1$  and  $g_1$ )
- Extract  $f_1$ ,  $g_1$  and  $h_{1L}$   $k_T$ -dependences from CLAS6 data
- Study models for nuclear PDFs and  $\text{NH}_3$  dilutions
- .....