Study of the low momentum compaction B-factory

Sam Heifets, and A. Novokhatski

SLAC

ICFA mini-workshop "Frontiers of short bunches in Storage rings"

Abstract

For a given rf frequency, the quasi-isochronous lattice allows, in principle, to double the number of bunches compared with the nominal lattice. We explore such a possibility considering the beam stability and luminosity of the PEP-II B-factory.

1 Introduction and motivation

- The luminosity $L = 10^{34}\ cm^{-2}s^{-1}$ has been achieved in the PEP-II B-factory by filling each other RF bucket.

- Further increase of L by increasing I_{bunch} is limited mostly by the beam-beam effect.

- For V_{rf} fixed, $\sigma_0 \propto \sqrt{\alpha}$. Then, L can be increased for small α,

$$L \propto \frac{1}{\sqrt{\alpha}}, \quad \text{provided } \beta_y \propto \sigma_y \propto \sqrt{\alpha}. \tag{1}$$

- The phase plane for small α looks like the upper plot in Fig. 1.

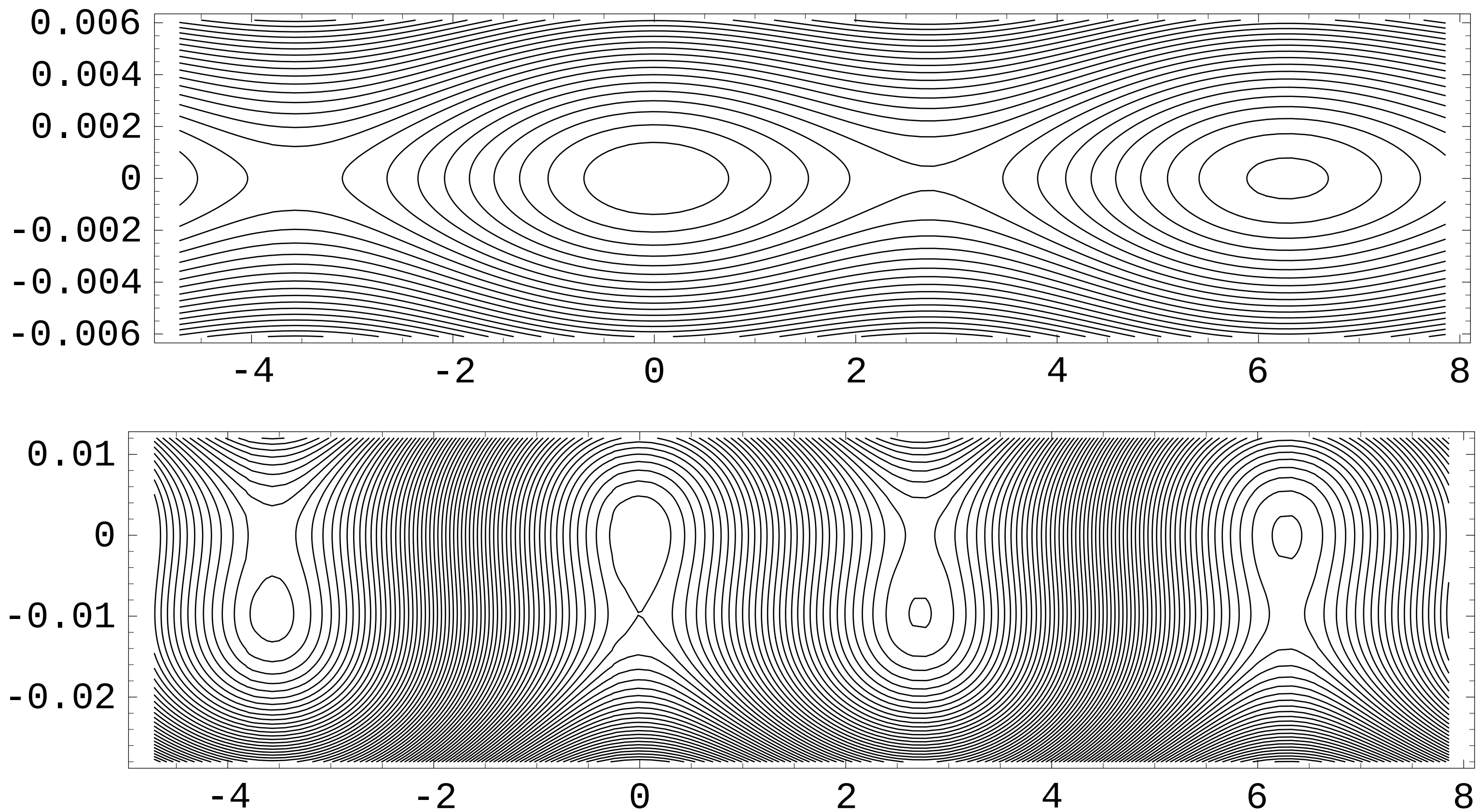


Figure 1: Phase plot for the Hamiltonian $H(\delta,\zeta) = \alpha_0\delta^2/2 + \alpha_1\delta^3/3 - \lambda(\sin[\phi - \zeta] - \sin[\phi]) - \lambda\zeta\cos[\phi]$. Parameters are $\lambda = 2.0E-7$, $\alpha_1 = 5.5E-2$, and $\alpha_0 = 5.0E-2$ (above), and $\alpha_0 = 5.34E-4$ (bottom). Note the different vertical scale for two plots.

- We notice that small α may alow doubling of the number of bunches per rf wave length using the quasi-isochronous lattice.

  There are many questions to be answered whether such approach can be used to increase luminosity.

  We considered some of them:

  a) lattice design,

  b) equilibrium bunch shape and rms

  c) parasitic crossings

  d) the rf stability

  e) microwave and the longitudinal HT instabilities.

  etc.

  Some results can be of general interest and are reported below.

2 Longitudinal dynamics with small α

- There are two stable fixed points (FP) in the phase space of the longitudinal motion:

$$(1.)\quad \{\frac{\omega_{rf} z_1}{c} = 0,\ \delta_1 = 0\},\quad (2.)\quad \{\frac{\omega_{rf} z_2}{c} = 2\phi_s,\ \delta_2 = -\frac{\alpha_0}{\alpha_1}\}.$$

- It is possible to have two bunches within the rf wave length centered at the 2FPs: trailing at (1) and leading at (2) FPs.
- The motion in the small vicinity of the FPs is stable (if $0 < \phi_s < \pi/2$) with the same $\Omega_s \propto \sqrt{\alpha_0}$,
- In the region with the D_x, the relative horizontal shift of the centroids is

$$\Delta x = -\frac{\alpha_0}{\alpha_1} D_x. \tag{2}$$

- The energy acceptance is defined by the α_0/α_1 requiring

$$\frac{\alpha_0}{\alpha_1} >\simeq 10\,\delta_0. \tag{3}$$

- For PEP-II, $\alpha_1 \simeq 0.05$, $\delta_0 = 6.1E-4$, and the high repetition rate of injection what relaxes requirements for the dynamic aperture and the energy acceptance.

  Acceptable $\alpha_0 = 5.344E-4$ are only by a factor 5 smaller than the nominal $\alpha_0 = 2.4E-3$.

3 Bunch profile

- We model the wake field of a point-like bunch for the LER PEP-II B-factory adding contributions of the experimentally measured modes of six RF cavities, resistive wall, and the inductive components of the ring.

- The steady-state longitudinal bunch profiles $\rho(z)$ for two sub-bunches are given by the Haissinski solution.

- The wake enters with the opposite sign. Therefore, the dynamics of the leading bunch is the same as the dynamics of a bunch in the lattice with the negative momentum compaction factor.

- The potential well distortion (PWD) makes the first (leading) bunch shorter and the second (trailing) bunch longer.

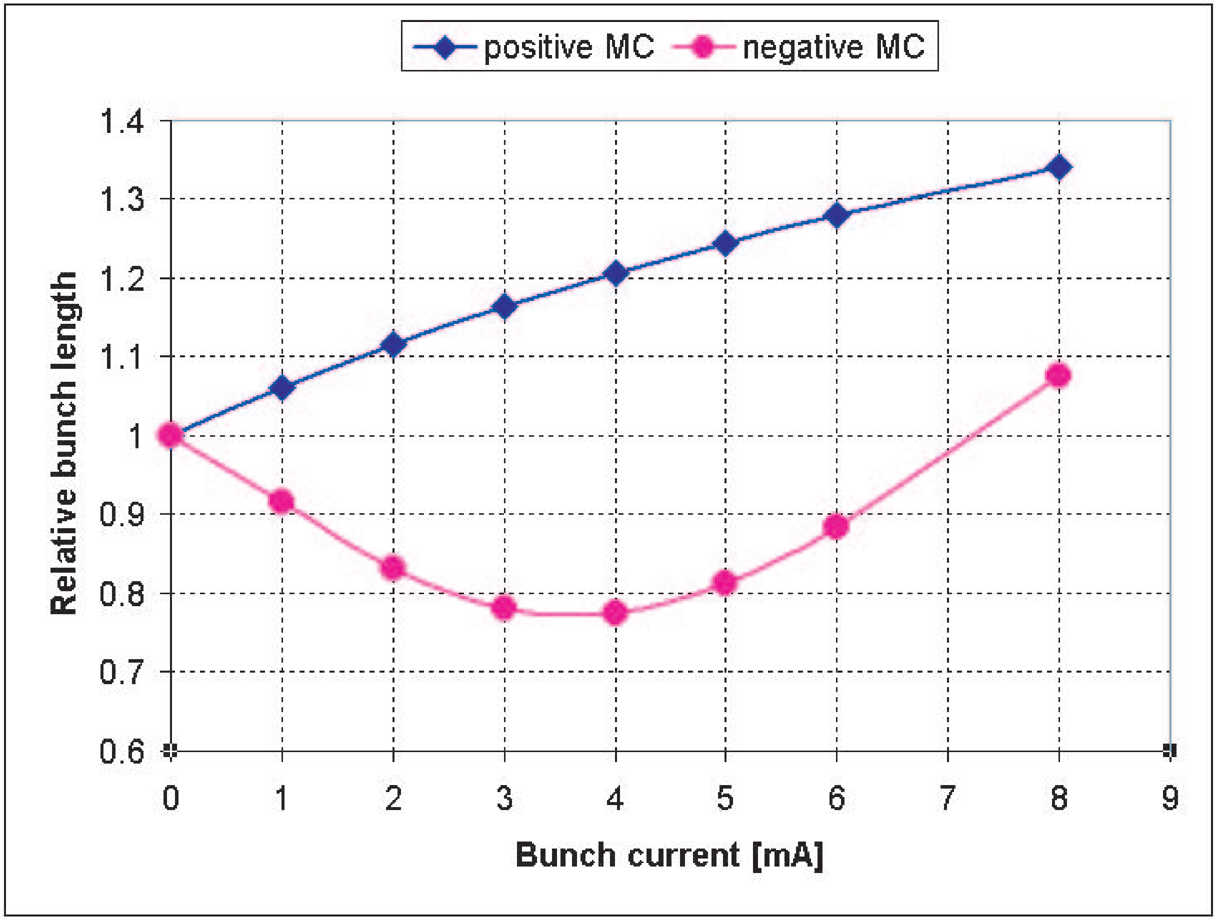


Figure 2: Bunch lengthening vs bunch current for positive and negative momentum compaction (MC) factors.

3.1 Other effects

- The only new additional parasitic crossings are the interaction at $\pm 3\lambda/4 = 45$ cm inside of the B1 magnet.

- For equal beam currents there is no need to detune the cavities. It can be shown (see the full text in the SLAC-PUB-11467, September 7, 2005) that there is no beam-loading instability.

- The threshold bunch current to Microwave Instability $\propto$ 20 mA for $\alpha_0 > 0$ is reduced to 5 mA for $\alpha_0 < 0$. However, even 5 mA bunch current is by a factor two higher that the present PEP-II bunch current and may be acceptable.

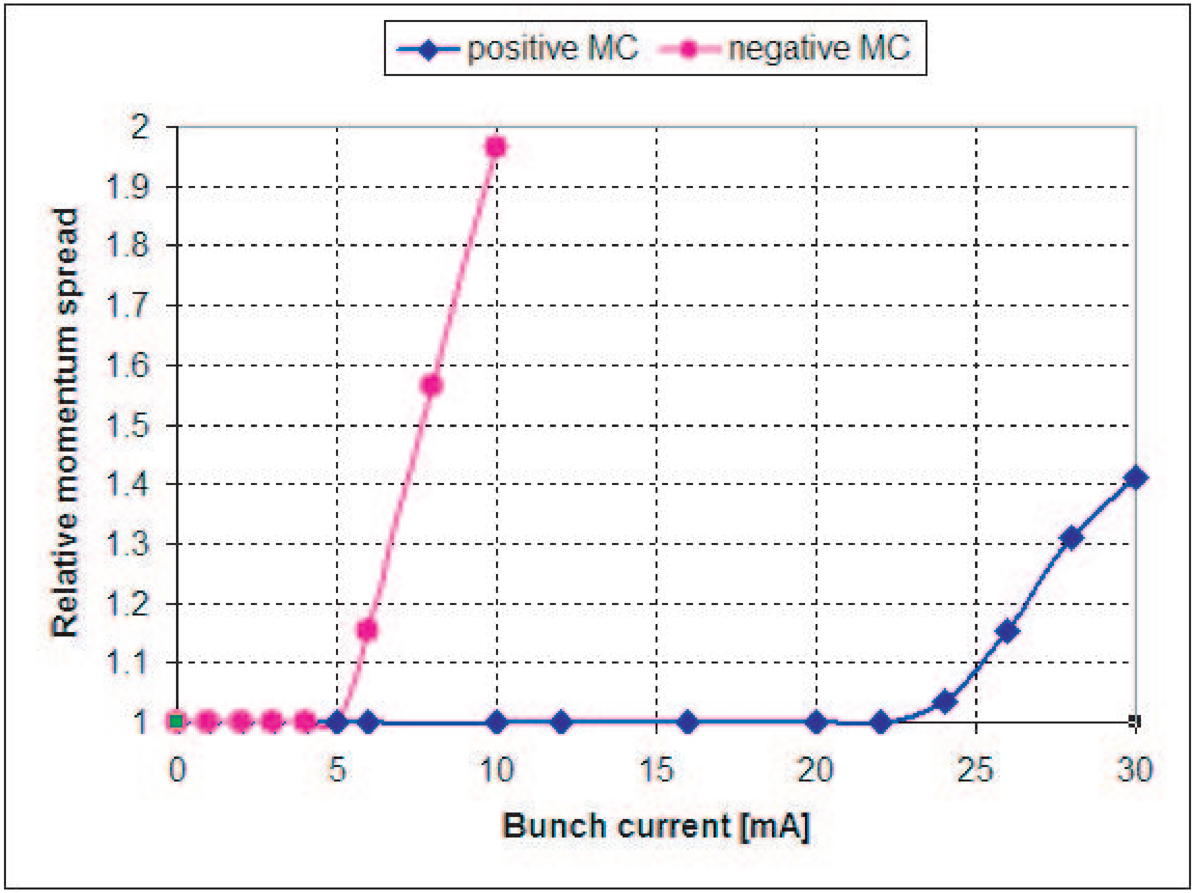


Figure 3: Energy spread vs beam current for positive and negative momentum compaction factors.

4 Longitudinal head-tail instability

The growth rate of instability $1/\tau \propto (\alpha_1/\alpha_0)$ and is important for small α_0.

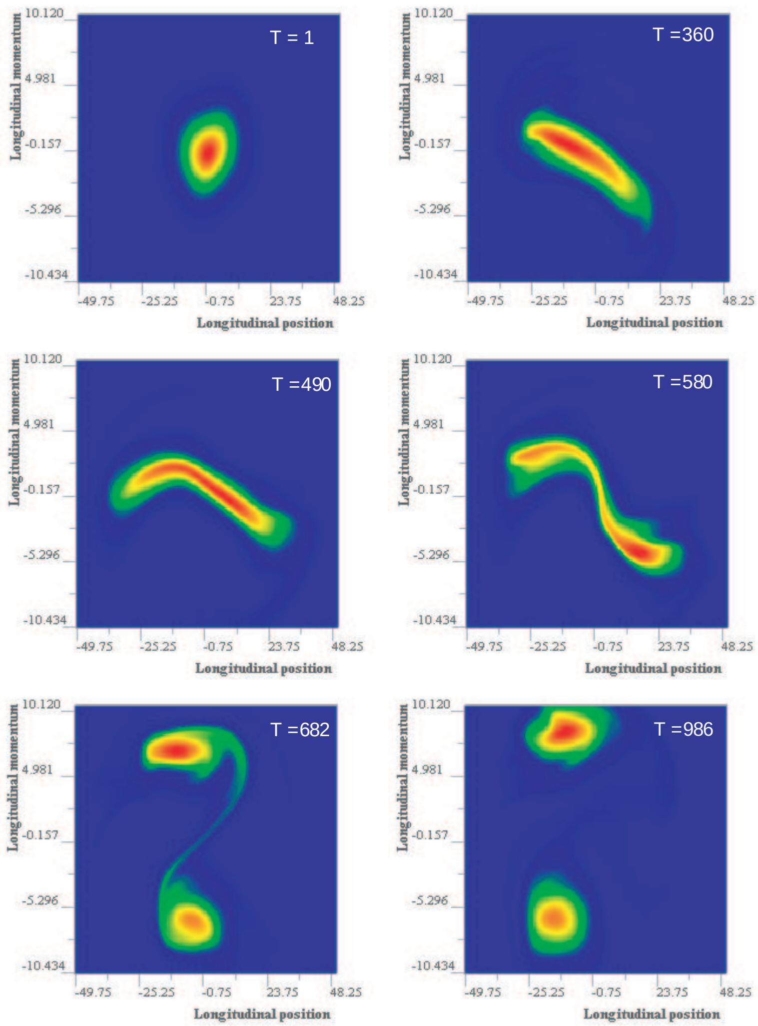


Figure 4: Dynamics of the head-tail single-bunch instability is shown in the phase plane (z, δ) for 2 mA bunch current. Time is indicated in the figure. At $t > 600$ turns, the bunch splits in halves.

We carried out two type of simulations.

**** In the simple simulations, we calculate trajectories of four particles $(i, j) = 1, 2, ..., 4$, solving with MATHEMATICA equations of motion

$$\frac{d\zeta_i}{d\tau} = -\delta_i\,(1 + \epsilon\delta_i),$$
$$\frac{d\delta_i}{d\tau} = \zeta_i - \sum_j \frac{I_{bunch}}{4}\,W[\zeta_i - \zeta_j], \tag{4}$$

where $\tau = \omega_s t$, and $\epsilon = (\alpha_1/\alpha_0)\delta_0$. Trajectories with initial conditions $\zeta_i = -0.33, -0.27, 0.27, 0.33$, $\delta_i = 0$, were calculated for the time interval up to 700 synchrotron periods. That allows fast study of the dynamics of the system including quantitative result for the emittance variation.

**** More elaborate simulations used the Fokker-Plank solver developed to study microwave instability.

Results of the simple simulations for the bunch current $I_{bunch} = 0.5$ mA are shown in Fig. 5. The system in clearly unstable although the growth rate is small.

- The instability is caused by the variation of the energy loss during the synchrotron period due to variation of the rms bunch length.
- Therefore, it seems that the instability might be stabilized by the longitudinal feedback system (FB).
- We model the FB generating a buffer with positions of the bunch centroid for the last 56 revolutions and interpolate data as

$$\zeta_f(\tau_k) = a_0 + a_1 \sin(\nu\tau_k + \phi_1) + a_2 \sin(2\nu\tau_k + \phi_2) + a_3 \sin(3\nu\tau_k + \phi_3), \tag{5}$$

where $\tau_k = \tau - (k-1)T_0$, $k = 1, ..., 56$, and a_j, $j = 0, 1, 2, 3$ and ϕ_j, $j = 1, 2, 3$ are fitting parameters.

The kick $\delta_i \rightarrow \delta_i + K$ is applied to each of four tracking particles where $K = 0.1\,[d\zeta_f/d\tau]_{k=1}$.

The result of tracking in Fig. (6) seems encouraging: the amplitude of the oscillations for each particle remain stable for 700 synchrotron periods.

- Unfortunately, the simulations with the Fokker-Plank solver do not confirm this conclusion, see Fig. (7) and Fig. (8).

  Although the results in many respect seems similar to the four-particle model, there is a systematic growth of emittance.

  The difference of two simulations is apparently due to the difference in the models: the two-particle model does not include fluctuations which are included in the Fokker-Plank equation.

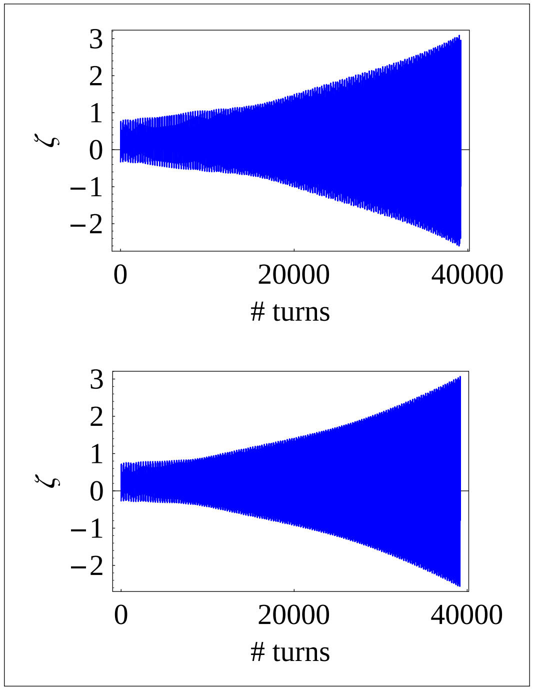


Figure 5: Tracking of four particles with the feedback off, $I_{bunch} = 0.5$ mA. The trajectories are unstable.

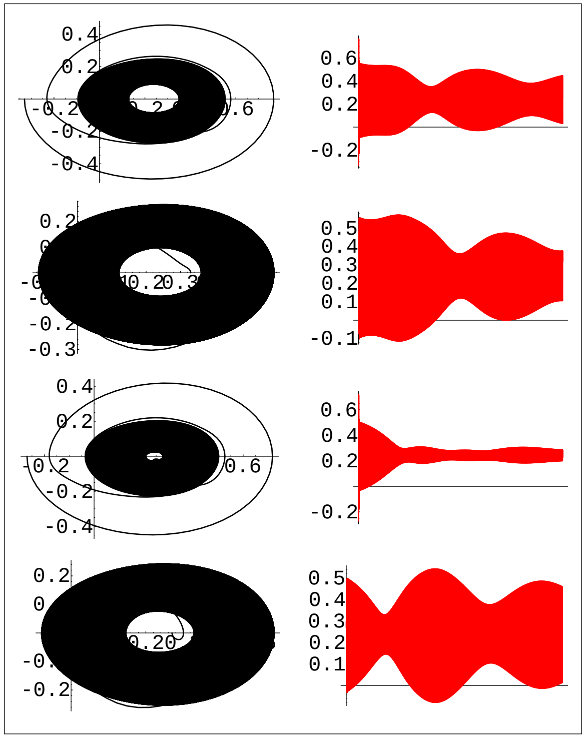
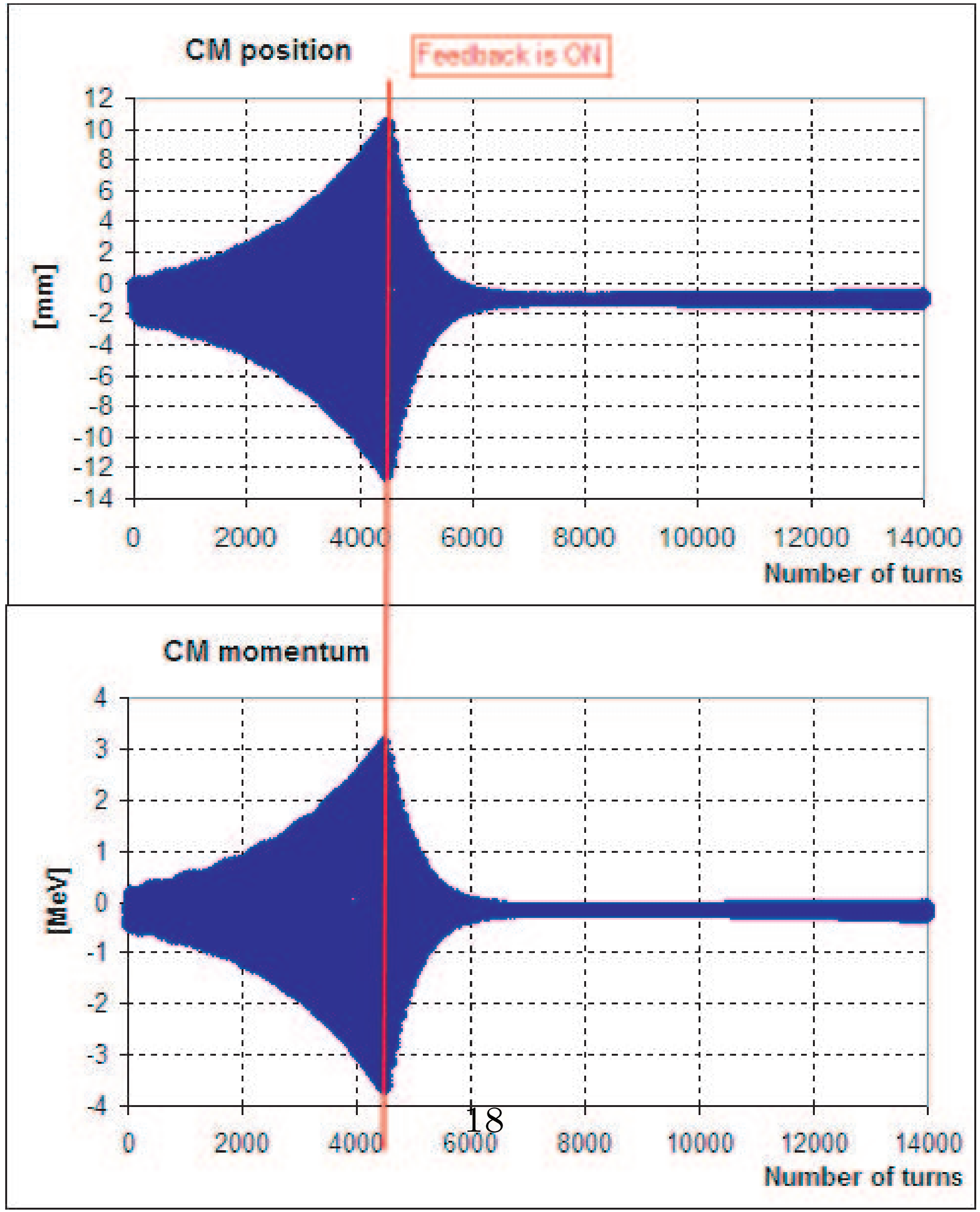
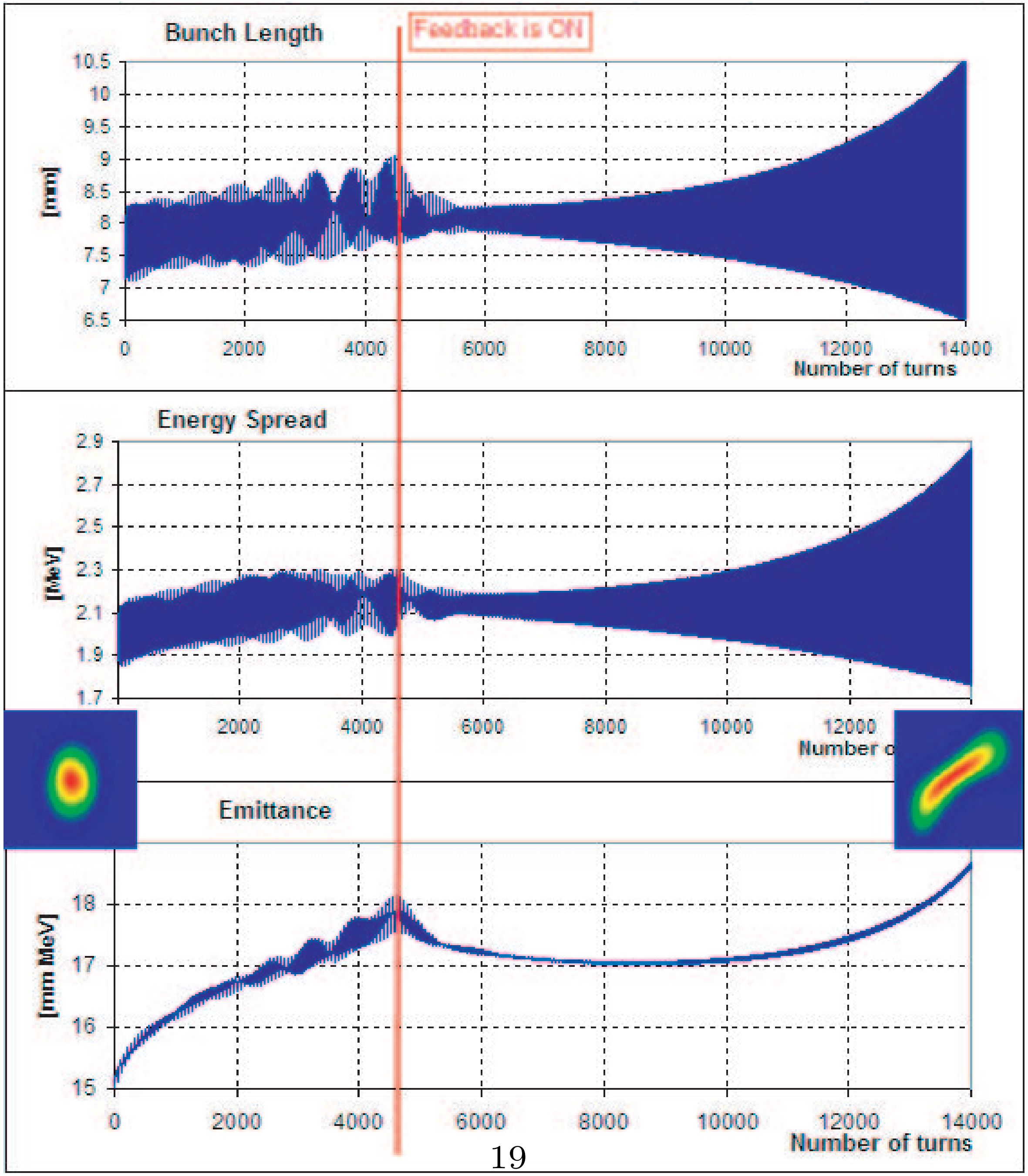


Figure 6: Tracking of four particles with the feedback on. The i-th row shows the trajectory in the phase plane (on the left) and time variation $\zeta_i(\tau)$ (on the right) for the $i = 1, 2, 3, 4$ particle. The trajectory after one or two turns finds the fix point and then remains stable for 700 synchrotron periods.


CM position
Feedback is ON
[mm]
Number of turns
CM momentum
[MeV]
Number of turns


Bunch Length
Feedback is ON
[mm]
10.5
10
9.5
9
8.5
8
7.5
7
6.5
0
2000
4000
6000
8000
10000
12000
14000
Number of turns
Energy Spread
[MeV]
2.9
2.7
2.5
2.3
2.1
1.9
1.7
Emittance
[mm MeV]
18
17
16
15
Number of turns

5 Summary

The quasi-isochronous ring with the reduced momentum compaction factor allows to have two stable bunches per rf bucket. It is tempting to increase the number of bunches per ring without increasing the rf frequency. The paper presents a preliminary study of this possibility. We consider the lattice design, the bunch lengthening and distortion, parasitic crossings, the rf beam stability, and microwave instability. The longitudinal head-tail instability makes the beam unstable and the feeback system can not stabilize it although the growth rate of instability is small.

Therefore, the statement that there are two stable fix points in the low alpha lattices is an illusion: the fluctuations make the particles in the second fixed point unstable.

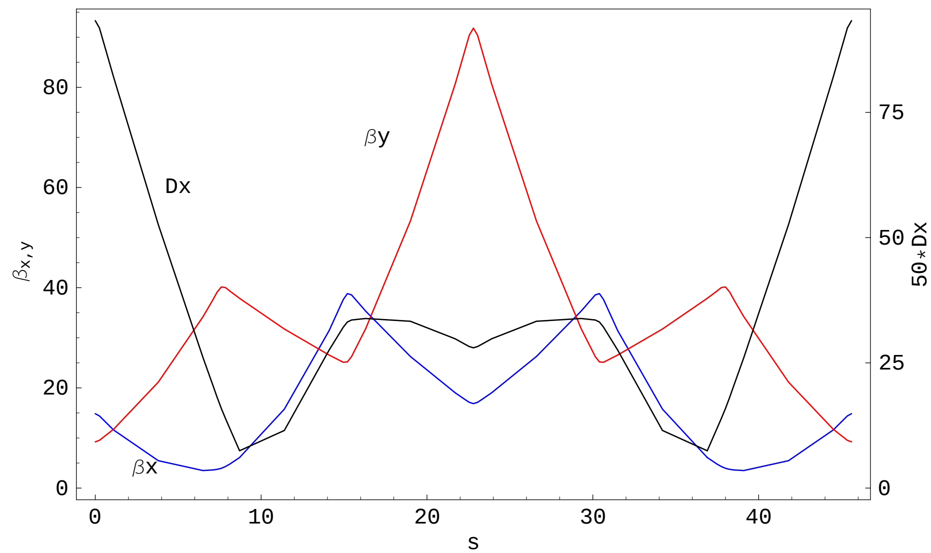


Figure 9: Twiss parameters for the quasi-isochronous lattice with $\alpha_0 = 5.3E-4$. The same lattice can be used to make the lattice isochronous and $\alpha < 0$ by varying a single quad strength.

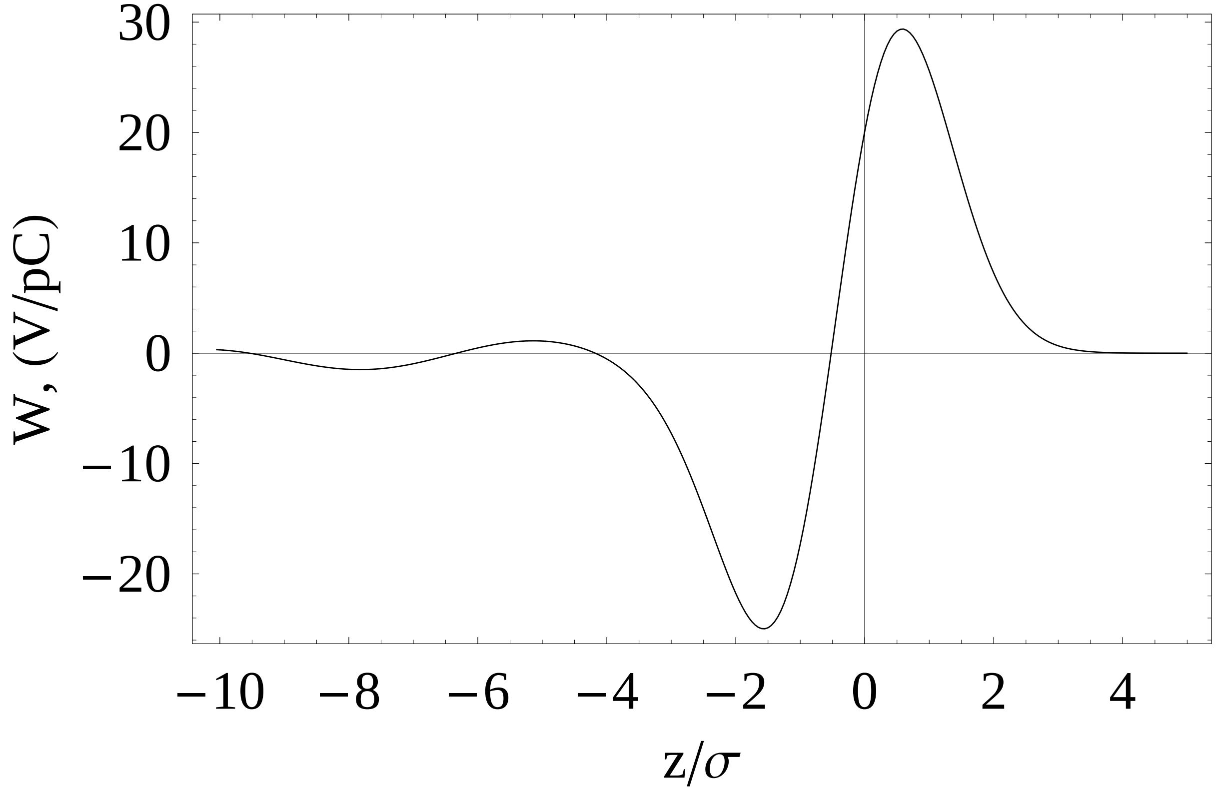


Figure 10: Wake field obtained by convolution of the $W\delta(z)$ with the $\sigma = 8$ mm Gaussian distribution.

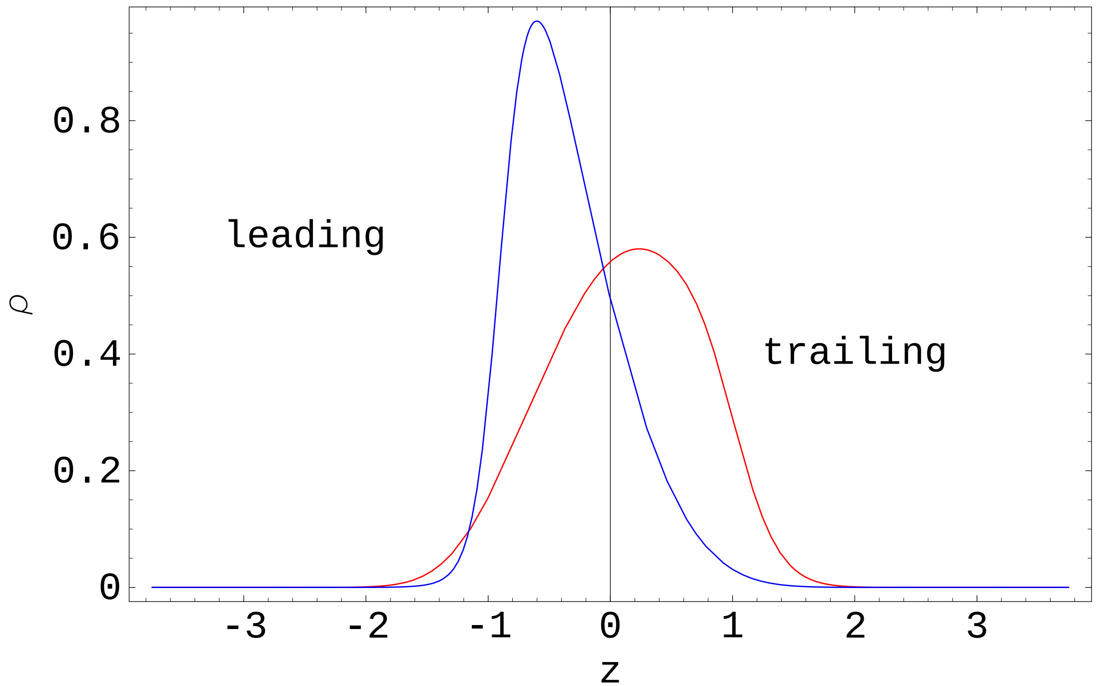


Figure 11: An example of the bunch profiles for the leading (the bunch centered at the 2nd FP, blue line) and trailing bunch(centered at the 1st FP, red line). Parameter $\alpha_0 = 0.8\,10 - 3$. The zero current $\sigma_0 = 1$ cm, the bunch current $I_B = 2.5$ mA.