Longitudinal Single Bunch Instability by Coherent Synchrotron Radiation

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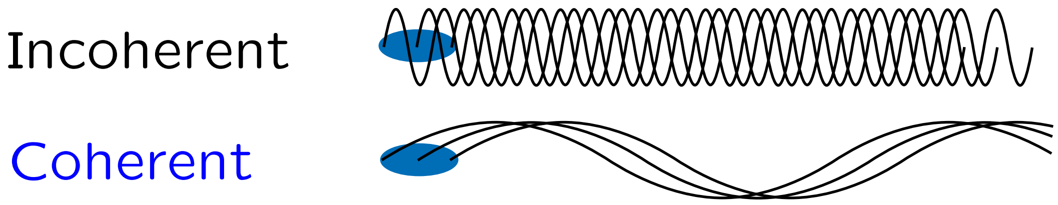
Contents

SBSR05 at Frascati, Nov.7-8, 2005

1. Introduction

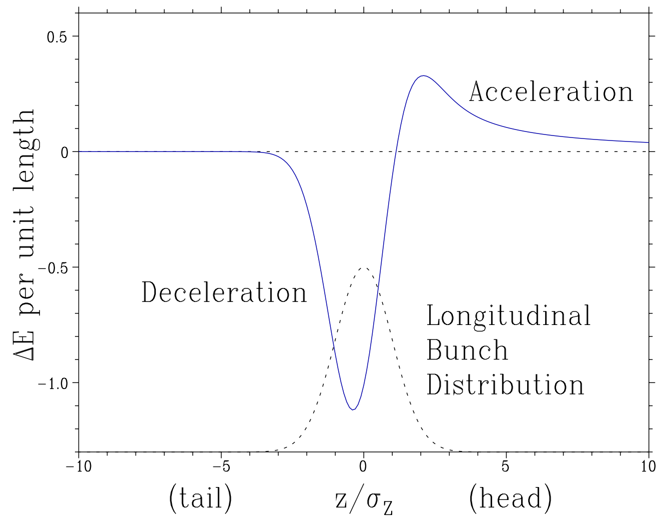
- In the spectrum of synchrotron radiation, the components such that $\lambda \gtrsim \sigma_z$ produce Coherent Synchrotron Radiation. (CSR)



- Energy change of particles

$\boxed{\text{Short range interaction}}$

$\Rightarrow$ Energy spread
$\Rightarrow$ Single bunch instability



CSR in storage rings

Bunch length $\sim$ a few mm

1. Strong shielding
   - Waves with wavelength $\lambda \gtrsim \sqrt{6h^3/\pi\rho}$ is suppressed.
     $\longrightarrow$ Vacuum chamber should be properly considered.

2. CSR field is transient due to finite magnet length.
   - CSR evolves in a dipole magnet (s-dependent).

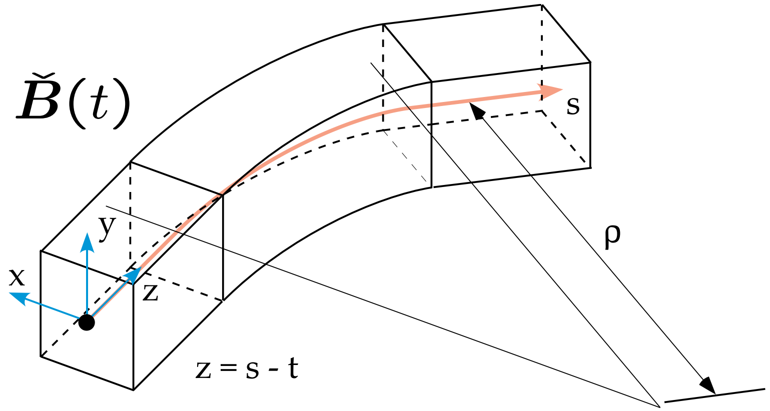
3. Variation of the bunch distribution
   - Fine structure in the bunch
     $\longrightarrow$ CSR with short wavelength is emitted.
     (Shielding effect is weak.)

2. Calculation of CSR using paraxial approximation

Mesh calculation of $(\boldsymbol{E}, \boldsymbol{B})$ in a beam pipe

(1) Begin with Maxwell equations in accel.coordinates (x, y, s)

$\check{\boldsymbol{E}}(t), \check{\boldsymbol{B}}(t)$



(2) Fourier transformation

$$\check{f}(t-s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, f(k)\, e^{-ik(t-s)}$$

$\Rightarrow$ frequency domain $\boldsymbol{E}(k), \boldsymbol{B}(k)$

(3) Approximate these equations

Paraxial Approximation

(4) Solve them numerically by finite difference (mesh calculation)

pipe = boundary condition

(5) Fourier transformation

$\Rightarrow$ time domain (GOAL)

- From Maxwell equations,

$$\nabla(\nabla\cdot\check{\boldsymbol{E}}) - \nabla\times(\nabla\times\check{\boldsymbol{E}}) - \frac{\partial^2\check{\boldsymbol{E}}}{\partial t^2} = \mu_0\left(\nabla\check{J}_0 + \frac{\partial\check{\boldsymbol{J}}}{\partial t}\right) \tag{1}$$

- Transform into frequency domain and neglect higher order terms $O(\epsilon^2)$,

$$\left(2ik\frac{\partial}{\partial s} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{2k^2}{\rho}x\right)E_x - \mu_0\frac{\partial J_0}{\partial x} = C_x - \frac{\partial^2 E_x}{\partial s^2} \tag{2}$$

Effect of magnet edge C_x is negligible.

$$C_x = \left(\frac{\partial}{\partial s}\frac{1}{\rho}\right)E_s + x\left(\frac{\partial}{\partial s}\frac{1}{\rho}\right)ikE_x + x\left(\frac{\partial}{\partial s}\frac{1}{\rho}\right)\frac{\partial E_x}{\partial s} \tag{3}$$

Assuming that $\boldsymbol{E}(x,y,s)$ depends on s weakly, then neglect $\partial^2\boldsymbol{E}/\partial s^2$,

$$\frac{\partial\boldsymbol{E}_\perp}{\partial s} = \frac{i}{2k}\left[\left(\nabla_\perp^2 + \frac{2k^2x}{\rho}\right)\boldsymbol{E}_\perp - \mu_0\nabla_\perp J_0\right] \tag{4}$$

– Equation of Evolution –

where $\boldsymbol{E}_\perp = (E_x, E_y)$, $\nabla_\perp = (\partial_x, \partial_y)$.

Mesh size

- Generally, mesh size must be

  $\Delta x \ll \lambda/2\pi$ (wavelength)

  in EM-field analysis.

- Our method ignores $\partial^2 \boldsymbol{E}/\partial s^2$

  $\Leftrightarrow e^{ik(s+t)}$ is ignored

  $\Rightarrow e^{ik(s-t)}$ can be factored out.

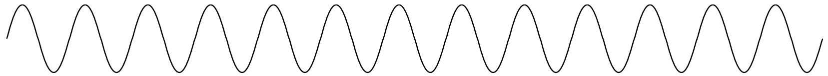
$$\check{\boldsymbol{E}} \propto \boldsymbol{E}(x, y, s; k)\, e^{ik(s-t)}$$

$\Rightarrow$ deal only with $\boldsymbol{E}(x, y, s; k)$

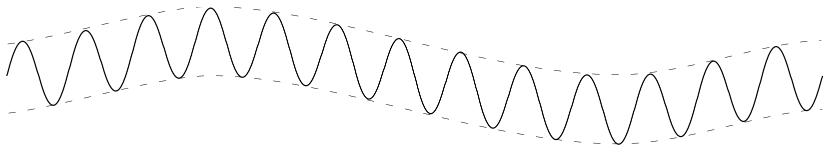
Mesh size can be larger than the actual wave length.

$$\Delta x, \Delta y, \Delta s > \lambda$$

$e^{ik(s-t)}$:sinusoidal wave

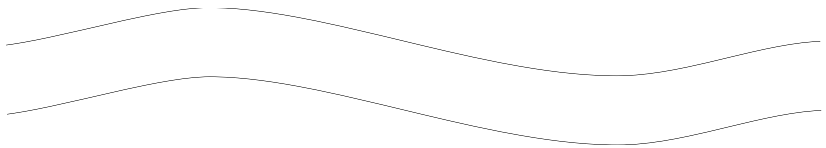


$\check{E}(\boldsymbol{x}; t)$:actual wave



$\Downarrow$ Fourier Transform

$E(\boldsymbol{x}; k)$



slow change

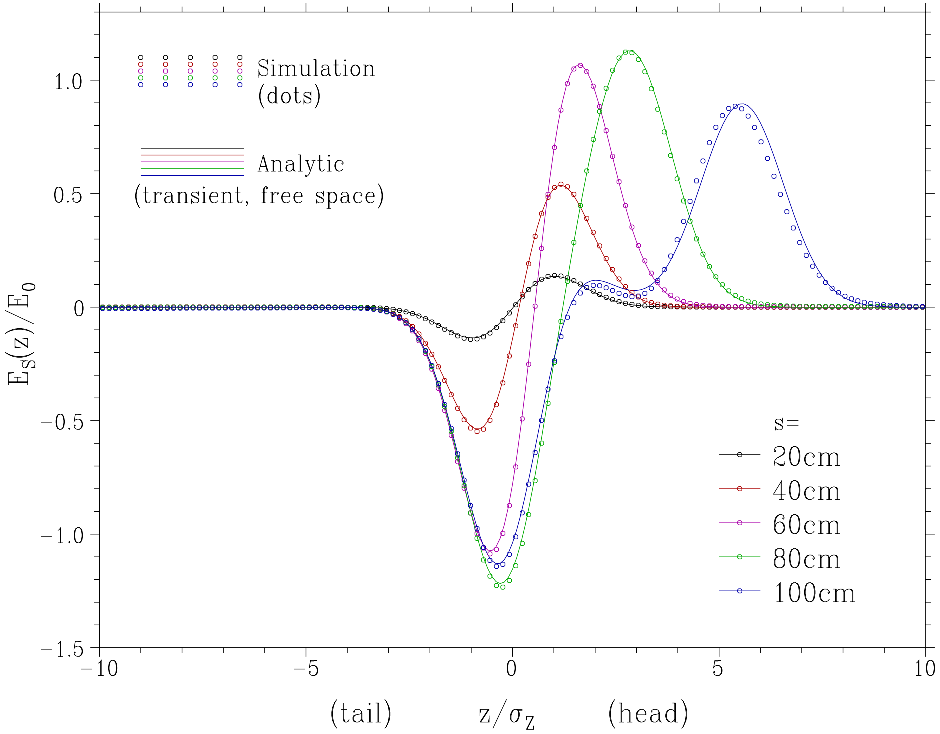
Longitudinal field

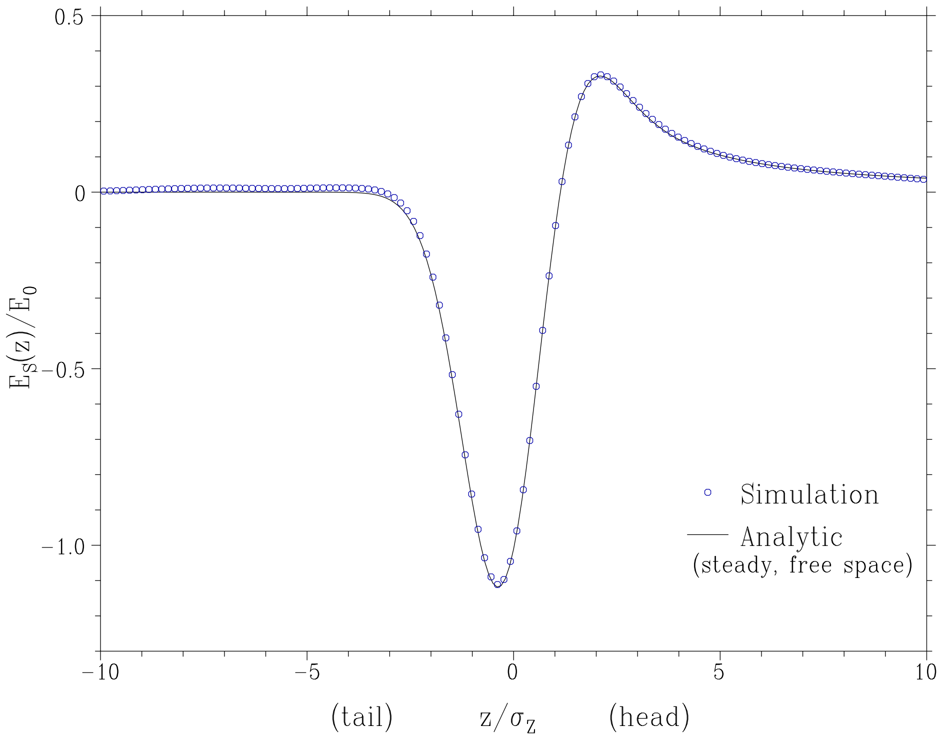
Good Agreement
with analytic solutions

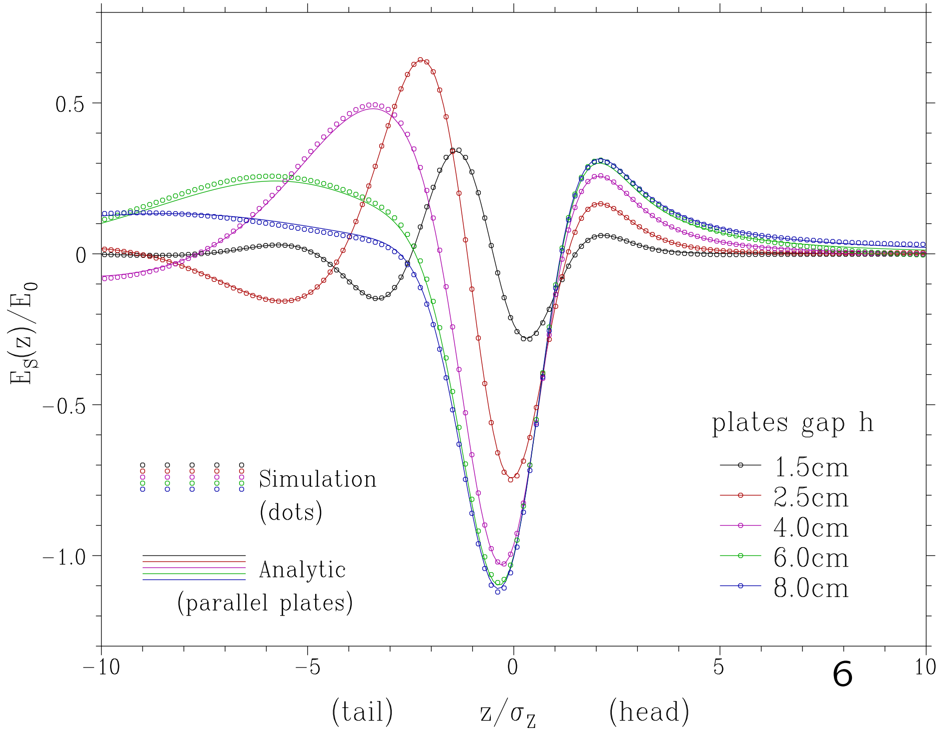
∗ Transient state →

∗ Shielded CSR ↘

∗ Steady CSR in free space ↓



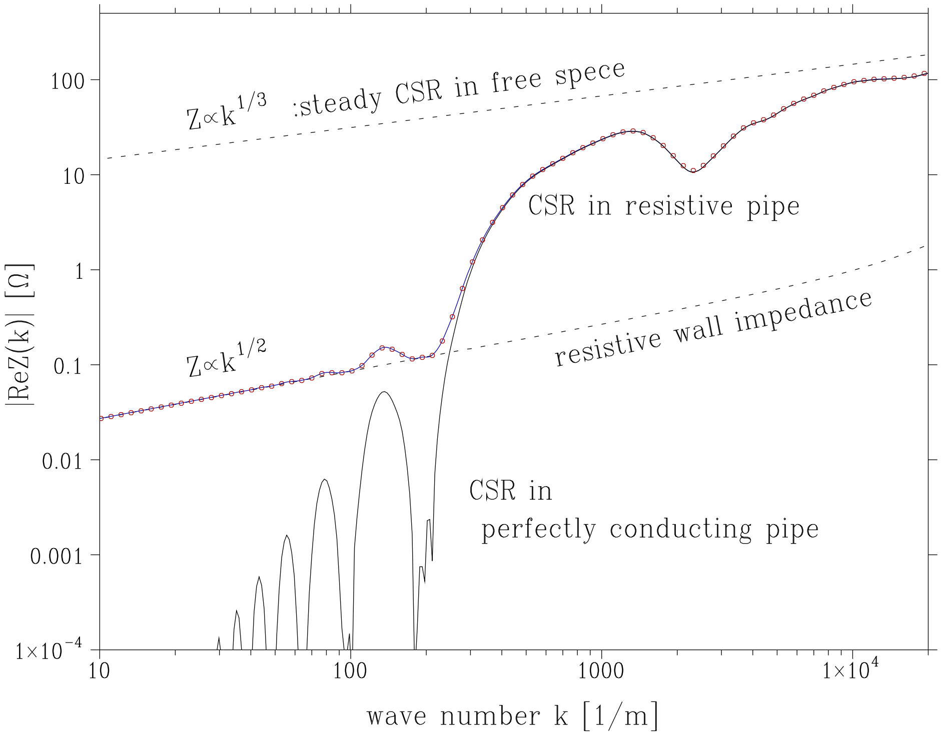




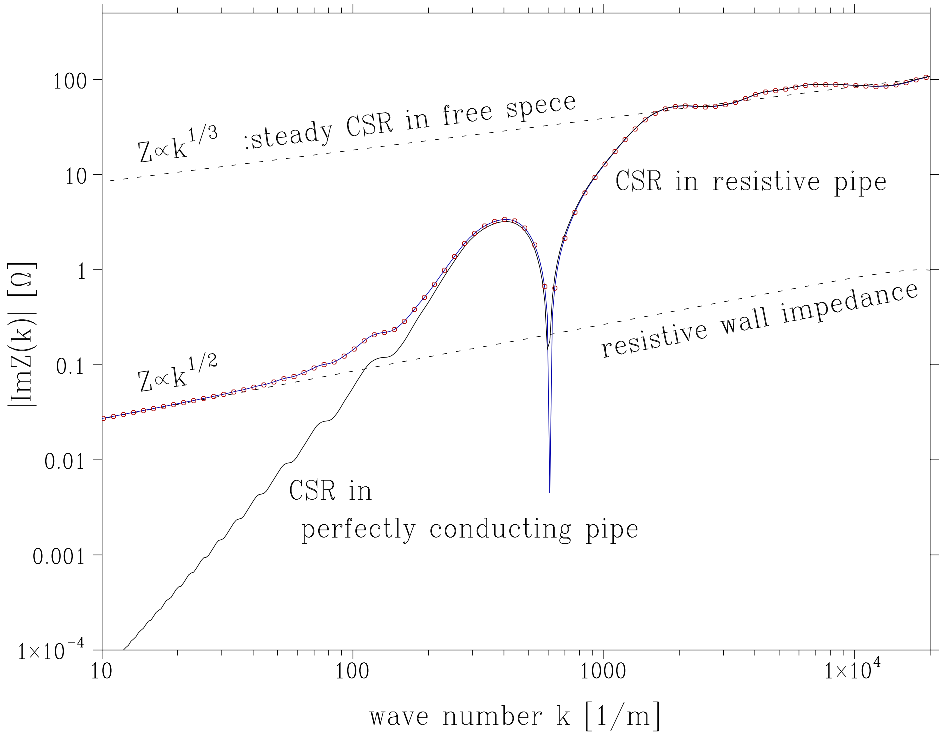
Impedance of CSR&ResistiveWall

Longitudinal impedance in a bent copper pipe (60mm square)

Real part



Imaginary part



Low $\omega \rightarrow$ Resistive wall: $Z_{||}(k) = \frac{\sqrt{Z_0/\sigma_c}}{2\pi r} e^{-i\pi/4}\sqrt{k}$

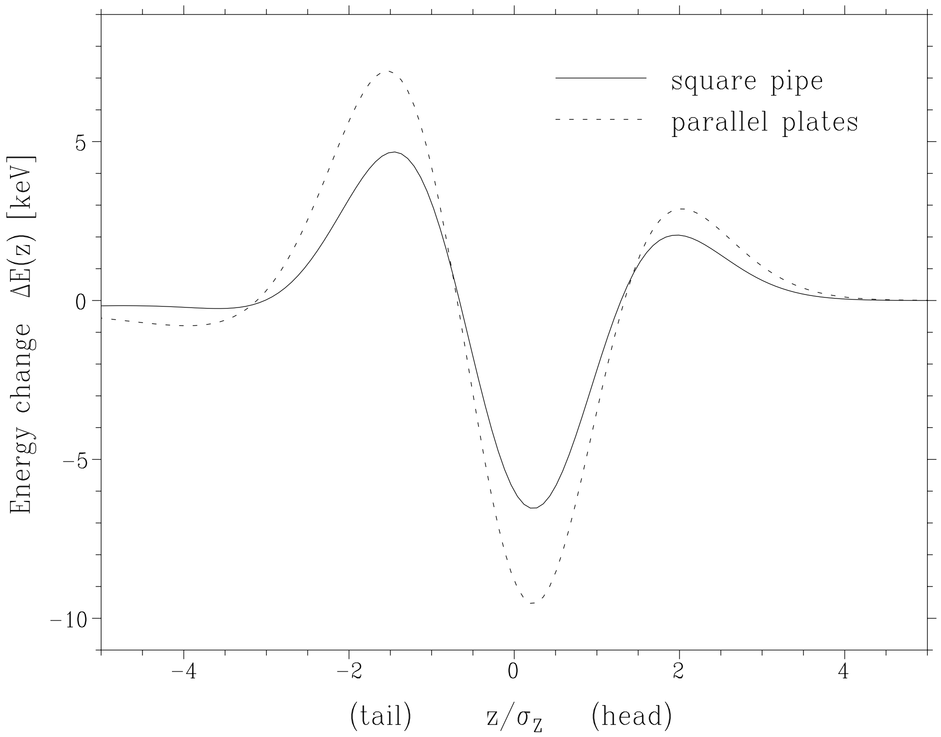
High $\omega \rightarrow$ CSR in free space: $Z_{||}(k) = \frac{Z_0}{2\pi}\Gamma\left(\frac{2}{3}\right) e^{i\pi/6}\left(\frac{k}{3\rho^2}\right)^{1/3}$

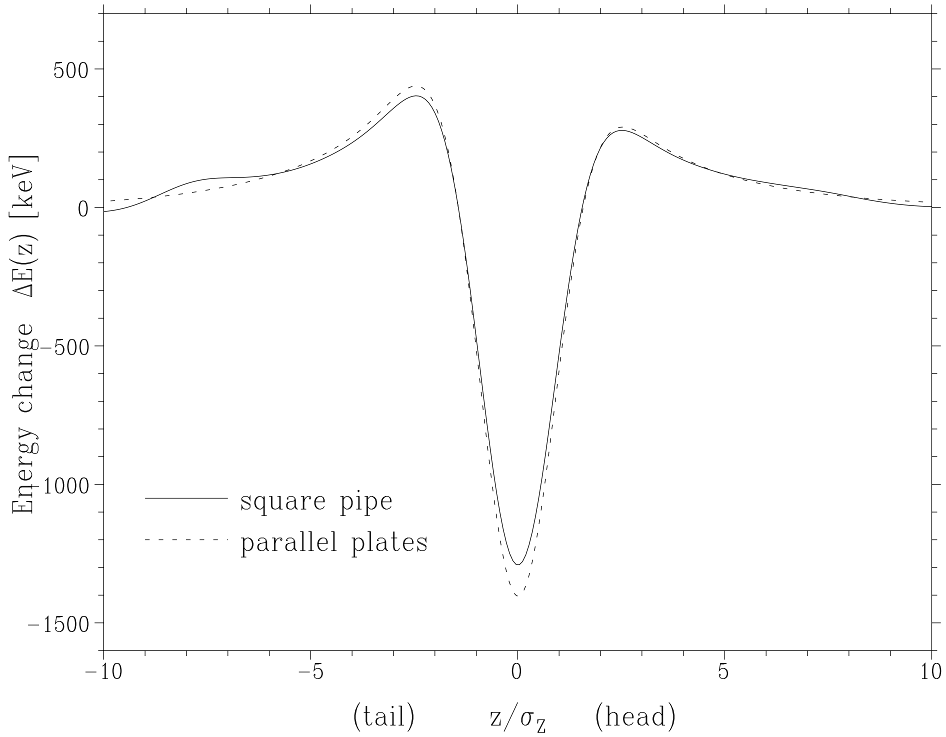
Square pipe and Parallel plates

chamber size: $w \times h = 94 \times 94$mm (square: solid line)

$w \times h = 400 \times 94$mm ($\sim$ paralell plates: dotted line)

bunch length: $\sigma_z = 3$mm $\sigma_z = 0.3$mm



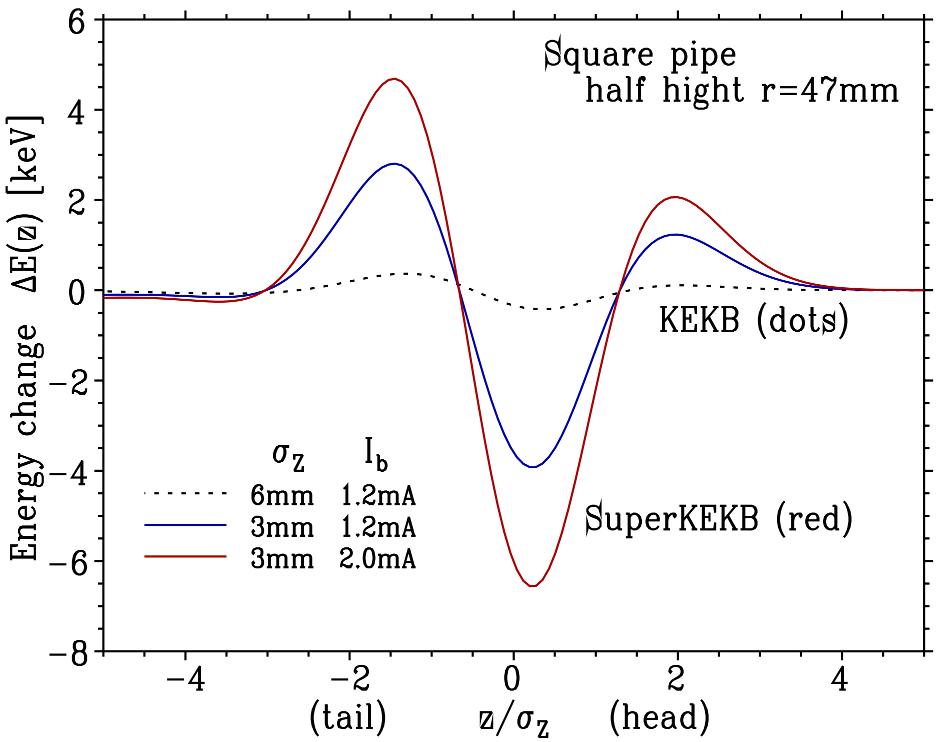


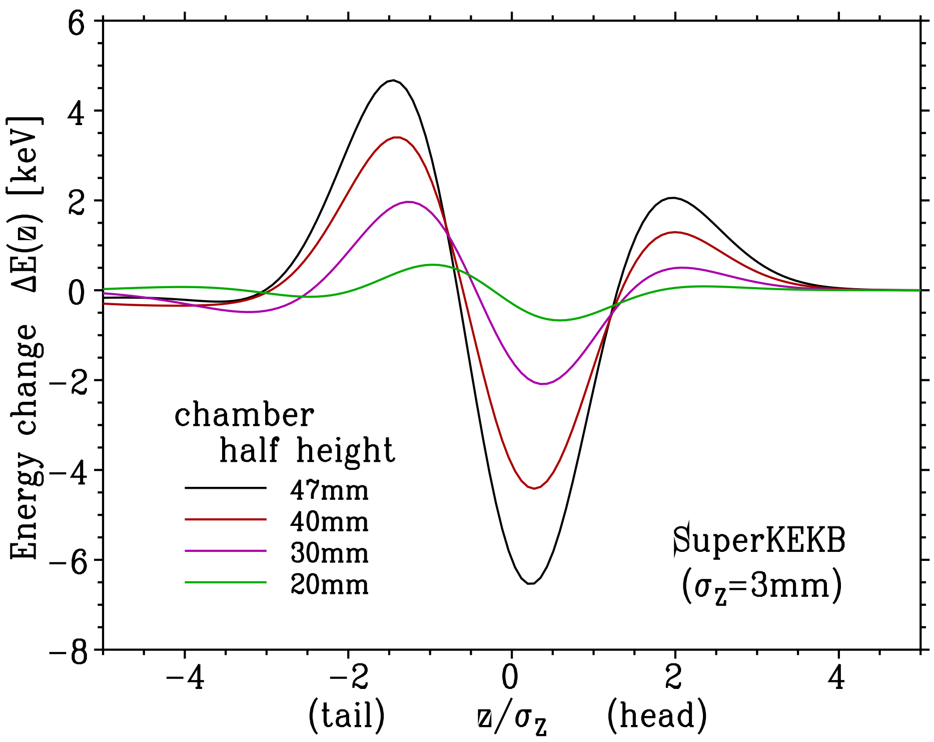
difference: $\Delta = 46\%$ $\Delta = 8.8\%$

CSR in SuperKEKB

	KEKB		SuperKEKB
Bunch length :	$\sigma_z = 6\text{mm}$	$\Rightarrow$	3mm
Bunch current :	$I_b = 1.2\text{mA}$	$\Rightarrow$	2mA ($\approx$ 20nC)

Energy change due to CSR in a bending magnet



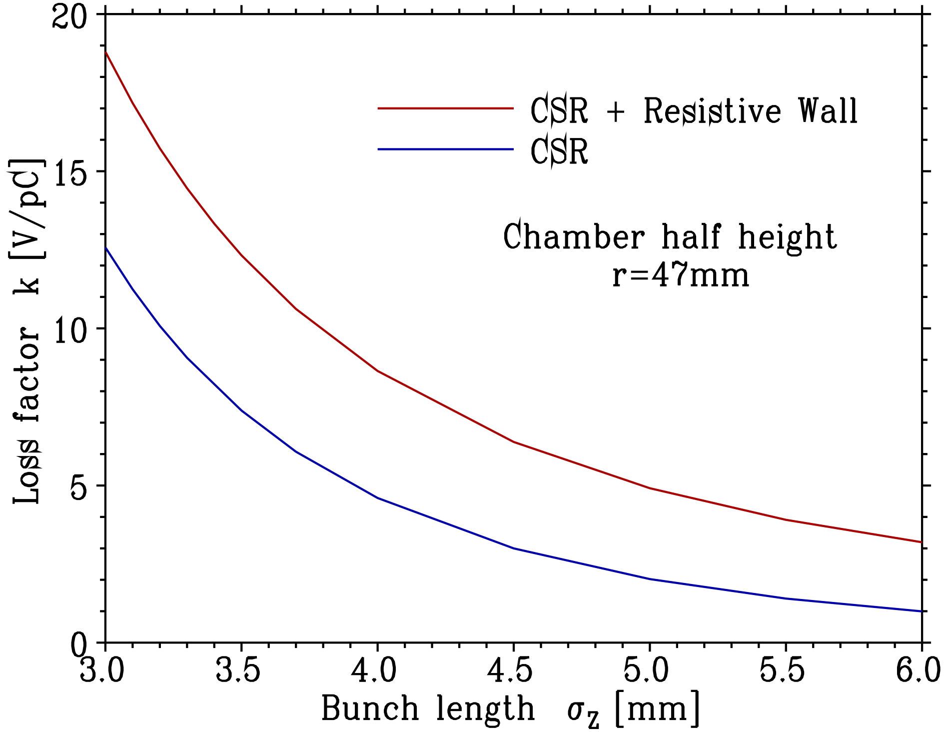


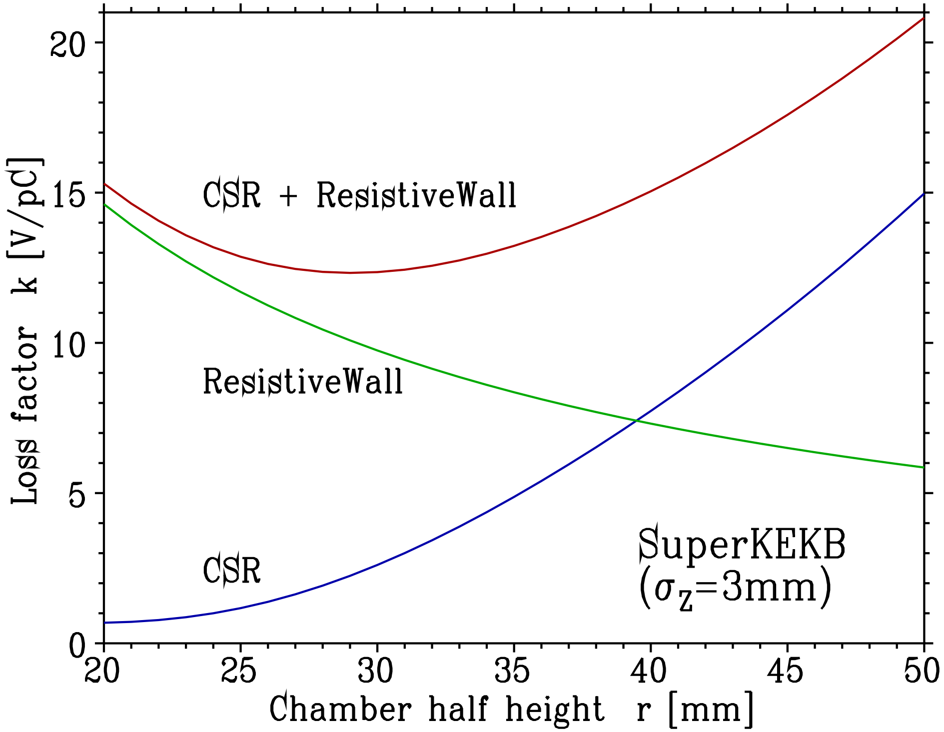
KEKB $\Rightarrow$ SuperKEKB

14 times larger ΔE

CSR can be suppressed by using chambers of small cross section.

Loss factor due to CSR and Resistive Wall wakefield



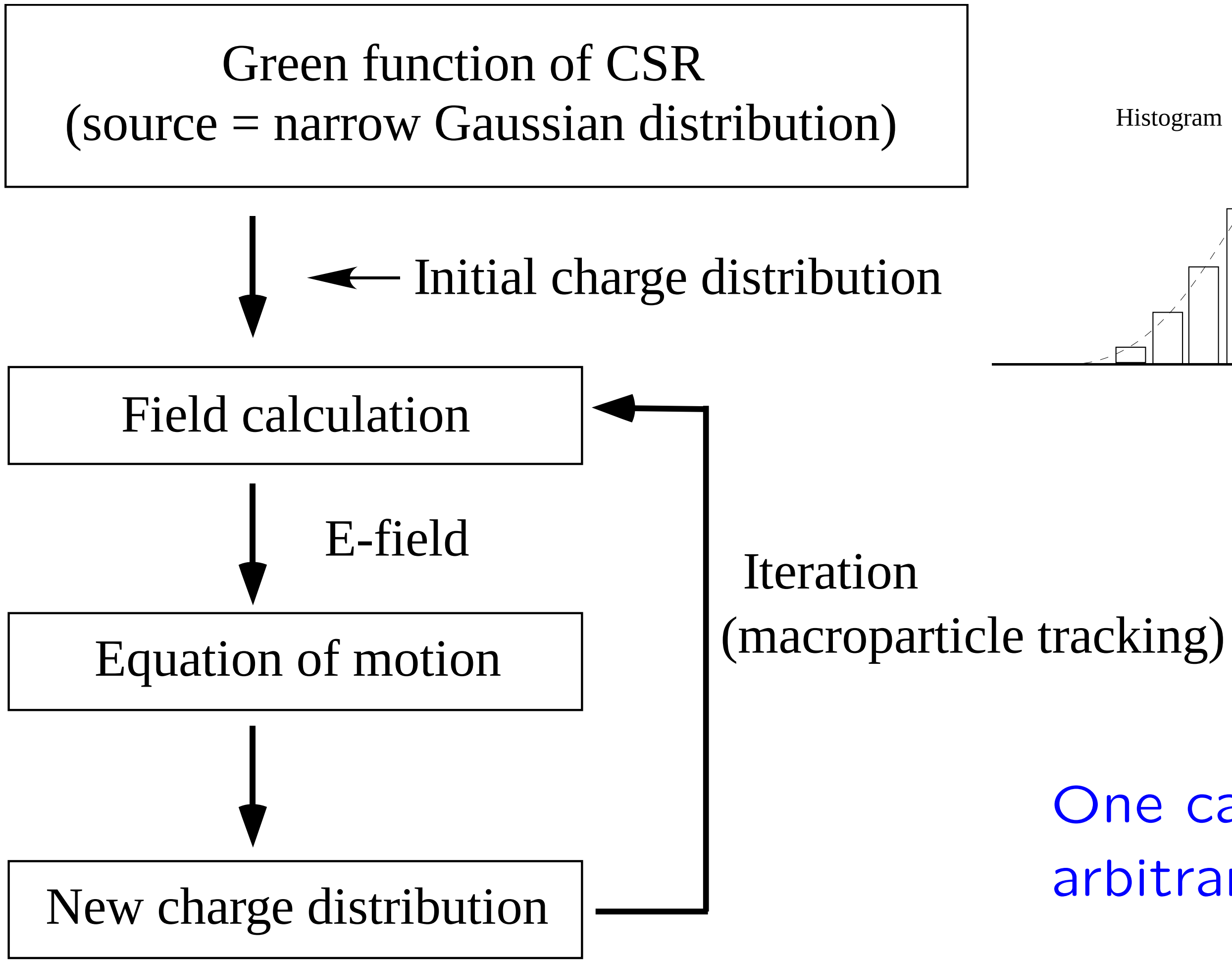


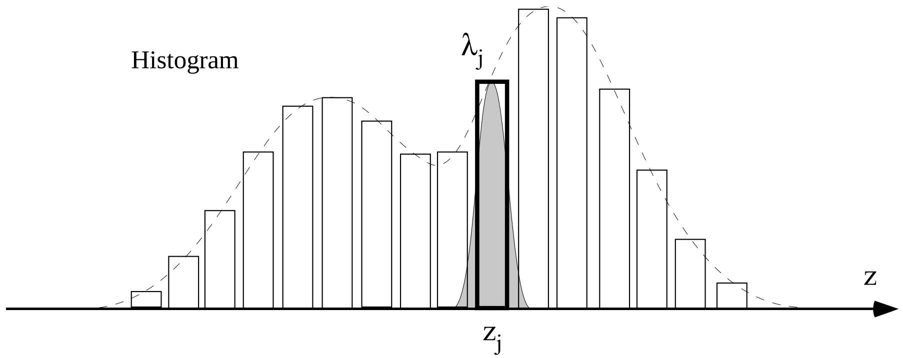
$\boxed{\sigma_z = 3\text{mm}}$	$\boxed{\sigma_z = 6\text{mm}}$
CSR	CSR
$k = 12.6\,\text{V/pC}$	$k = 1.0\,\text{V/pC}$
CSR + RW	CSR + RW
$k = 18.8\,\text{V/pC}$	$k = 3.2\,\text{V/pC}$

Loss factor due to CSR+RW is always larger than 12.3 V/pC.

The minimum value is determined by the dipole magnets (ρ, ℓ_m).

Variation of bunch distribution





One can calculate CSR for arbitrary bunch distribution.

Longitudinal Instability in SuperKEKB LER

- Field calculation of CSR = Paraxial Approximation in a beam pipe
  T.Agoh, K.Yokoya, Phys.Rev.ST-AB, 7, 054403 (2004)
- Equations of Longitudinal Motion

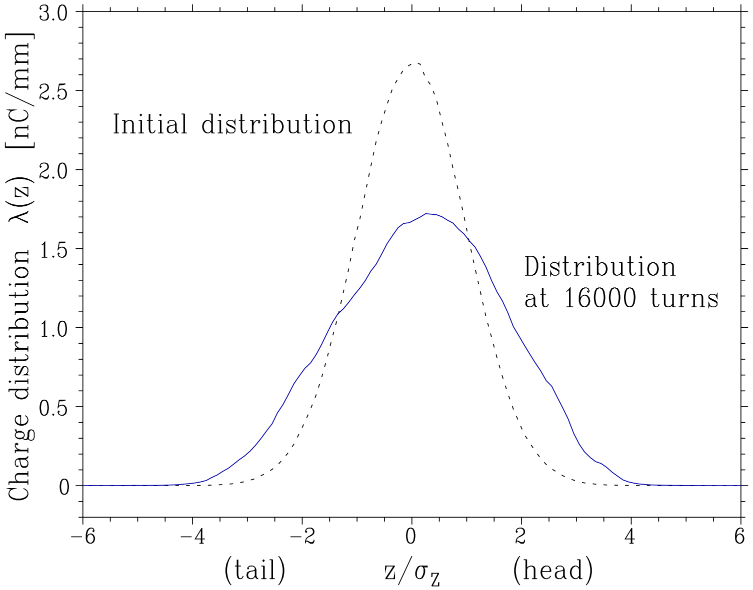
$$\begin{cases} z' = -\eta\delta \\ \delta' = \dfrac{(2\pi\nu_s)^2}{\eta C^2} z - \dfrac{2U_0}{CE_0}\delta + Q + \mathsf{CSR} + (\mathsf{RW}) \end{cases}$$

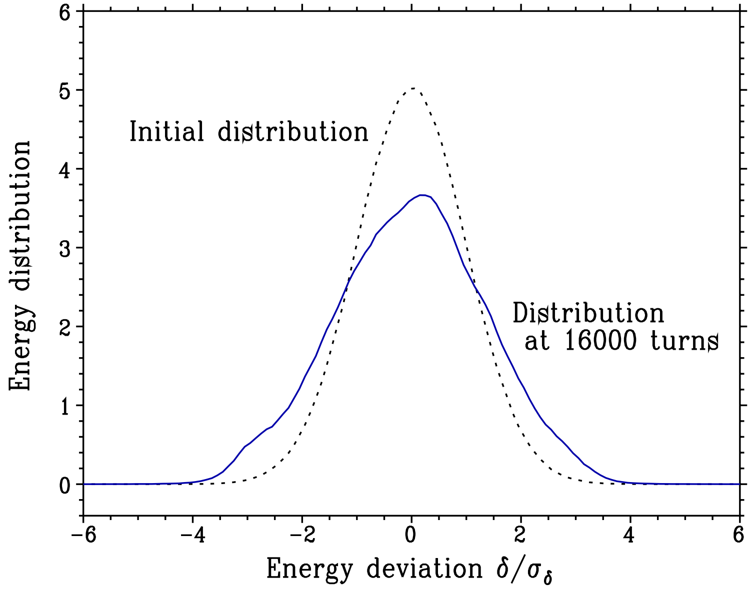
- 134 bends in the arc section are considered for CSR, but CSR in wiggler is ignored.
  (It should be considered.)
- Wiggler is taken into account in computing the radiation damping U_0.
- Copper pipe of square cross section
  (Actual one is round.)
- RW = Resistive Wall wakefield
  in the straight section
- Initial condition = Equilibrium without CSR, RW

- parameters

$E_0 = 3.5\,\mathrm{GeV}$
$C = 3016.26\,\mathrm{m}$
$\sigma_z = 3\,\mathrm{mm}$
$\sigma_\delta = 7.1 \times 10^{-4}$
$V_{\mathrm{rf}} = 15\,\mathrm{MV}$
$\omega_{\mathrm{rf}} = 508.887\,\mathrm{Hz}$
$h = 5120$
$\alpha = 2.7 \times 10^{-4}$
$U_0 = 1.23\,\mathrm{MeV/turn}$
$\nu_s = 0.031$

• Bunch distribution • Energy spread $(r = 47\text{mm})$





$W_{||} = \text{CSR}$

Initial

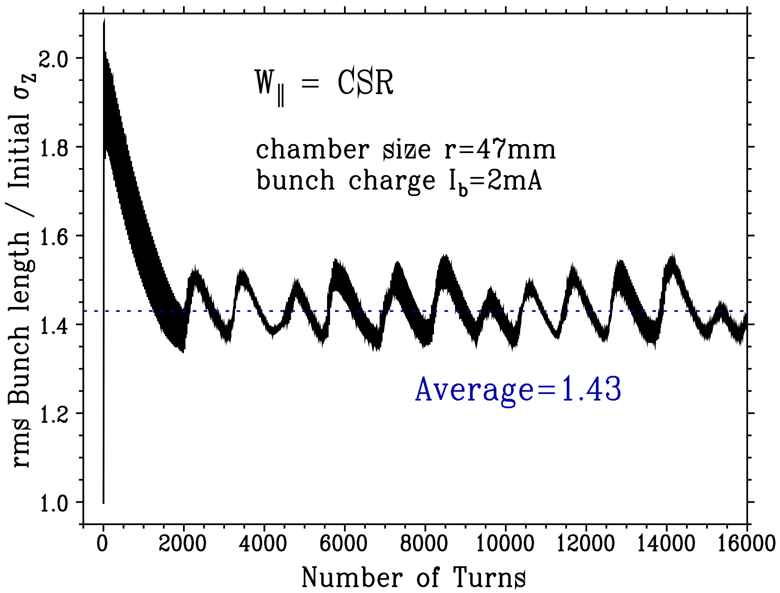
Bunch length

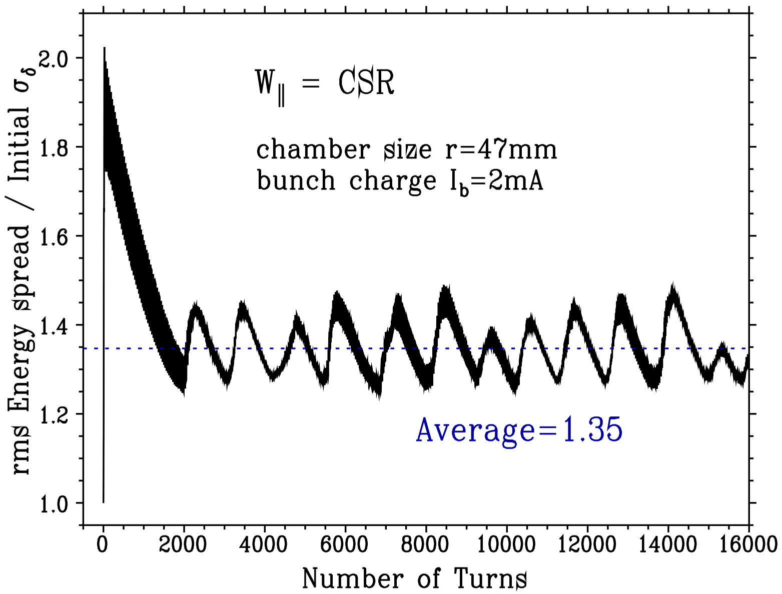
$\sigma_z = 3.0\text{mm}$

Initial

Energy spread

$\sigma_\delta = 7.1 \times 10^{-4}$





Average

Bunch length

$\sigma_z \sim 4.3\text{mm}$

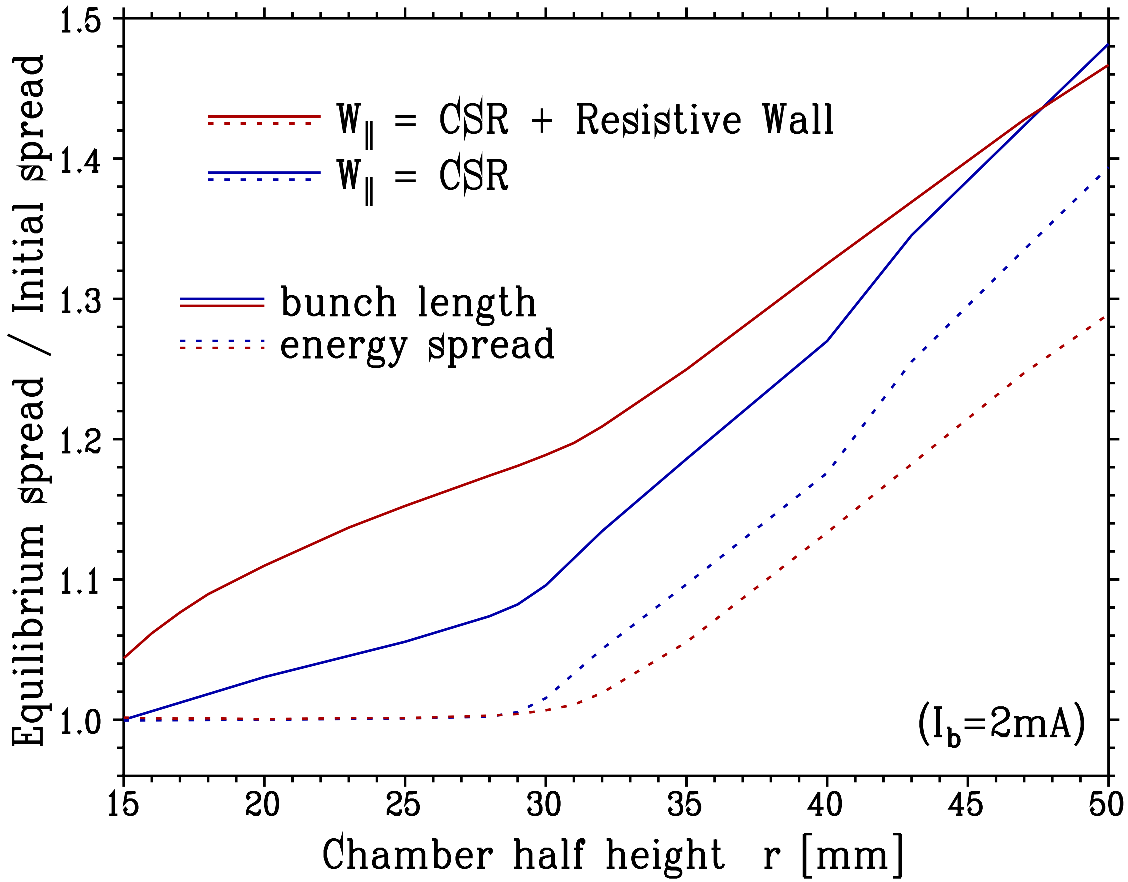
Average

Energy spread

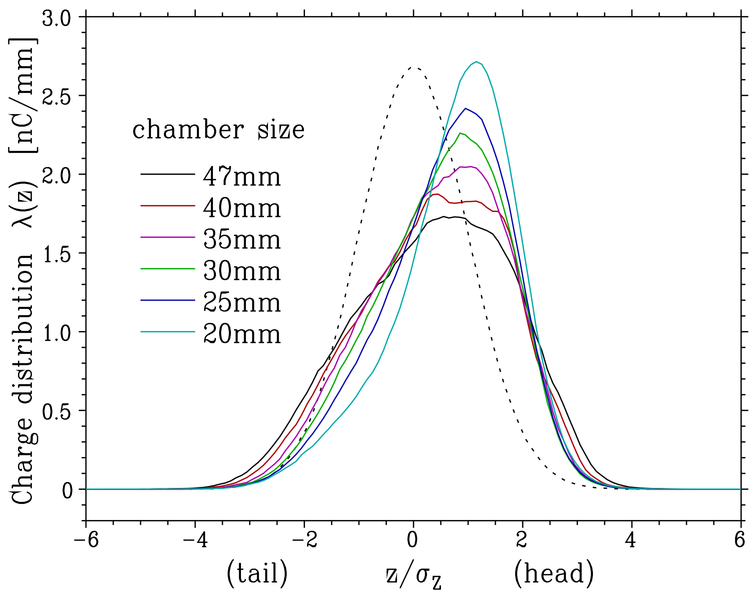
$\sigma_\delta \sim 9.6 \times 10^{-4}$

Threshold for chamber size

- rms Bunch length and Energy spread



- Longitudinal bunch distribution

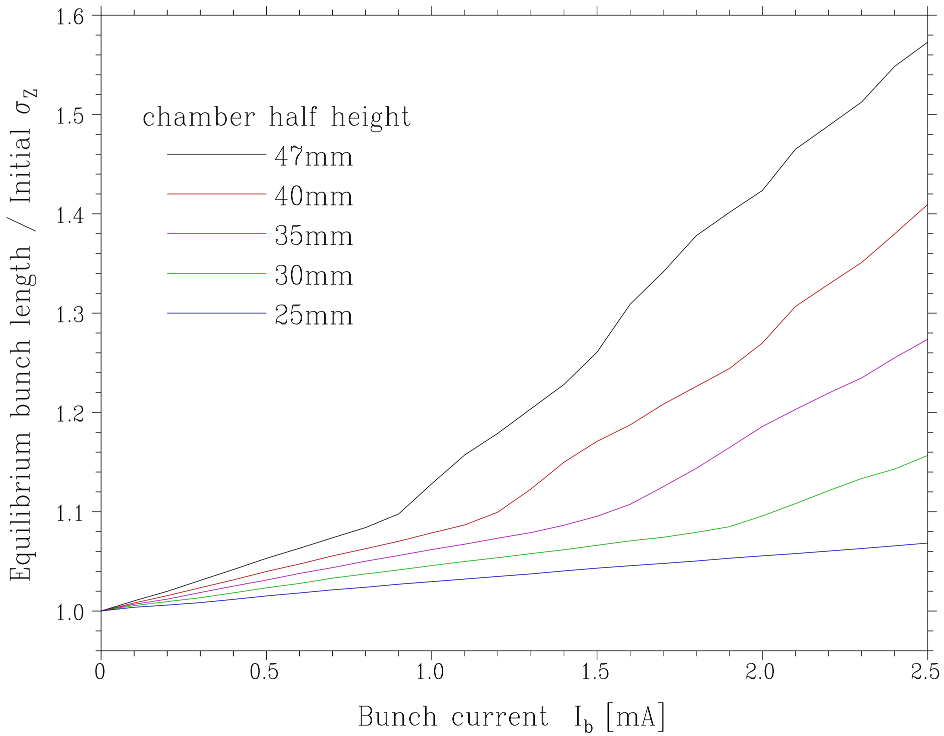


The bunch leans forward because of the energy loss due to the resistive wall.

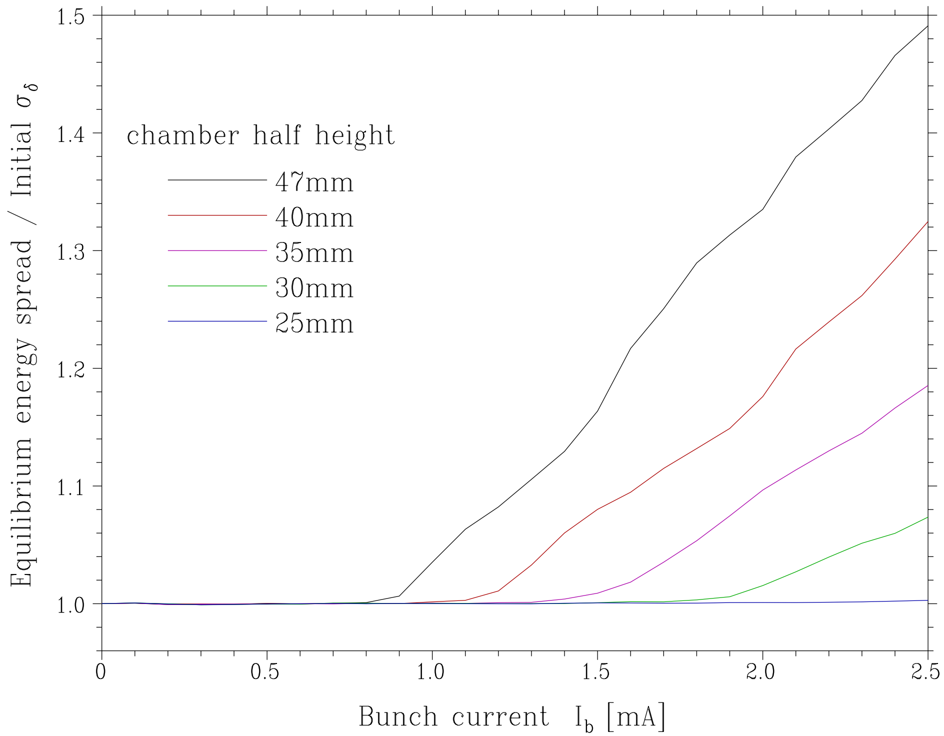
Threshold for the chamber half height is $r_{\text{th}} \sim 30$mm, when the bunch current is $I_b = 2$mA ($Ne \sim 20\,$nC).

Threshold of longitudinal instability

Bunch length vs Bunch current
Initial $\sigma_z = 3\text{mm}$



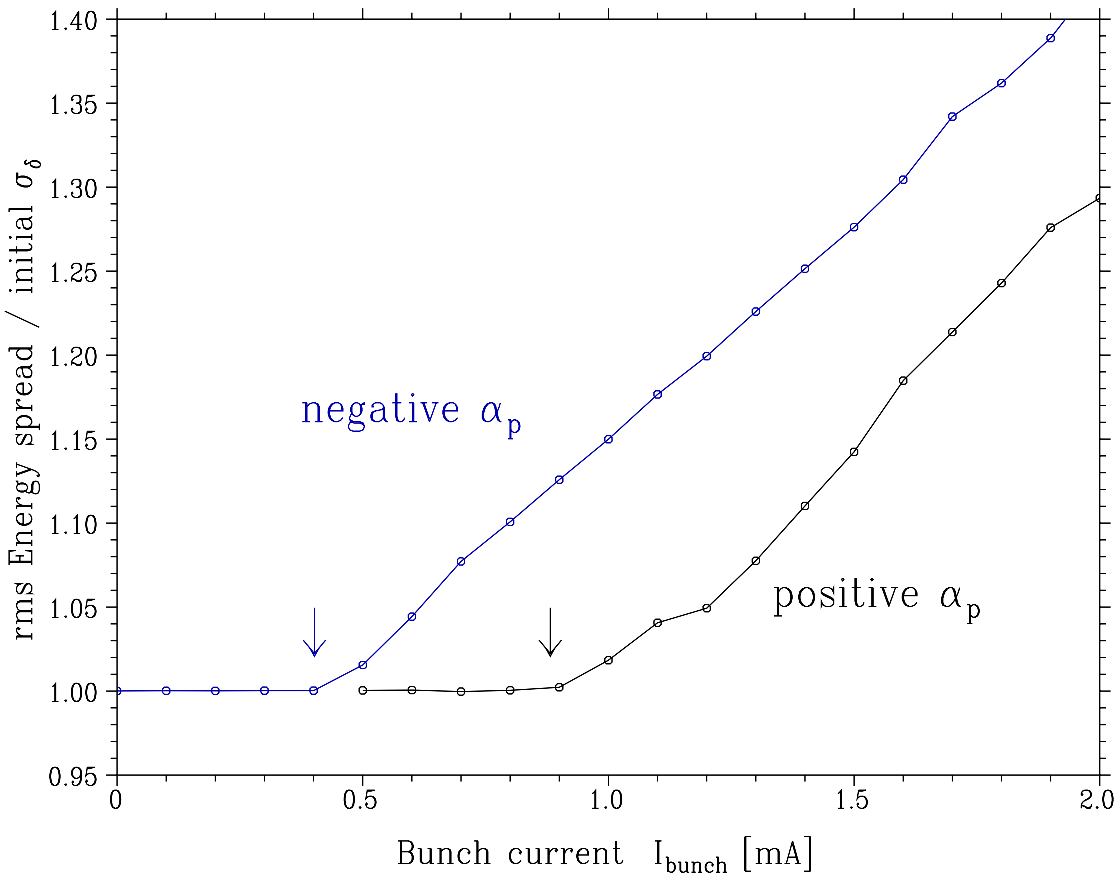
Energy spread vs Bunch current
Initial $\sigma_\delta = 7.1 \times 10^{-4}$



The length increases fast, and the energy spread starts increasing above a threshold which is determined by the chamber size.

The limit current is 0.9mA ($Ne \sim$ 9nC) in the chamber of $r = 47\text{mm}$.

Negative momentum compaction factor

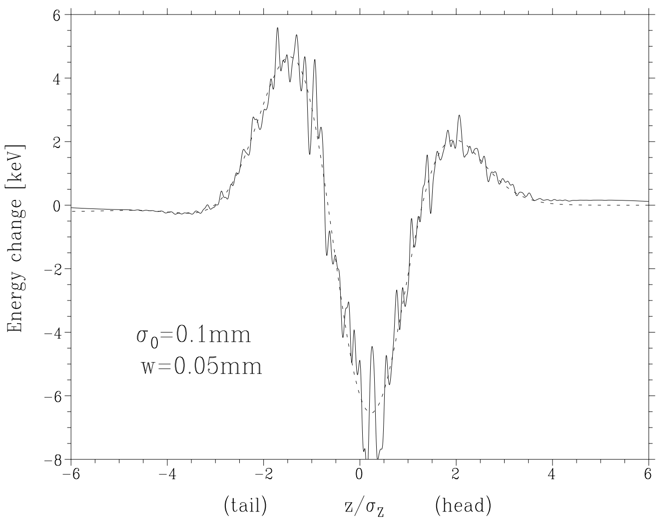


Though we do not understand the mechanism, negative α_p may not work well.

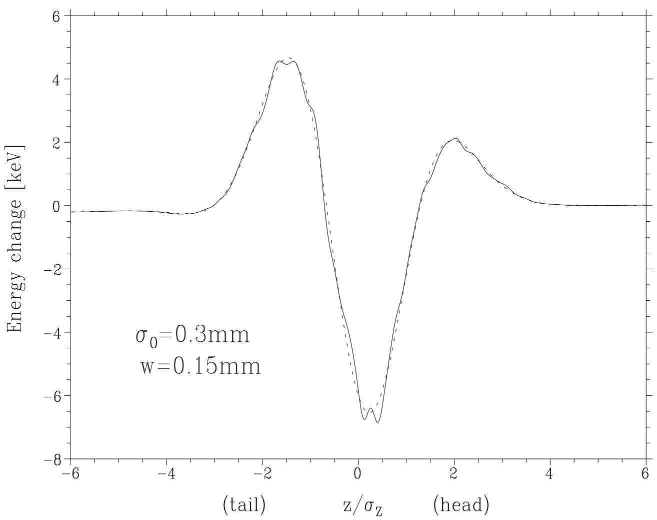
4. Numerical problem

Superposition of Gauusian Green function

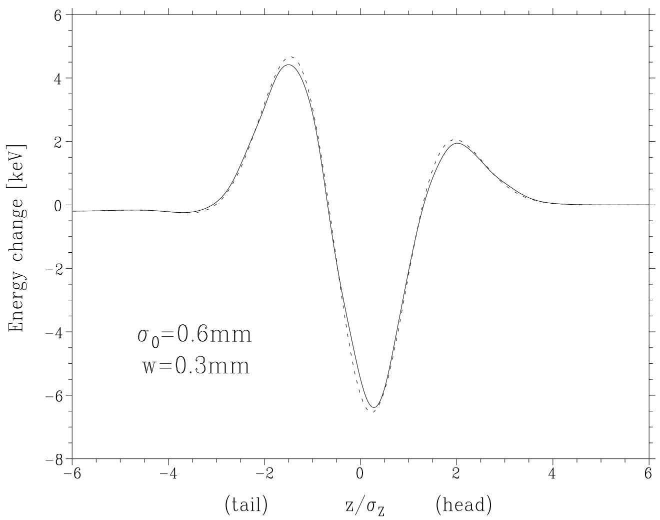
Number of macroparticles: $N = 10^6$
bunch length: $\sigma_z = 3\text{mm}$



$\sigma_0 = 0.1\text{mm}$
$\Delta b = 0.05\text{mm}$



$\sigma_0 = 0.3\text{mm}$
$\Delta b = 0.15\text{mm}$

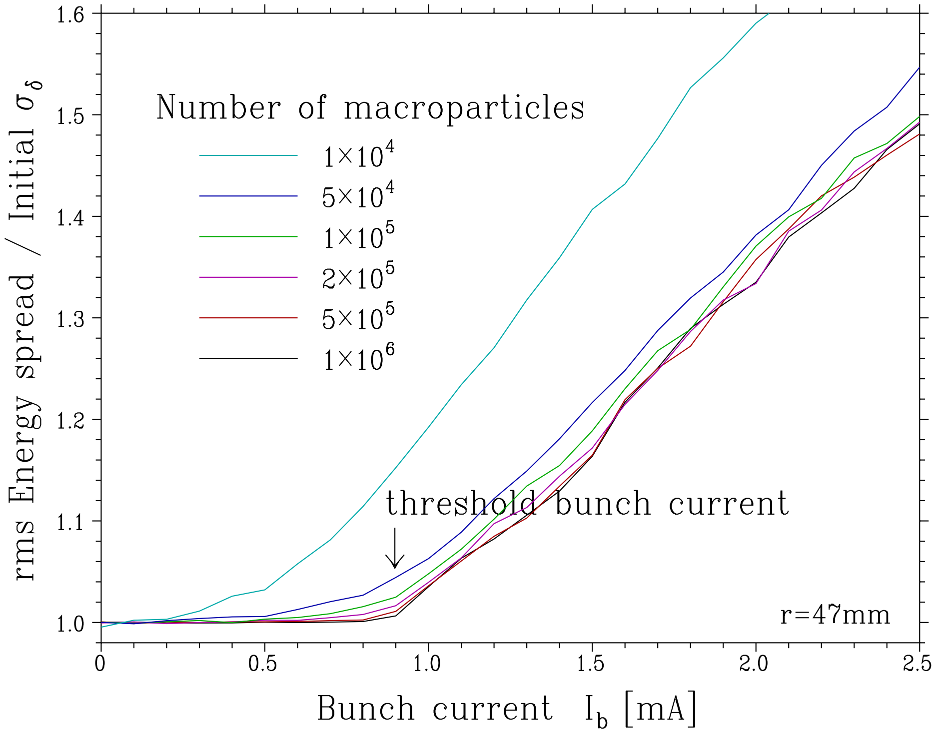


$\sigma_0 = 0.6\text{mm}$
$\Delta b = 0.3\text{mm}$

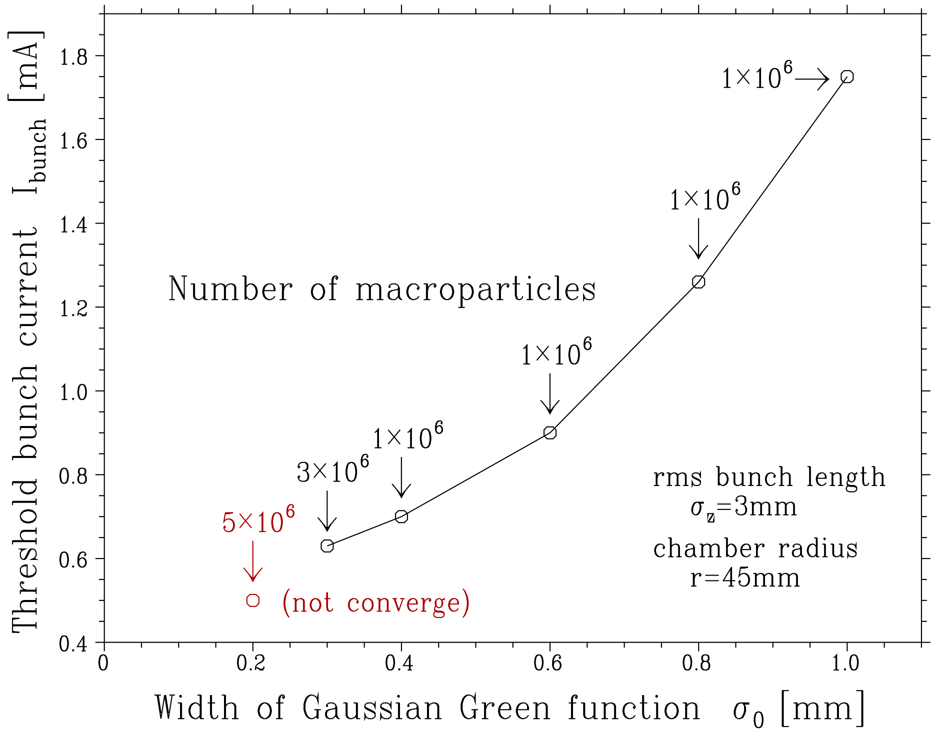
Statistical fluctuation must be avoided.

Convergence for width of Green function

Width of Green function
$\sigma_0 = 0.6\text{mm}\ (= \sigma_z/5)$



Width of Green function
vs Threshold current



Threshold current does not converge for the width of Green function due to the computing cost. (PC, CPU = 3.2GHz)

We cannot distinguish between statistical fluctuation and physical structure in the bunch which is induced by CSR.

5. Summary

- Giving a bunch distribution, we can obtain the CSR which is shielded by a pipe-shaped vacuum chamber.
  Transient state, Resistive wall

- Because of strong shielding effect by the vacuum chamber, model using parallel plates for the vacuum chamber is not valid for calculation of CSR in storage rings.

- Macroparticle tracking simulation does not work for CSR caused longitudinal instability due to the statistical noise. (failure report)
  $*$ We will try to solve Vlasov equation.

- Though instability threshold for the bunch current is not specified, at least it is lower than 0.63mA($Ne \sim$ 6nC).
  CSR will limit the performance of SuperKEKB LER.