Longitudinal Single Bunch Instability by Coherent Synchrotron Radiation

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SBSR05 at Frascati, Nov.7-8, 2005
1. Introduction

- In the spectrum of synchrotron radiation, the components such that $\lambda \gtrsim \sigma_z$ produce **Coherent Synchrotron Radiation**. (CSR)

- Energy change of particles
  - Short range interaction
  - $\Rightarrow$ Energy spread
  - $\Rightarrow$ Single bunch instability
CSR in storage rings

Bunch length \( \sim \) a few mm

1. Strong shielding
   - Waves with wavelength \( \lambda \gtrsim \sqrt{6\hbar^3/\pi \rho} \) is suppressed.
     \( \rightarrow \) Vacuum chamber should be properly considered.

2. CSR field is transient due to finite magnet length.
   - CSR evolves in a dipole magnet \((s\text{-dependent})\).

3. Variation of the bunch distribution
   - Fine structure in the bunch
     \( \rightarrow \) CSR with short wavelength is emitted.
       (Shielding effect is weak.)
2. Calculation of CSR using paraxial approximation

Mesh calculation of \((E, B)\) in a beam pipe

(1) Begin with Maxwell equations in accel. coordinates \((x, y, s)\)

(2) Fourier transformation

\[
\tilde{f}(t - s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \, f(k) \, e^{-ik(t-s)}
\]

\Rightarrow \text{frequency domain } E(k), B(k)

(3) Approximate these equations

Paraxial Approximation

(4) Solve them numerically by finite difference (mesh calculation)

\text{pipe} = \text{boundary condition}

(5) Fourier transformation

\Rightarrow \text{time domain (GOAL)}
• From Maxwell equations,

\[ \nabla (\nabla \cdot \vec{E}) - \nabla \times (\nabla \times \vec{E}) - \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \left( \nabla \vec{J}_0 + \frac{\partial \vec{J}}{\partial t} \right) \]  

(1)

• Transform into frequency domain and neglect higher order terms \(O(\epsilon^2)\),

\[ \left( 2ik \frac{\partial}{\partial s} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{2k^2}{\rho} x \right) E_x - \mu_0 \frac{\partial J_0}{\partial x} = C_x - \frac{\partial^2 E_x}{\partial s^2} \]  

(2)

Effect of magnet edge \(C_x\) is negligible.

\[ C_x = \left( \frac{\partial}{\partial s \rho} \right) E_s + x \left( \frac{\partial}{\partial s \rho} \right) ik E_x + x \left( \frac{\partial}{\partial s \rho} \right) \frac{\partial E_x}{\partial s} \]  

(3)

Assuming that \(E(x, y, s)\) depends on \(s\) weakly, then neglect \(\partial^2 E/\partial s^2\),

\[ \frac{\partial E_\perp}{\partial s} = \frac{i}{2k} \left[ \left( \nabla_\perp^2 + \frac{2k^2 x}{\rho} \right) E_\perp - \mu_0 \nabla_\perp J_0 \right] \]  

(4)

– Equation of Evolution –

where \(E_\perp = (E_x, E_y)\), \(\nabla_\perp = (\partial_x, \partial_y)\).
Mesh size

- Generally, mesh size must be \( \Delta x \ll \lambda/2\pi \) (wavelength) in EM-field analysis.

- Our method ignores \( \partial^2 E/\partial s^2 \)
  \( \Leftrightarrow e^{ik(s+t)} \) is ignored
  \( \Rightarrow e^{ik(s-t)} \) can be factored out.

\[ \tilde{E} \propto E(x, y, s; k) e^{ik(s-t)} \]

\( \Rightarrow \) deal only with \( E(x, y, s; k) \)

Mesh size can be larger than the actual wave length.

\( \Delta x, \Delta y, \Delta s > \lambda \)
Longitudinal field

Good Agreement with analytic solutions

* Transient state $\rightarrow$

* Shielded CSR $\downarrow$

* Steady CSR in free space $\downarrow$
Impedance of CSR & Resistive Wall

Longitudinal impedance in a bent copper pipe (60mm square)

Low $\omega \rightarrow$ Resistive wall:  
$$Z_\parallel(k) = \frac{\sqrt{Z_0/\sigma c}}{2\pi r} e^{-i\pi/4} \sqrt{k}$$

High $\omega \rightarrow$ CSR in free space:  
$$Z_\parallel(k) = \frac{Z_0}{2\pi} \Gamma\left(\frac{2}{3}\right) e^{i\pi/6} \left(\frac{k}{3\rho^2}\right)^{1/3}$$
Square pipe and Parallel plates

chamber size: \( w \times h = 94 \times 94 \text{mm} \) (square: solid line)
\( w \times h = 400 \times 94 \text{mm} \) (\( \sim \) parallell plates: dotted line)

bunch length: \( \sigma_z = 3 \text{mm} \)

\( \sigma_z = 0.3 \text{mm} \)

difference: \( \Delta = 46\% \)
\( \Delta = 8.8\% \)
CSR in SuperKEKB

**KEKB**
- Bunch length: $\sigma_z = 6\text{mm}$
- Bunch current: $I_b = 1.2\text{mA}$

**SuperKEKB**
- $\sigma_z = 3\text{mm}$
- $I_b = 2\text{mA}$ ($\approx 20\text{nC}$)

Energy change due to CSR in a bending magnet

![Graphs showing energy change due to CSR in KEKB and SuperKEKB](image)

KEKB $\Rightarrow$ SuperKEKB
14 times larger $\Delta E$

CSR can be suppressed by using chambers of small cross section.
Loss factor due to CSR and Resistive Wall wakefield

\[ \sigma_z = 3\text{mm} \]

CSR

\[ k = 12.6 \text{ V/pC} \]

CSR + RW

\[ k = 18.8 \text{ V/pC} \]

\[ \sigma_z = 6\text{mm} \]

CSR

\[ k = 1.0 \text{ V/pC} \]

CSR + RW

\[ k = 3.2 \text{ V/pC} \]

Loss factor due to CSR+RW is always larger than 12.3 V/pC.

The minimum value is determined by the dipole magnets \((\rho, \ell_m)\).
Variation of bunch distribution

Green function of CSR
(source = narrow Gaussian distribution)

Initial charge distribution

Field calculation

E-field

Equation of motion

Iteration
(macroparticle tracking)

New charge distribution

One can calculate CSR for arbitrary bunch distribution.
Longitudinal Instability in SuperKEKB LER

- Field calculation of CSR = Paraxial Approximation in a beam pipe

- Equations of Longitudinal Motion

\[
\begin{align*}
  z' &= -\eta \delta \\
  \delta' &= \left(\frac{2\pi \nu_s}{\eta C^2}\right)^2 z - \frac{2U_0}{CE_0} \delta + Q + \text{CSR} + (RW)
\end{align*}
\]

- 134 bends in the arc section are considered for CSR, but CSR in wiggler is ignored. (It should be considered.)

- Wiggler is taken into account in computing the radiation damping \( U_0 \).

- Copper pipe of square cross section (Actual one is round.)

- \( RW \) = Resistive Wall wakefield in the straight section

- Initial condition = Equilibrium without CSR, RW

- parameters
  \( E_0 \) = 3.5 GeV
  \( C \) = 3016.26 m
  \( \sigma_z \) = 3 mm
  \( \sigma_\delta \) = \( 7.1 \times 10^{-4} \)
  \( V_{rf} \) = 15 MV
  \( \omega_{rf} \) = 508.887 Hz
  \( h \) = 5120
  \( \alpha \) = \( 2.7 \times 10^{-4} \)
  \( U_0 \) = 1.23 MeV/turn
  \( \nu_s \) = 0.031
- Bunch distribution

\( B_{\text{dist}} \)

\( r = 47 \text{mm} \)

- Energy spread

\( W_{\parallel} = \text{CSR} \)

Initial

Bunch length

\( \sigma_z = 3.0 \text{mm} \)

Initial

Energy spread

\( \sigma_\delta = 7.1 \times 10^{-4} \)

Average

Bunch length

\( \sigma_z \sim 4.3 \text{mm} \)

Average

Energy spread

\( \sigma_\delta \sim 9.6 \times 10^{-4} \)
Threshold for chamber size

- **rms Bunch length and Energy spread**

  - $W_{\|} = \text{CSR} + \text{Resistive Wall}$
  - $W_{\|} = \text{CSR}$

  ![Graph showing Equilibrium spread / Initial spread vs Chamber half height](image)

  - **bunch length**
  - **energy spread**

  ($I_b = 2\text{mA}$)

- **Longitudinal bunch distribution**

  ![Graph showing Charge distribution vs $(\sigma/\sigma_z)$](image)

  - Chamber size: 47mm, 40mm, 35mm, 30mm, 25mm, 20mm

  The bunch leans forward because of the energy loss due to the resistive wall.

Threshold for the chamber half height is $r_{th} \sim 30\text{mm}$, when the bunch current is $I_b = 2\text{mA}$ ($N_e \sim 20\text{nC}$).
Threshold of longitudinal instability

**Bunch length vs Bunch current**

Initial $\sigma_z = 3\text{mm}$

**Energy spread vs Bunch current**

Initial $\sigma_\delta = 7.1 \times 10^{-4}$

The length increases fast, and the energy spread starts increasing above a threshold which is determined by the chamber size.

The limit current is $0.9\text{mA}$ ($N_e \sim 9\text{nC}$) in the chamber of $r = 47\text{mm}$.  

Negative momentum compaction factor

Though we do not understand the mechanism, negative $\alpha_p$ may not work well.
4. Numerical problem

Superposition of Gaussian Green function

Number of macroparticles: $N = 10^6$

bunch length: $\sigma_z = 3\text{mm}$

Statistical fluctuation must be avoided.
Convergence for width of Green function

Width of Green function
\[ \sigma_0 = 0.6\text{mm} \quad (= \sigma_z/5) \]

Threshold current does not converge for the width of Green function due to the computing cost. (PC, CPU = 3.2GHz)

We cannot distinguish between statistical fluctuation and physical structure in the bunch which is induced by CSR.
5. Summary

- Giving a bunch distribution, we can obtain the CSR which is shielded by a pipe-shaped vacuum chamber.
  
  Transient state, Resistive wall

- Because of strong shielding effect by the vacuum chamber, model using parallel plates for the vacuum chamber is not valid for calculation of CSR in storage rings.

- Macroparticle tracking simulation does not work for CSR caused longitudinal instability due to the statistical noise. (failure report)
  
  * We will try to solve Vlasov equation.

- Though instability threshold for the bunch current is not specified, at least it is lower than 0.63 mA ($N_e \sim 6 \text{nC}$).
  
  CSR will limit the performance of SuperKEKB LER.