Probing the topological exciton condensate *via* Coulomb drag

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"Quantum Field Theory Aspects of Condensed Matter Physics"
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Collaborators

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Giovanni Vignale (UMO, USA)

This talk mainly based on:

- M.P. Mink, H.T.C. Stoof, R.A. Duine, M. Polini, and G. Vignale, arXiv:1108.2298v1 (2011)

See also:

- R.A. Duine, M. Polini, H.T.C. Stoof, and G. Vignale, Phys. Rev. Lett. **104**, 220403 (2010)
- R.A. Duine, M. Polini, A. Raoux, H.T.C. Stoof, and G. Vignale, New J. Phys. **13**, 045010 (2011)

Universiteit Utrecht

MISSOURI

Outline

- **Drag-type experiments in CMP and cold atoms**
  - Coulomb drag between closely spaced electronic circuits ~ 1977
  - Coulomb drag and semiconductor-bilayer exciton condensates ~ 1996
  - Many-body effects in spin-polarized transport: spin drag ~ 2000
  - Spin drag in electron liquids and cold atom gases: experiments ~ 2005-2011
- **Theory of Coulomb drag in the vicinity of exciton condensation**
  - Exciton condensates in topological-insulator thin films (and double-layer graphene)
  - Model Hamiltonian and Boltzmann equation
  - Effective interactions: the Bethe-Salpeter equation
- **Numerical results and discussion**
  - Drag resistivity close to the critical temperature
  - Mean-field critical temperature with static screening
  - Comparison of these results with those for spin drag close to a ferromagnetic instability
- **Conclusions and future perspectives**

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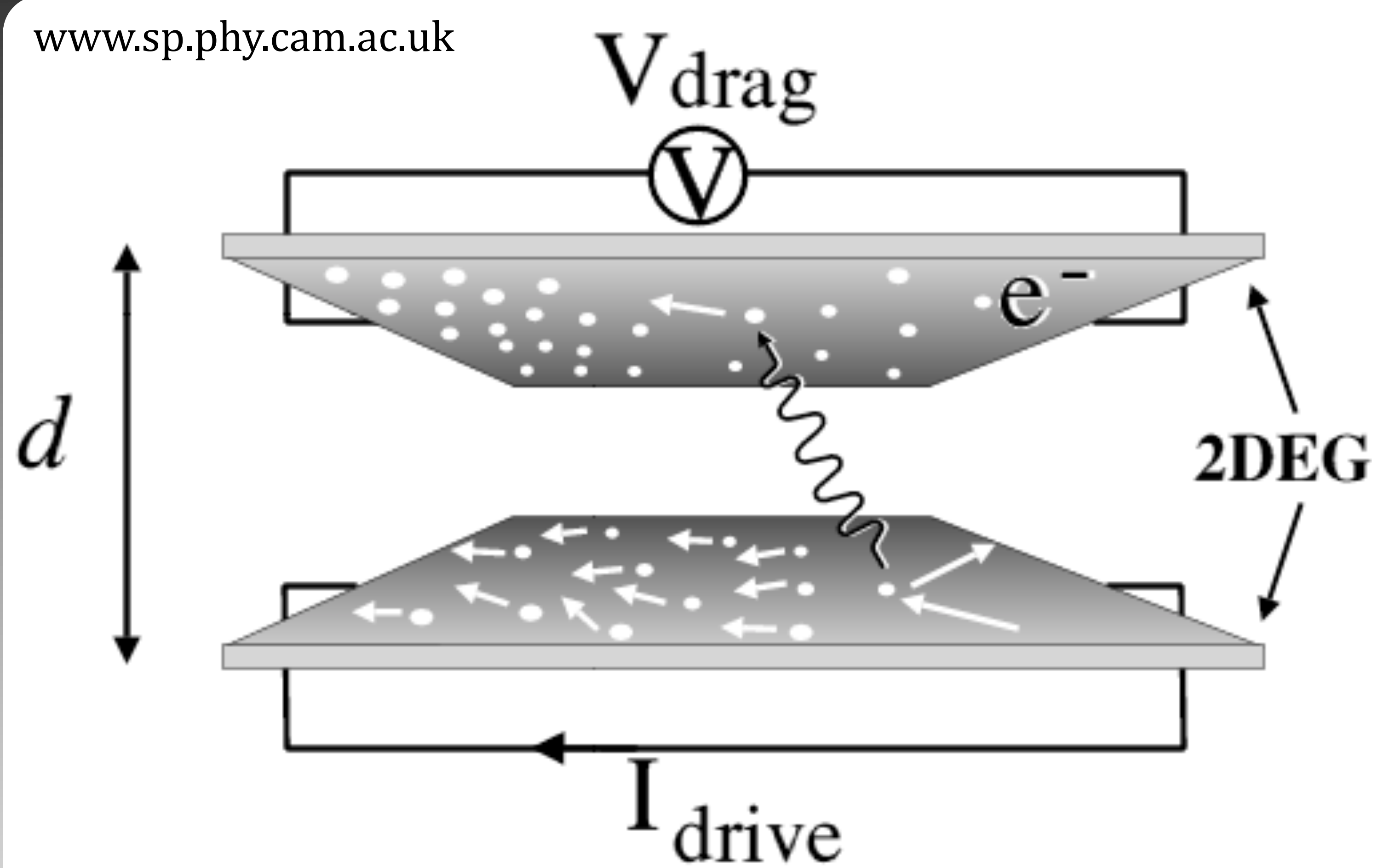
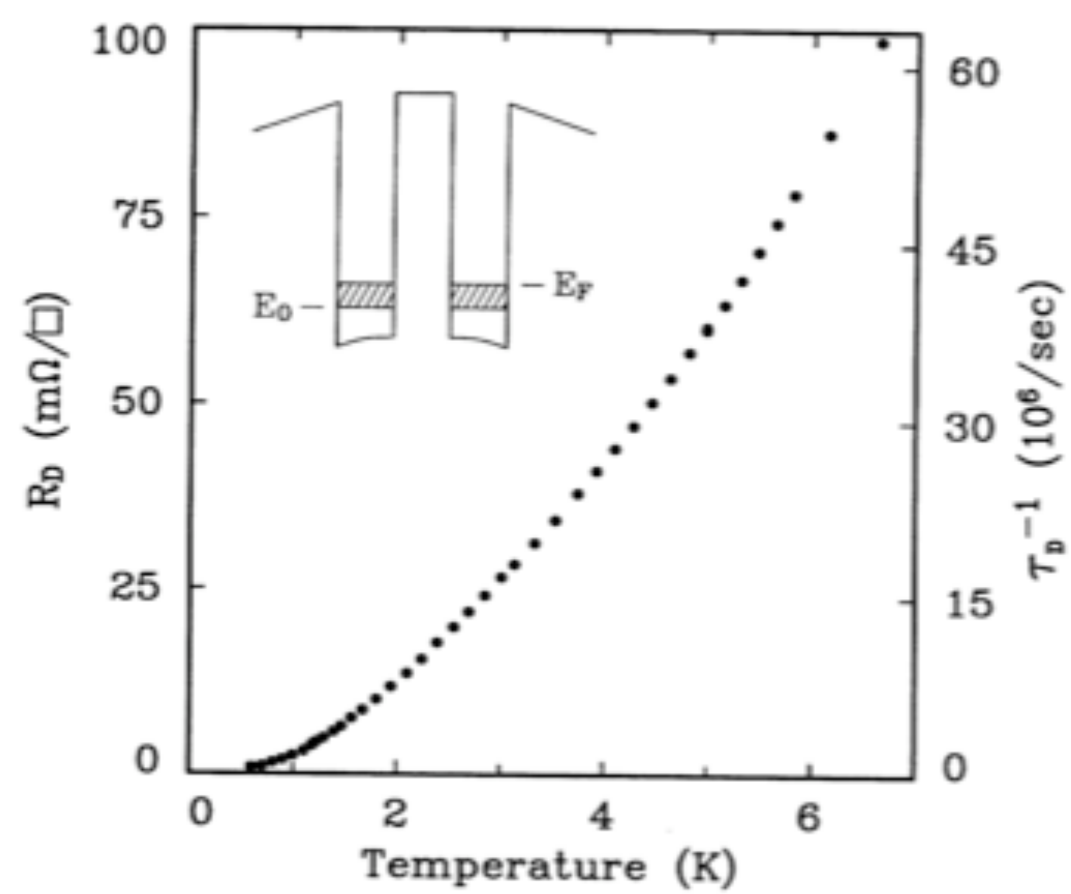
Coulomb drag





T.J. Gramila *et al.*, Phys. Rev. Lett. **66**, 1216 (1991)

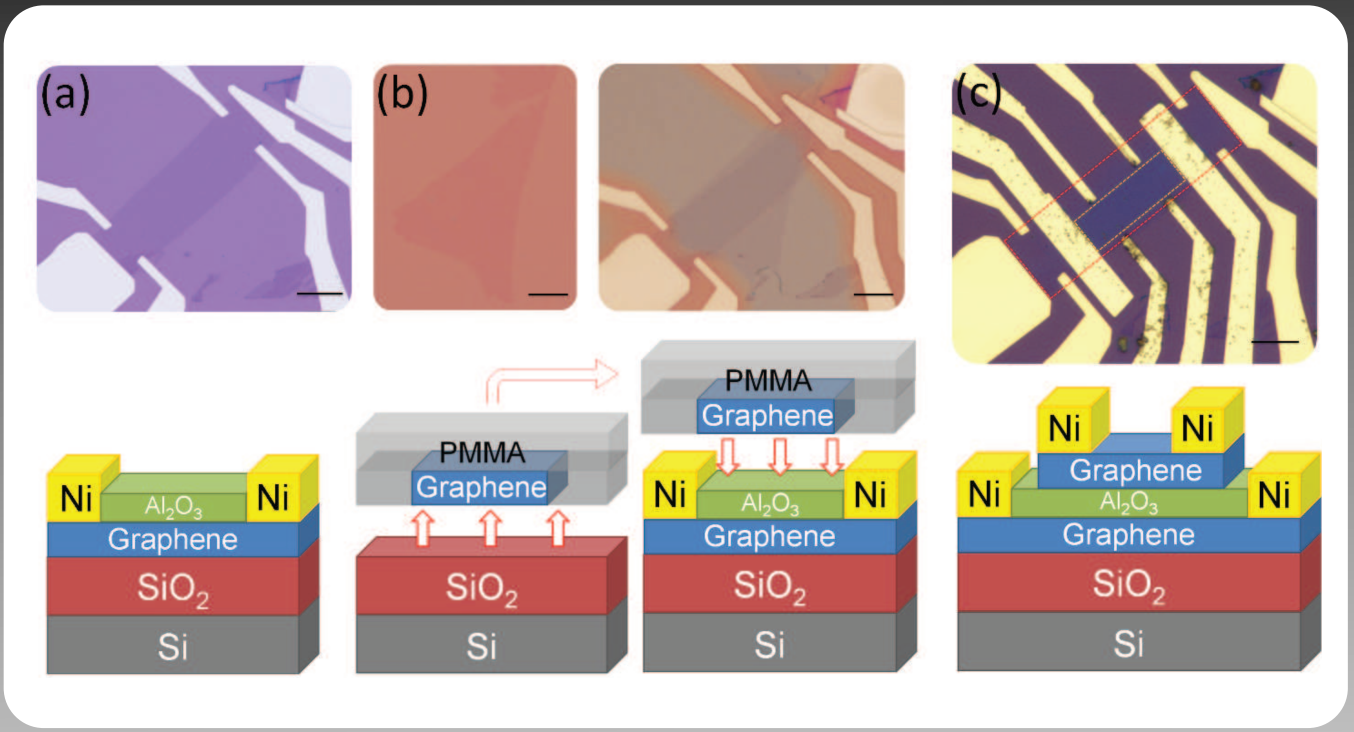
M.B. Pogrebinskii, Sov. Phys. Semicond. **11**, 372 (1977)
P.J. Price, Physica **117B**, 750 (1983)
L. Zheng and A.H. MacDonald, Phys. Rev. B **48**, 8203 (1993)
A.-P. Jauho and H. Smith, Phys. Rev. B **47**, 4420 (1993)
A. Kamenev and Y. Oreg, Phys. Rev. B **52**, 7516 (1995)
...

In a Fermi liquid:

$$R_{\mathrm{D}} \equiv \frac{V_{\mathrm{drag}}}{I_{\mathrm{drive}}} \propto \frac{1}{\tau_{\mathrm{D}}} \sim T^2$$

... what else ?

Coulomb drag in double-layer graphene



This slide contains experimental data from the **UT Austin** group [S. Kim *et al.*, Phys. Rev. B **83**, 161401(R) (2011)] but there are also (yet unpublished, I believe) data from the **Manchester** and the **Columbia** groups

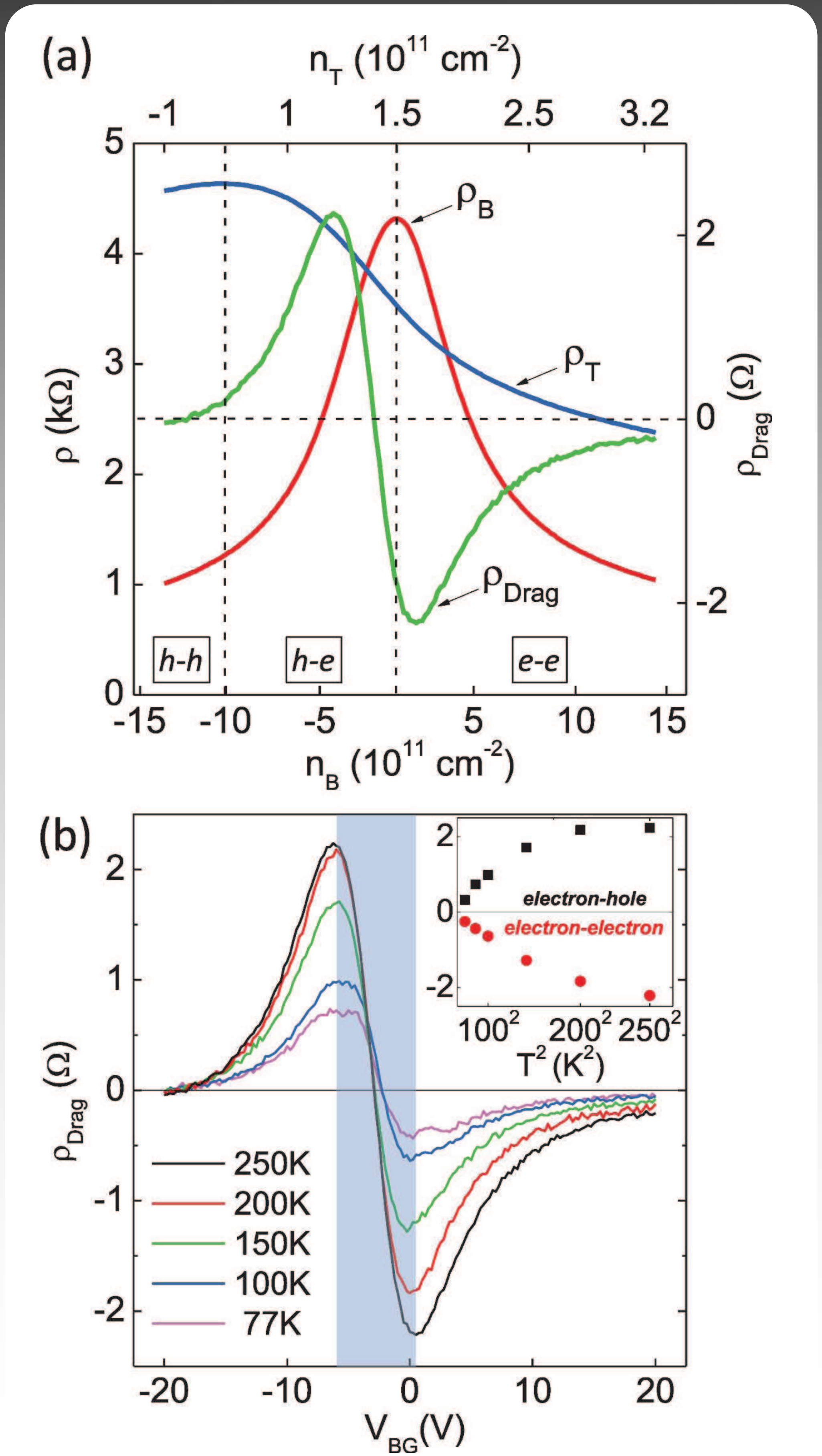
Thick limit (*i.e.* weak coupling)

$$\rho_D \propto n^{-3} d^{-4}$$

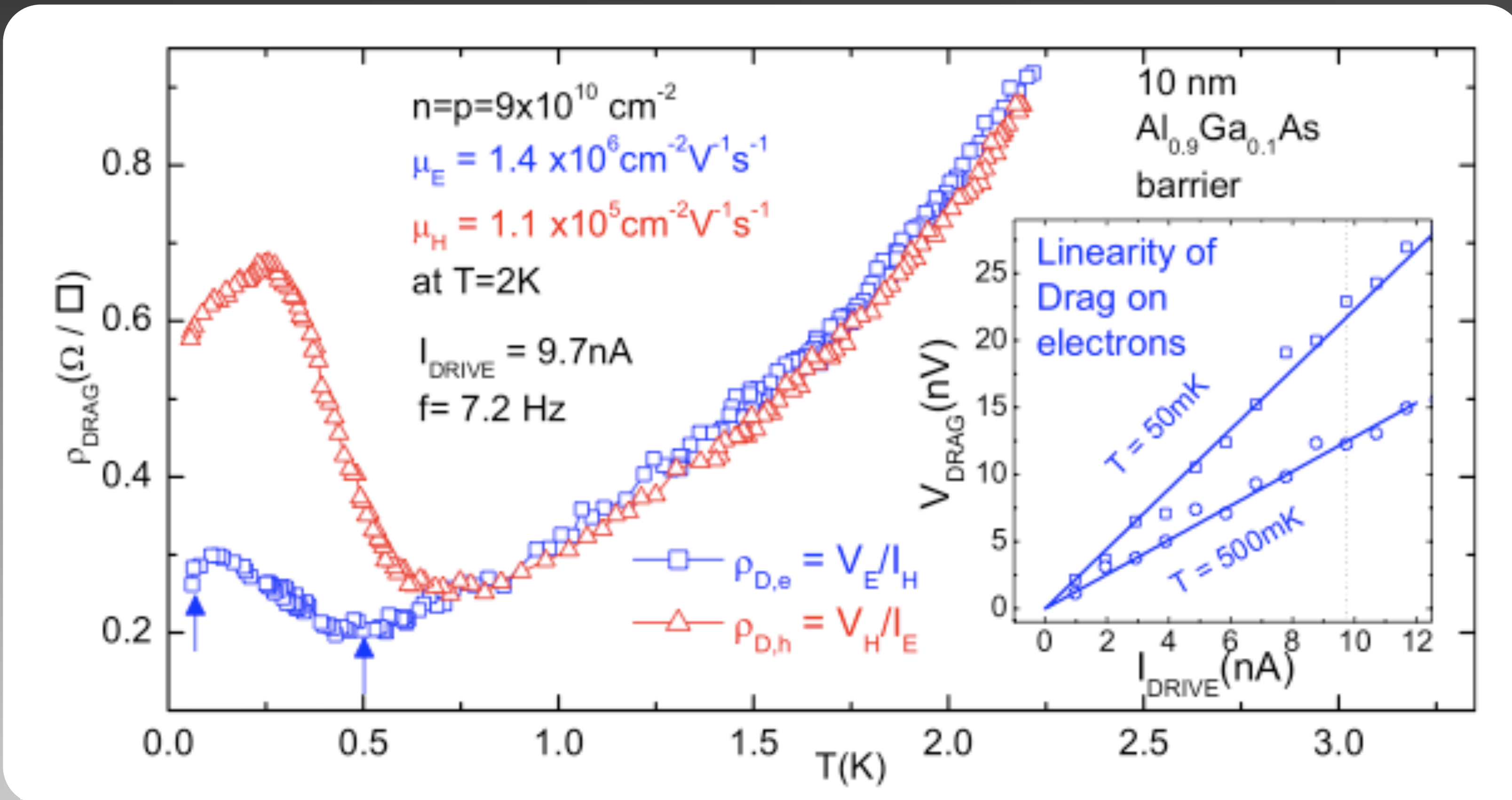
Thin limit (*i.e.* strong coupling)

$$\rho_D \propto n^{-1} |\ln(nd^2)|$$

See *e.g.* M.I. Katsnelson, Phys. Rev. B **84**, 041407(R) (2011) and other recent articles by Peres, Castro Neto, Das Sarma, etc

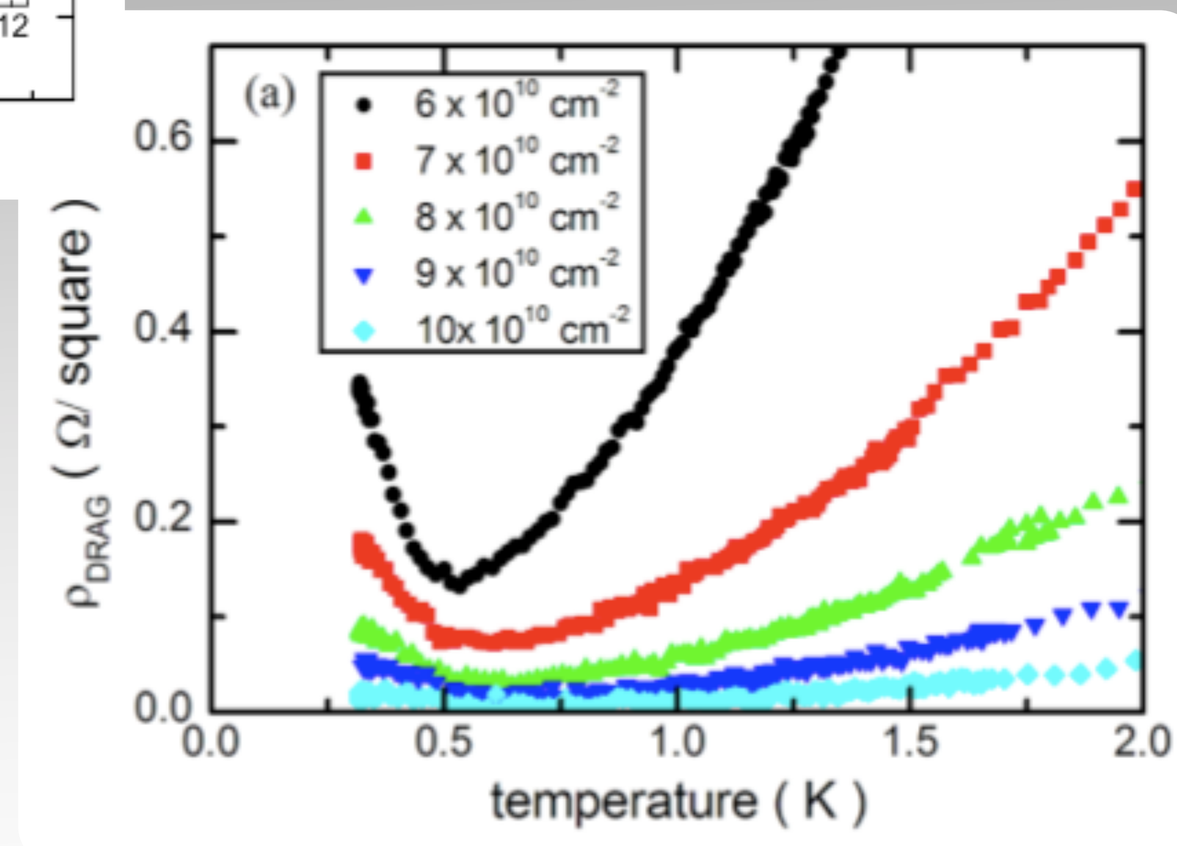


Coulomb drag in an electron-hole bilayer



A.F. Croxall *et al.*, Phys. Rev. Lett. **101**, 246801 (2008)

Note the anomalous upturn at low T
(signature of exciton condensation ?)



J.A. Seamons *et al.*, Phys. Rev. Lett. **102**, 026804 (2009)

For a theoretical discussion of drag in an e-h bilayer see:
G. Vignale and A.H. MacDonald, Phys. Rev. Lett. **76**, 2786 (1996): in the condensed phase
B.Y-.K. Hu, Phys. Rev. Lett. **85**, 820 (2000): above T_c (pairing fluctuations)

Friction in spin-polarized transport: spin Coulomb drag

Force between particles (electrons, atoms, etc) with antiparallel (pseudo)spin

$$F_{\sigma\bar{\sigma}} = -m\frac{n_{\bar{\sigma}}}{n}\frac{v_\sigma - v_{\bar{\sigma}}}{\tau_{\rm sd}}$$

Rate of change of spin-up momentum

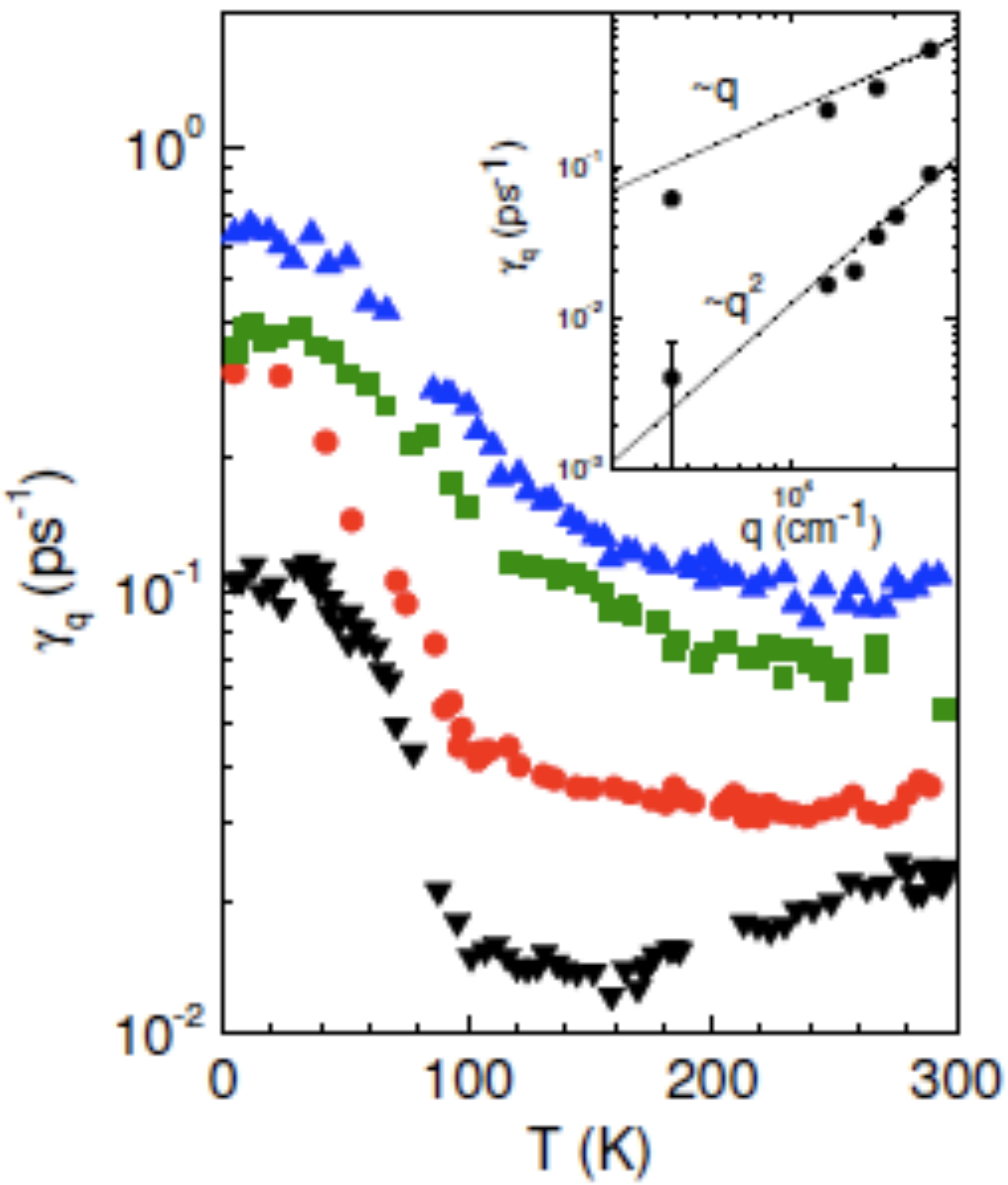
$$\frac{dP_\uparrow}{dt} = -\frac{1}{\tau_{\rm sd}}P_\uparrow$$

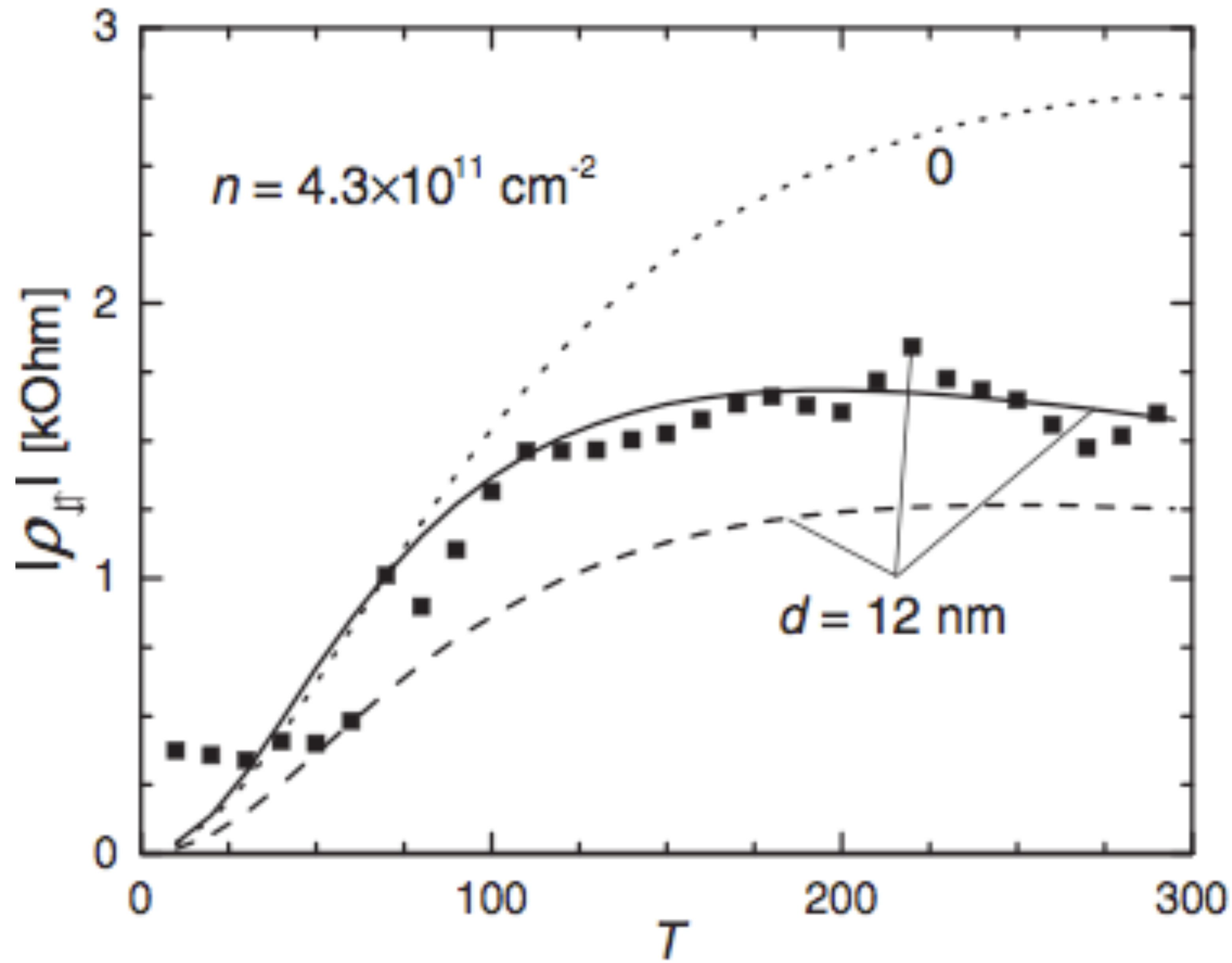
Leading term in the spin drag relaxation rate starts at second order

$$\frac{1}{\tau_{\rm sd}} \propto \frac{n}{n_\uparrow n_\downarrow k_{\rm B}T}\int\frac{d^D\boldsymbol{q}}{(2\pi)^D}\;\frac{q^2}{D}v_q^2\int_0^{+\infty}\frac{d\omega}{\pi}\;\frac{\Im m\chi_\uparrow^{(0)}(q,\omega)\Im m\chi_\downarrow^{(0)}(q,\omega)}{\sinh^2[\hbar\omega/(2k_{\rm B}T)]}$$

I. D'Amico and G. Vignale, Phys. Rev. B **62**, 4853 (2000)

Spin Coulomb drag: experimental (semiconductor quantum wells)

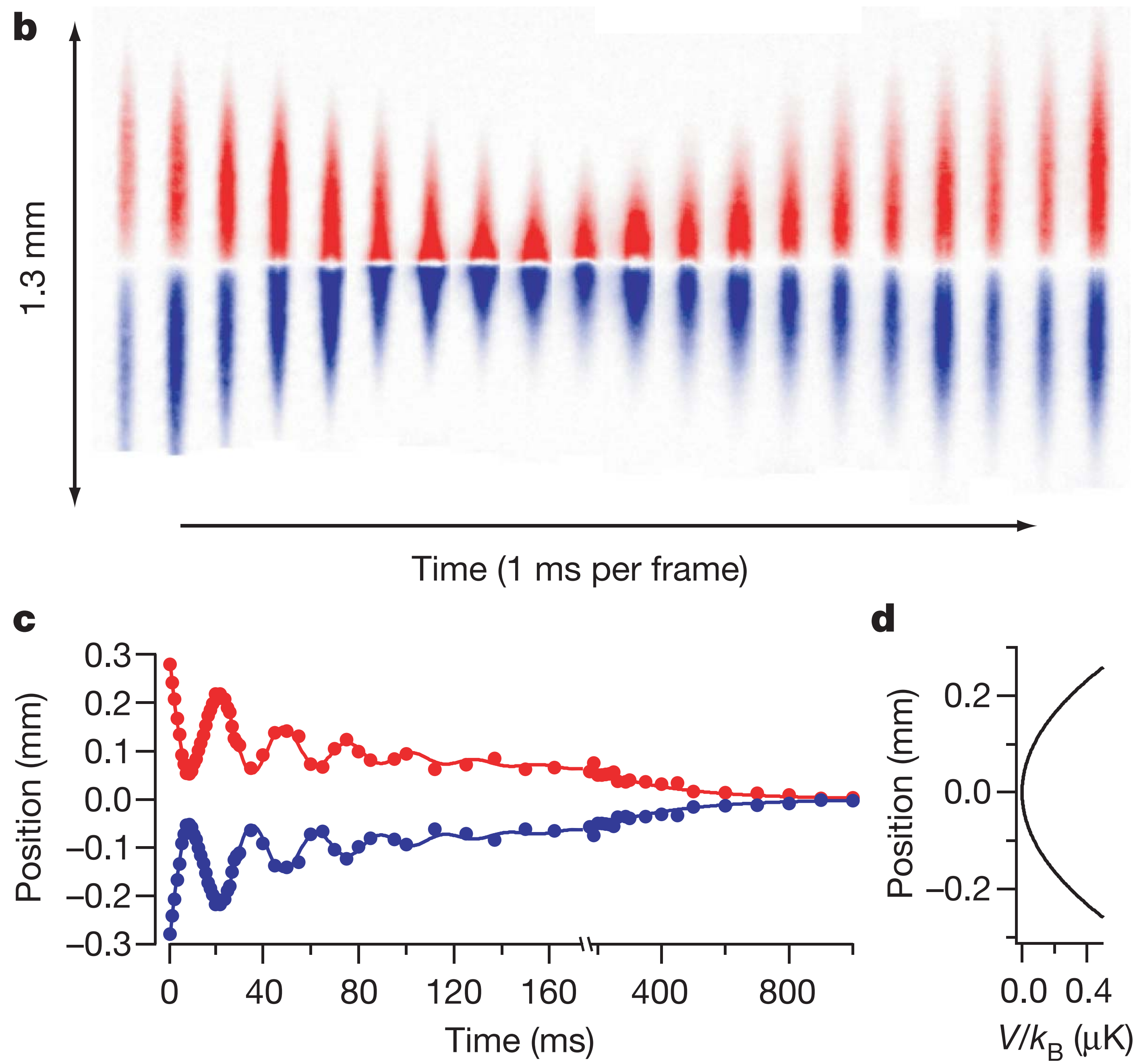




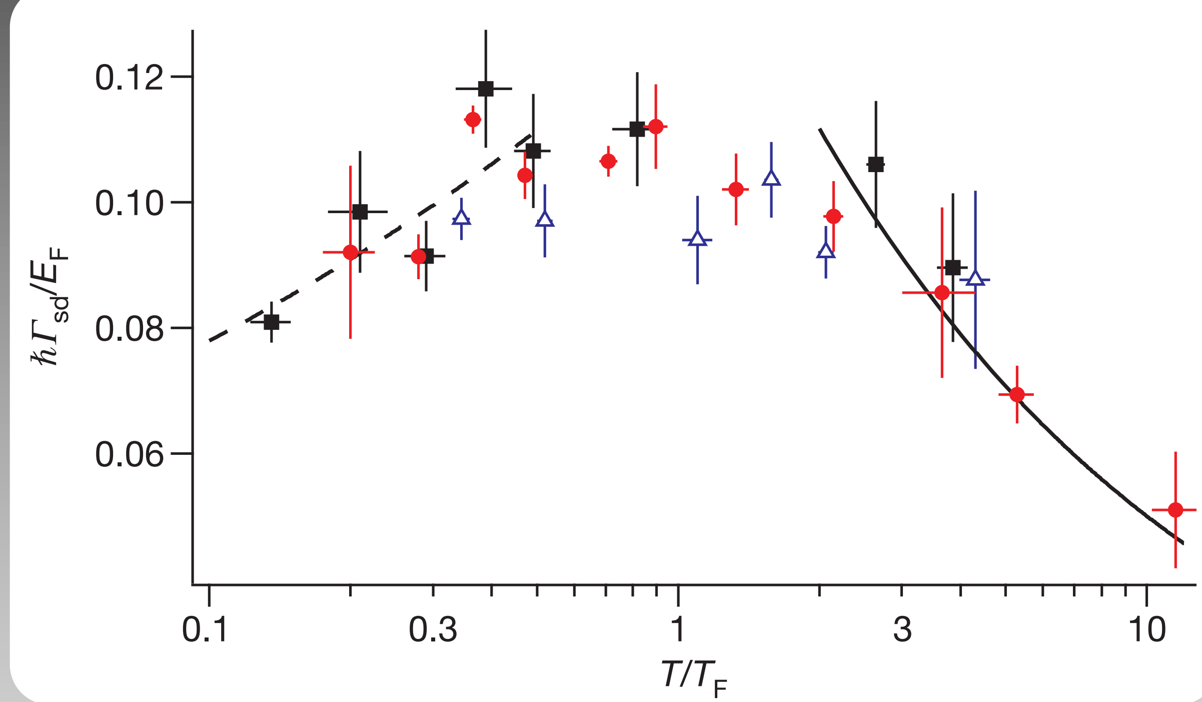
Experimental: C.P. Weber *et al.*, Nature **437**, 1330 (2005)
Theory: S.M. Badalyan, C.S. Kim, and G. Vignale, Phys. Rev. Lett. **100**, 016603 (2008)

Spin drag: experimental (cold Fermi gases at unitarity)

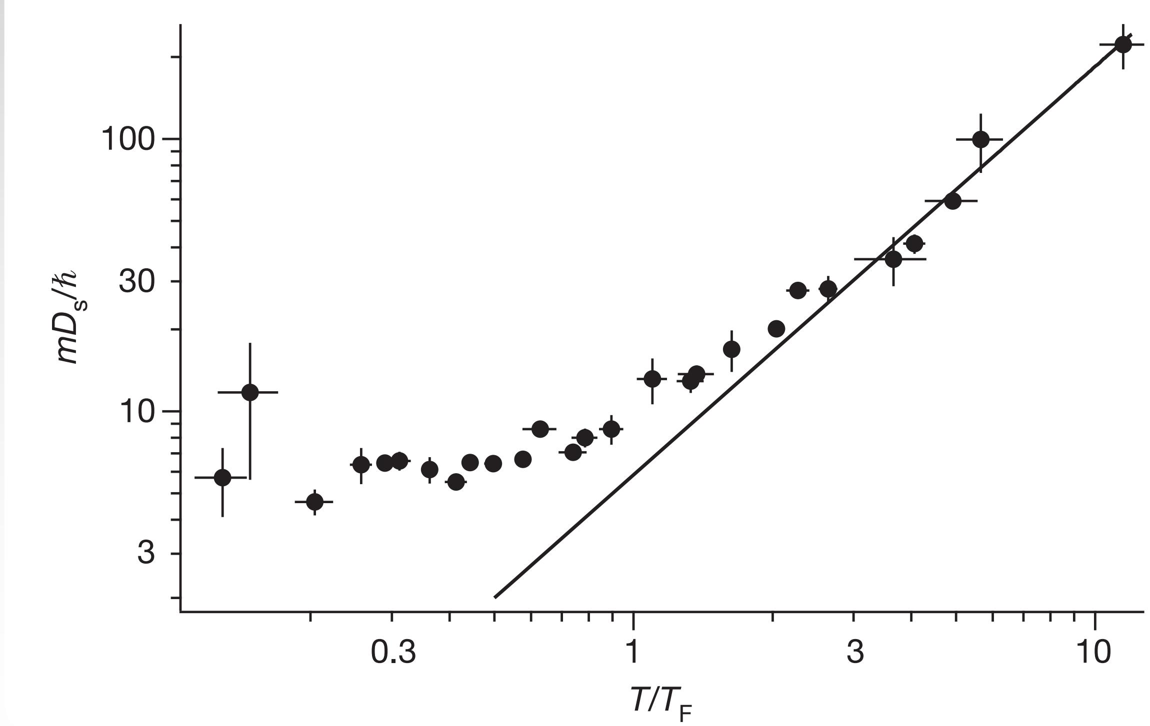
A. Sommer *et al.*, Nature **472**, 201 (2011)



$$\Gamma_{\rm sd}\partial_t d + \omega_z^2 d = 0$$



Spin-drag relaxation rate measured at equilibrium



Spin diffusion constant measured at equilibrium

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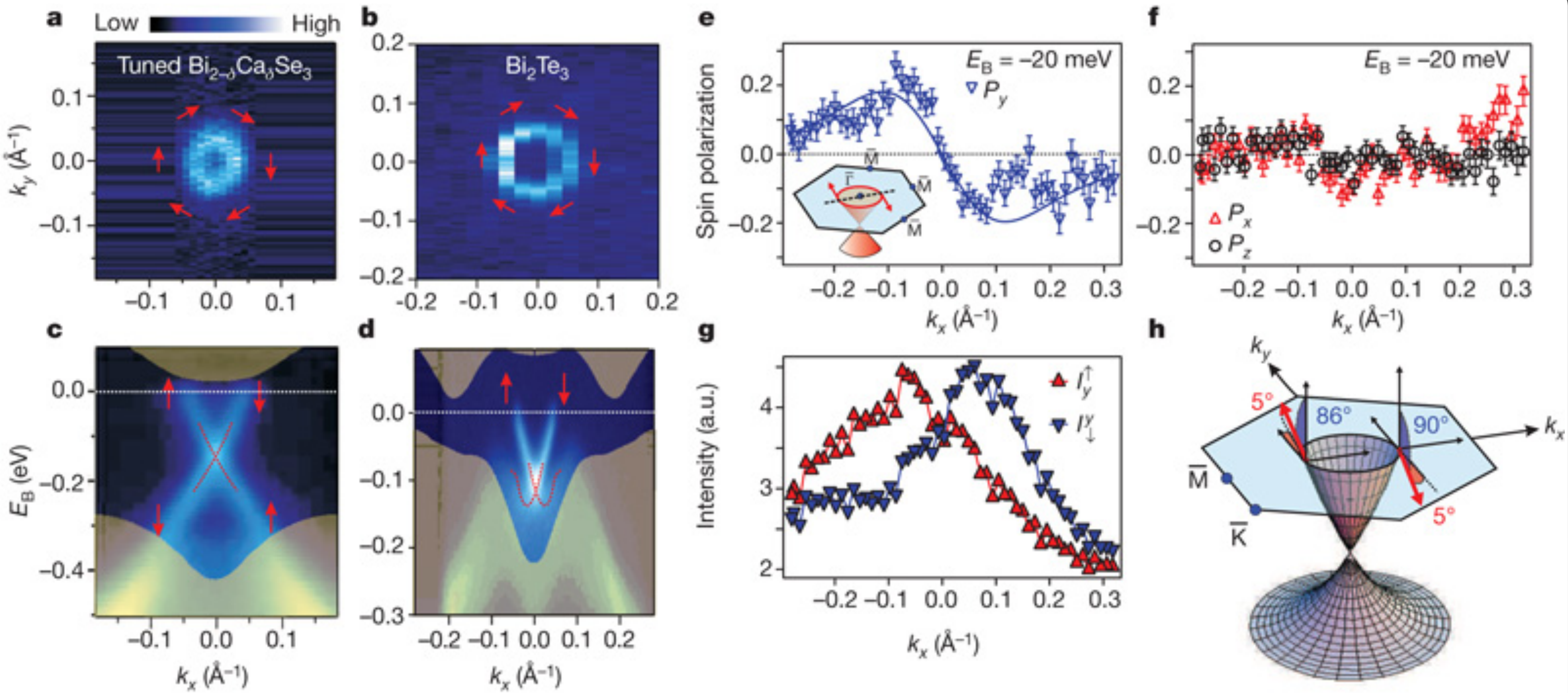
3D topological insulators



D. Hsieh *et al.*, Nature **460**, 1101 (2009) (Hasan group)

Concrete examples of topological insulators:

$Bi_{1-x}Sb_x$, Bi_2Se_3, Bi_2Te_3, etc

See *e.g.* M.Z. Hasan and C.L. Kane, Rev. Mod. Phys. **82**, 3045 (2010);
X.-L. Qi and S.-C. Zhang, arXiv:1008.2026

Topological exciton condensates

PRL **103,** 066402 (2009) PHYSICAL REVIEW LETTERS week ending 7 AUGUST 2009

Exciton Condensation and Charge Fractionalization in a Topological Insulator Film

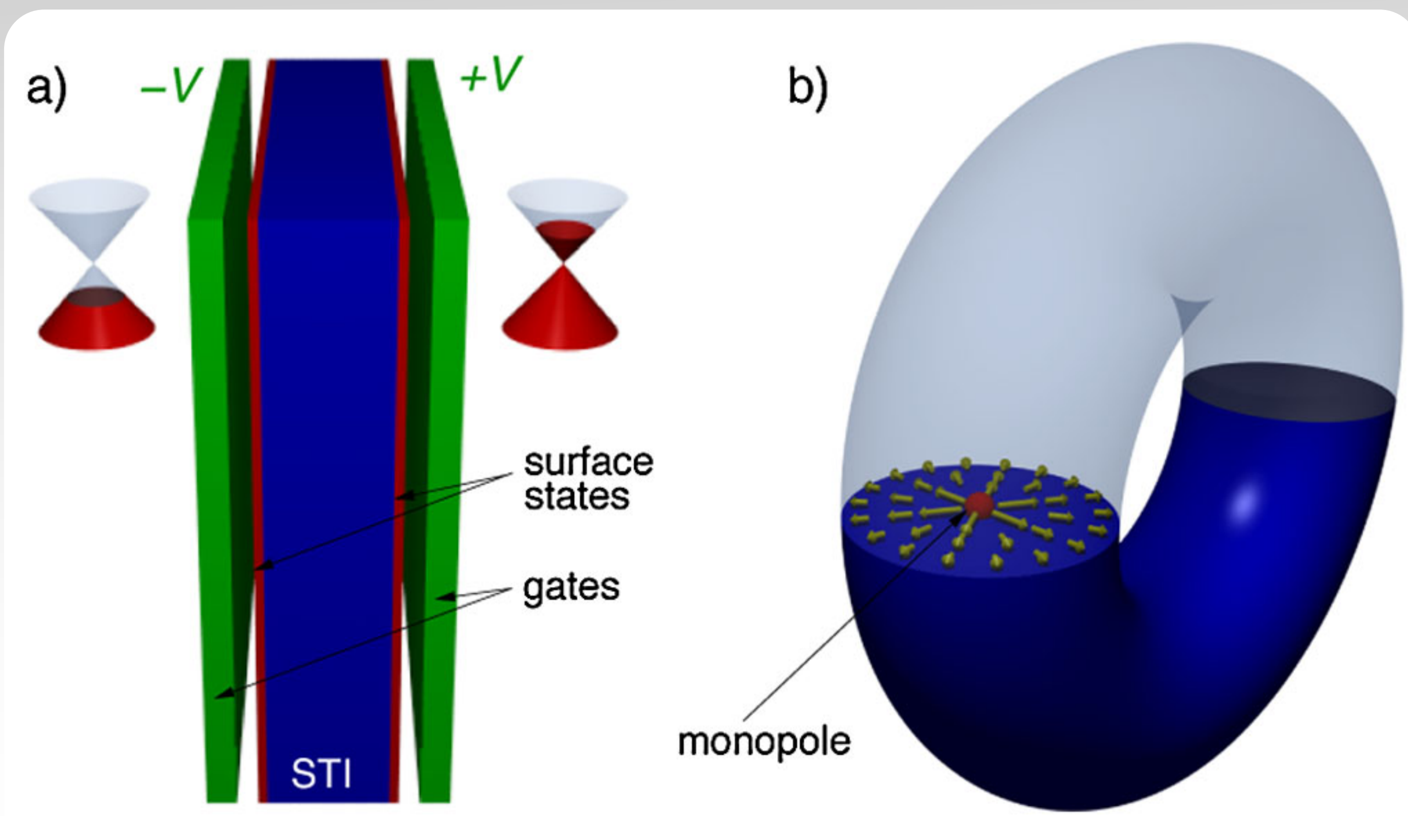
B. Seradjeh,[1] J.E. Moore,[2,3] and M. Franz[4]

[1]*Department of Physics, University of Illinois, 1110 West Green Street, Urbana, Illinois 61801-3080, USA*
[2]*Department of Physics, University of California, Berkeley, California 94720, USA*
[3]*Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[4]*Department of Physics and Astronomy, University of British Columbia,Vancouver, BC, Canada V6T 1Z1*

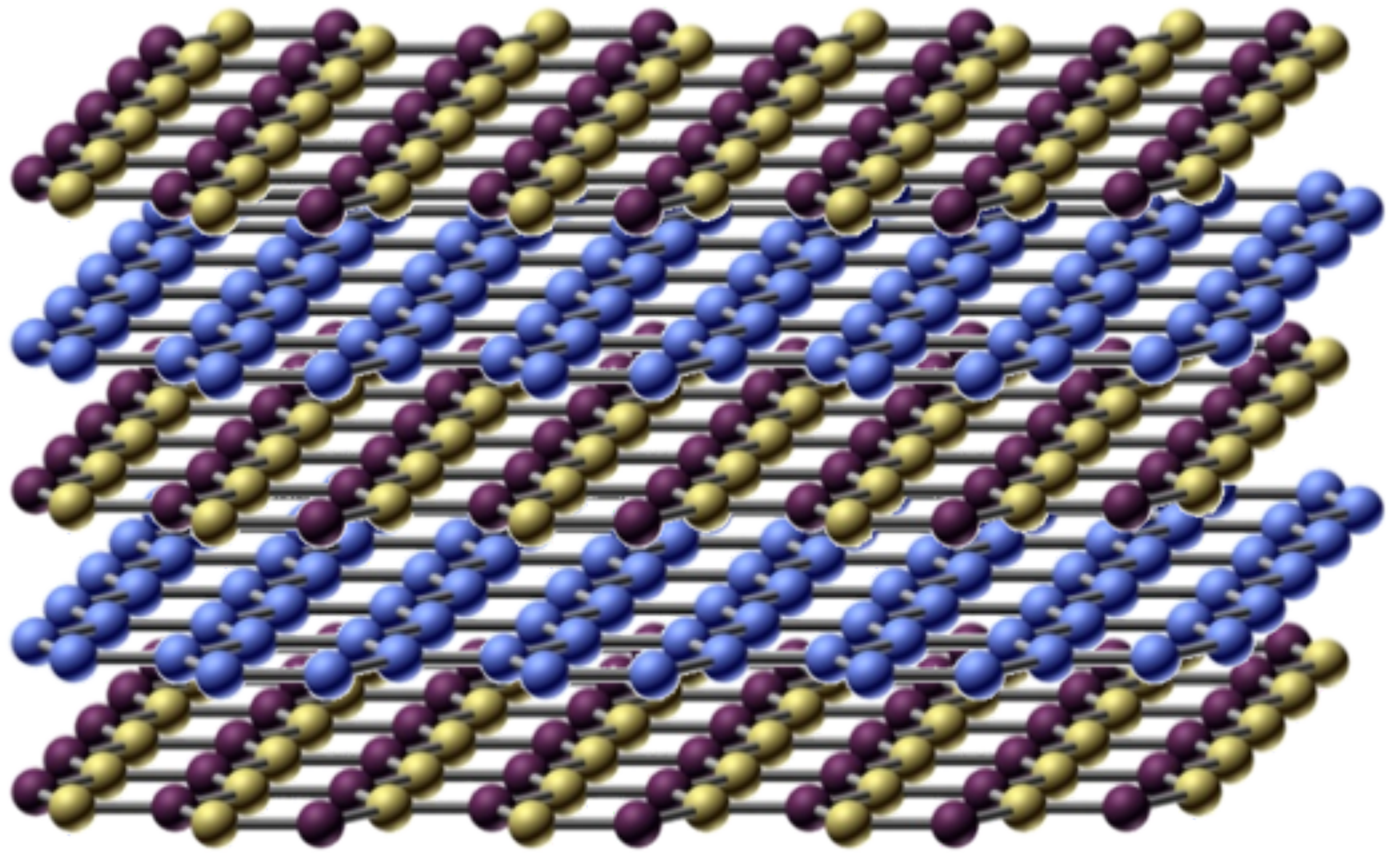
(Received 10 February 2009; published 7 August 2009)

An odd number of gapless Dirac fermions is guaranteed to exist at a surface of a strong topological insulator. We show that in a thin-film geometry and under external bias, electron-hole pairs that reside in these surface states can condense to form a novel exotic quantum state which we propose to call "topological exciton condensate" (TEC). This TEC is similar in general terms to the exciton condensate recently argued to exist in a biased graphene bilayer, but with different topological properties. It exhibits a host of unusual properties including a stable zero mode and a fractional charge $\pm e/2$ carried by a singly quantized vortex in the TEC order parameter.

DOI: 10.1103/PhysRevLett.103.066402 PACS numbers: 71.35.−y, 71.10.Pm, 73.20.−r



Other candidate system: Graphene-Boron Nitride heterostructures

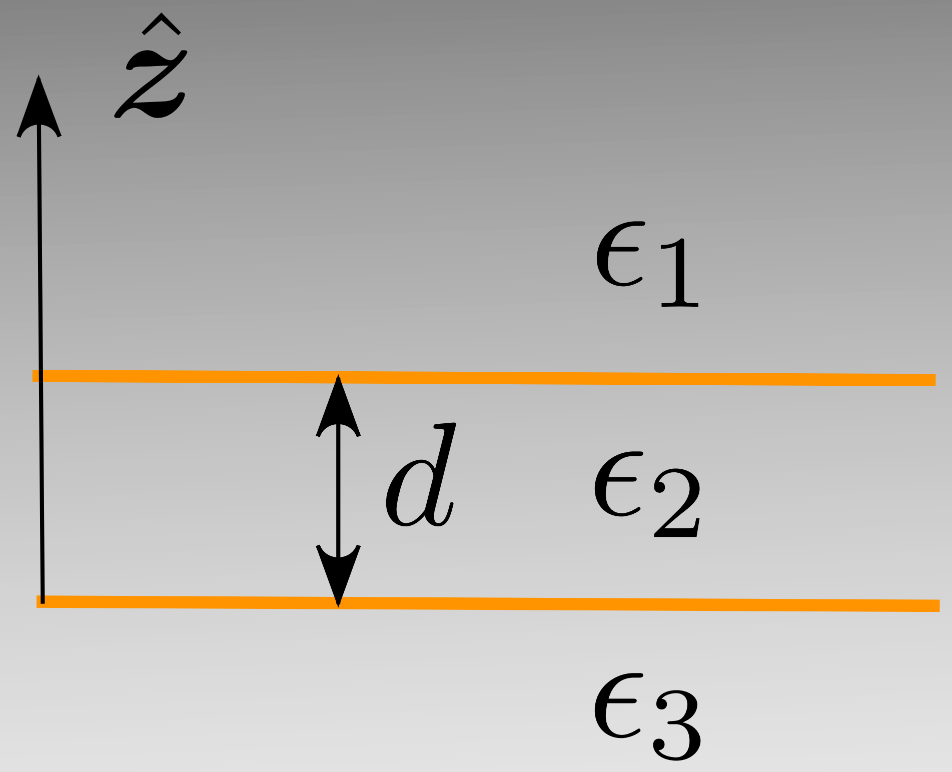


courtesy of K. Novoselov

- Dramatic quality improvement with respect to conventional samples on *e.g.* SiO_2 (major sources of scattering seem to be absent)
- Complete tunability (inter-layer tunneling, inter-layer distance, etc)

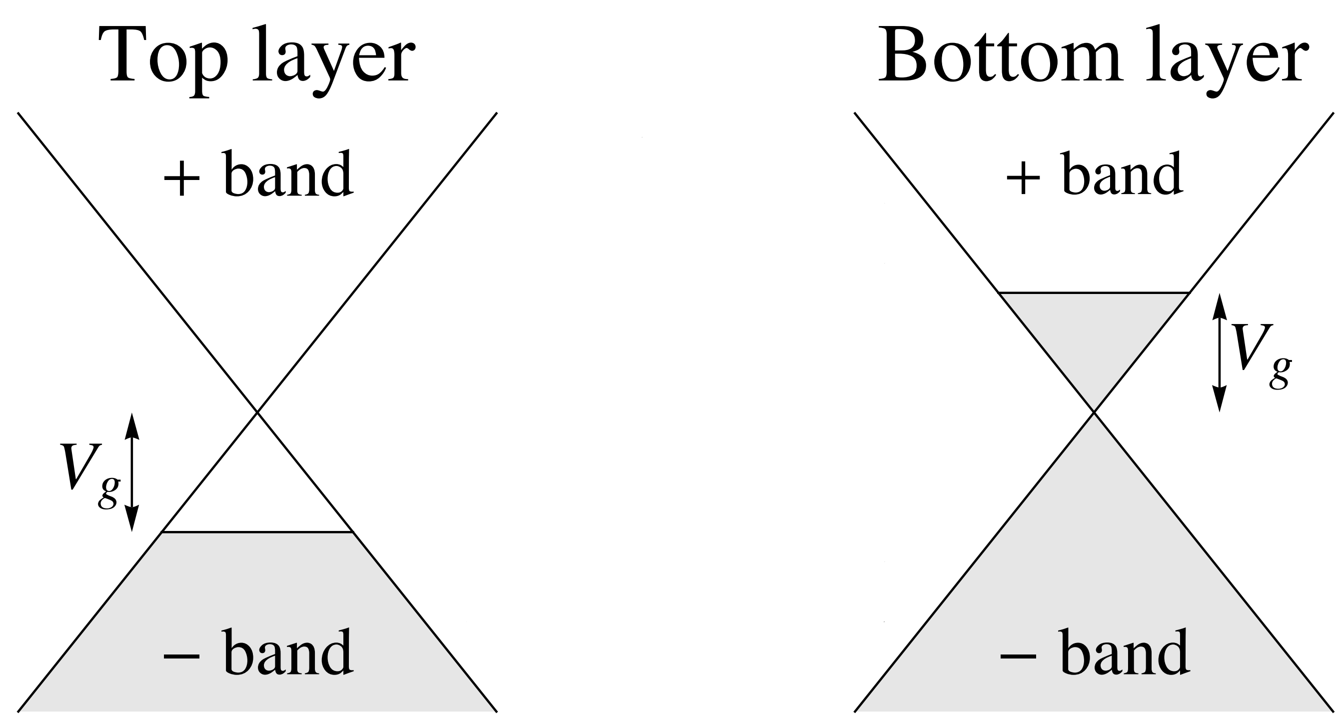
Yu. E. Lozovik and A.A. Sokolik, JETP Lett. **87**,55 (2008)
H. Min *et al.*, Phys. Rev. B **78**, 121401(R) (2008)
C.-H. Zhang and Y.N. Joglekar, Phys. Rev. B **77**, 205426 (2008)

Model Hamiltonian and "closed-band" approximation



$$\epsilon_2 = \epsilon_{\mathrm{TI}} \gg \epsilon_1, \epsilon_3$$

Two surface states described by a massless Dirac fermion model

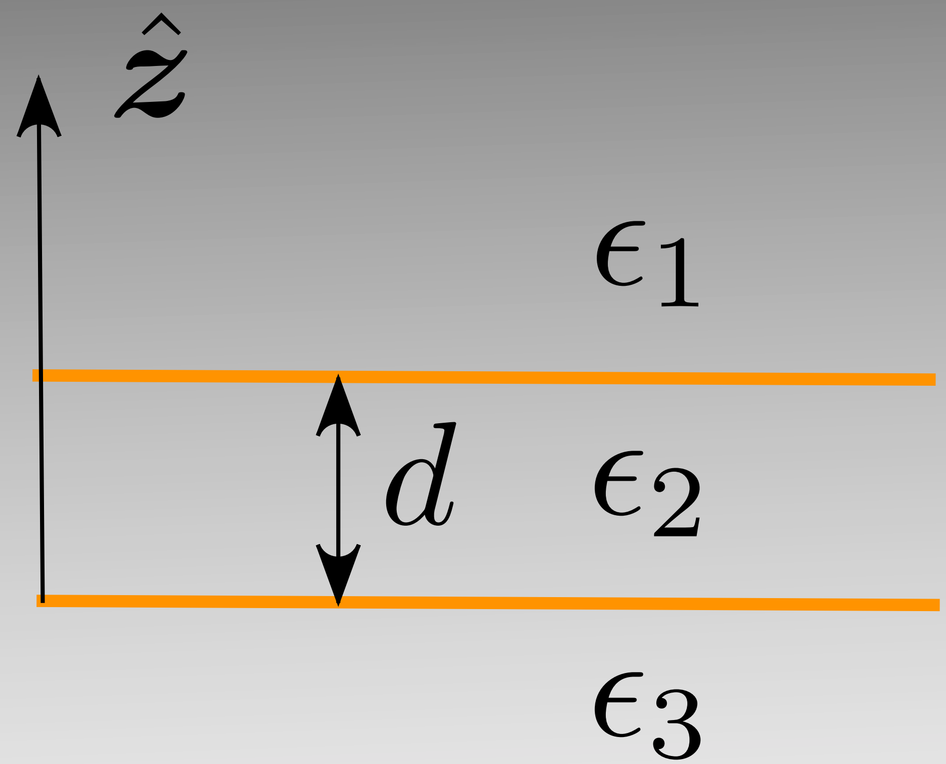


M.P. Mink, H.T.C. Stoof, R.A. Duine, M. Polini, and G. Vignale, arXiv:1108.2298

1) Inter- and intra-layer **screened** Coulomb interactions calculated *e.g.* in R.E.V. Profumo, M. Polini, R. Asgari, R. Fazio, and A.H. MacDonald, Phys. Rev. B **82**, 085443 (2010)

2) For a study of the influence of **remote bands** see: M.P. Mink, H.T.C. Stoof, R.A. Duine, and A.H. MacDonald, arXiv:1107.4477

Model Hamiltonian and "closed-band" approximation



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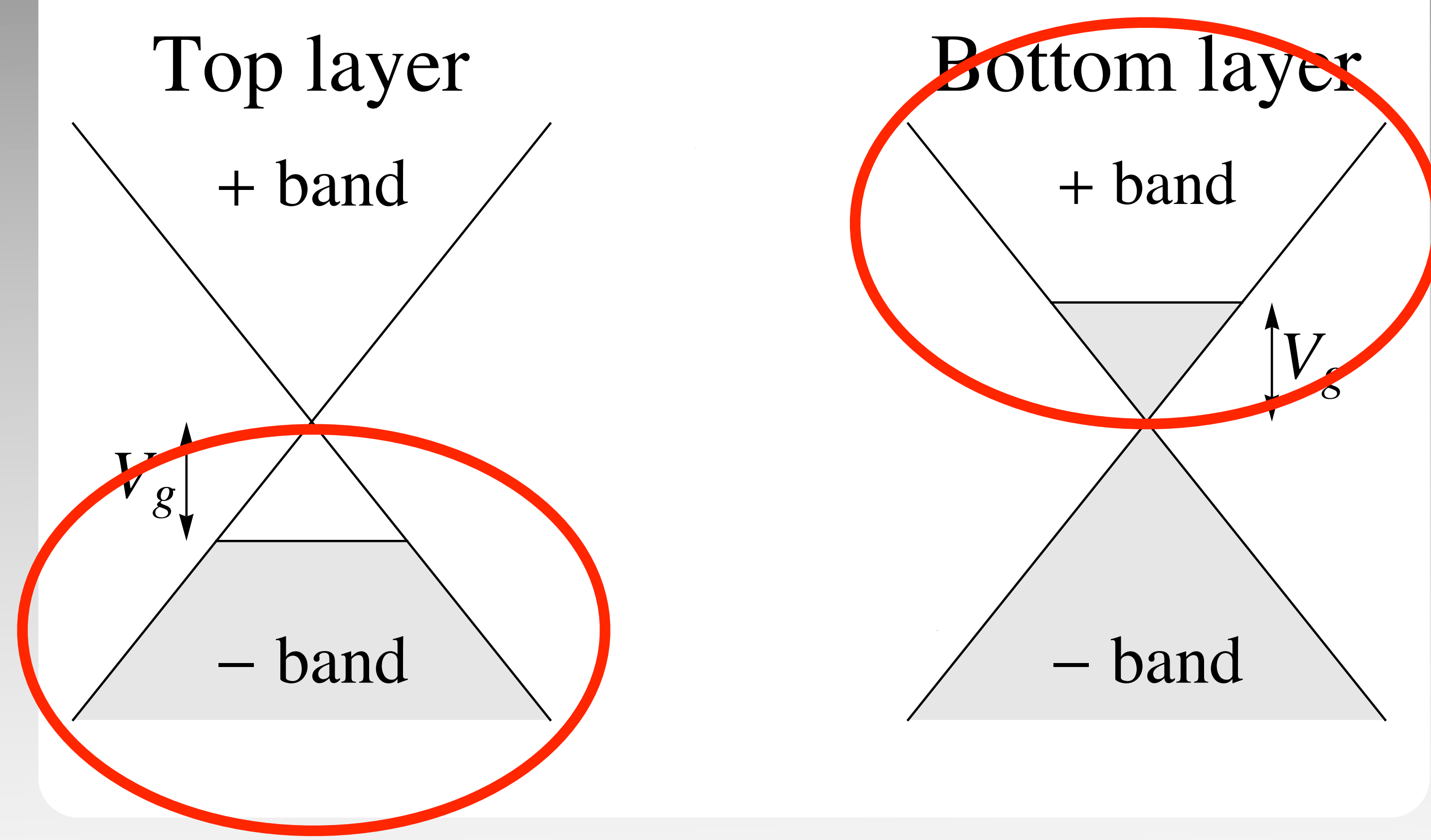

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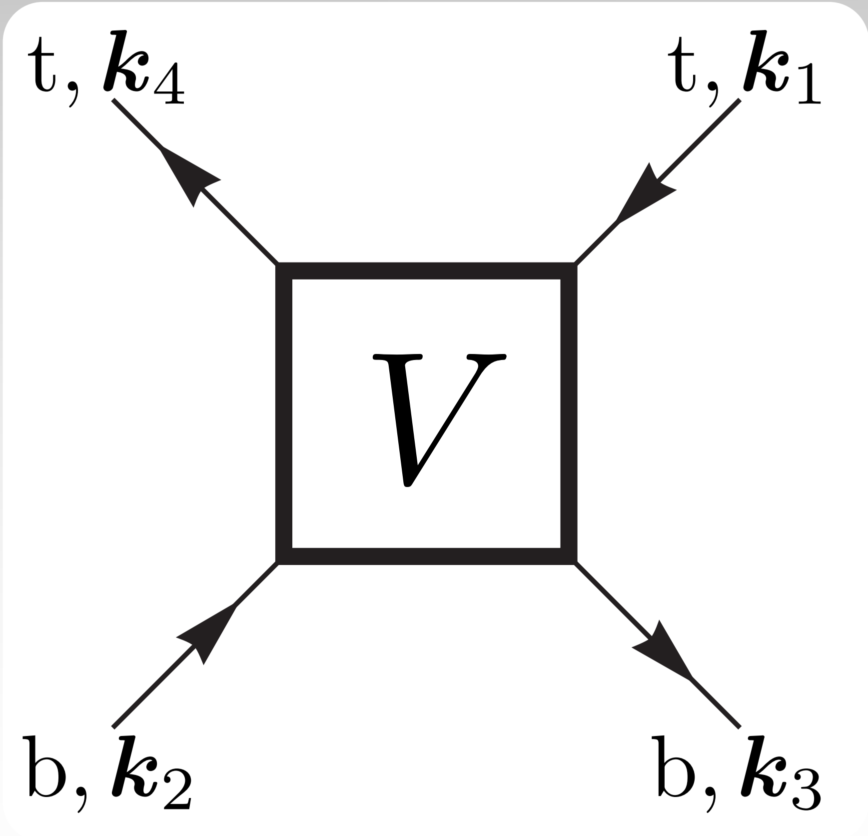
Boltzmann-equation approach to the drag resistivity (I)

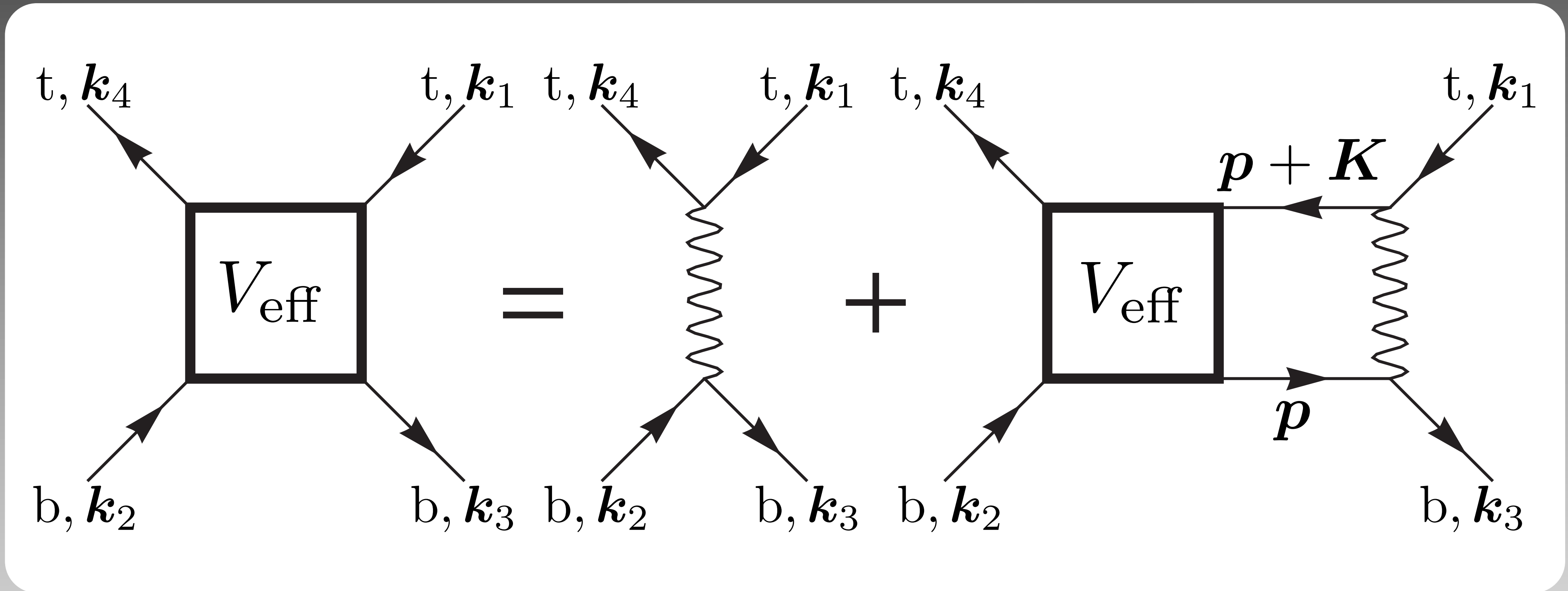
$$\begin{aligned}\rho_{\mathrm{D}} \quad &\propto \quad \sum_{\boldsymbol{k}_i} |V(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)|^2 \delta(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}_3-\boldsymbol{k}_4)\delta(\epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4) \\ &\times \quad n_1 n_2 (1-n_3)(1-n_4)(\boldsymbol{v}_1-\boldsymbol{v}_4)\cdot(\boldsymbol{v}_2-\boldsymbol{v}_3)\end{aligned}$$

Key ingredient in the Boltzmann approach: the scattering amplitude



M.P. Mink, H.T.C. Stoof, R.A. Duine, M. Polini, and G. Vignale, arXiv:1108.2298

The effective interaction relevant to the excitonic instability



$$\boldsymbol{K} = \boldsymbol{k}_1 - \boldsymbol{k}_3 \qquad \Omega = \epsilon_{\mathrm{b}}(\boldsymbol{k}_1) - \epsilon_{\mathrm{t}}(\boldsymbol{k}_3)$$

Approximate scattering amplitude

$$V(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) \simeq V_{\text{eff}}(\boldsymbol{K}, \Omega) \equiv \frac{U}{1 - U\Xi(\boldsymbol{K}, \Omega)}$$

Pairing susceptibility

$$\Xi(\boldsymbol{K}, \Omega) = \frac{1}{A} \sum_{\boldsymbol{k}} \frac{n(\epsilon_{\mathrm{t}}(\boldsymbol{k} + \boldsymbol{K})) - n(\epsilon_{\mathrm{b}}(\boldsymbol{k}))}{\epsilon_{\mathrm{b}}(\boldsymbol{k}) - \epsilon_{\mathrm{t}}(\boldsymbol{k} + \boldsymbol{K}) - \Omega - i0^+}$$

M.P. Mink, H.T.C. Stoof, R.A. Duine, M. Polini, and G. Vignale, arXiv:1108.2298

Final equation for the drag resistivity (II)

$$\rho_{\mathrm{D}} = -\frac{\beta}{2(2\pi)^6 e^2 n v^2} \int d\boldsymbol{K} d\Omega \frac{|V_{\mathrm{eff}}(\boldsymbol{K},\Omega)|^2}{\sinh^2(\beta\Omega/2)}$$
$$\times \int d\boldsymbol{k} d\boldsymbol{k}' \Im m \left[b(\boldsymbol{k};\boldsymbol{K},\Omega)\right] \Im m \left[b(\boldsymbol{k}';\boldsymbol{K},\Omega)\right]$$
$$\times \left[\boldsymbol{v}_{\mathrm{t}}(\boldsymbol{k}'+\boldsymbol{K}) - \boldsymbol{v}_{\mathrm{t}}(\boldsymbol{k}+\boldsymbol{K})\right] \cdot \left[\boldsymbol{v}_{\mathrm{b}}(\boldsymbol{k}') - \boldsymbol{v}_{\mathrm{b}}(\boldsymbol{k})\right]$$

Group velocities

$$\boldsymbol{v}_{\mathrm{t(b)}}(\boldsymbol{k}) \equiv \boldsymbol{\nabla}\epsilon_{\mathrm{t(b)}}(\boldsymbol{k})$$

Same integrand that controls the pairing susceptibility introduced in the previous slide

$$b(\boldsymbol{k};\boldsymbol{K},\Omega) \equiv \frac{n(\epsilon_{\mathrm{t}}(\boldsymbol{k}+\boldsymbol{K})) - n(\epsilon_{\mathrm{b}}(\boldsymbol{k}))}{\epsilon_{\mathrm{b}}(\boldsymbol{k}) - \epsilon_{\mathrm{t}}(\boldsymbol{k}+\boldsymbol{K}) - \Omega - i0^+}$$

M.P. Mink, H.T.C. Stoof, R.A. Duine, M. Polini, and G. Vignale, arXiv:1108.2298

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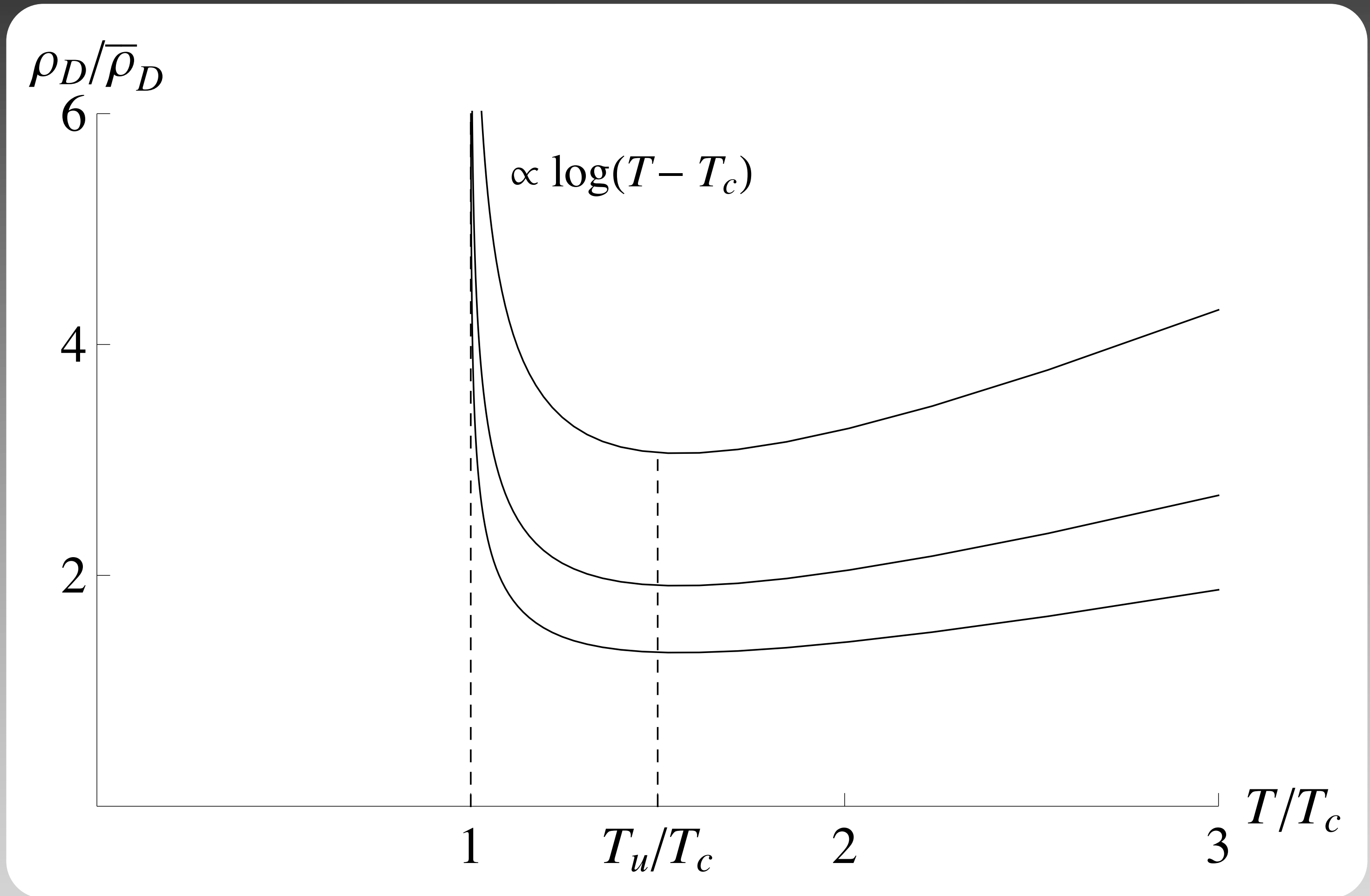
Drag resistivity close to T_c



Expansion of the effective interaction close to T_c (the coefficient a(T) is ultraviolet convergent!)

$$1 - U\Xi(\boldsymbol{K}, \Omega) \simeq \alpha(T) + a(T)U\nu_0(\beta v K)^2 + iU\nu_0\beta\Omega/4$$

Drag resistivity close to T_c

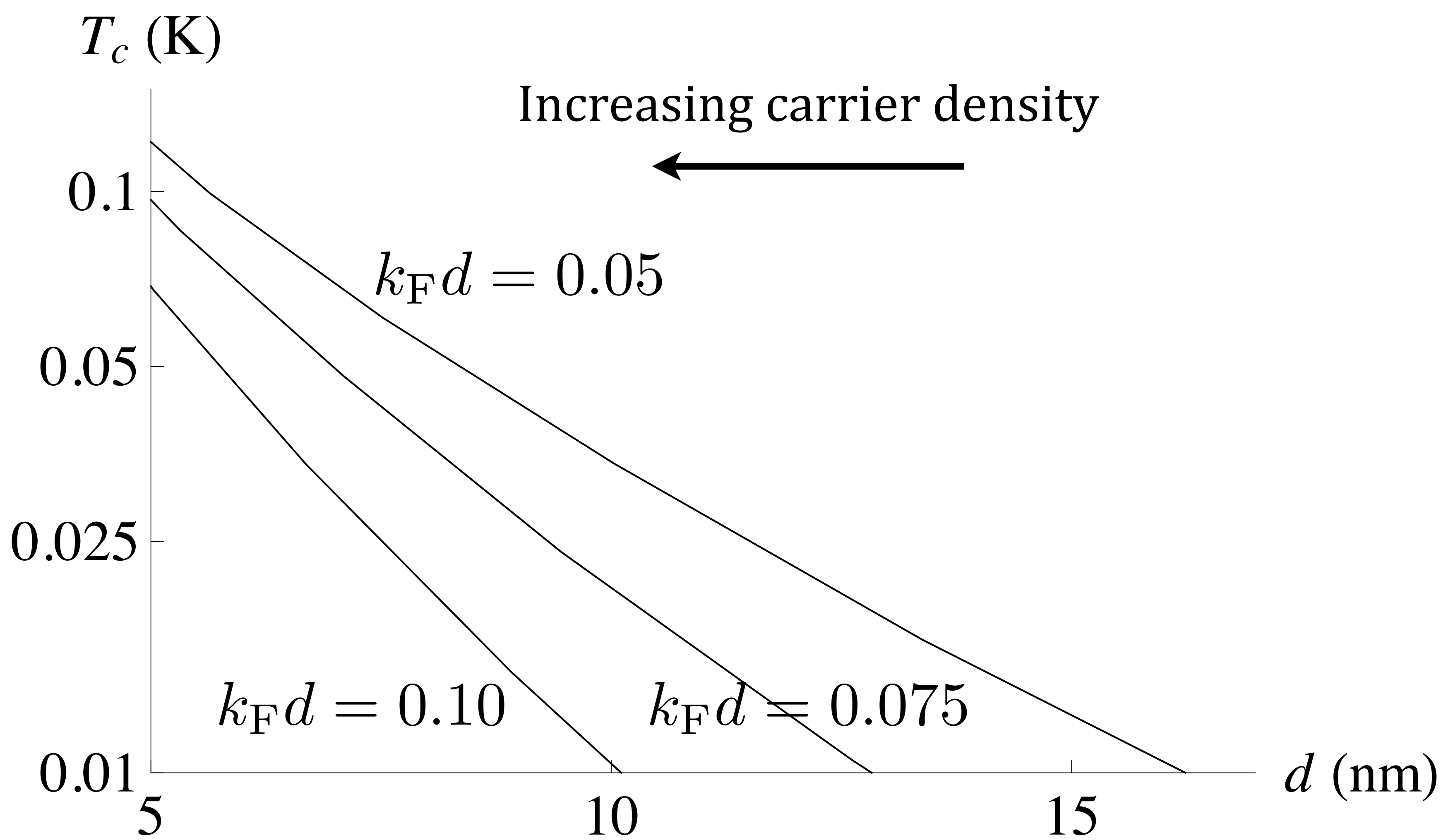
$$\rho_{\mathrm{D}} \propto \int dK d\Omega \frac{(\beta\Omega/4)^2}{\sinh^2(\beta\Omega/2)} \frac{(\nu_0 U)^2 K}{[\alpha(T) + aU\nu_0(\beta v K)^2]^2 + (U\nu_0\beta\Omega/4)^2}$$

which is immediately seen to diverge **logarithmically** when T approaches T_c

Mean-field critical temperature (with static screening)

Mean-field critical temperature determined from the pole of the effective interaction in a **separable** approximation (no need for ultraviolet cut-off) and with **static** screening:

$$1 = \int d^2\boldsymbol{k} V_{\rm sep}^2(k) \frac{n(\epsilon_{\rm t}(\boldsymbol{k})) - n(\epsilon_{\rm b}(\boldsymbol{k}))}{\epsilon_{\rm b}(\boldsymbol{k}) - \epsilon_{\rm t}(\boldsymbol{k})}$$



Comparison with our recent theory for the spin-drag relaxation rate close to a ferromagnetic instability

R.A. Duine, MP, H.T.C. Stoof, and G. Vignale, Phys. Rev. Lett. **104**, 220403 (2010)

Stoner ferromagnetism (I)

Minimal model: competition between kinetic energy and short-range repulsive interactions between antiparallel-spin fermions:

$$\hat{\mathcal{H}} = \int d^3\boldsymbol{x} \sum_{\alpha\in\{\uparrow,\downarrow\}} \hat{\psi}^\dagger_\alpha(\boldsymbol{x}) \left(-\frac{\hbar^2 \nabla_{\boldsymbol{x}}{}^2}{2m} - \mu\right) \hat{\psi}_\alpha(\boldsymbol{x}) + U \int d^3\boldsymbol{x}\, \hat{\psi}^\dagger_\uparrow(\boldsymbol{x})\hat{\psi}^\dagger_\downarrow(\boldsymbol{x})\hat{\psi}_\downarrow(\boldsymbol{x})\hat{\psi}_\uparrow(\boldsymbol{x})$$

Density-density linear-response function

$$\chi_{nn}(q,\omega) = \frac{\chi_0(q,\omega)}{1 - \frac{U}{2}\chi_0(q,\omega)}$$

Spin-spin linear-response function

$$\chi_{S_z S_z}(q,\omega) = \frac{\chi_0(q,\omega)}{1 + \frac{U}{2}\chi_0(q,\omega)}$$

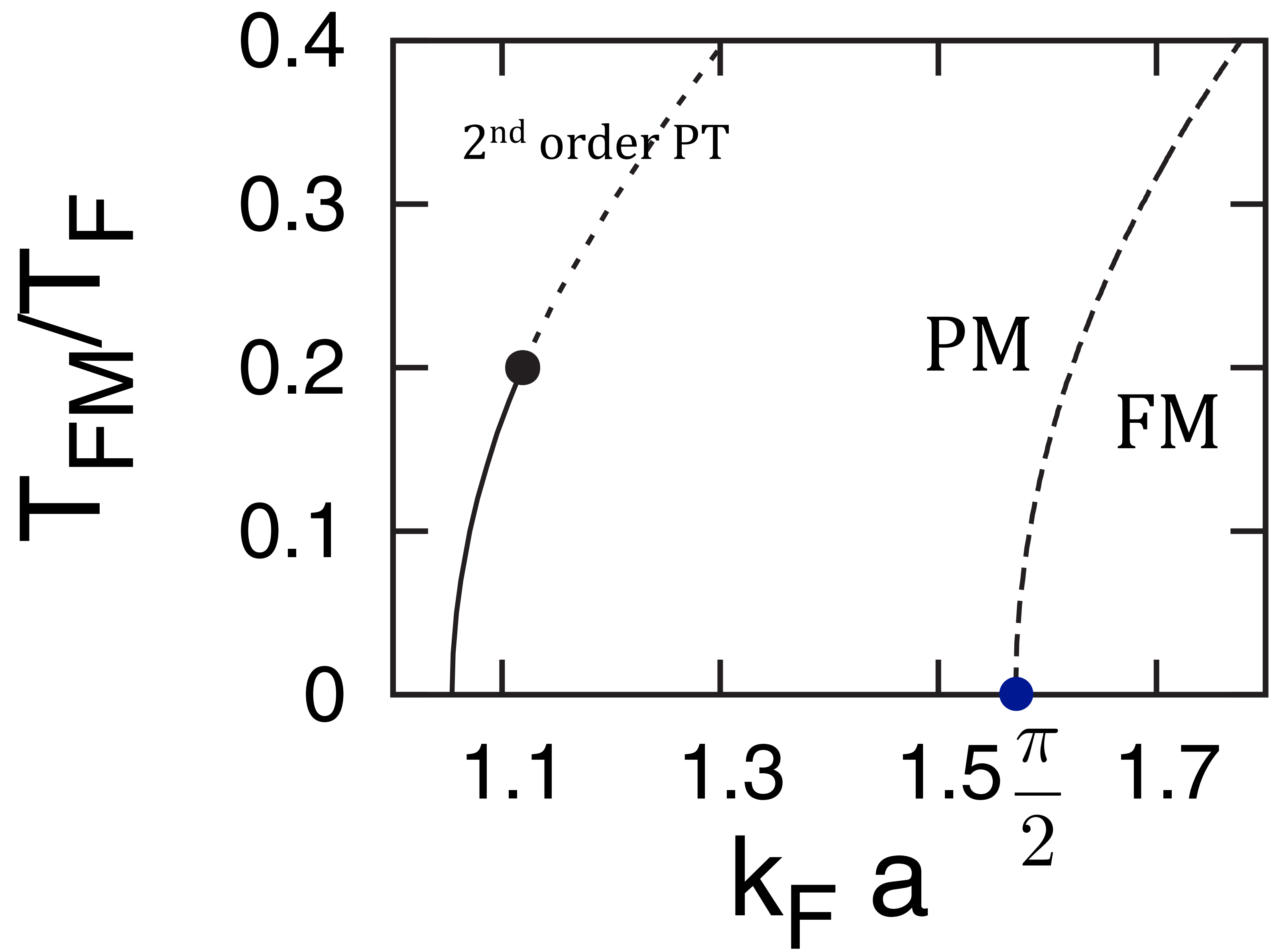
Stoner criterion for ferromagnetism:

$$1 + \frac{U}{2} \lim_{q\to 0} \lim_{\omega\to 0} \chi_0(q,\omega) = 0$$

Stoner ferromagnetism (II)

Stoner criterion for ferromagnetism:

$$1 + \frac{U}{2} \lim_{q \to 0} \lim_{\omega \to 0} \chi_0(q, \omega) = 0$$



R.A. Duine and A.H. MacDonald, Phys. Rev. Lett. **95**, 230403 (2005)

Spin-drag relaxation rate

Boltzmann transport and collision integral

Scattering amplitude

$$I_{\rm coll}[f_{\boldsymbol{k},\uparrow}] \propto \int \frac{d^D\boldsymbol{k}'}{(2\pi)^D} \int \frac{d^D\boldsymbol{q}}{(2\pi)^D} \int_{-\infty}^{+\infty} d\omega \; |A_{\uparrow\downarrow}(q,\omega)|^2 [f_{\boldsymbol{k},\uparrow}(1-f_{\boldsymbol{k}+\boldsymbol{q},\uparrow}) f_{\boldsymbol{k}',\downarrow}(1-f_{\boldsymbol{k}'-\boldsymbol{q},\downarrow}) - f_{\boldsymbol{k}+\boldsymbol{q},\uparrow}(1-f_{\boldsymbol{k},\uparrow}) f_{\boldsymbol{k}'-\boldsymbol{q},\downarrow}(1-f_{\boldsymbol{k}',\downarrow})] \delta(\omega - \varepsilon_{\boldsymbol{k}+\boldsymbol{q},\uparrow} + \varepsilon_{\boldsymbol{k},\uparrow}) \delta(\omega + \varepsilon_{\boldsymbol{k}'-\boldsymbol{q},\downarrow} - \varepsilon_{\boldsymbol{k}',\downarrow})$$

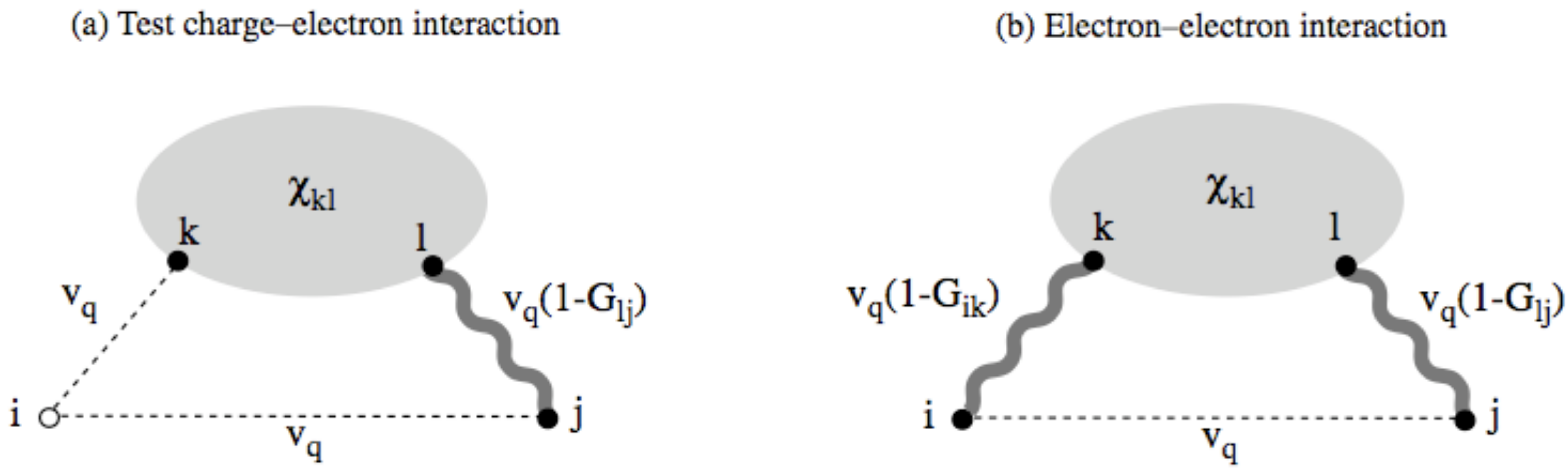
Rate of change of spin-up momentum

$$\frac{d\boldsymbol{P}_\uparrow}{dt} = \sum_{\boldsymbol{k}} \boldsymbol{k} \; I_{\rm coll}[f_{\boldsymbol{k},\uparrow}]$$

Spin-drag relaxation rate above critical temperature

$$\frac{1}{\tau_{\rm sd}(T)} = \frac{1}{4Mnk_{\rm B}T} \int \frac{d^D\boldsymbol{q}}{(2\pi)^D} \frac{q^2}{D} \int_{-\infty}^{+\infty} \frac{d\omega}{\pi} |A_{\uparrow\downarrow}(q,\omega)|^2 \frac{[\Im m \; \chi^{(0)}(q,\omega)]^2}{\sinh^2[\omega/(2k_{\rm B}T)]}$$

Effective interactions

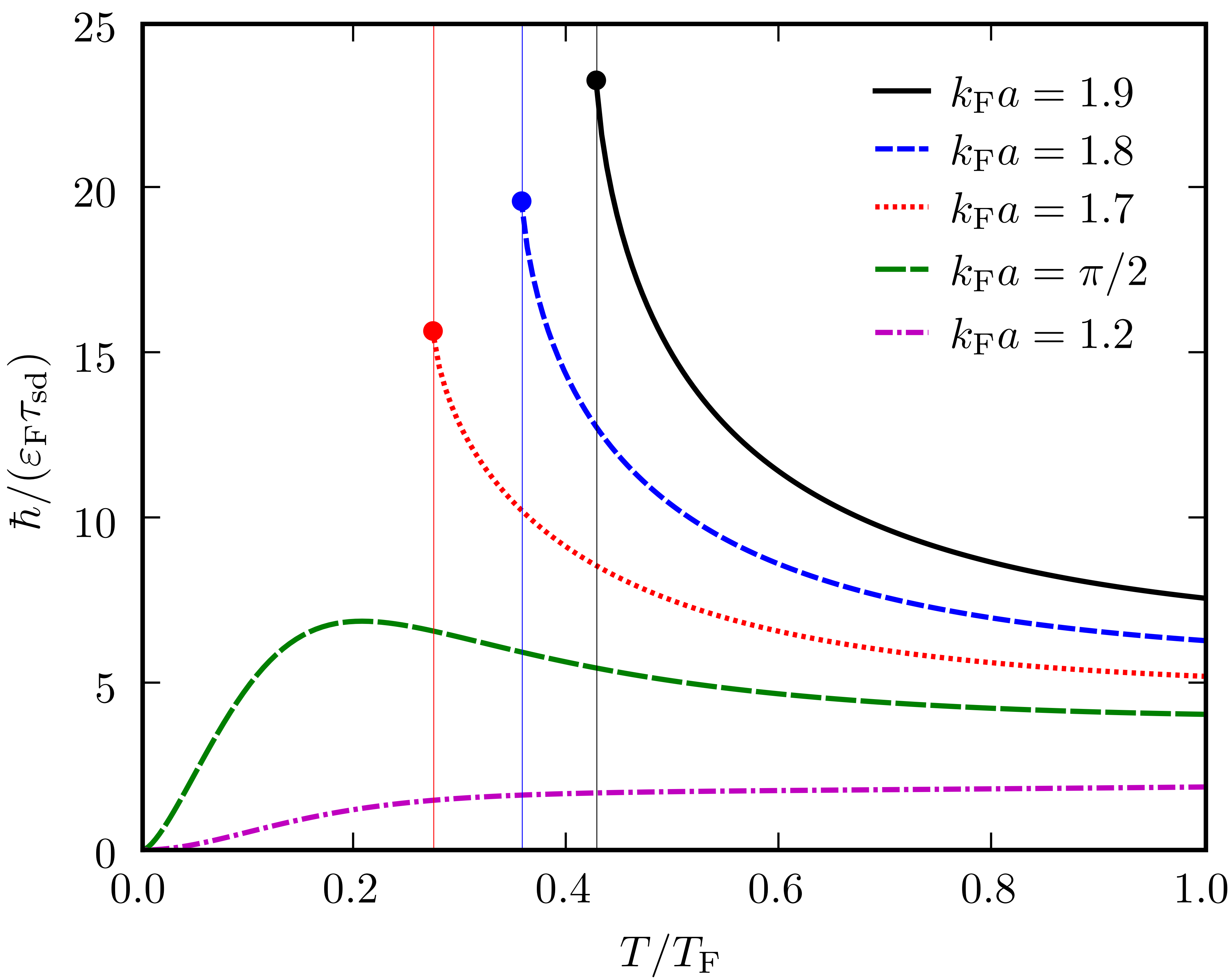


Scattering amplitude: density, longitudinal and transverse spin fluctuations

$$A_{\uparrow\downarrow}(q,\omega) = \underbrace{U}_{\text{direct term}} + \underbrace{\frac{U^2}{4}\,\chi_{\rho\rho}(q,\omega)}_{\text{density fluctuations}} - \underbrace{\frac{U^2}{4}\,\chi_{S_zS_z}(q,\omega)}_{\parallel\ \text{spin fluctuations}} - \underbrace{2\frac{U^2}{4}\,\chi_{S_zS_z}(q,\omega)}_{\perp\ \text{spin fluctuations}}$$

C.A. Kukkonen and A.W. Overhauser, Phys. Rev. B **20**, 550 (1979)
G.F. Giuliani and G. Vignale, Quantum Theory of the Electron Liquid (CUP, Cambridge, 2005)
see also A.V. Chubukov and D.L. Maslov, Phys. Rev. Lett. **103**, 216401 (2009)

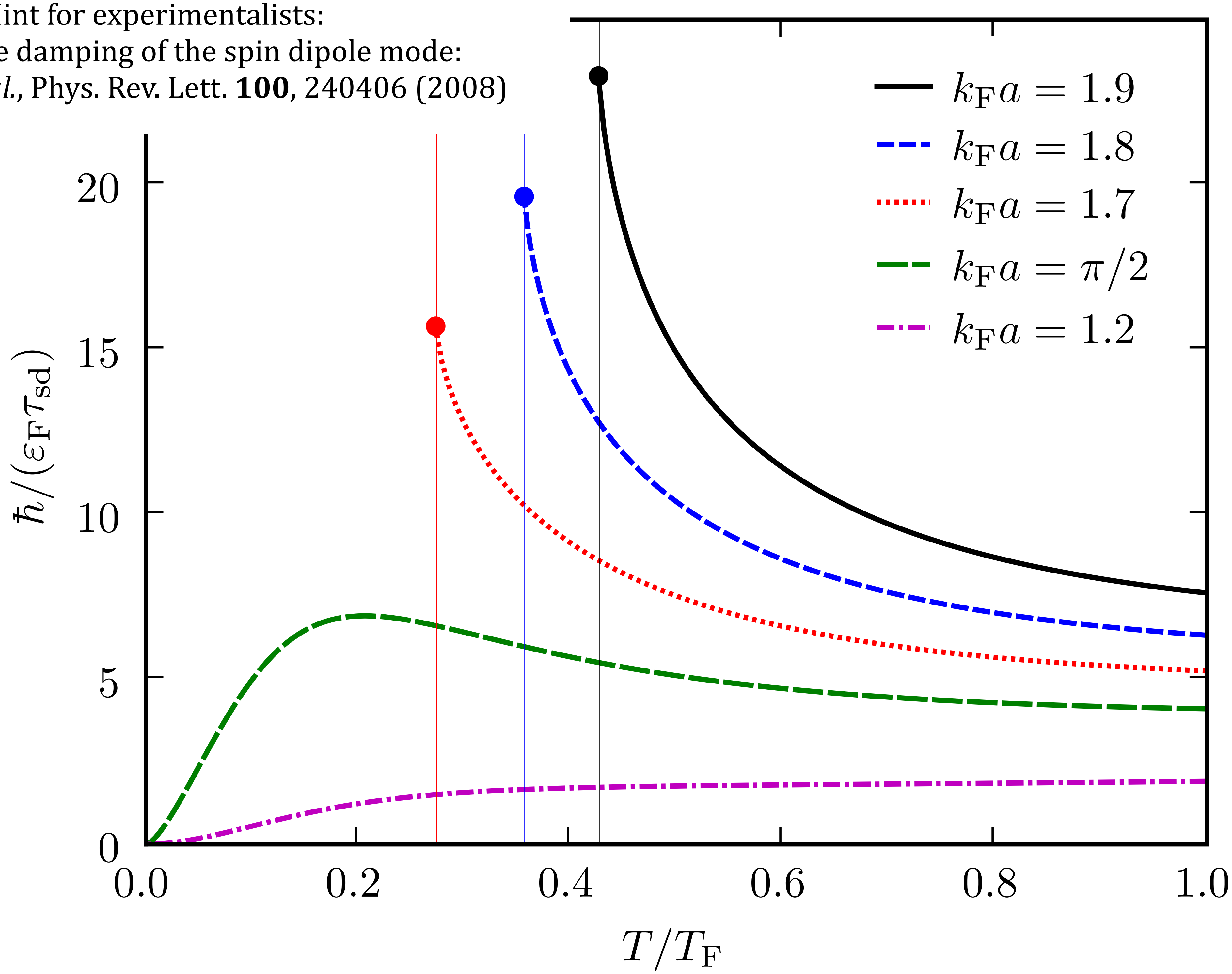
Temperature dependence of the spin-drag relaxation rate



R.A. Duine, MP, H.T.C. Stoof, and G. Vignale, Phys. Rev. Lett. **104**, 220403 (2010)

Temperature dependence of the spin-drag relaxation rate

Hint for experimentalists:
measure the damping of the spin dipole mode:
G.M. Bruun *et al.*, Phys. Rev. Lett. **100**, 240406 (2008)



R.A. Duine, MP, H.T.C. Stoof, and G. Vignale, Phys. Rev. Lett. **104**, 220403 (2010)

Summary

- Topological-insulator thin films and double-layer graphene sheets embedded in BN might be ideal hosts for superfluids of spatially-separated electrons and holes
- We have calculated a (pessimistic) upper bound for the critical temperature taking into account static screening
- We have presented a theory of Coulomb drag in double-layer-based exciton condensates of massless Dirac fermions demonstrating that it is logarithmically enhanced close to the critical temperature when the condensed phase is approached from above
- What's next? Contact-less probes of exciton condensation are very welcome: **ARPES**!

Thank you for your attention!

M.P. Mink, H.T.C. Stoof, R.A. Duine, MP, and G. Vignale, arXiv:1108.2298v1