Solid state models and emergent relativistic description

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- It is useful quantitatively understand the relation between lattice models and emerging QFT description and to keep fully into account the lattice. Methods of Constructive QFT are sometimes suitable for that.
- 1) fermionic chains (benchmark)
- 2)Hubbard models on the honeycomb lattice
- 3)A lattice gauge theory for graphene

1.

$$H = -\frac{1}{2}\sum_x [a_x^+ a_{x+1}^- + a_{x+1}^+ a_x^-] + \mu \sum_x \rho_x + \lambda \sum_{x,y} v(x-y)\rho_x \rho_y$$

where $a_x^{\pm}$ are the fermion creation or annihilation operators and $\rho_x = a_x^+ a_x^-$. $|v(x-y)| \le Ce^{-\kappa|x-y|}$.

Fermionic chains

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3. $\mathbf{x} = (x_0, x)$, $O_\mathbf{x} = e^{Hx_0} O_x e^{-Hx_0}$ and, if $A = O_{\mathbf{x}_1} \ldots O_{\mathbf{x}_n}$, $\langle A \rangle = \frac{\mathrm{Tr} e^{-\beta H} \mathbf{T}(A)}{\mathrm{Tr} e^{-\beta H}}|_T$, $\mathbf{T}$ being the time order product and T denoting truncation. $< a_{\mathbf{x}_1}^{\varepsilon_1} \ldots a_{\mathbf{x}_n}^{\varepsilon_n} >$ Schwinger functions.

Some Physical observables

1. $\mathbf{p} = (p_0, p)$ ($p_0 = \frac{2\pi n}{\beta}$ also called ω_n) Susceptibility
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2. The Drude weight

$$D = \lim_{p_0\to 0}\lim_{p\to 0} -\Delta- < \hat{J}_{\mathbf{p}}\hat{J}_{-\mathbf{p}} > \equiv \lim_{p_0\to 0}\lim_{p\to 0}\hat{D}(p_0, p)$$

J_x is the Paramagnetic current $J_x = \frac{1}{2i}[a^+_{x+1}a^-_x - a^+_x a^-_{x+1}]$, $\Delta = -\frac{1}{2} < \tau_x >$, $\tau_x = a^+_x a^-_{x+1} + a^+_{x+1}a^-_x$ is the Diamagnetic current.

The conductivity is $\sigma = \lim_{\omega\to 0}\lim_{\delta\to 0}\frac{\hat{D}(-i\omega+\delta, 0)}{-i\omega+\delta}$.

Ward Identities

1. From $\frac{\partial \rho_x}{\partial x_0} = -i[J_{x,x_0} - J_{x-1,x_0}]$ we get
2. Vertex WI
   $-ip_0 < \hat{\rho}_{\mathbf{p}} a^-_{\mathbf{k}} a^+_{\mathbf{k}+\mathbf{p}} > -i(1-e^{-ip}) < \hat{J}_{\mathbf{p}} a^-_{\mathbf{k}} a^+_{\mathbf{k}+\mathbf{p}} >=$
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3. Density and current WI

$$-ip_0 < \hat{\rho}_{\mathbf{p}} \hat{\rho}_{-\mathbf{p}} > -i(1-e^{-ip}) < \hat{J}_{\mathbf{p}} \hat{\rho}_{-\mathbf{p}} >= 0$$
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4. If the correlations are finite

$$< \hat{\rho}_{\mathbf{p}} \hat{\rho}_{-\mathbf{p}} >_{p_0,0} = 0 \quad \hat{D}(0,p) = 0$$

Exponents as convergent series in the spin chain

1. Benfatto, Mastropietro CMP (2002). For λ small enough

$$\langle \rho_{\mathbf{x}} \rho_{\mathbf{0}} \rangle \sim \frac{\cos(2p_F x)(1+O(\lambda))}{2\pi^2[x^2+(v_F x_0)^2]^{X_+}} + \frac{1+O(\lambda)}{2\pi^2[x^2+(v_F x_0)^2]}$$

where $p_F = \cos^{-1}(\mu) + O(\lambda)$ ($\mu = 0$ $p_F = \pi/2$),
$v_F = \sin p_F + O(\lambda)$, X_+ convergent expansion

$$X_+ = 1 - \frac{[\hat{v}(0) - \hat{v}(2p_F)]}{(\pi \sin p_F)} \lambda + O(\lambda^2)$$

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2. X_- is the Cooper pair 2-point function $a^+_{\mathbf{x}} a^+_{\mathbf{x}'}$ exponent

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Luttinger liquid relations

1. In the solvable Luttinger model

$$\kappa = \frac{X_+}{\pi v_F} \quad D = \frac{v_F X_+}{\pi} \quad v_F^2 = D/\kappa$$

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2. True in the XXZ model; by the Bethe ansatz $\cos\bar{\mu} = -\lambda$, $v_F = \frac{\pi}{\bar{\mu}}\sin\bar{\mu}$, $\kappa = [2\pi(\pi/\bar{\mu} - 1)\sin\bar{\mu}]^{-1}$ and $v_F^2 = D/\kappa$. Note that $X_+ = (2(1-\frac{\bar{\mu}}{\pi}))^{-1} = 1 - \frac{2\lambda}{\pi} + O(\lambda^2)$ $(\hat{v}(p) = e^{ip})$ (cfr before $\hat{v}(p) = e^{ip}$).

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$$\varepsilon(\omega k) = (\omega k - p_F) + \frac{1}{2m}(\omega k - k_F)^2 + \lambda \frac{1}{12m^2 v_F}(\omega k - k_F)^3$$

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4. Regularity properties (from conv. exp.)+lattice WI+Emergent WI→ LL Relations.

Theorem

(Benfatto,Mastropietro CMP 2009, JSP 2010) For λ small enough, there exists K analytic $K = 1 - \lambda\frac{\hat{v}(0)-\hat{v}(2p_F)}{\pi \sin p_F} + O(\lambda^2)$ such that

$$X_+ = K \quad , \quad X_- = K^{-1} \quad 2\eta = K + K^{-1} - 2$$

and

$$D = \frac{v_F K}{\pi} \quad \kappa = \frac{K}{\pi v_F}$$

Equivalence with a QFT model

- We introduce the QFT model, if $j_\mu = \bar{\psi}_{\mathbf{x}} \gamma_\mu \psi_{\mathbf{x}}$

$$\int P(d\psi^{(\leq N)}) e^{\tilde{\lambda}_\infty \int d\mathbf{x} v(\mathbf{x}-\mathbf{y}) j_{\mu,\mathbf{x}} j_{\mu,\mathbf{y}}}$$

where $\psi = \psi_+, \psi_-$, $k_\mu = k_0, ck$, $P(d\psi^{(\leq N)})$ have propagator $\chi_N(\mathbf{k}) \frac{\not{\mathbf{k}}}{|\mathbf{k}|^2}$ with a smooth cut-off function vanishing for $|\mathbf{k}| \geq 2^N$ and $v(\mathbf{x}-\mathbf{y})$ a short range symmetric interaction.

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- A multiscale integration is now necessary also in the ultraviolet region to perform the limit $N \to \infty$.

Equivalence with a QFT model

It is possible to choose $\tilde{\lambda}_\infty$ and c functions of λ $c = v_F$, the exponents coincide and, for $\kappa \leq |\mathbf{k}|, |\mathbf{k}, \mathbf{k} + \mathbf{p}| \leq \kappa$

$$< \hat{\rho}_{\mathbf{p}} \hat{a}^{+}_{\mathbf{k}+\mathbf{p}_F^\omega} \hat{a}^{-}_{\mathbf{k}+\mathbf{p}+\mathbf{p}_F^\omega} >= \frac{Z^{(3)}}{Z^2} < \hat{j}_{0,\mathbf{p}} \hat{\psi}^{+}_{\mathbf{k},\omega} \hat{\psi}^{-}_{\mathbf{k}+\mathbf{p},\omega} > (1 + r_1)$$

$$< \hat{J}_{\mathbf{p}} \hat{a}^{+}_{\mathbf{k}+\mathbf{p}_F^\omega} \hat{a}^{-}_{\mathbf{k}+\mathbf{p}+\mathbf{p}_F^\omega} >= \sin p_F \frac{\tilde{Z}^{(3)}}{Z^2} < \hat{j}_{1,\mathbf{p}} \hat{\psi}^{+}_{\mathbf{k},\omega} \hat{\psi}^{-}_{\mathbf{k}+\mathbf{p},\omega} > (1 + r_2)$$

with $|r_1|, |r_2| \leq C\kappa^\theta$ (contribution from the irrelevant terms)

$$\frac{\tilde{Z}^{(3)}}{Z^{(3)}} = 1 + \frac{[\hat{v}(0) - \hat{v}(2p_F)]}{\pi \sin p_F} \lambda + O(\lambda^2)$$

Rigorous relations connecting the fermionic chain (l.h.s) with the QFT model (r.h.s.); essential that the QFT is regularized with a momentum cut-off and the fact that there is a line of fixed points.

1. Moreover

$$< \hat{\rho}_{\mathbf{p}} \hat{\rho}_{-\mathbf{p}} > = [\frac{Z^{(3)}}{Z}]^2 < \hat{j}_{0,\mathbf{p}} \hat{j}_{0,-\mathbf{p}} > + \hat{A}_{\rho,\rho}(\mathbf{p})$$

$$< \hat{J}_{\mathbf{p}} \hat{J}_{-\mathbf{p}} > = (\sin p_F)^2 [\frac{\tilde{Z}^{(3)}}{Z}]^2 < \hat{j}_{1,\mathbf{p}} \hat{j}_{i,-\mathbf{p}} > + \hat{A}_{j,j}(\mathbf{p})$$

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$$|A_{\rho,\rho}(\mathbf{x})|, |A_{j,j}(\mathbf{x})| \leq \frac{C}{|\mathbf{x}|^{2+\theta}}$$

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2. Therefore $A_{\rho,\rho}(\mathbf{p}), A_{j,j}(\mathbf{p})$ are continuous
3. What we have gained? The QFT model has a symmetry more (the chiral one) so more WI, and this implies more relations (emergent WI) for the chain model.

Ward Identities

- $$\mathbf{p}_\mu < \hat{j}_{\mu,\mathbf{p}} \hat{\psi}^+_{\mathbf{k}} \hat{\psi}^-_{\mathbf{k}+\mathbf{p}} > = < \hat{\psi}^+_{\mathbf{k},\omega} \hat{\psi}^-_{\mathbf{k},\omega} > - < \hat{\psi}^+_{\mathbf{k}+\mathbf{p}} \hat{\psi}^-_{\mathbf{k}+\mathbf{p}} > + \Delta_N$$
  $$\lim_{N\to\infty} \Delta_N(\mathbf{k},\mathbf{p}) = i\nu \mathbf{p}_\mu < j_{\mu,\mathbf{p}} \psi_{\mathbf{k},\omega} \bar{\psi}_{\mathbf{k}+\mathbf{p},\omega} >$$
  with $\nu = \frac{\tilde{\lambda}_\infty}{4\pi c} \hat{v}(\mathbf{p})$

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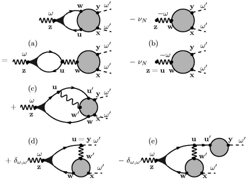
  with $\nu = \frac{\tilde{\lambda}_\infty}{4\pi c} \hat{v}(\mathbf{p})$
- ν is linear in $\tilde{\lambda}_\infty$. Non perturbative 1+1 analogue of anomaly non renormalization in QED4. Note: with local interaction high order corrections $\nu = \frac{\tilde{\lambda}_\infty}{4\pi c} + b\tilde{\lambda}^2_\infty + \ldots$. (CFR Adler-Bardeen (1969), Jackiw-Johnson (1969))



Fixing parameters

- In the limit $N \to \infty$ the QFT WI has the form (Johnson 1961 recovered)

$$\gamma_\mu \mathbf{p}_\mu < j_{\mu,\mathbf{p}} \psi_{\mathbf{k},\omega} \bar{\psi}_{\mathbf{k}+\mathbf{p}} >= \frac{1}{1-\nu} [< \psi_{\mathbf{k}} \bar{\psi}_{\mathbf{k}} > - < \psi_{\mathbf{k}+\mathbf{p}} \bar{\psi}_{\mathbf{k}+\mathbf{p}} >]$$

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- Implies a WI for the chain

$$ip_0 < \hat{\rho}_{\mathbf{p}} a^-_{\mathbf{k}} a^+_{\mathbf{k+p}} > -p \frac{v_F}{\sin p_F} \frac{Z^{(3)}}{\tilde{Z}^{(3)}} < \hat{J}_{\mathbf{p}} a^-_{\mathbf{k}} a^+_{\mathbf{k+p}} >=$$

$$\frac{Z^{(3)}}{Z(1-\nu)}[< a^-_{\mathbf{k}} a^+_{\mathbf{k}} > - < a^-_{\mathbf{k+p}} a^+_{\mathbf{k+p}} >]$$

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$$\frac{Z^{(3)}}{Z(1-\nu)} [< a^-_{\mathbf{k}} a^+_{\mathbf{k}} > - < a^-_{\mathbf{k}+\mathbf{p}} a^+_{\mathbf{k}+\mathbf{p}} >]$$

- It must coincide with the one found via continuity equation therefore

$$\frac{Z^{(3)}}{(1-\nu) Z} = 1 \qquad \frac{v_F}{\sin p_F} \frac{Z^{(3)}}{\tilde{Z}^{(3)}} = 1$$

Emerging WI due to chiral phase. The invariance under $\psi_\pm \to e^{i\alpha_\pm}\psi_\pm$ of the QFT implies, if $j_0 = \rho_+ + \rho_-$, $j_1 = \rho_+ - \rho_-$, implies if $D_\pm = -ip_0 \pm v_F p$

$$< \hat\rho_{\mathbf{p}} \hat\rho_{-\mathbf{p}} >= [\frac{Z^{(3)}}{Z}]^2 \frac{1}{4\pi v_F} \frac{1}{1-\nu^2} [\frac{D_-(\mathbf{p})}{D_+(\mathbf{p})} + \frac{D_+(\mathbf{p})}{D_-(\mathbf{p})} - 2] + \hat A_{\rho,\rho}(\mathbf{p})$$

$$< \hat J_{\mathbf{p}} \hat J_{-\mathbf{p}} >= (\sin p_F)^2 [\frac{\tilde Z^{(3)}}{Z}]^2 \frac{1}{4\pi v_F} \frac{1}{1-\nu^2} [\frac{D_-(\mathbf{p})}{D_+(\mathbf{p})} + \frac{D_+(\mathbf{p})}{D_-(\mathbf{p})} - 2] + \hat A_{j,j}(\mathbf{p})$$

The LL relations

- The value of $\hat{A}_{\rho,\rho}(\mathbf{0})$ (exists by continuity) is fixed by the lattice WI $< \hat{\rho}_{\mathbf{p}} \hat{\rho}_{-\mathbf{p}} > |_{p,0} = 0$ (irrelevant if you neglect wrong result!).
- With this value, using $\frac{Z^{(3)}}{(1-\nu)Z} = 1$, if $K = \frac{1-\nu}{1+\nu}$

$$< \hat{\rho}_{\mathbf{p}} \hat{\rho}_{-\mathbf{p}} >= \frac{K}{\pi v_F} \frac{v_F^2 p^2}{p_0^2 + v_F^2 p^2} + O(\mathbf{p})$$

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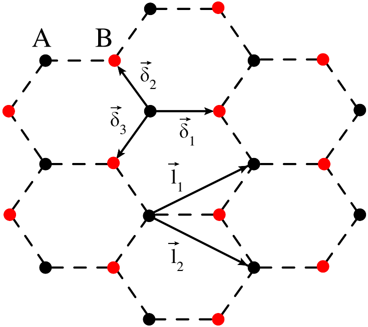
- Similarly, from $D(0,p) = 0$ (due to lattice WI) and $v_F \frac{Z^{(3)}}{\sin p_F \tilde{Z}^{(3)}} = 1$ we get

$$D(\mathbf{p}) = \frac{K v_F}{\pi} \frac{p_0^2}{p_0^2 + v_F^2 p^2} + O(\mathbf{p})$$

implying the LL relation for the non solvable chain $v_F^2 = D/\kappa$.
- The irrelevant terms plays a crucial role

Honeycomb lattice



$$H = -\bar{t} \sum_{\vec{x}\in\Lambda, i=1,2,3} \sum_{\sigma=\uparrow\downarrow} \left(a^+_{\vec{x},\sigma} b^-_{\vec{x}+\vec{\delta}_i,\sigma} + b^+_{\vec{x}+\vec{\delta}_i,\sigma} a^-_{\vec{x},\sigma} \right)$$

$$+U \sum_{\vec{x}\in\Lambda_A} \prod_{\sigma=\uparrow\downarrow} (a^+_{\vec{x},\sigma} a^-_{\vec{x},\sigma} - \frac{1}{2}) + U \sum_{\vec{x}\in\Lambda_B} \prod_{\sigma=\uparrow\downarrow} (b^+_{\vec{x},\sigma} b^-_{\vec{x},\sigma} - \frac{1}{2})$$

where $\Lambda_A = \Lambda = \{n_1\vec{l}_1 + n_2\vec{l}_2 \ : \ n_1, n_2 = 0, \ldots, L-1\}$ be a periodic triangular lattice of period L, with basis vectors: $\vec{l}_1 = \frac{1}{2}(3, \sqrt{3})$, $\vec{l}_2 = \frac{1}{2}(3, -\sqrt{3})$ and

$$\vec{\delta}_1 = (1,0) \ , \quad \vec{\delta}_2 = \frac{1}{2}(-1, \sqrt{3}) \ , \quad \vec{\delta}_3 = \frac{1}{2}(-1, -\sqrt{3})$$

Zero temperature optical conductivity

$$\sigma_{lm} = \lim_{p_0 \to 0} \lim_{\beta \to \infty} -\frac{2}{3\sqrt{3}} \frac{1}{p_0} \left[\hat{K}_{lm}(p_0, 0) + \Delta_{lm} \right] ,$$

where $\hat{K}_{l,m}(\mathbf{p})$ is the Fourier transform of $\langle J_{\mathbf{x},l}; J_{\mathbf{y},m} \rangle_\beta$, with $J_{\mathbf{x}} \equiv v_F^{(0)} j_{\mathbf{x}}$ the paramagnetic current

$$\vec{J}_{\vec{p}} = ie\bar{t} \sum_{\substack{\vec{x} \in \Lambda \\ \sigma, j}} e^{-i\vec{p}\vec{x}} \vec{\delta}_j \eta^j_{\vec{p}} \left(a^+_{\vec{x},\sigma} b^-_{\vec{x}+\vec{\delta}_j,\sigma} - b^+_{\vec{x}+\vec{\delta}_j,\sigma} a^-_{\vec{x},\sigma} \right)$$

with $v_F^{(0)} = \frac{3}{2}\bar{t}$, $\eta^j_{\vec{p}} = \frac{1 - e^{-i\vec{p}\vec{\delta}_j}}{i\vec{p}\vec{\delta}_j}$; sum of the three bond currents. Δ_{lm} is the diamagnetic term.

The free case

1. The 2-point function $S(\mathbf{k}) =< \Psi^-_{\mathbf{k}} \Psi^+_{\mathbf{k}} >$

$$S(\mathbf{k}) = \frac{1}{k_0^2 + |v(\vec{k})|^2} \begin{pmatrix} ik_0 & -v^*(\vec{k}) \\ -v(\vec{k}) & ik_0 \end{pmatrix},$$

$v(\vec{k}) = \sum_{i=1}^{3} e^{i\vec{k}(\vec{\delta}_i - \vec{\delta}_1)} = 1 + 2e^{-i3/2k_1} \cos \frac{\sqrt{3}}{2} k_2.$

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2. If $\vec{p}_F^{\,\pm} = (\frac{2\pi}{3}, \pm \frac{2\pi}{3\sqrt{3}})$ close to a Dirac propagator

$$S(\mathbf{k} + \mathbf{p}_F^{\pm}) \sim \begin{pmatrix} ik_0 & v_F^{(0)}(ik_1' \mp k_2') \\ v_F^{(0)}(-ik_1' \mp k_2') & ik_0 \end{pmatrix}^{-1},$$

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4. Same results found for free Dirac fermions (Ludwig et al (1994)).

- Indeed recent optical measurements in graphene (Nair et al. Nat. Mat. (2007)) show that at half-filling and small temperatures, if the frequency is above the temperature, the conductivity is essentially constant and equal, up to a few percent, to $\sigma_0 = \frac{e^2}{h}\frac{\pi}{2}$.

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- Of course, interaction effects could produce modifications to this theoretical value, obtained by neglecting interactions; There are interaction corrections to conductivity ?

Effective QFT model

1. Effective Nambu Jona-Lasinio model in $d = 2 + 1$

$$\int d\mathbf{x} \bar{\Psi}_{\mathbf{x}} \gamma_\mu \partial_\mu \Psi_{\mathbf{x}} + U \int d\mathbf{x} (\bar{\Psi}_{\mathbf{x}} \gamma_\mu \Psi_{\mathbf{x}})(\bar{\Psi}_{\mathbf{x}} \gamma_\mu \Psi_{\mathbf{x}})$$

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2. An ultraviolet regularizations is necessary to avoid uv divergences
3. With a momentum cut-off (somewhat more realistic) one gets non vanishing corrections to the conductivity; with dimensional cut-off (Juricic, Vafek, Herbut PRB 2010) no corrections at second order. Cut-off dependence; which is the correct answer?

Universality of the conductivity

Theorem

Giuliani-Mastropietro-Porta PRB 2010. There exists a constant $U_0 > 0$ *such that, for* $|U| \leq U_0$ *and any fixed* p_0*,* $\sigma^{\beta}_{lm}(p_0)$ *is analytic in* U *uniformly in* β *and*

$$\sigma_{lm} = \lim_{p_0 \to 0^+} \lim_{\beta \to \infty} \sigma_{lm}(p_0) = \frac{e^2}{h} \frac{\pi}{2} \delta_{lm} \ .$$

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- Close to the limit we have $\beta^{-1} \ll p_0 \ll t$, which corresponds to the range of frequencies investigated with optical techniques.

Ward Identities

1. The WI are derived by the continuity equation

$$-ie\partial_{x_0}\rho_{(x_0,\vec{p})} + i\vec{p}\cdot\vec{J}_{(x_0,\vec{p})} = 0$$

If $\hat{G}_{2,1;\mu}(\mathbf{k},\mathbf{p})$ is the vertex $\mu = 0$ density, $\mu = 1, 2$ current) and $\hat{K}_{\mu,\nu}$ the density-density $\mu = \nu = 0$ or current correlations

$$p^{\mu}\hat{G}_{2,1;\mu}(\mathbf{k},\mathbf{p}) = -e\hat{S}(\mathbf{k}+\mathbf{p}) + e\hat{S}(\mathbf{k})$$

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2. $$p^\mu \hat{K}^\beta_{\mu 0}(\mathbf{p}) = 0$$

$$p^\mu \hat{K}_{\mu m}(\mathbf{p}) = -\lim_{L\to\infty}\frac{1}{L^2}\left[\vec{p}\cdot\langle\hat{\Delta}_{\vec{p},-\vec{p}}\rangle\right]_m \qquad m = 1,2$$

Convergence of the series expansion

1. Giuliani-Mastropietro (CMP 2007):

$$S(\mathbf{k}) = \frac{1}{Z}\begin{pmatrix} -ik_0 & -v_F\Omega^*(\mathbf{k}) \\ -v_F\Omega(\mathbf{k}) & -ik_0 \end{pmatrix}^{-1}(1 + O(|\mathbf{k} - \mathbf{p}_F^\omega|^\theta)) ,$$

where

$$Z = Z(U) = 1 + O(U^2) \quad v_F = v_F(U) = \frac{3t}{2} + O(U^2)$$

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   where

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   are given by convergent series (again by determinant bounds).

2. v_F is different from $v_F^{(0)}$ (increases $v_F(U) > v_F(0)$); isotropy of the Dirac cones follows from the lattice symmetries. Non universal

1. If $0 < |\mathbf{p}| \ll |\mathbf{k} - \mathbf{p}_F^\omega| \ll 1$

$$\hat{G}_{2,1;\mu} = eZ_\mu S(\mathbf{k}+\mathbf{p})\Gamma_\mu(\vec{p}_F^{\pm}, \vec{0})S(\mathbf{k})\left(1 + O(|\mathbf{k} - \mathbf{p}_F^\omega|^\theta)\right) ,$$

where $Z_\mu = Z_\mu(U)$ are analytic in U and $0 < \theta < 1$.
$Z_1 = Z_2$

The vertex and current function

1. If $0 < |\mathbf{p}| \ll |\mathbf{k} - \mathbf{p}_F^\omega| \ll 1$

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   where $Z_\mu = Z_\mu(U)$ are analytic in U and $0 < \theta < 1$.
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2. $$\hat{K}_{lm}(\mathbf{p}) = \frac{Z_l Z_m}{Z^2}\langle \hat{\jmath}_{\mathbf{p},l}; \hat{\jmath}_{-\mathbf{p},m}\rangle_{0,v_F} + \hat{R}_{lm}(\mathbf{p})$$

   where $\langle \cdot \rangle_{0,v_F}$ is the average associated to a non-interacting system with Fermi velocity $v_F(U)$ and

   $$|R_{lm}(\mathbf{x},\mathbf{y})| \leq \frac{C}{1 + |\mathbf{x}-\mathbf{y}|^{4+\theta}}$$

   with $0 < \theta < 1$, so that $\hat{R}_{lm}(p_0, \vec{0})$ is continuous and differentiable at $\mathbf{p} = \mathbf{0}$ (CFR 1d chain; similar but here with free QFT).

Implications of WI

1. By the lattice WI and the fact that the 2 and vertex functions have vanishing corrections at the FS

$$Z_0 = Z \ , \qquad Z_1 = Z_2 = v_F Z \ .$$

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Implications of WI

1. As

$$|K_{\mu,\nu}(\mathbf{x})| \leq \frac{C}{1+|\mathbf{x}|^4} \ ,$$

$\hat{K}_{\mu\nu}(\mathbf{p})$ is continuous at $\mathbf{p}=\mathbf{0}$; from the WI

$$\begin{aligned} i\frac{p_0}{p_1}\hat{K}_{0m}(p_0,p_1,0) &= \Big[\hat{K}_{1m}(p_0,p_1,0) + \\ &+ \lim_{\beta,L\to\infty}\frac{1}{L^2}\langle[\hat{\Delta}_{(p_1,0),(-p_1,0)}]_{1m}\rangle_{\beta,L}\Big] \ . \end{aligned}$$

Taking first the limit $p_0 \to 0$ and then the limit $p_1 \to 0$ we get that the l.h.s. is vanishing in the limit.

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Taking first the limit $p_0 \to 0$ and then the limit $p_1 \to 0$ we get that the l.h.s. is vanishing in the limit.
2. Therefore by continuity
$$\sigma_{lm} = -\frac{2}{3\sqrt{3}} \lim_{p_0\to 0^+} \lim_{\beta\to\infty} \frac{1}{p_0}\Big[\hat{K}_{lm}(p_0, \vec{0}) - \hat{K}_{lm}(\mathbf{0})\Big] \, .$$

Finally

$$\sigma_{lm} = -\frac{2}{3\sqrt{3}} \lim_{p_0 \to 0^+} \frac{1}{p_0} \Big[\big(\hat{R}_{lm}(p_0, \vec{0}) - \hat{R}_{lm}(\mathbf{p}) \big)$$
$$+ \Big(v_F^2 \big\langle \hat{\jmath}_{(p_0,\vec{0}),l} ; \hat{\jmath}_{(-p_0,\vec{0}),m} \big\rangle_{0,v_F} - v_F^2 \big\langle \hat{\jmath}_{\mathbf{0},l} ; \hat{\jmath}_{\mathbf{0},m} \big\rangle_{0,v_F} \Big) \Big] \, .$$

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$$+ \Big(v_F^2 \langle \hat{\jmath}_{(p_0,\vec{0}),l} ; \hat{\jmath}_{(-p_0,\vec{0}),m} \rangle_{0,v_F} - v_F^2 \langle \hat{\jmath}_{\mathbf{0},l} ; \hat{\jmath}_{\mathbf{0},m} \rangle_{0,v_F} \Big) \Big] .$$

The first term is differentiable and even hence vanishing, while the first term is identical to the free one (it does not depend from v_F)

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Lattice Gauge theory for graphene

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- The interaction with the gauge field is obtained via the Peierls substitution

$$H_0 = -\bar{t} \sum_{\substack{\vec{x}\in\Lambda_A \\ j=1,2,3}} \sum_{\sigma=\uparrow\downarrow} a^+_{\vec{x},\sigma} b^-_{\vec{x}+\vec{\delta}_j,\sigma} e^{ie \int_0^1 \vec{\delta}_j \cdot \vec{A}(\vec{x}+s\vec{\delta}_j,0)\, ds} + c.c.$$

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- $A(x) = (\vec{A}(x), A_3(x))$ is the quantized vector potential in the Coulomb gauge, H_A is the energy
- $$V_C = \frac{e^2}{2} \sum_{\vec{x},\vec{y}\in\Lambda_A\cup\Lambda_B} (n_{\vec{x}} - 1)\varphi(\vec{x} - \vec{y})(n_{\vec{y}} - 1) \ ,$$

  where $\hat{\varphi}_{\vec{p}} := \int \frac{dp_3}{(2\pi)} \frac{\chi(|\vec{p}|^2+p_3^2)}{|\vec{p}|^2+p_3^2}$ is a regularized version of the static Coulomb potential.

Functional integral representation

- The Schwinger functions can be obtained by the following generating functional

$$e^{\mathcal{W}^{\xi}(\Phi,J,\lambda)} = \int P(d\Psi) P^{\xi,h^*}(dA) e^{V(\Psi,A+J)+\mathcal{B}(\Psi,A+J,\Phi)+(\lambda,\Psi)}$$

where $P(d\Psi)$ is the fermionic integration and $P(dA)$ is the gauge field integration in the ξ gauge with an infrared cut-off 2^{h^*}

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$$\partial_{\xi}\frac{\int P(d\Psi)P^{\xi,h^*}(dA)e^{V(\Psi,A)}F(A,\Psi)}{\int P(d\Psi)P^{\xi,h^*}(dA)e^{V(\Psi,A)}} = 0$$

  where $F(\Psi, A) = F(e^{ie\alpha}\Psi, A + \partial\alpha)$. The most convenient gauge is the Feynman $\xi = 0$.

Ward Identities

- $$0 = \frac{\partial}{\partial \hat{\alpha}_{\mathbf{p}}} \mathcal{W}^{\xi}(\Phi, J + \partial\alpha, \lambda e^{ie\alpha})\Big|_{\hat{\alpha}=0}$$

  and making derivatives with respect to the external fields WI are derived.

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  and making derivatives with respect to the external fields WI are derived.
- WI with a bosonic cut-off at scale 2^h to get information on the effective couplings of the theory with no cut-offs and at scale h.

The flow of the effective couplings

1. The honeycomb lattice symmetries has to be exploited carefully to show that a number of possible terms are indeed irrelevant.

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3. Thanks to the WI

$$\nu_h = O(e^2 2^h) \quad e_h \to e_{-\infty} = e + O(e^5)$$
$$v_h \to c \quad Z_h \sim 2^{-\eta h}$$

where η is a critical exponent. Vanishing of the beta function of the charge; vanishing of the photon mass; line of fixed points.

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4. Emergent relativistic symmetry.

Cancellations at lowest orders

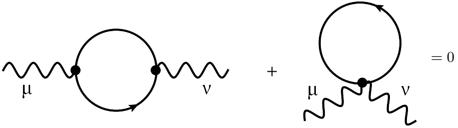

FIG. 1:

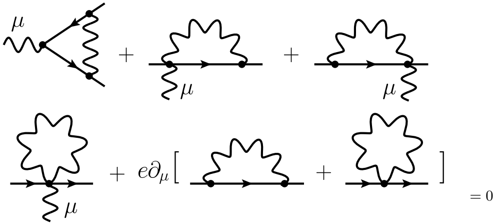

FIG. 2:

$$< \Psi^- \Psi^+ >_{\mathbf{k}' + \mathbf{p}_F^\omega} \sim$$

$$-\frac{1}{Z(\mathbf{k}')} \begin{pmatrix} ik_0 & v(\mathbf{k}')(-ik_1' + \omega k_2') \\ v(\mathbf{k}')(ik_1' + \omega k_2') + & ik_0 \end{pmatrix}^{-1}$$

where

$$Z(\mathbf{k}') \simeq |\mathbf{k}'|^{-\eta} , \qquad 1 - v(\mathbf{k}') \simeq (1-v)|\mathbf{k}'|^{\tilde{\eta}} ,$$

where

$$\eta = \frac{e^2}{12\pi^2} + O(e^4) , \qquad \tilde{\eta} = \frac{2e^2}{5\pi^2} + O(e^4) ,$$

The Fermi velocity increases up to the light velocity and the wave function renormalization vanishes.

1. The fact that the Fermi velocity flows up to the light velocity was predicted first by Gonzalez-Guinea-Vozmediano model (Nucl Phys 1994) in the effective model

$$\int d\mathbf{x}\bar{\Psi}_{\mathbf{x}}(\gamma_0\partial_0 + v\vec{\gamma}\vec{\partial} + \not{A})\psi_{\mathbf{x}}$$

with $v \neq c$

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with $v \neq c$

2. Dimensional regularization is however necessary to achieve this; if momentum regularizations is used in the GGV model

$$v_h \to c - ae^2 + \ldots$$

$a > 0$ No emergent Lorentz symmetry with momentum cut-off. If gauge invariance is lost no emergent symmetry.

Anomalous exponents

1. We consider responses to sever bilinear, in particular Kekule' (K), Charge Density waves (CDW), Neel antiferromagnetism (N), Superconductivity (S), Haldane currents (H) responses. If $e = 0$ they decays as $|\mathbf{x} - \mathbf{y}|^{-4}$.

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2. The interaction produces anomalous exponents; the response is enhanced for K, CDW, N, H; exponents $4 - \xi$ with

$$\xi^{(K)} = \frac{4e^2}{3\pi^2} + O(e^4); \quad \xi^{(CDW)} = \frac{4e^2}{3\pi^2} + O(e^4)$$

$$\xi^{(AF)} = \frac{4e^2}{3\pi^2} + O(e^4); \quad \xi^{(H)} = \frac{4e^2}{3\pi^2} + O(e^4)$$

Anomalous exponents

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4. $\xi^J = 0$ at all orders, by the WI.

Anomalous exponents

$$
\begin{aligned}
\zeta^{K}_{\mathbf{x},j} &= \sum_\sigma \left(e^{ie\int_0^1 ds\, \delta_j \vec{A}_{\mathbf{x}+s\delta_j}} a^+_{\mathbf{x},\sigma} b^-_{\mathbf{x}+\delta_j,\sigma} + c.c. \right) \\
\zeta^{CDW}_{\mathbf{x},j} &= \sum_\sigma \left(a^+_{\mathbf{x},\sigma} a^-_{\mathbf{x},\sigma} - b^+_{\mathbf{x}+\delta_j,\sigma} b^-_{\mathbf{x}+\delta_j,\sigma} \right) \\
\zeta^{AF}_{\mathbf{x},j} &= \sum_\sigma \sigma \left(a^+_{\mathbf{x},\sigma} a^-_{\mathbf{x},\sigma} - b^+_{\mathbf{x}+\delta_j,\sigma} b^-_{\mathbf{x}+\delta_j,\sigma} \right) \\
\zeta^{D}_{\mathbf{x},j} &= \sum_\sigma \left(a^+_{\mathbf{x},\sigma} a^-_{\mathbf{x},\sigma} + b^+_{\mathbf{x}+\delta_j,\sigma} b^-_{\mathbf{x}+\delta_j,\sigma} \right) \\
\zeta^{J}_{\mathbf{x},j} &= \sum_\sigma \left(ie^{ie\int_0^1 ds\, \vec{\delta}_j \vec{A}_{\mathbf{x}+s\delta_j}} a^+_{\mathbf{x},\sigma} b^-_{\mathbf{x}+d_j,\sigma} + c.c. \right) \\
\zeta^{H}_{\mathbf{x},j} &= \sum_\sigma \left(ie^{ie\int_0^1 ds\, \vec{m}_j \vec{A}_{\mathbf{x}+s\mathbf{m}_j}} a^+_{\mathbf{x},\sigma} a^-_{\mathbf{x}+\mathbf{m}_j,\sigma} - \right. \\
& \left. e^{-ie\int_0^1 ds\, \vec{m}_j \vec{A}_{\mathbf{x}+s\mathbf{m}_j}} b^+_{\mathbf{x}+\delta_j,\sigma} b^-_{\mathbf{x}+\delta_j+\mathbf{m}_j,\sigma} + c.c. \right)
\end{aligned}
$$

where in the last line $m_1 = \delta_2 - \delta_3$, $m_2 = \delta_3 - \delta_1$ and $m_3 = \delta_1 - \delta_2$ indicate next to nearest neighbor vectors.

Anomalous exponents

$$R_{ij}^{(K)}(\mathbf{x}) = \frac{27}{8\pi^2} A_K \frac{\cos\left(\vec{p}_F^{+}(\vec{x} - \vec{\delta}_i + \vec{\delta}_j)\right)}{|\mathbf{x}|^{4-\xi^{(K)}}} + r_{ij}^{(K)}(\mathbf{x}) \ ,$$

$$R_{ij}^{(CDW)}(\mathbf{x}) = \frac{27}{8\pi^2} A_{CDW} \frac{1}{|\mathbf{x}|^{4-\xi^{(CDW)}}} + r_{ij}^{(CDW)}(\mathbf{x}) \ ,$$

$$R_{jj'}^{(AF)}(\mathbf{x}) = \frac{27}{8\pi^2} A_{AF} \frac{1}{|\mathbf{x}|^{4-\xi^{(AF)}}} + r_{ij}^{(AF)}(\mathbf{x}) \ ,$$

$$R_{jj'}^{(H)}(\mathbf{x}) = \frac{27}{8\pi^2} A_{H} \frac{1}{|\mathbf{x}|^{4-\xi^{(H)}}} + r_{ij}^{(H)}(\mathbf{x}) \ ,$$

where $A_{\#} = 1 + O(1-v) + O(e^2))$. Moreover, the correction terms $r_{ij}^{(a)}(\mathbf{x})$ are subdominant contributions, decaying at infinity faster than $|\mathbf{x}|^{-4+\xi^{(a)}}$.

Mass terms

- If we allow distortions of the honeycomb lattice, the hopping becomes a function of the bond length $\ell_{\vec{x},j}$

$$t_{\vec{x},j} = t + g(\ell_{\vec{x},j} - \bar{\ell})$$

with $\bar{\ell}$ the equilibrium length of the bonds. A Kekulé dimerization pattern corresponds to, for any $j_0 \in \{1,2,3\}$, if $\phi_{\vec{x},j} = g(\ell_{\vec{x},j} - \bar{\ell})$

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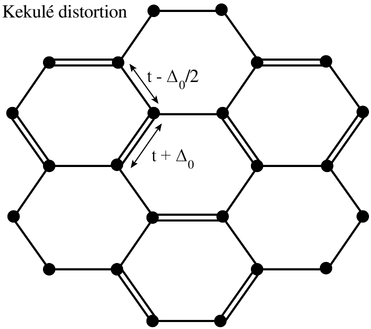
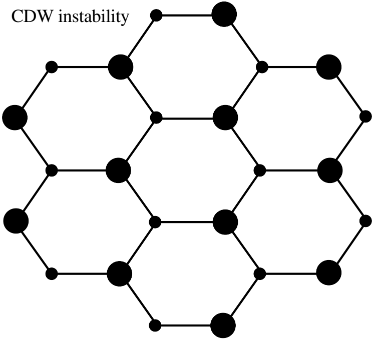
- Similarly we can add a term describing an electronic density asymmetry between the two sublattices

$$\Delta_0 \sum_{\sigma\vec{x}} [a^+_{\vec{x}} a^-_{\vec{x}} - b^+_{\vec{x}+\vec{\delta}1} a^-_{\vec{x}+\vec{\delta}_2}]$$

Mass terms





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- In the absence of interaction such terms produce a mass Δ_0

$$||\langle\psi^{-}_{\mathbf{k}'+\mathbf{p}^{\pm}_F}\psi^{-}_{\mathbf{k}'+\mathbf{p}^{\pm}_F}\rangle|| \sim \frac{1}{\sqrt{|\mathbf{k}'|^2+\Delta_0^2}}$$

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$$\Delta(\mathbf{k}_F^\pm) = \Delta_0^{1/(1+\eta^K)} \quad \eta^K = 2e^2/(3\pi^2) + \cdot$$

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- Increasing of mass with momentum resembles mass generation in Gorbar, Gusynin,Miransky PRB (2002); here the bare mass cannot vanish as lattice acts as a uv cut-off.

Gap equation

- We can let $\underline{\phi} = \{\phi_{\vec{x},j}\}_{\vec{x}\in\Lambda}^{j=1,2,3}$ be a classical field to be fixed self-consistently, so that the total energy $E_0(\underline{\phi}) + \frac{\kappa}{2g^2}\sum_{\substack{\vec{x}\in\Lambda \\ j}} \phi_{\vec{x},j}^2$ is minimal.

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- The Kekulé distortion pattern is a stationary point, provided that $\phi_0 = c_0 g^2/\kappa + \cdots$ for a suitable constant c_0 and that Δ_0 satisfies the following non-BCS gap equation:

$$\Delta_0 \simeq 6\frac{g^2}{\kappa} \int_{\Delta \lesssim |\mathbf{k}'| \lesssim 1} d\mathbf{k}' \frac{Z^{-1}(\mathbf{k}')\Delta(\mathbf{k}')}{k_0^2 + v^2(\mathbf{k}')|\Omega(\vec{k}' + \vec{k}_F^\omega)|^2 + |\Delta(\mathbf{k}')|^2} ,$$

where $\Delta = \Delta_0^{1/(1+\eta_\Delta)}$ and $Z(\mathbf{k}') \sim |\mathbf{k}'|^{-\eta}$, $v(\mathbf{k}') \sim 1 - (1-v)|\mathbf{k}'|^{\tilde{\eta}}$ and $\Delta(\mathbf{k}') \sim \Delta_0\,|\mathbf{k}'|^{-\eta_\Delta}$, $\eta^\Delta = \xi^K/2$.

Gap equation

- Our gap equation has the same qualitative properties of the simpler equation:

$$1 = g^2 \int_\Delta^1 d\rho \frac{\rho^{\eta - \eta_\Delta}}{1 - (1 - v)\rho^{\tilde{\eta}}}$$

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- For small e $g_c \sim \sqrt{v}$, with v the free Fermi velocity, even though the effective Fermi velocity tends to the speed of light ($e = 0$ reduces to the one in Hou-Chamon-Mudry PRL 2007;Chamon,Hou,Jackiw,Mudry,Pi,Semenoff PRB 2008)

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- Interactions facilitate the formation of a Kekulé pattern. If $\eta_\Delta - \eta = \frac{7e^2}{12\pi^2} + \cdots$ exceeds 1, then the integrand in the r.h.s. of the gap equation diverges as $\Delta \to 0$ so $g_c \to 0$ (Spontaneous distortion).

Transverse conductivity

- If we consider a bilayer graphene, the intra-planar e.m. interaction decreases the inter-planar transverse conductivity; if $H = H_1 + H_2 + tP$ with $P = -t \sum_{\vec{x} \in \Lambda_A} [a^+_{\vec{x},1} a^-_{\vec{x},2} + a^+_{\vec{x},2} a^-_{\vec{x},1} + \{a \to b\}]$

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- Transverse conductivity

$$\sigma^{\perp}_{\beta}(\omega_n) = \frac{1}{\omega_n}\lim_{p\to 0}[-\left\langle \hat{j}^{P,\perp}_{\mathbf{p}};\hat{j}^{P,\perp}_{-\mathbf{p}}\right\rangle + \langle j^{D,\perp}_x\rangle]$$

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- CFR with transverse conductivity between Luttinger liquids in the same range $\sigma^{\perp}_{\beta}(\omega_n) \sim t^2\omega_n^{2\eta-1}$

Open problems

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4. Transverse conductivity at lower temperatures
5. Of course: large U, gap generation ...

Some references

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