Anderson localization in quark-gluon plasma

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Analogy: Dirac operator of QCD — random Hamiltonian

$T < T_c$

- Statistics of small Dirac eigenvalues:
  random matrix statistics (Verbaarschot, Shuryak,...)
    - Symmetries $\Rightarrow$ Effective σ-model (Gasser & Leutwyler,...)
    - Lattice (numerical)

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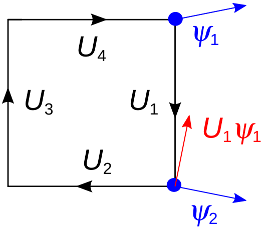
$T > T_c$

- No analytic result available
- Is the Dirac op. a "random matrix"?

Introduction: the lattice Dirac operator



- ψ_i quark fields on lattice sites
- $U_i \in SU(N_c)$ $(N_c = 3$ in QCD)
  - vector potential $\rightarrow$ $U \approx e^{iA}$
  - U_i's dynamical var's on links

Discretization

Derivative:	$\partial_\mu \psi$	$\rightarrow$	$\frac{1}{a}(\psi_2 - \psi_1)$
Covariant derivative:	$D_\mu \psi$	$\rightarrow$	$\frac{1}{a}(\psi_2 - U_1\psi_1)$
Gauge action:	$\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$	$\rightarrow$	$-\frac{1}{g^2}\,\mathrm{tr}(U_1U_2U_3U_4)$

Long distance theory — Lorentz invariant QCD

Lattice Dirac operator

- Partition function (integrating out quarks):

$$Z = \int \mathscr{D}\psi\, \mathscr{D}\bar{\psi}\, \mathscr{D}U \; e^{-S_g[U] \; - \; \bar{\psi}\{D[U]+M\}\psi}$$

$$= \int \mathscr{D}U \;\; det\{D[U]+M\} \cdot e^{-S_g[U]}$$

- Statistical physics system (4-dimensional, Euclidean)
- Dynamical variables: $U_i \in SU(N_c)$ on lattice links
- Temperature: $T = \frac{1}{N_t a}$ (N_t: extension in Euclidean time)

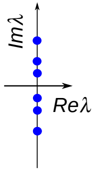
Dirac operator: $D[U]$

- Discretized differential operator depending on U-s
- $(D[U]+M)^{-1}$ appears in physical quantities
- Small eigenvalues (eigenvectors) physically important

The structure of the Dirac operator

- Symmeries:

$$\left.\begin{array}{l}\{\gamma_5, D\} = 0 \\ D^\dagger = -D\end{array}\right\} \Rightarrow \quad D = \begin{pmatrix} 0 & iC \\ iC^\dagger & 0 \end{pmatrix}$$



- $\Rightarrow$ Spectrum imaginary, symmetric around 0

- probability distribution of $[U]$ $\Rightarrow$ random $D[U]$ with given distribution
- distribution of $D[U]$ $\Rightarrow$ physical quantities
- eigenvalue ststistics of $D[U]$ $\Rightarrow$ bulk termodynamics

What do we know about the spectral statistics of $D[U]$?

Is the detailed dynamics important or it is already given by the symmetries?

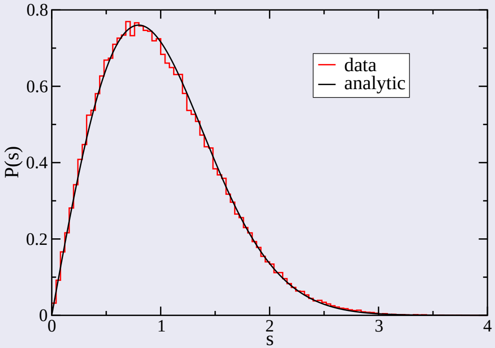
Random matrix theory (RMT)

- $N \times N$-es matrices ($N \gg 1$), iid. random elements
- Statistical properties of the spectrum are largely universal
- Depends on: certain symmetries of the matrices
- Within a given class, largely independent of:
  - Distribution of matrix elements
  - Detailed structure of the matrix (eg.: which elements vanish, etc.)
- Analytically calculable if matrix elements are Gaussian distributed
- No preferred basis $\Rightarrow$ typical eigenvectors "delocalized"

Example: chiral orthogonal ensemble

- Matrices of the form $\begin{pmatrix} 0 & iW \\ iW^\dagger & 0 \end{pmatrix}$ with $W \in \mathbb{R}^{n\times n}$
- W_{ij} distributed independently and uniformly in $[-1, 1]$

Distribution of nearest neighbor level spacings ("unfolded"=scaled)

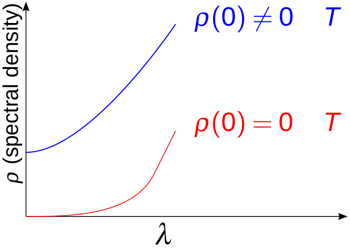


5000 matrices of size 100×100

$$s = \frac{\lambda_{i+1} - \lambda_i}{\langle \lambda_{i+1} - \lambda_i \rangle}$$

Is the Dirac operator a “random matrix”?

- $\lambda = 0$ special point (symmetry).
- Transition at $T_c \approx 200$MeV :



$\rho(0) \neq 0$ $T < T_c$ hadronic phase

$\rho(0) = 0$ $T > T_c$ quark-gluon plasma

$\rho(0) \neq 0 \quad \Rightarrow$ statistics of low eigenvalues of $D[U]$ described by random matrix theory

analytically (σ-model) $+$ numerically (lattice QCD)

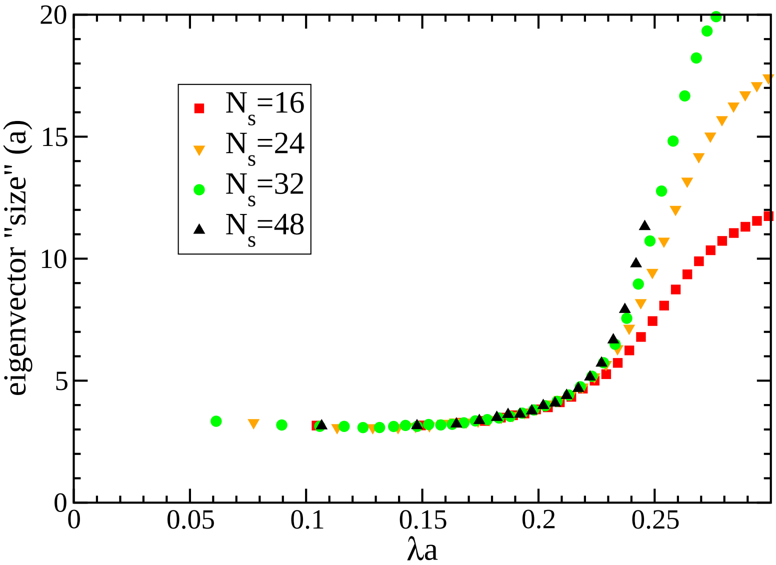
Lattice simulation (this work)

TGK, PRL **104** (2010) 031601
TGK & F. Pittler, PRL **105** (2010) 192001
F. Bruckmann, TGK and S. Schierenberg, PRD **84** (2011) 034505 $\leftarrow$ talk by S.S.

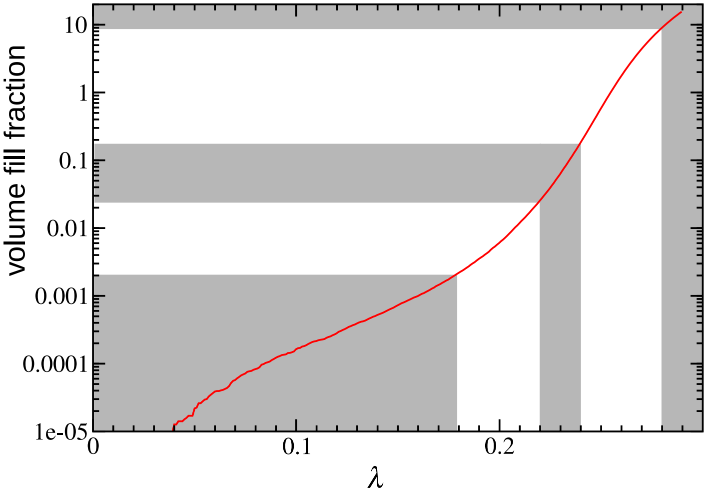
- Number of colors: $N_c = 2, 3$
  $\Rightarrow$ gauge symmetry: $SU(2), SU(3)$
- $T = 2.6 T_c$
- Various lattice spacings: $N_t = 4, 6, 8 \quad (T = \frac{1}{aN_t})$
- Various spatial volumes: $N_s = 12 - 48$
- Different discretizations:
  overlap, staggered (similar results).
- "Quenched" (ignoring $det(D+m)$) and
  $N_f = 2+1$ dynamical (similar results)

Average "spatial size" of eigenvectors

based on participation ratio (staggered, $N_t = 4$)



Volume fill fraction of eigenvectors



spectral density increases

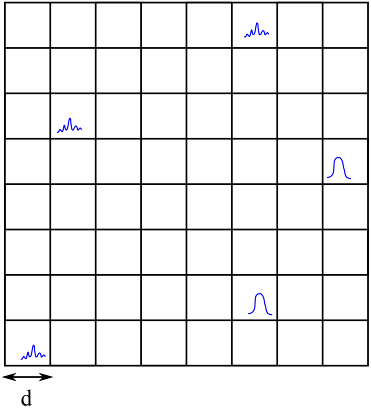
eigenvectors become delocalized

$\longrightarrow$

Typical eigenvectors

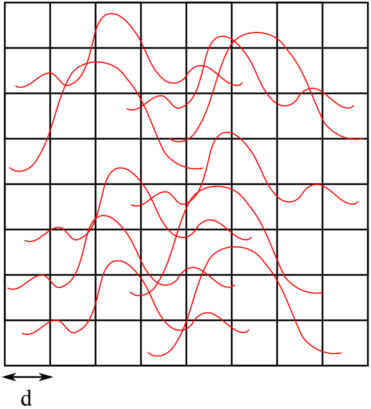
lowest eigenmodes

volume fill fraction $\ll 1$



higher up in the spectrum

volume fill fraction $\gg 1$

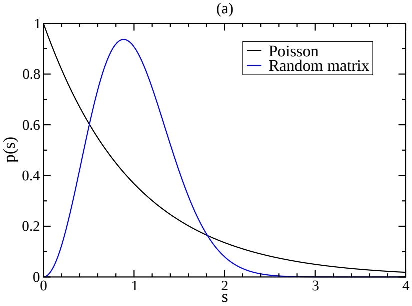


Simple picture $T > T_c$

- Eigenmodes at the low end of the spectrum are localized in small subvolumes of $\approx d^3$.
- Avg. number of small modes per subvolume of $d^3 \quad \ll 1$.
- $\Rightarrow$ Modes sample different random gauge backgrounds
  $\Rightarrow$ they are statistically independent.
- Is that reflected in the spectral statistics?

Unfolded level spacing distribution

- Level spacing: $\lambda_{n+1} - \lambda_n$
- Unfolding: rescaling $s = \frac{\lambda_{n+1}-\lambda_n}{\langle \lambda_{n+1}-\lambda_n \rangle}$
- Two extreme possibilities:
  - λ_n statistically independent $\Rightarrow$ $p(s) = \exp(-s)$
  - Eigenmodes mix maximally $\Rightarrow$ $p(s)$ random matrix stat.



Statistics of Dirac eigenvalues above T_c

$SU(3)$, $N_f = 2+1$, staggered, $N_t = 4$ $N_s = 32$,

unfolded level spacing distribution $p(s)$ $\quad s = \frac{\lambda_{n+1} - \lambda_n}{\langle \lambda_{n+1} - \lambda_n \rangle}$

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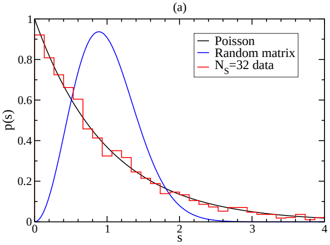

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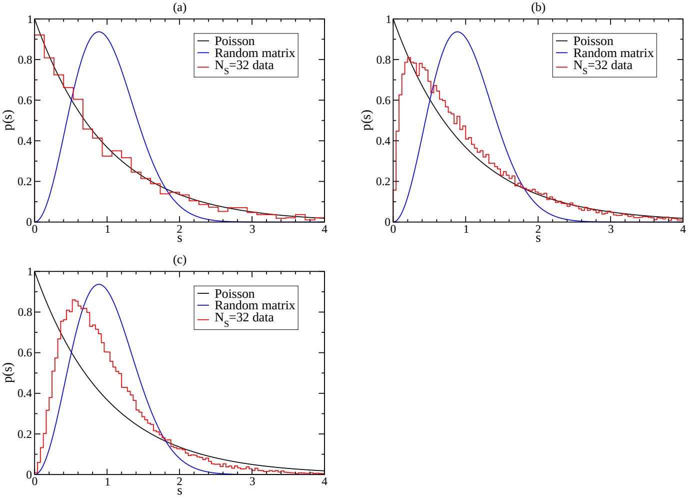

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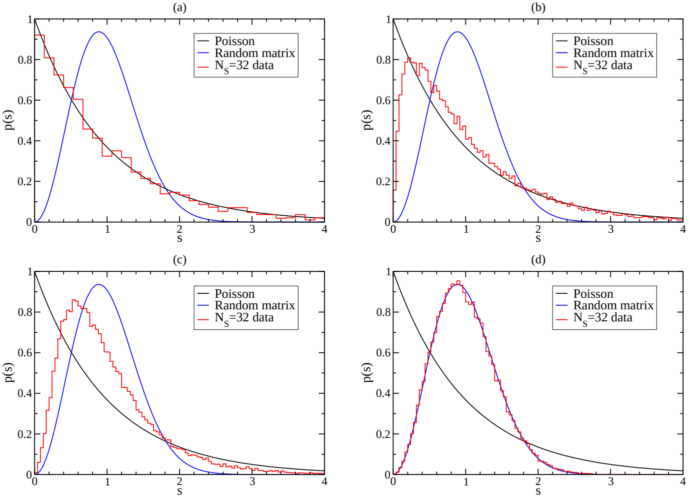
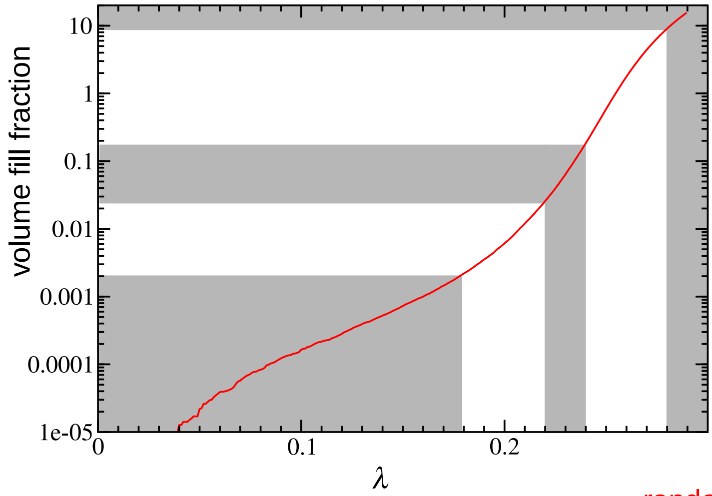


Spectral statistics at $T > T_c$



Poisson transition random matrix →

Analogy: Anderson localization

Is the hadron → quark-gluon plasma transition an Anderson transition? (Garcia-Garcia, Osborn, Phys. Rev. **D75**, 034503 2007)

Anderson localization in solid state physics:

- Perfect periodic crystal → Delocalized electron states: bands
- Defects (disorder) → 1-electron H-operator "random matrix".
- Strong disorder → localized states appear at the band edge.

But!
No on-site disorder. Disorder is in hopping terms (gauge field).

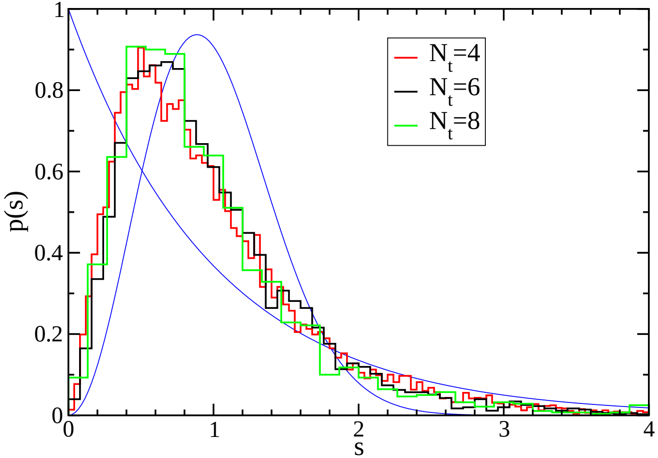
Continuum limit

- Tune system to 2^{nd} order phase transition
- $\xi \to \infty$ (in lattice units)
- Change physical lattice spacing a such that $\xi a = \text{const.}$
- Lattice spacing: $a \to 0$ since $\frac{1}{M_{\text{phys}}} \approx \xi a$
- Keep physical volume and temperature fixed.

$$\Rightarrow \quad N_t, N_s \propto \frac{1}{a}$$

Continuum limit: level spacing distribution (unfolded)

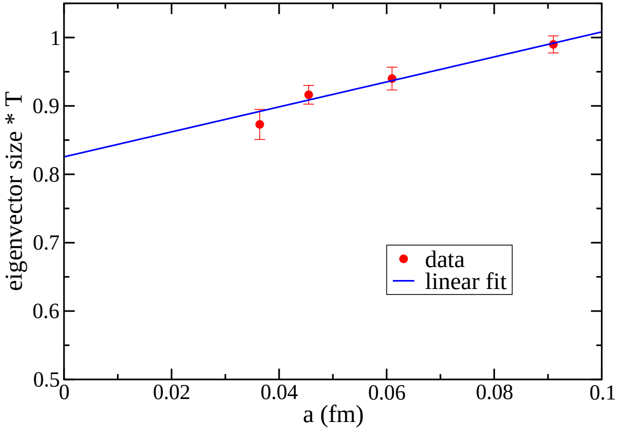
same physical volume and T, same physical location in the spectrum



Transition in the spectrum at the same physical point:
$\lambda \approx 500\text{MeV}$

Continuum limit: linear size of eigenmodes

SU(2) quenched, overlap, $N_t = 4 - 10$, $T = 2.6T_c$



$\Rightarrow$ physical objects, not “dislocations”

Conclusions, further questions

- Poisson $\rightarrow$ RMT transition in Dirac spectrum at $T > T_c$: generic feature of 4d non-Abelian gauge theories
- Density and linear size of localized modes scales
- What is their physical origin?
- Analogy to Anderson localization
  (Garcia-Garcia and Osborn, PRD 2007)
- Any measurable effects e.g. in heavy ion collisions?