# Aharonov-Bohm interferences from local deformations in graphene

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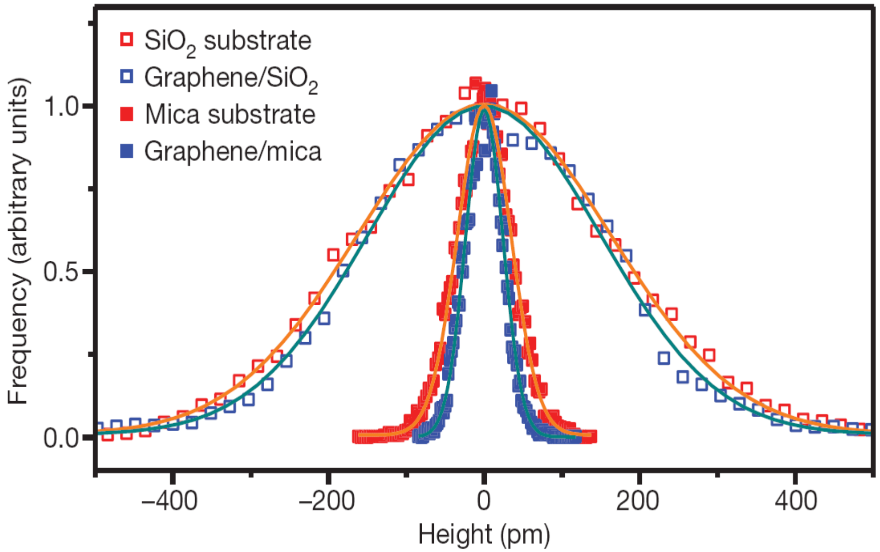
Nature Physics (2011)
DOI: 10.1038/NPHYS2034

# Graphene

Castro Neto, Guinea, Peres, Physics World (Nov 2006)

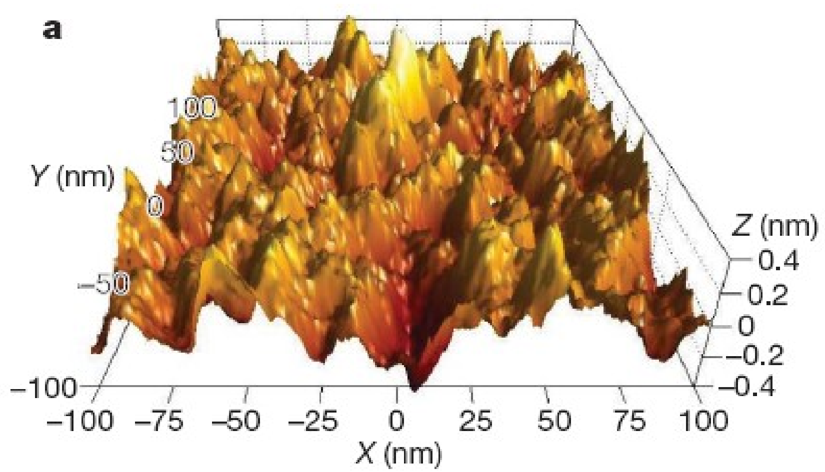
- Two dimensional crystal
- One of the strongest materials ever measured: Young modulus of TPa
- Yet, corrugations (and 3D structure in general) ubiquitous
- It supports large values of strain (20%)
- Interplay of electronics and structure!

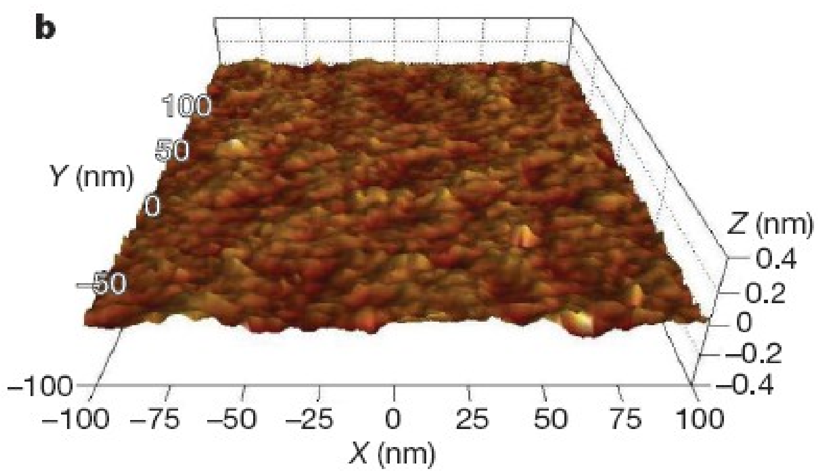
# Substrate induced curvature



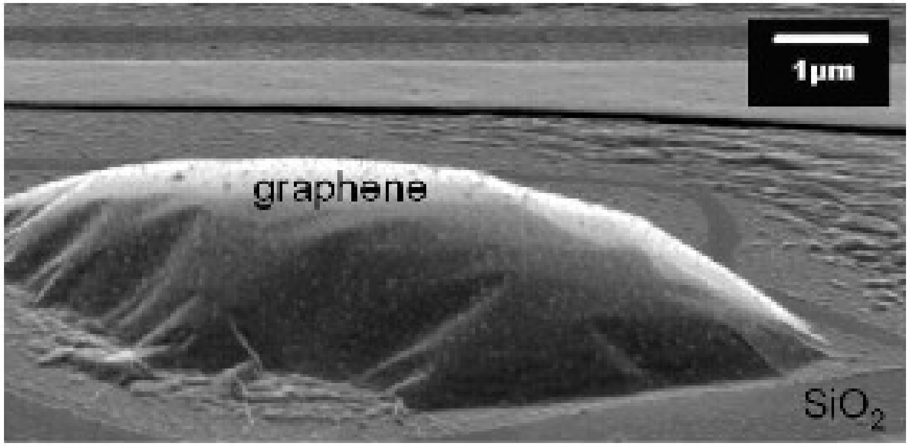

AFM experiments: Ripples correlate with substrate morphology

Lui *et al.* , Nature **462** 339 (2009)

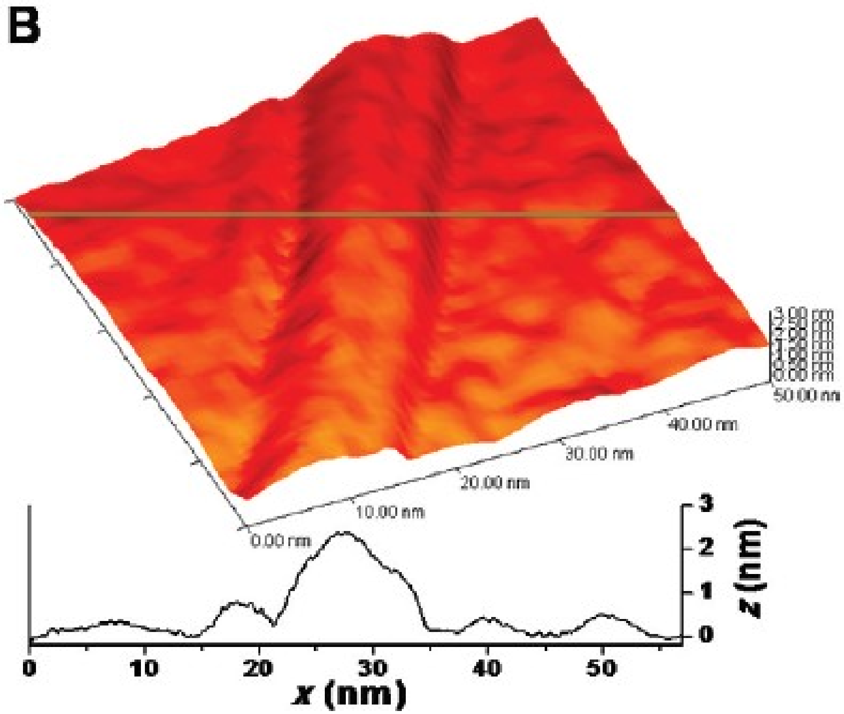


# Bubbles and wrinkles

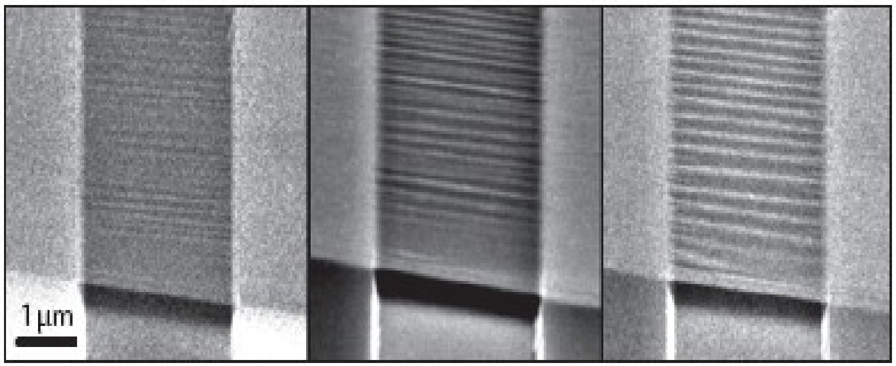



“Observation of Graphene Bubbles and Effective Mass Transport under Graphene Films” Stolyarova *et al*. , Nanolett.. **9** 332 (2009).

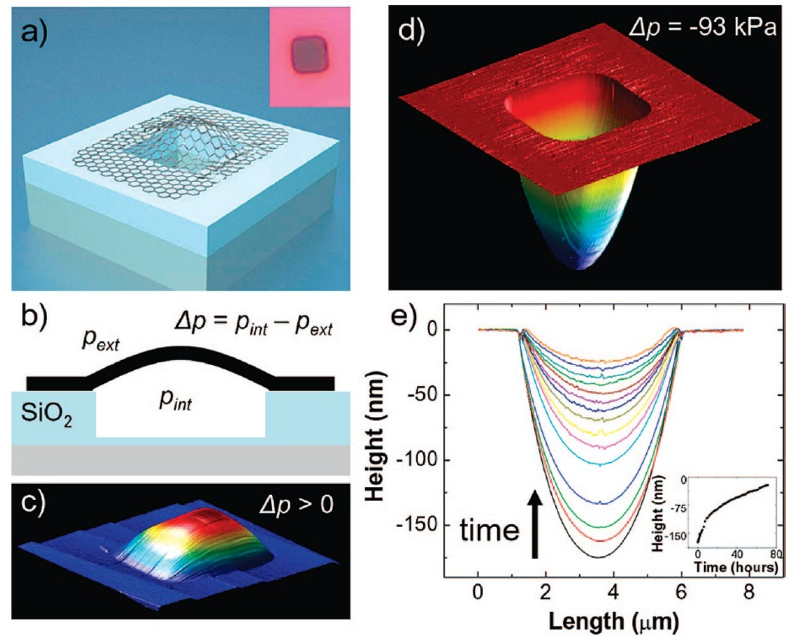



“Scanning Tunneling Microscopy Characterization of the Electrical Properties of Wrinkles in Exfoliated Graphene Monolayers”, Xu *et al*. , Nanolett. **9** 4446 (2009)

# Controlling strain




"**Controlled ripple texturing** of suspended graphene and ultrathin graphite membranes" Bao *et al.* , Nat. Nanotech. **4** 562 (2009)




"Impermeable atomic membranes from graphene sheets" Scott Bunch *et al.*, Nanolett. **8**, 2458 (2008)

- "**Introducing Nonuniform Strain** to Graphene Using Dielectric Nanopillars" (cond-mat/1106.1507 Tomori *et al.*)
- "Graphene bubbles with **controllable curvature**" (Cond-mat/1108.1701, Manchester group )
- "Topological properties of artificial graphene assembled by atom manipulation", where they produced **atomically engineered strains**. (Manoharan group, APS 2011)
- And the list goes on...

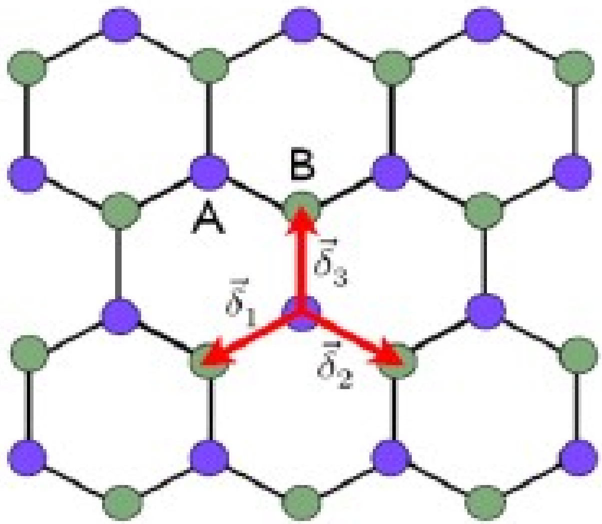
# Electrons in graphene

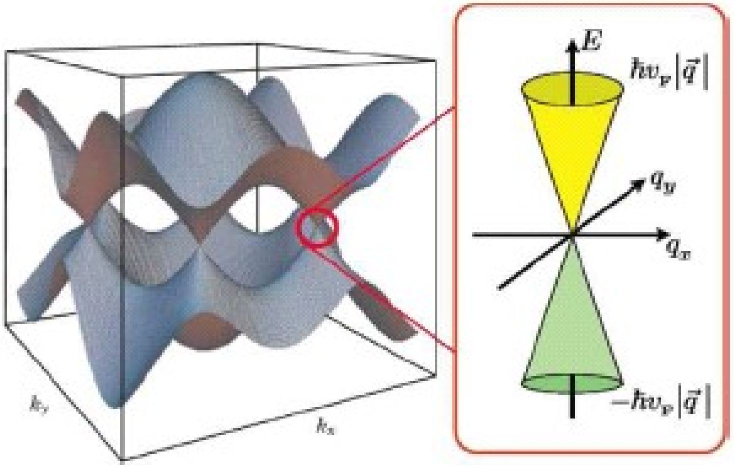
**Tight binding band structure:**

- Nearest neighbour hopping: t ~2.7 eV
- Two atoms per unit cell
- 2x2 Hamiltonian

$$H = -t \sum_{<ij>} a_i^\dagger b_j + cc.$$

$$= -t \sum_{n=1}^{3} \begin{pmatrix} 0 & e^{-i\vec{k}\cdot\vec{\delta}_n} \\ e^{i\vec{k}\cdot\vec{\delta}_n} & 0 \end{pmatrix}$$



# Strain and gauge fields

$$H = -\sum_{n=1}^{3}(t+\delta t_n)\begin{pmatrix} 0 & e^{-i(\vec{K}+\vec{q})\cdot\vec{d}_n} \\ e^{i(\vec{K}+\vec{q})\cdot\vec{d}_n} & 0 \end{pmatrix}$$

-Guinea, Horowitz, Le Doussal, Phys. Rev. B **77,** 205421 (2008)
-Vozmediano, Katsnelson, Guinea Phys. Rep. **496**, 109–148 (2010)

$$\begin{pmatrix} 0 & e^{-i\vec{K}\cdot\vec{d}_n} \\ e^{i\vec{K}\cdot\vec{d}_n} & 0 \end{pmatrix} \equiv \vec{\sigma}\times\vec{d}_n$$

**Dirac fermions: expand H in q**

$$\approx -t\sum_{n=1}^{3}\begin{pmatrix} 0 & e^{-i\vec{K}\vec{d}_n}(1-i\vec{d}_n\vec{q}) \\ e^{i\vec{K}\vec{d}_n}(1+i\vec{d}_n\vec{q}) & 0 \end{pmatrix}$$

$$= -t\sum_{n=1}^{3}(\vec{\sigma}\times\vec{d}_n)i\sigma_z(\vec{q}\cdot\vec{d}_n)$$

$$= -t\sum_{n=1}^{3}(\vec{\sigma}\cdot\vec{d}_n)(\vec{q}\cdot\vec{d}_n) = -t\,\vec{\sigma}\cdot\vec{q}$$

**Gauge field: expand H in $\delta t_n$**

$$H = \sum_{n=1}^{3}\delta t_n\vec{\sigma}\times\vec{d}_n$$

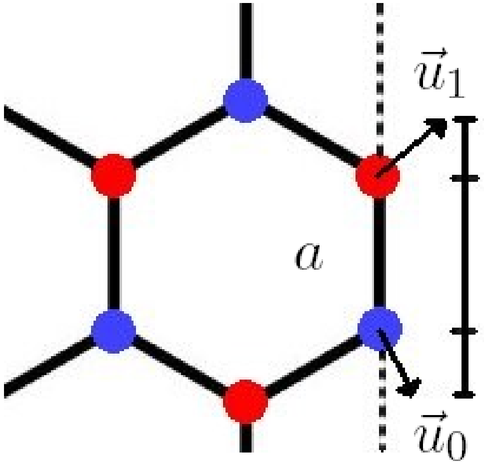
$$A_1 = \frac{\sqrt{3}}{2}(\delta t_1 - \delta t_2)$$

$$A_2 = \frac{1}{2}(\delta t_1 + \delta t_2 - 2\delta t_3)$$

$$H = \vec{A}\cdot\vec{\sigma}$$

# A general strain tensor

$$H = \sum_{n=1}^{3} \delta t_n \vec{\sigma} \times \vec{d}_n$$




$$\beta = -\frac{\partial \ln t}{\partial \ln a} \simeq 2$$

$$\delta t_n = \frac{\beta t}{a^2} (\vec{u}_n - \vec{u}_0) \vec{\delta}_n$$

$$\frac{\vec{u}_n - \vec{u}_0}{a} = \left(\vec{d}_n \cdot \vec{\nabla}\right) \vec{u}(r)$$

$$H = \frac{\beta t}{a} \sum_{n=1}^{3} \vec{\sigma} \times \vec{d}_n \left(\vec{d}_n \cdot \vec{\nabla}\right) \left(\vec{u}(r) \cdot \vec{d}_n\right)$$

$$H = \vec{A} \cdot \vec{\sigma}$$

$$A^i = \frac{\beta t}{a} f^{ijk} u^{jk}$$

$$f^{ijk} = \sum_{n=1}^{3} \epsilon^{il} d_n^l d_n^j d_n^k$$

$$A_1 = \frac{\beta t}{a} (u^{xx} - u^{yy})$$

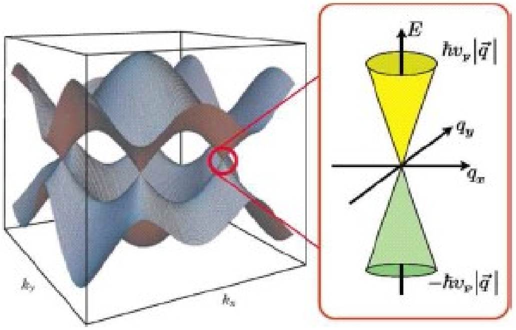
$$A_2 = \frac{\beta t}{a} (-2u^{xy})$$

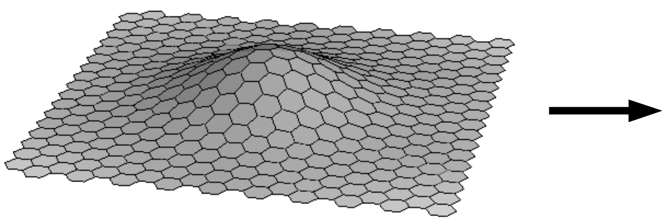
# Strain induced magnetic fields

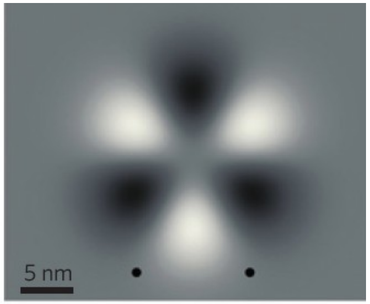
- These are physically real pseudo-magnetic fields!

$$B = \vec{\nabla} \times \vec{A}$$

- But time-reversal is preserved because the two valleys have opposite magnetic fields





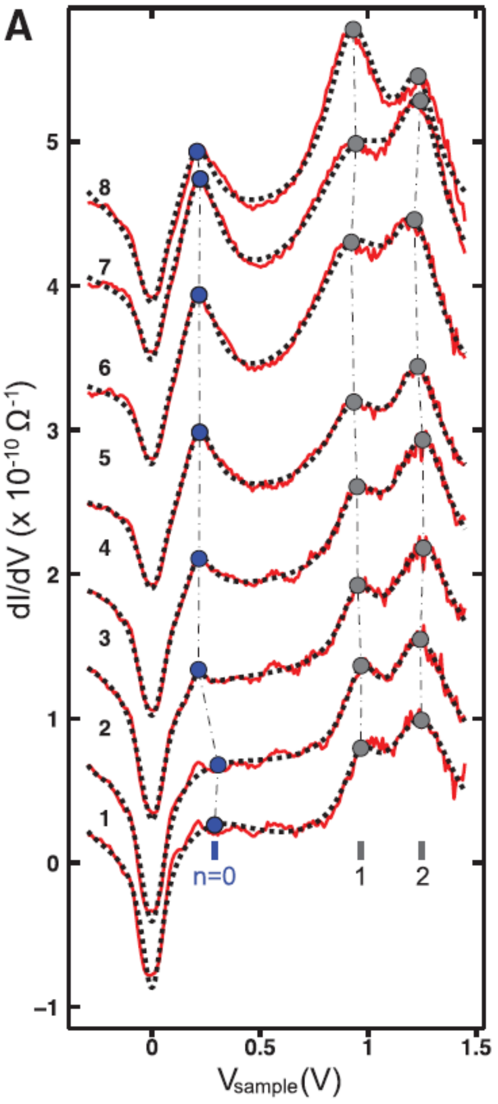



- They inherit the trigonal symmetry of the lattice!


- Morpurgo, & Guinea, "Intervalley scattering, long-range disorder, and effective time reversal symmetry breaking in graphene".. Phys. Rev. Lett. **97**, 196804 (2006).
- Guinea, Katsnelson, Geim, "Energy gaps, topological insulator state and zero-field quantum Hall effect in graphene by strain engineering". Nature Phys. **6**, 30–33 (2010).

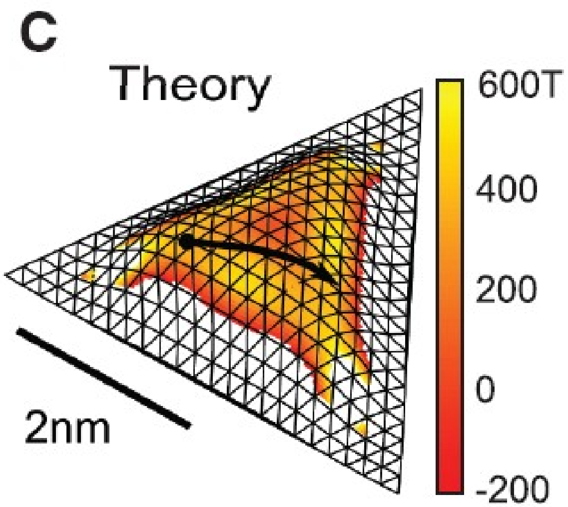

*"We believe that the suggested strategies to observe the pseudo-Landau gaps and QHE are completely attainable and will be realized sooner rather than later"*

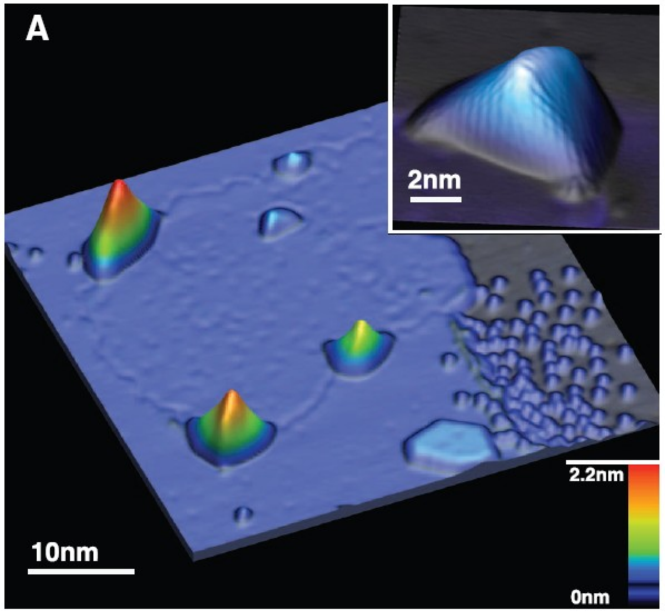
# Direct evidence of gauge fields I




**Strain-Induced Pseudo–Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles**

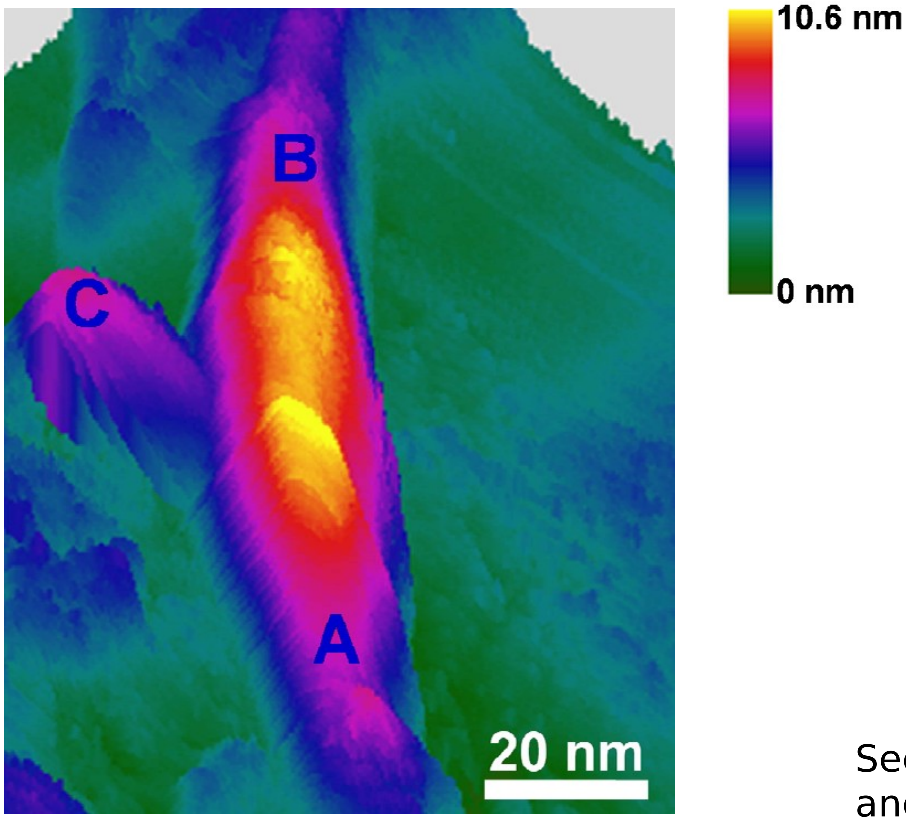
N. Levy, *et al.*
*Science* **329**, 544 (2010);

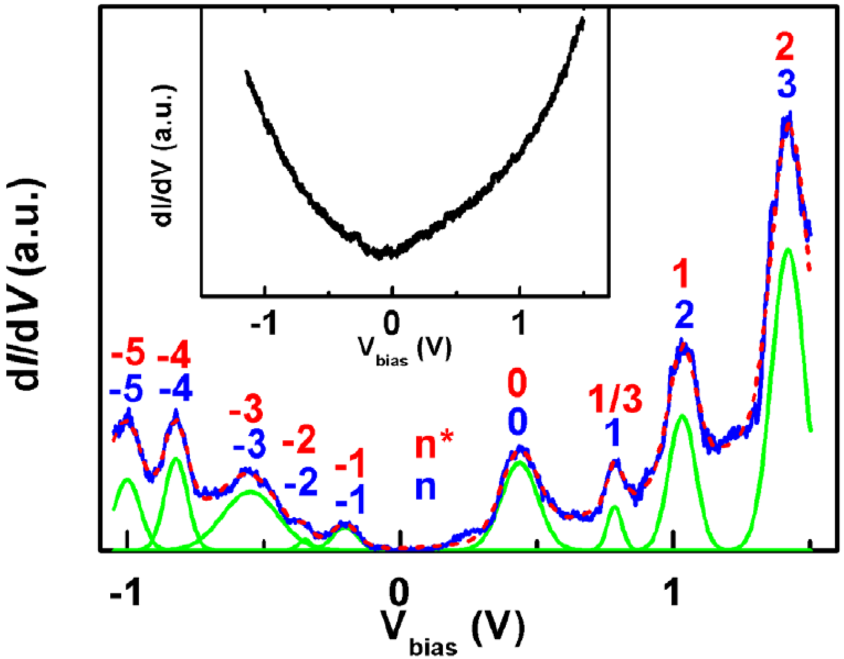


# Direct evidence of gauge fields II

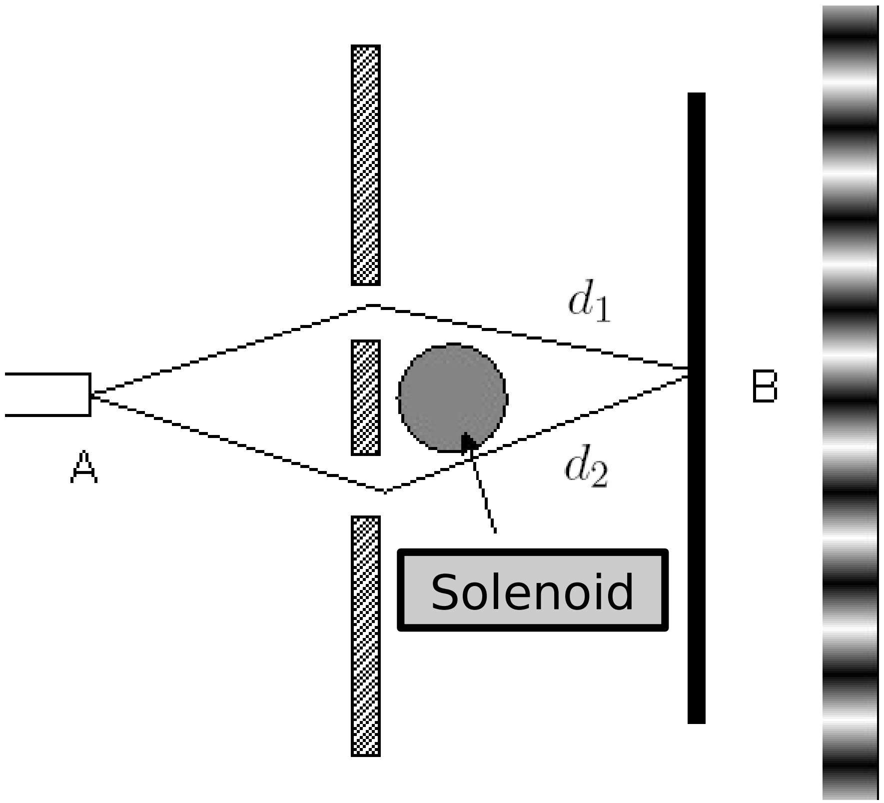
"Observation of Landau level-like quantizations at 77 K along a strained-induced graphene ridge" He *et al.* , cond-mat/1108.1016 (2011)




See also: "Strain-induced pseudo-magnetic fields and charging effects on CVD-grown graphene"
Yeh *et al*. Surf. Sci. **605**, 1649 (2010)

# Aharonov-Bohm interferences




$$|\psi|^2 \propto |e^{ikd_1} + e^{ikd_2}|^2$$

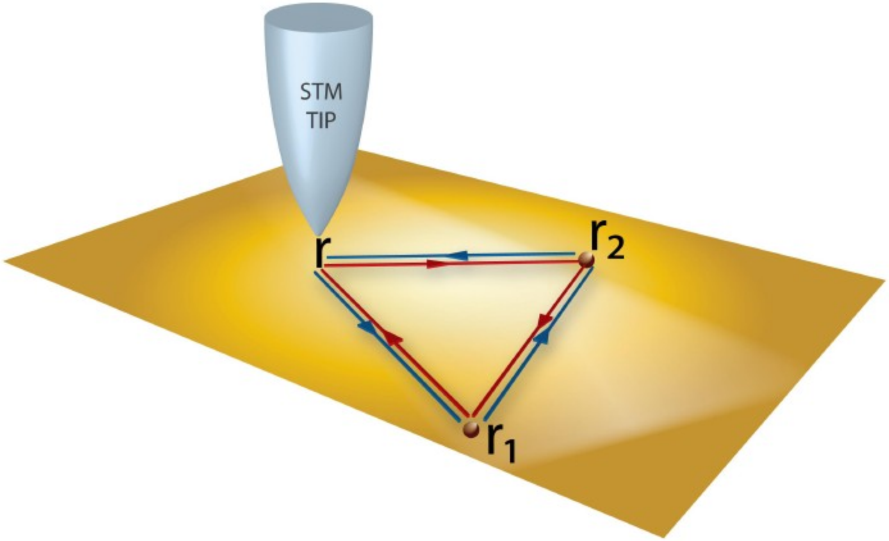
$$\propto 1 + \cos(k(d_1 - d_2))$$

Add magnetic flux through the solenoid:

$$\propto 1 + \cos(k(d_1 - d_2) + \phi)$$

- Probability of measuring an electron in B depends on magnetic flux through the region enclosed by the path

# Quantum interference in the LDOS




"Aharonov Bohm oscillations in the local density of states" A. Cano and I. Paul, Phys. Rev. B **80** 153401 (2009)

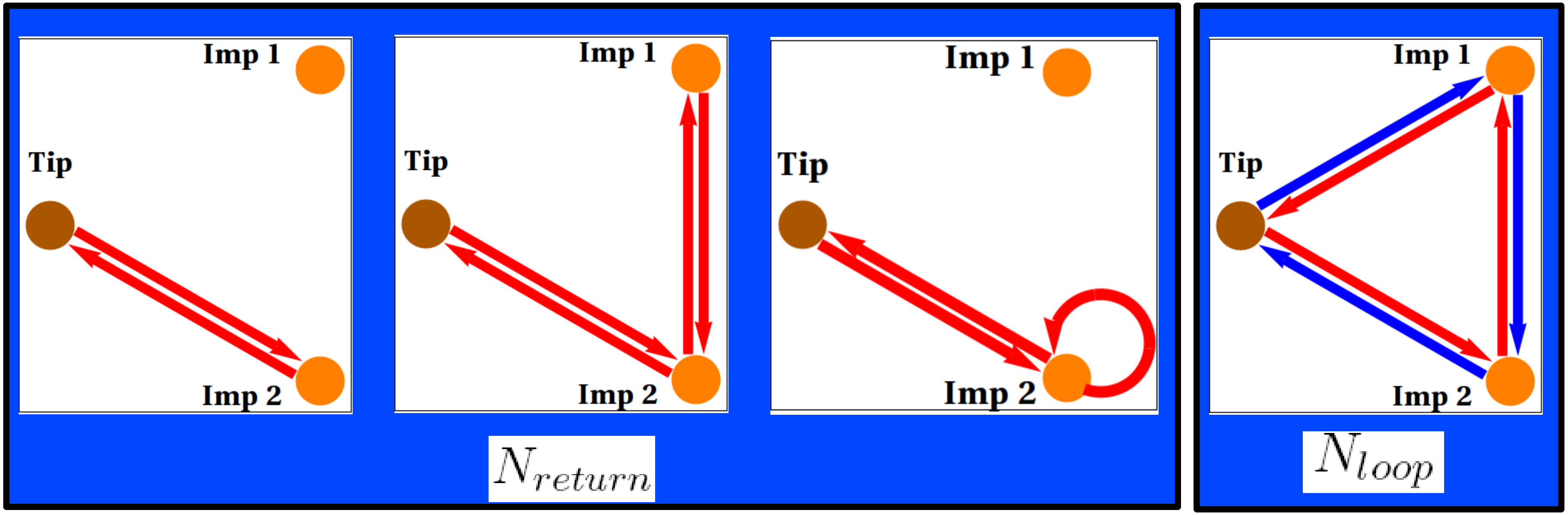
- Same AB physics in the LDOS
- Impurity scattering sets the path
- Semiclassical approximation required

$$N(\mathbf{r},\omega) = -\frac{2}{\pi}\mathrm{Im}\, G^R(\mathbf{r},\mathbf{r};\omega)$$

$$G(\mathbf{r},\mathbf{r}) = G_0(\mathbf{r},\mathbf{r}) + \int \mathrm{d}r' G_0(\mathbf{r},\mathbf{r}')U(\mathbf{r}')G_0(\mathbf{r}',\mathbf{r}) + \cdots$$

$$G_0(\mathbf{r}-\mathbf{r}') = \exp\left(i\frac{\pi}{\Phi_0}\int_{\mathbf{r}}^{\mathbf{r}'} \mathbf{A}(\mathbf{l})\cdot d\mathbf{l}\right) G_{00}(\mathbf{r}-\mathbf{r}')$$

# Quantum interference in the LDOS




$$U_0 \to \widetilde{U}_0 = \frac{U_0}{1 - U_0 G_0(0)}$$

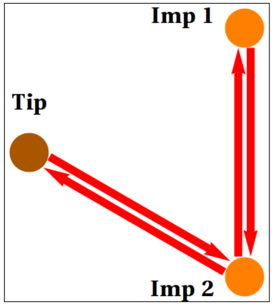
Multiple scattering from the same impurity is resummed in the T-matrix.

$$\delta G_{\rm loop}(\mathbf{r},\mathbf{r}) = W^2 G_0(\mathbf{r}-\mathbf{r}_1) G_0(\mathbf{r}_1-\mathbf{r}_2) G_0(\mathbf{r}_2-\mathbf{r})$$

$$W^2 = \frac{\widetilde{U}_0^2}{1 - \widetilde{U}_0^2 G_0(\mathbf{r}_1-\mathbf{r}_2) G_0(\mathbf{r}_2-\mathbf{r}_1)}$$

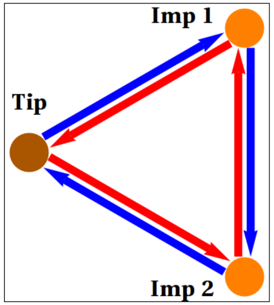
Multiple back and forth processes in the loop contribution are resummed in an analog function W.

# Add a magnetic field




$$N_{A=0} = N_{return} + N_{loop}$$

A magnetic flux will modify the loop terms:




$$N = N_{return} + N_{loop}\cos(\frac{\pi\Phi}{\Phi_0})$$

$$\delta G^{(2)}(\mathbf{r},\mathbf{r}) = U_0^2 \Big[G_0(\mathbf{r},\mathbf{r}_1)G_0(\mathbf{r}_1,\mathbf{r}_2)G_0(\mathbf{r}_2,\mathbf{r})e^{i\frac{\pi\Phi}{\Phi_0}} + G_0(\mathbf{r},\mathbf{r}_2)G_0(\mathbf{r}_2,\mathbf{r}_1)G_0(\mathbf{r}_1,\mathbf{r})e^{-i\frac{\pi\Phi}{\Phi_0}}\Big]$$

$$\delta G^{(2)}(\mathbf{r},\mathbf{r}) = U_0^2\left[G_0(\mathbf{r},\mathbf{r}_1)G_0(\mathbf{r}_1,\mathbf{r}_2)G_0(\mathbf{r}_2,\mathbf{r})2\cos\frac{\pi\Phi}{\Phi_0}\right]$$

$$N(\omega,r) = N_{A=0}(\omega,r) + N_{loop}(\omega,r)\left[\cos(\frac{\pi\Phi(r)}{\Phi_0}) - 1\right]$$

# Dirac fermions and interference

- Dirac fermions have a matrix Green's function

$$G_0(\mathbf{r}_1,\mathbf{r}_2;\omega) = -\frac{i\omega}{4v_F}\left[H_0(\omega|\mathbf{r}_1-\mathbf{r}_2|) + i\frac{\boldsymbol{\sigma}(\mathbf{r}_1-\mathbf{r}_2)}{|\mathbf{r}_1-\mathbf{r}_2|}H_1(\omega|\mathbf{r}_1-\mathbf{r}_2|)\right]$$

1

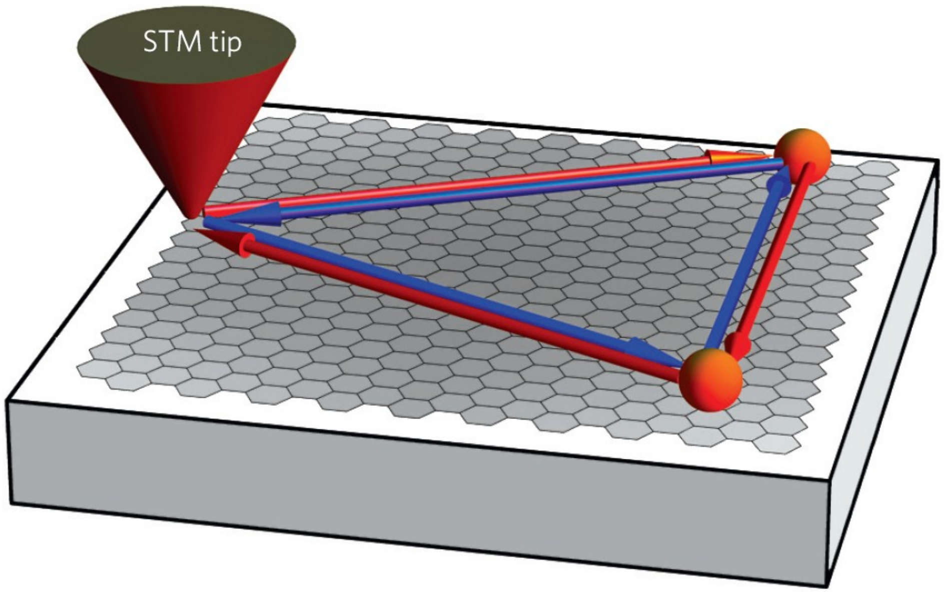
- The previous manipulations require to commute them: non trivial. But all commutators proportional to $\sigma_3$ and vanish after the trace (note this is spoiled for gapped graphene!).

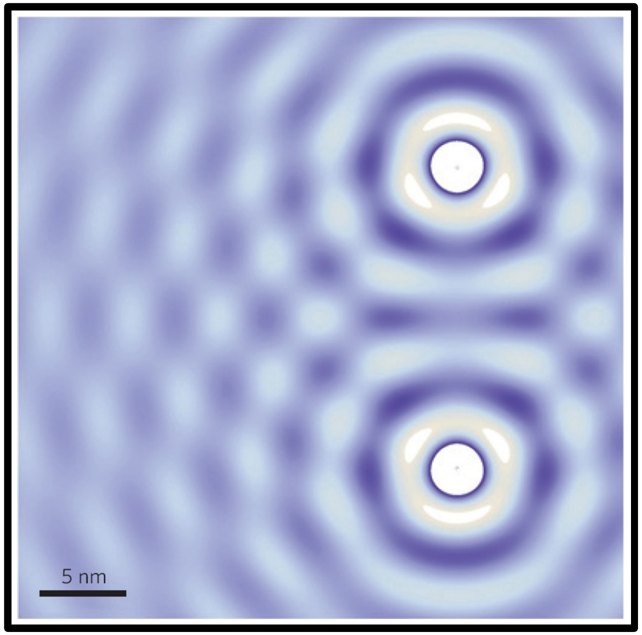
2

- Valley degree of freedom: very short range impurities may induce intervalley scattering. Pick longer ranged ones.

$$N(\omega,r) = N_{A=0}(\omega,r) + N_{loop}(\omega,r)\left[\cos\left(\frac{\pi\Phi(r)}{\Phi_0}\right) - 1\right]$$

# An experimental proposal

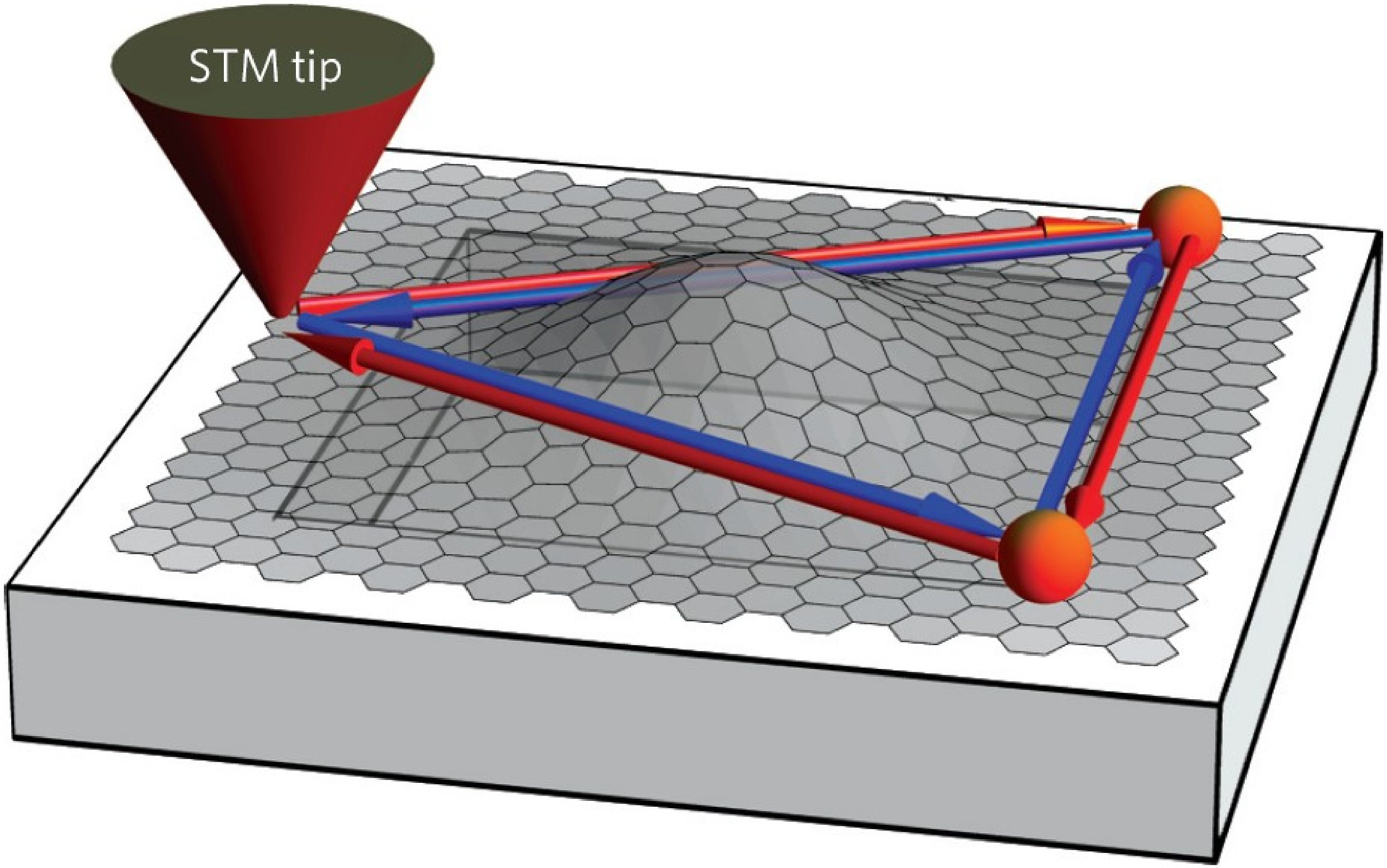



$N_{A=0}(\omega, r)$

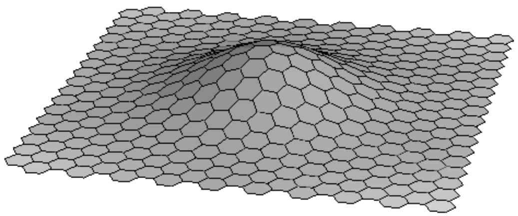
- In the flat sample, the STM tip measures the usual standing wave patterns.

# Induce controlled strain



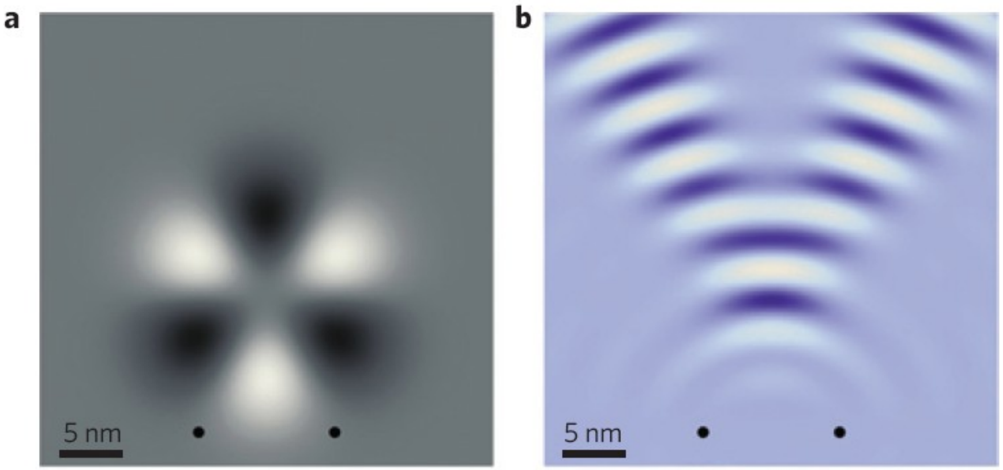
# Strain-induced interference

- Circular perturbation



$$h(r) = A\exp(-r^2/2\sigma^2)$$

**Magnetic field** **LDOS**

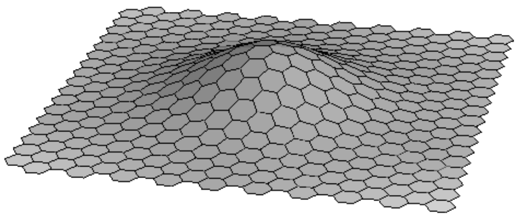



- In the curved sample, and after substraction of $N_{A=0}$, we see a new standing wave pattern (Nloop) modulated by the cosine of the flux through the triangle.

$$N_{loop}(\omega, r)\left[\cos\left(\frac{\pi\Phi(r)}{\Phi_0}\right) - 1\right]$$

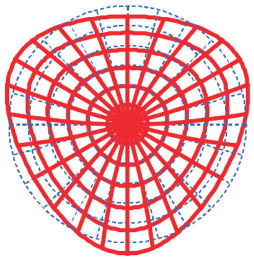
# Strain-induced interference

- Circular perturbation



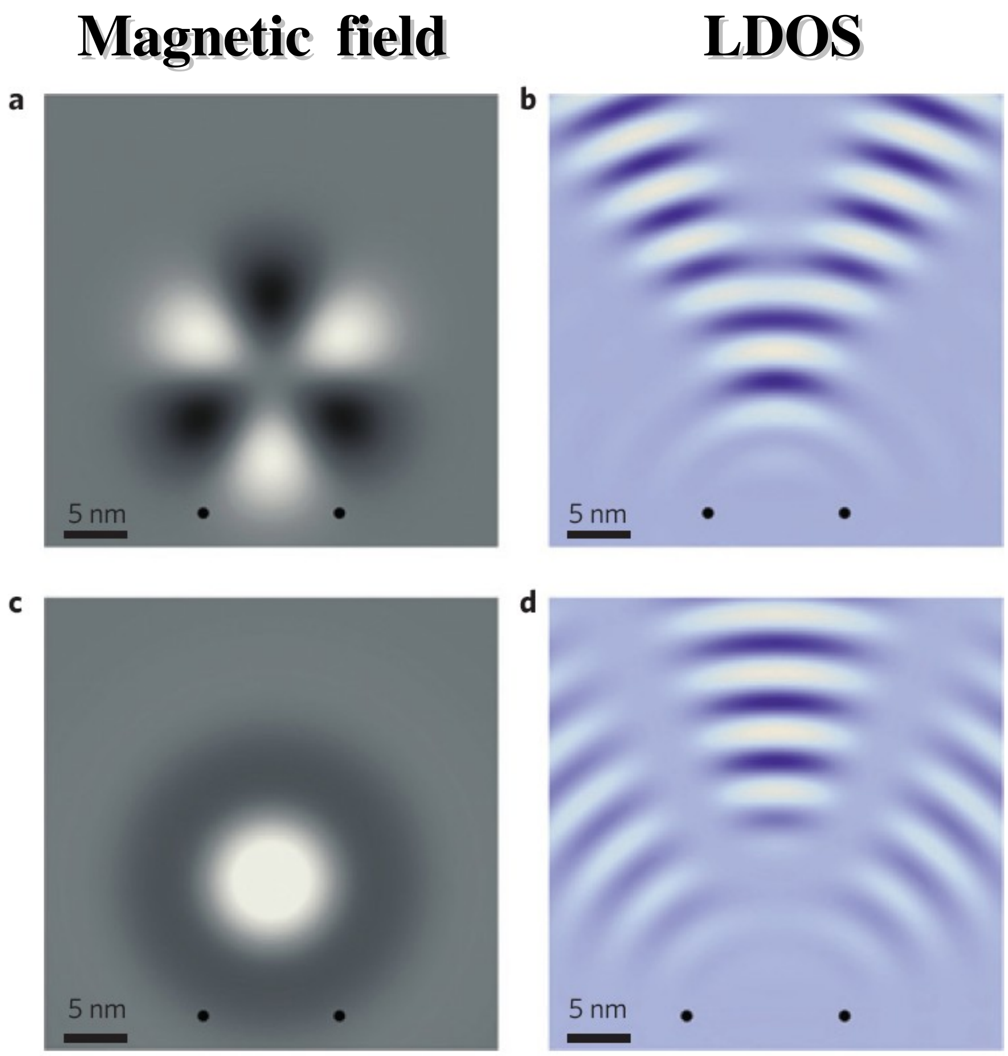
$$h(r) = A\exp(-r^2/2\sigma^2)$$

- Three-fold symmetric perturbation



$$u_r = u_0 r^2 \sin 3\theta$$
$$u_\theta = u_0 r^2 \cos 3\theta$$
$$u_0 = (u_{00}/\sigma^2)\exp(-r^2/2\sigma^2)$$

Guinea *et al.* , Nat. Phys.. **6** 30 (2010).



# Conclusions and future

- Strain induces effective pseudo-magnetic fields which are physically very real!

- These produce Aharonov-Bohm interferences in the LDOS which can be observed with STM

- The required strain is low but the measurement may still be challenging.

- The effect could be potentially used to measure strain locally by interferometry.

**Thanks for your attention!**

# AB in topological insulators

Aharonov-Bohm oscillations in the local density of topological surface states

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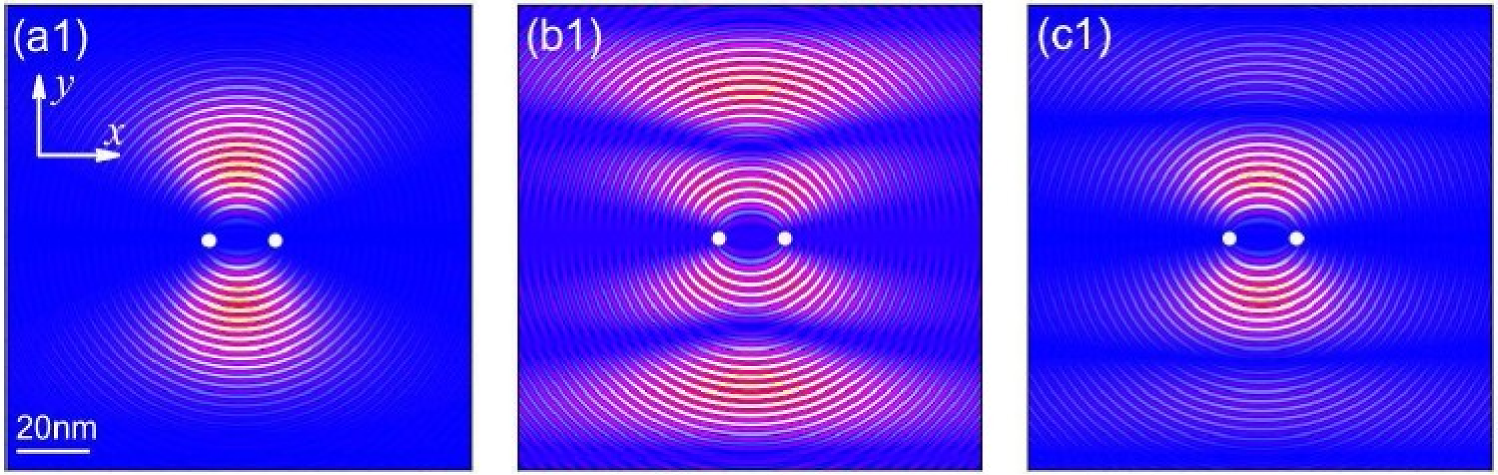


arXiv:1103.1710v1