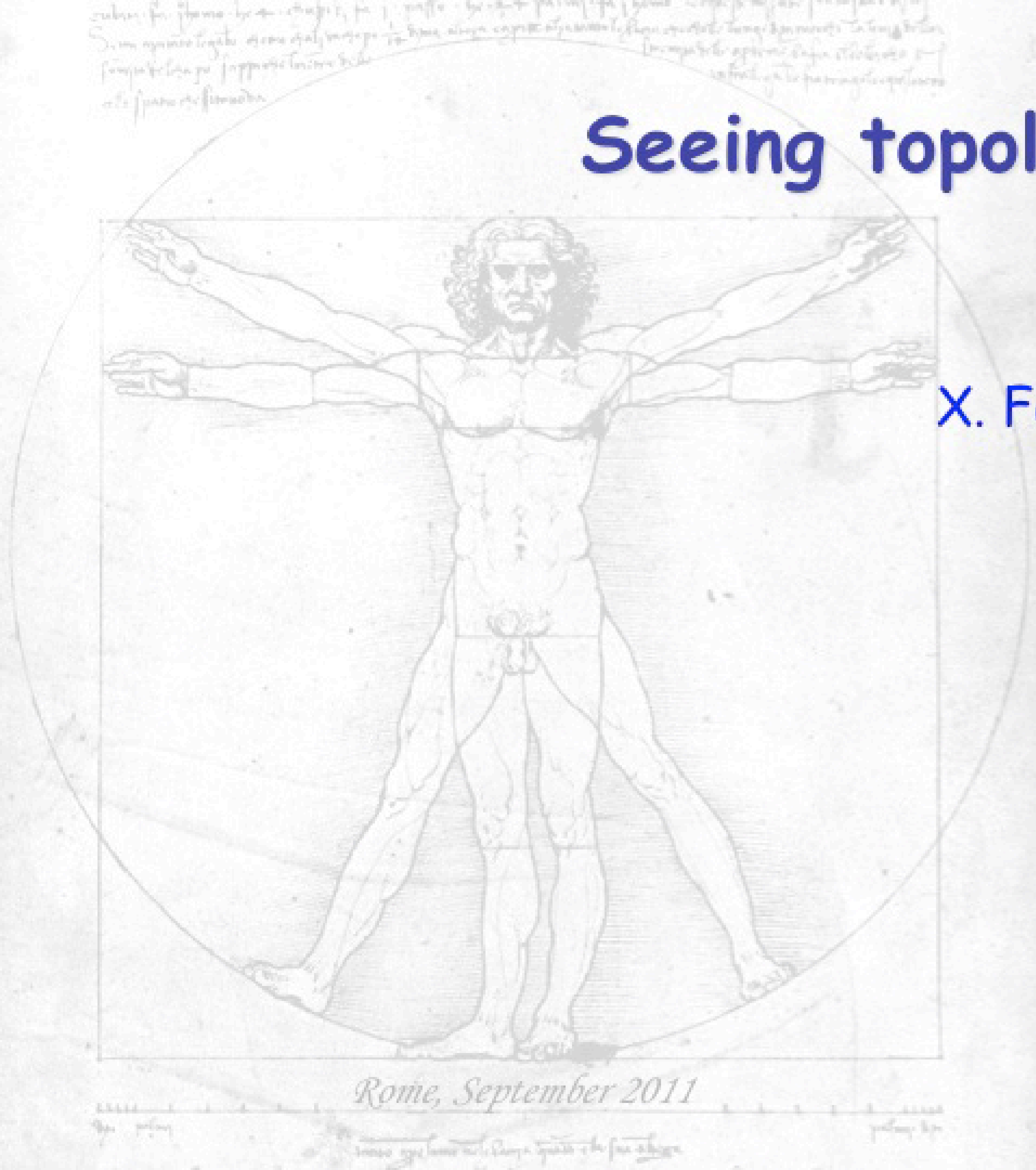

Seeing topological order

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[arXiv:1105.2775]

Rome, September 2011

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Introduction

Quantum order:

Assume a Hamiltonian with well defined **ground state** and **energy gap** above it

Zero temperature properties of matter identified by **local operators**: magnetisation $\langle \sigma_z \rangle$, identifies superconductivity...

Topological quantum order:

Identified by **non-local operators** *VERY ILLUSIVE*

The ground state can be described by **winding number**

Cold atoms:

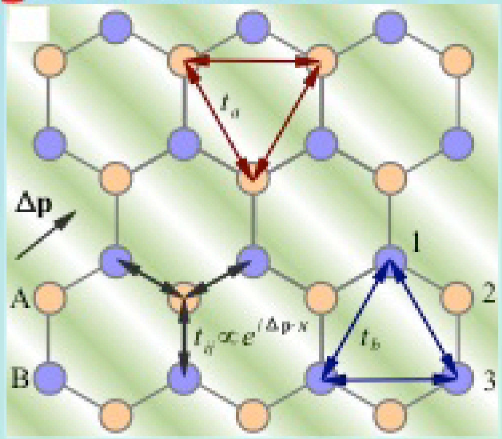
Simulate the **Haldane topological model** with **cold atoms**

Measure *Topo Order with experimentally plausible system*

Overview

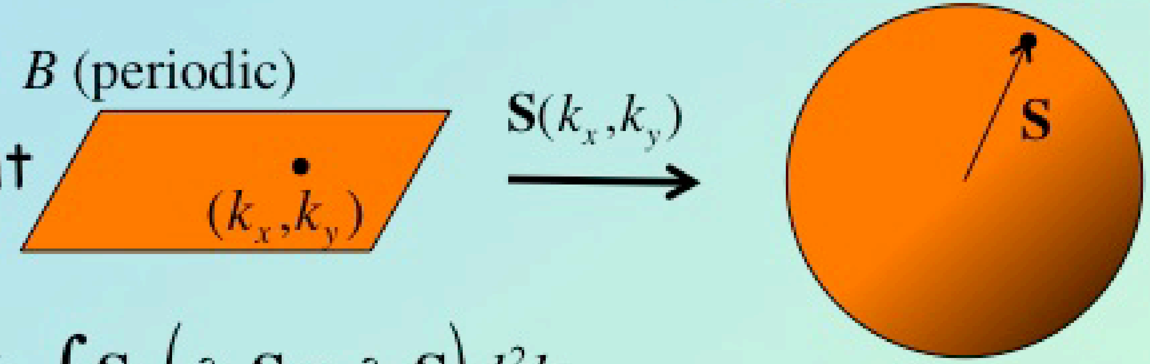
Cold atom simulation of Haldane's topological model

Two triangular fermionic lattices of different atom species.
Its ground state can have non-trivial **winding number**



Winding number

Direct measurement of topo order



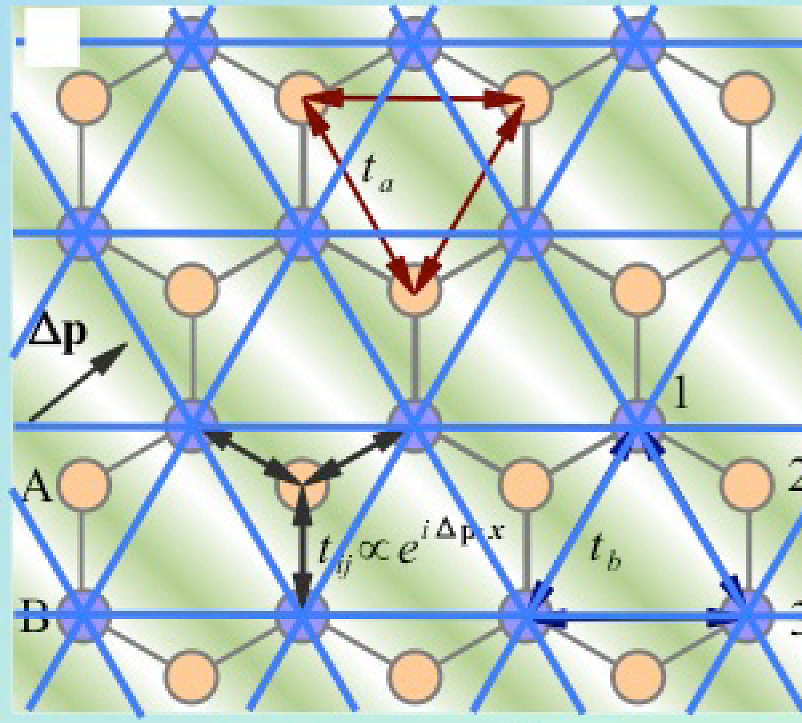
$$\nu = \frac{1}{4\pi}\int_B \mathbf{S}\cdot\left(\partial_{k_x}\mathbf{S}\times\partial_{k_y}\mathbf{S}\right)d^2k$$

(solid angle)

Counts how many times the vector $\mathbf{S}$ winds around the sphere when the argument (k_x, k_y) spans B.

Haldane's model with cold atoms

Two triangular sublattices: A, B

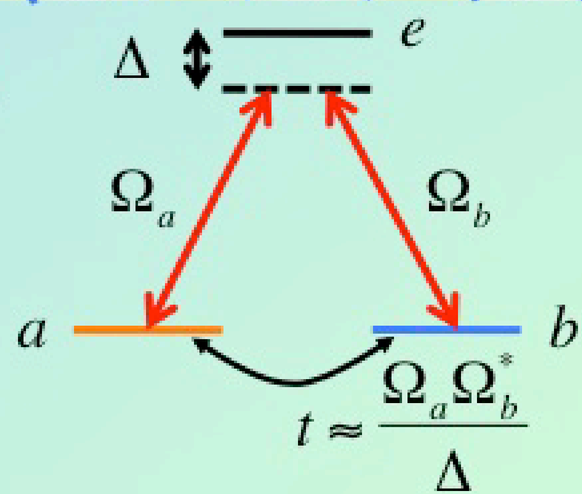


Each loaded with fermionic atoms in internal states a, b

Hamiltonian:

$$H = \sum_{\langle j,k \rangle} (t_{jk} b_j^+ a_k + t_{jk}^* a_k^+ b_j)$$

$$+ \sum_j \varepsilon (a_j^+ a_j - b_j^+ b_j) + \sum_{\langle\langle j,k \rangle\rangle} t_a a_j^+ a_k + \sum_{\langle\langle j,k \rangle\rangle} t_b b_j^+ b_k$$



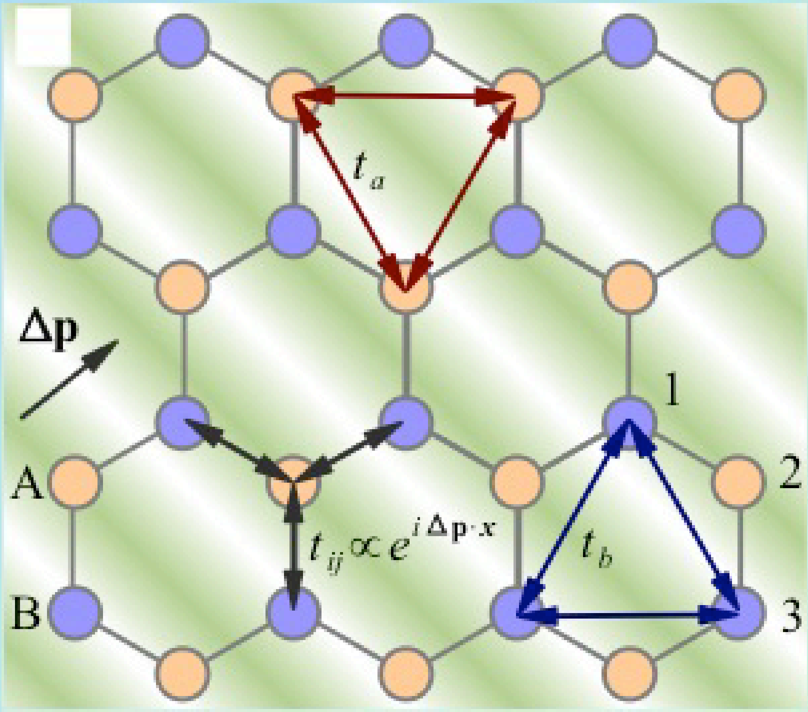
Interspieces tunneling $t_{jk} = te^{i\varphi}$
via **Raman transitions**
(can be complex)

Haldane's model with cold atoms

Linearisation of Hamiltonian around Fermi points

$$H \approx -E\sum_k (a_k^+ \; b_k^+)\,\mathbf{S}(k)\cdot\sigma\begin{pmatrix}a_k\\ b_k\end{pmatrix}$$

$$\mathbf{S}(k)\cdot\sigma \approx k_x\sigma_x + k_y\sigma_y + m\sigma_z$$



Topological properties of the model can be read of from vector $\mathbf{S}(k) = \langle\sigma\rangle$

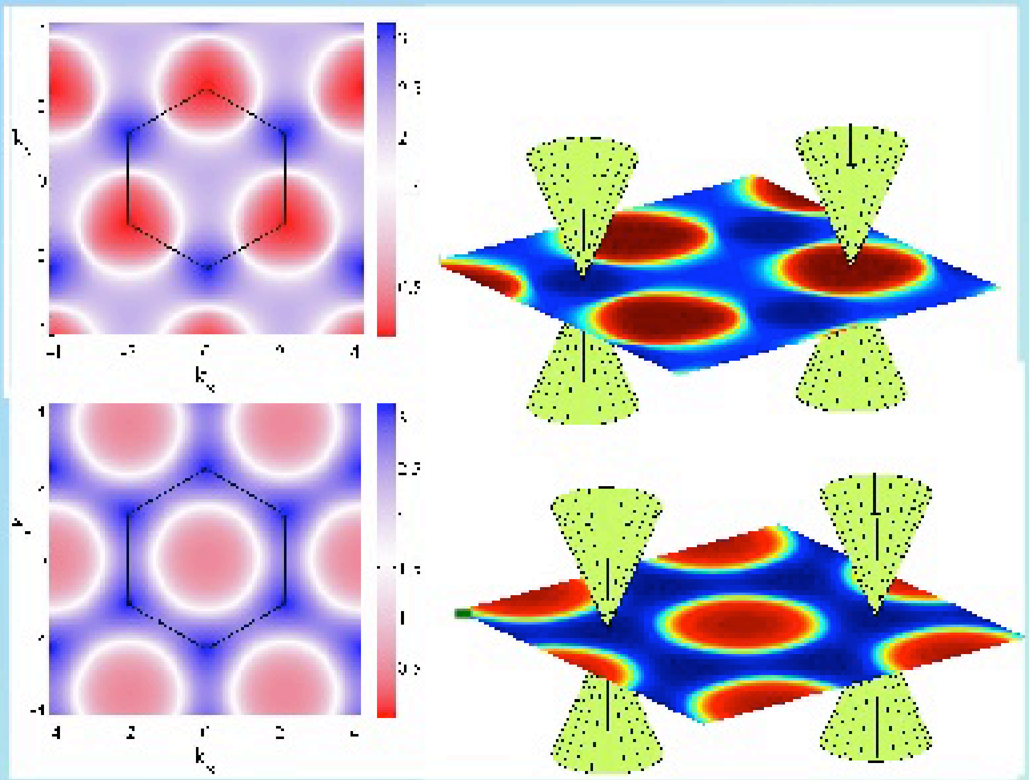
Its components are determined by the populations of atoms in state a and state b -> *possible with two species!*

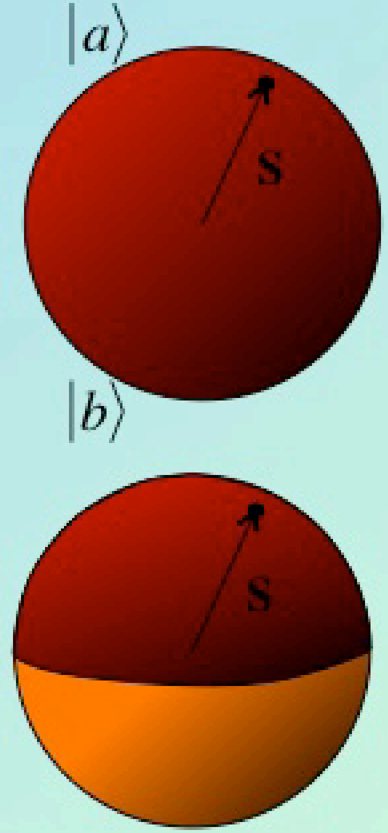
t gives rise to $k_x\sigma_x + k_y\sigma_y$

t_a, t_b, ε give rise to $m\sigma_z$ The mass m(k) can be +ve or -ve

Haldane's model with cold atoms

Position of Fermi points with respect to mass landscape (two Fermi points in Brillouin zone, B)





$\nu = \pm 1$

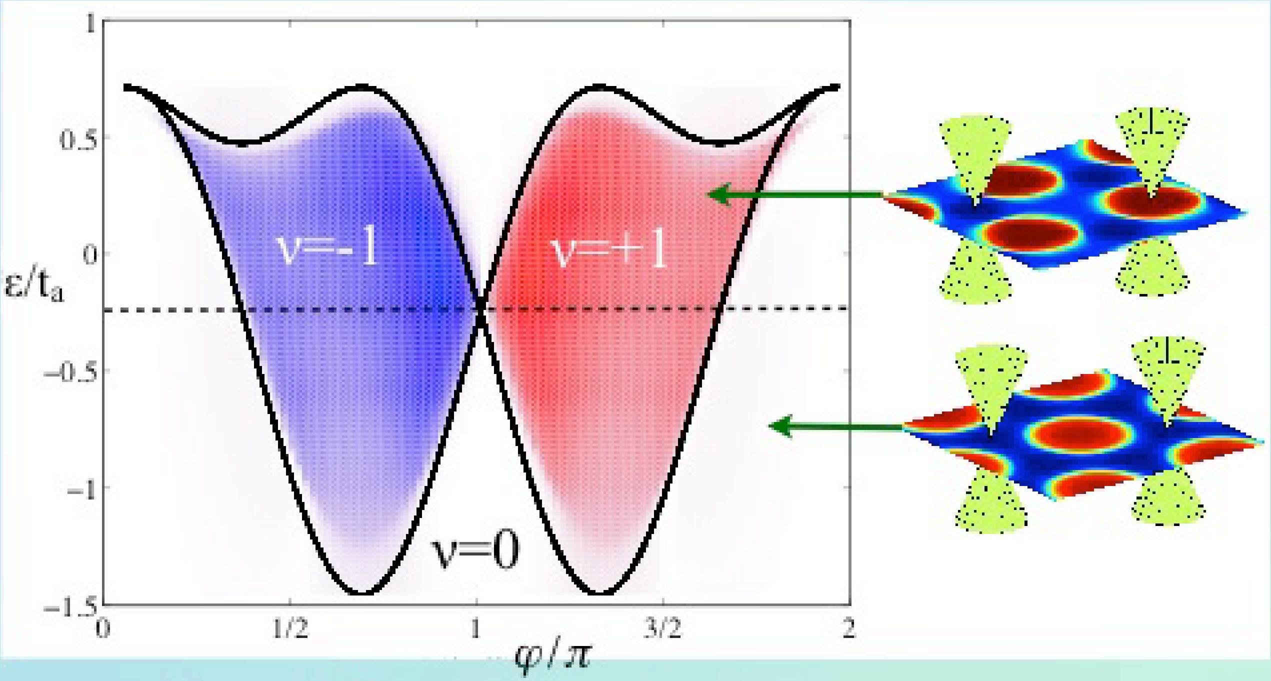
$\nu = 0$

$$\mathbf{S}(k)\cdot\sigma \approx k_x\sigma_x + k_y\sigma_y + m\sigma_z$$

$$\nu = \frac{1}{4\pi}\int_B \mathbf{S}\cdot\left(\partial_{k_x}\mathbf{S}\times\partial_{k_y}\mathbf{S}\right)d^2k$$

Haldane's model with cold atoms

Position of Fermi points with respect to mass landscape (two Fermi points in Brillouin zone, B)



Winding number (Chern number)

$$\nu = \frac{1}{4\pi}\int_B \mathbf{S}\cdot\left(\partial_{k_x}\mathbf{S}\times\partial_{k_y}\mathbf{S}\right)d^2k$$

Skyrmion images

Time-of-flight images:

If the optical lattice and trapping is removed then the atoms move freely.

Their spatial distribution matches their initial momentum distribution. So the **Brillouin zone** becomes the **spatial distribution** of the atomic cloud.

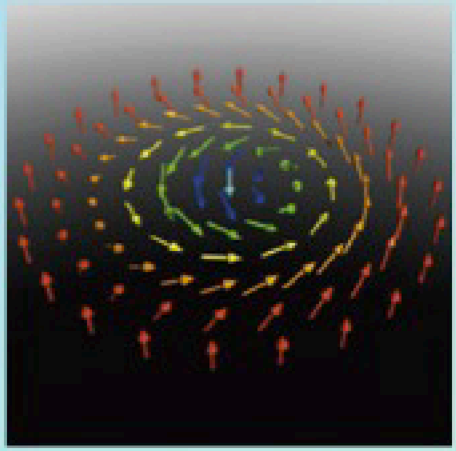
Obtain densities of species $n_a(k), n_b(k)$

Calculate

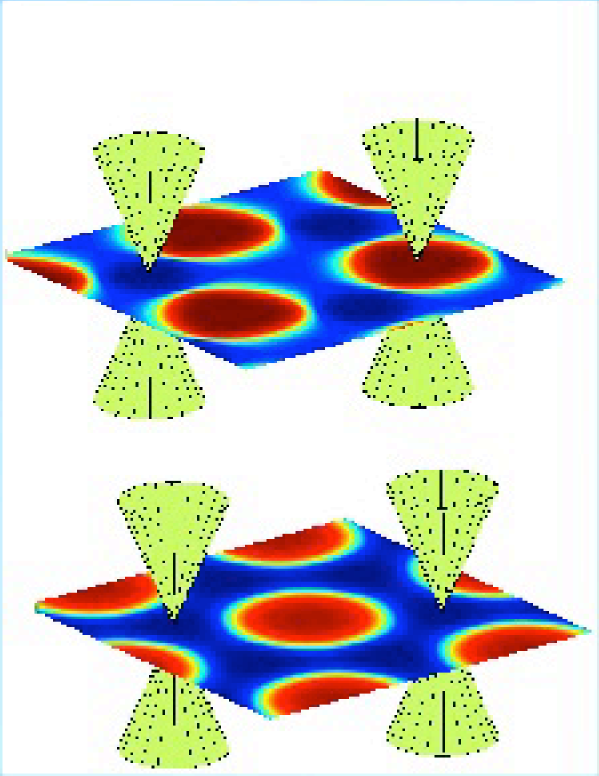
$$S_z(k) = \frac{1}{2}\frac{n_a(k) - n_b(k)}{n_a(k) + n_b(k)}$$

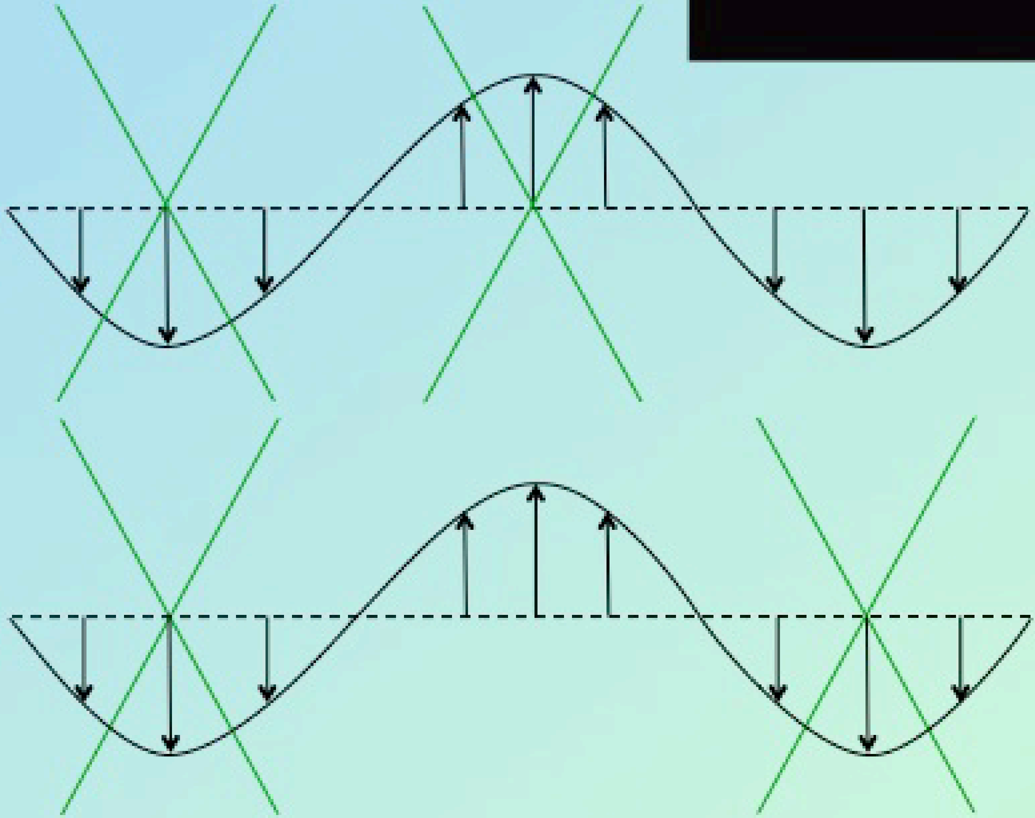
By Raman rotations between a and b during cloud expansion we can calculate $S_x(k), S_y(k)$ by mapping them to $S_z(k)$

Skyrmion images



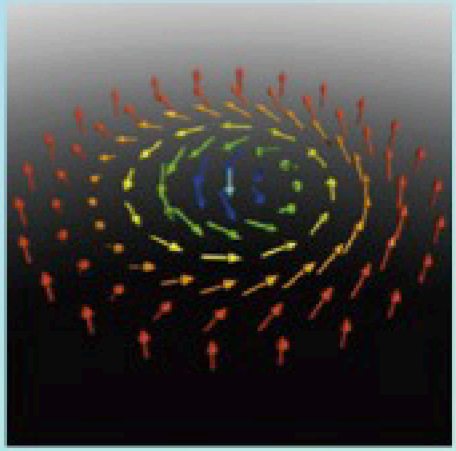
Time-of-flight images reveal **skyrmions**:





$$\mathbf{S}(k)\cdot \sigma \approx k_x\sigma_x + k_y\sigma_y + m\sigma_z$$

Skyrmion images



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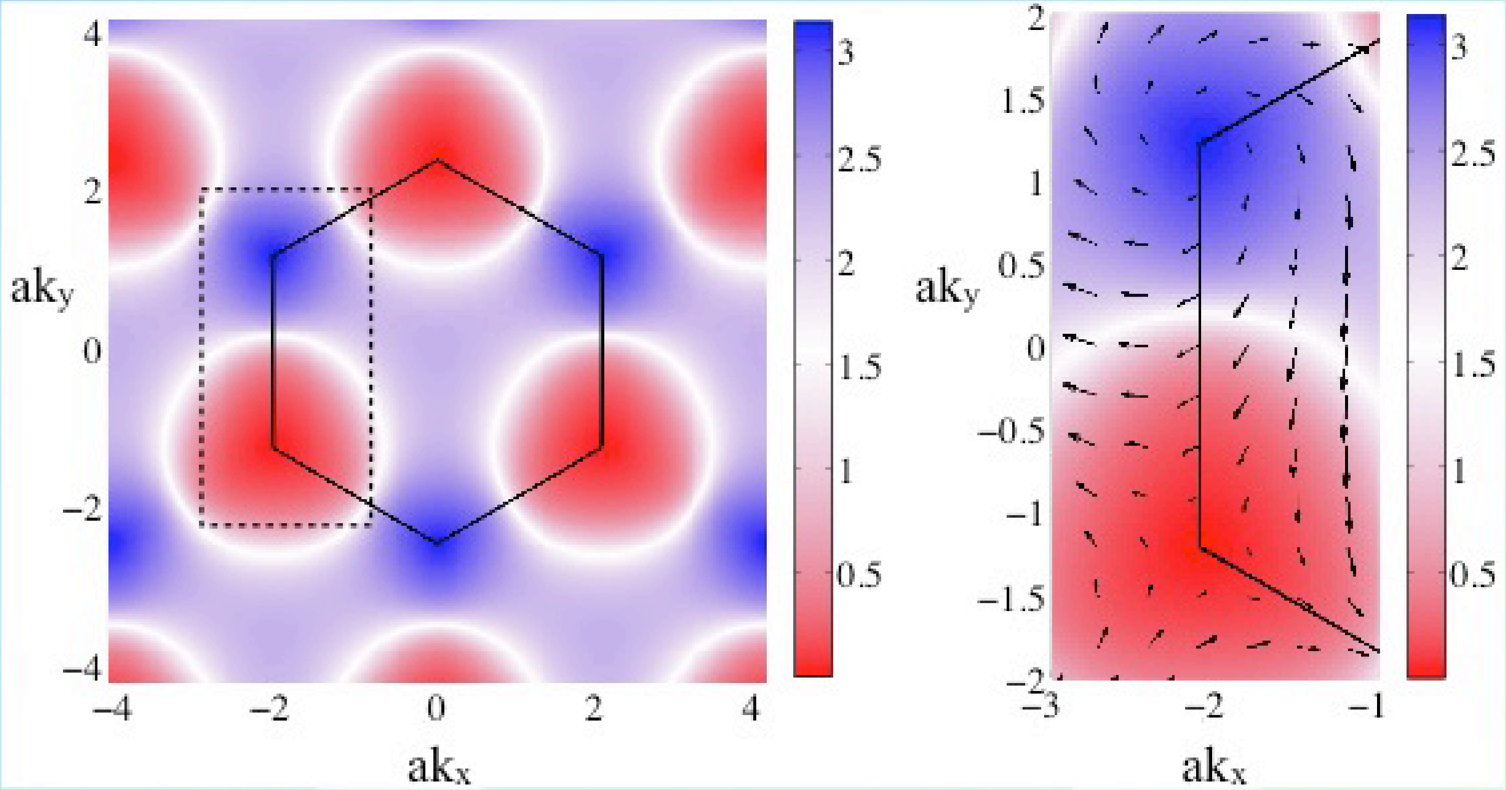
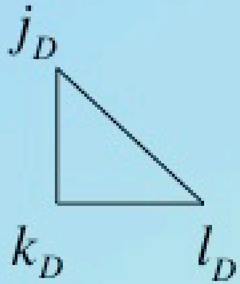
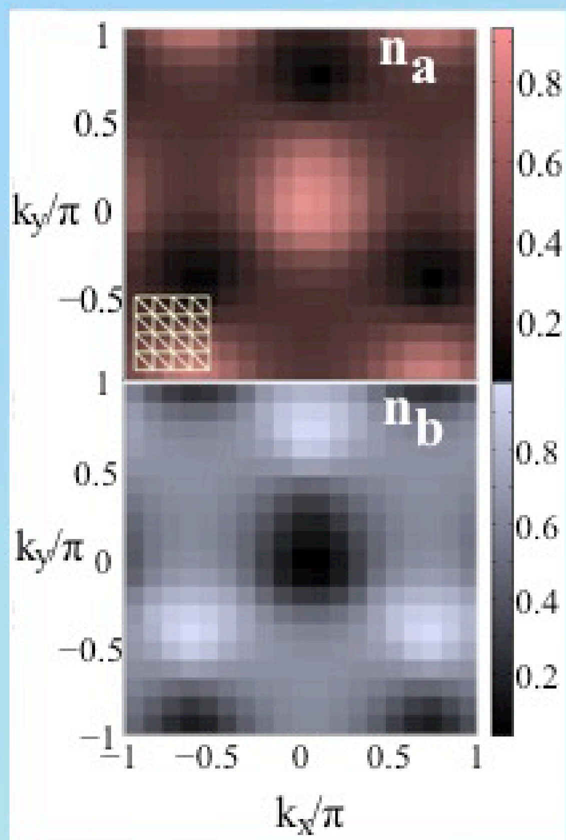


Resilience to experimental conditions

Pixelisation of images: $L \times L$ gives $\{\mathbf{S}_m\}_{m=1}^{L\times L}$

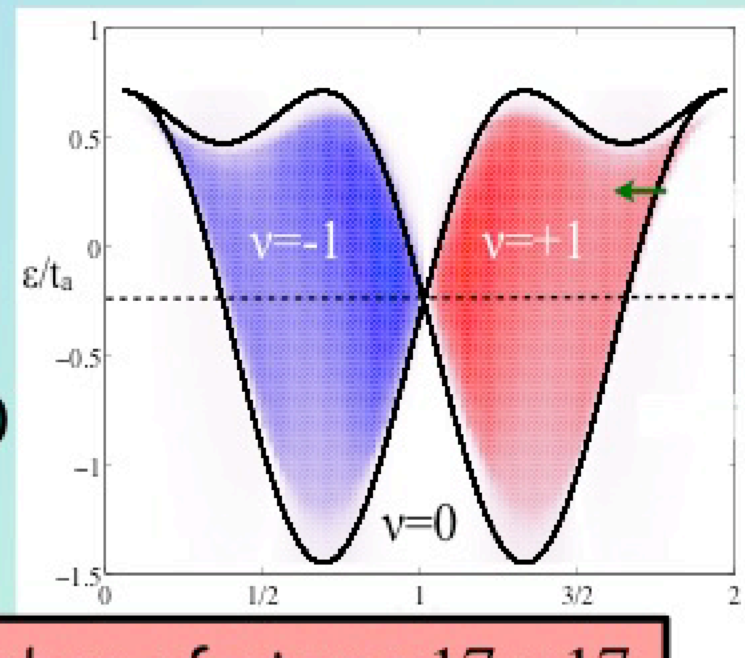


$$\nu_D = \frac{1}{8\pi} \sum_{\langle j_D k_D l_D \rangle} \mathbf{S}_{j_D} \cdot \mathbf{S}_{k_D} \times \mathbf{S}_{l_D} \approx \nu$$



$L = 20$

$L = 20$



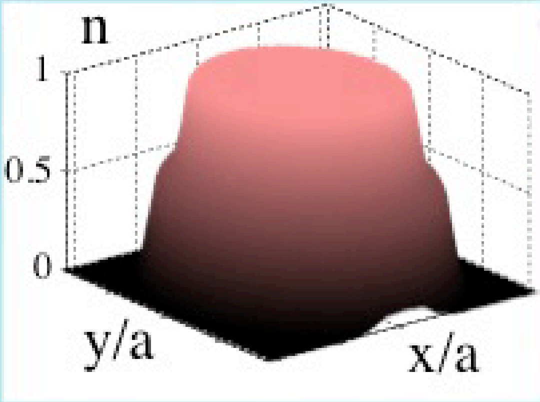
Also number of atoms: 17×17

Same results for 10000 atoms

Resilience to experimental conditions

How does harmonic **confining potential** affects topological behaviour?

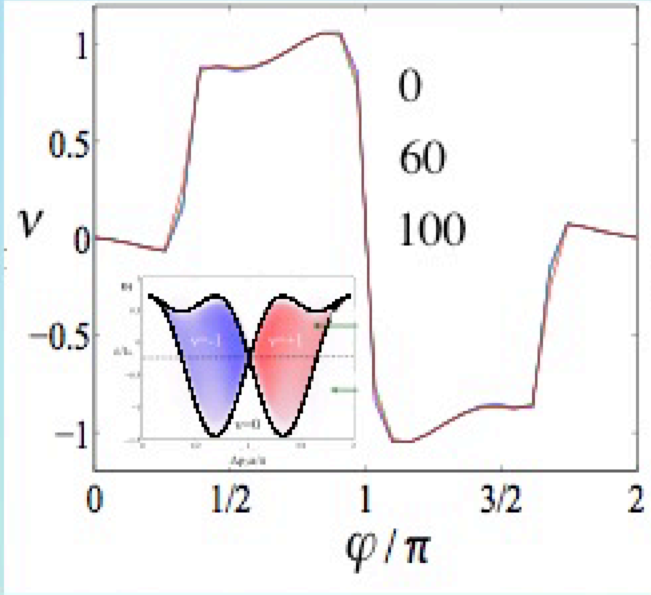
$$V_i(x_i) = \frac{1}{2} m\omega^2 x_i^2$$



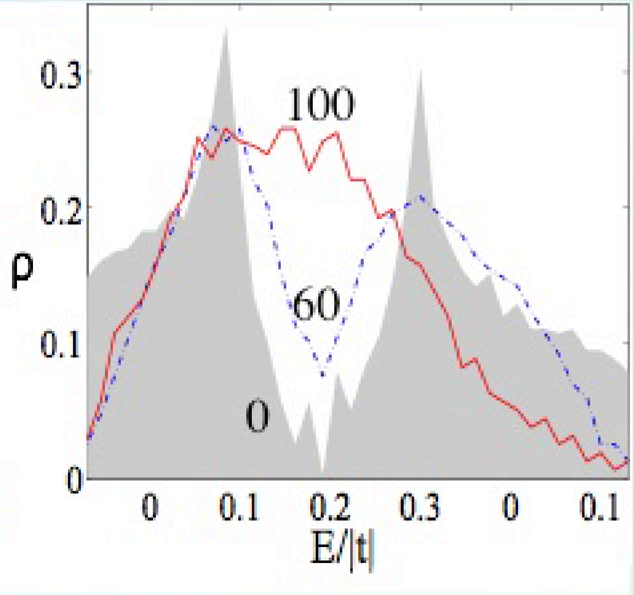
For Ca atoms

$$\frac{\omega}{2\pi} = 0,\ 60,\ 100\text{Hz}$$

Only topological component ***contributes*** *to winding number*



Winding number



Density of states

How did we do it?

Main ingredients that allow us to “see” topo order:

- Cold atoms: preparation/phase factors/read-out
  (Haldane’s model was considered “unphisical”)
- Two species: actual spin
  (not only mathematical tool)
- Time-of-flight: mapped momentum space to position space
  (**non-local** observables became **local**)
- Single point correlation measurements
  (density)
- Topology is on our side
  (resilience to many local characteristics)

Conclusions

- **Resilience** of topological signature in the cold atom simulation of Haldane's model:
  - Small number of atoms
  - Coarse grained measurement
  - Inhomogeneous potential on top of opt lat
  - Errors in exact values of:
    - chemical potential
    - # atoms
    - finite temperature

Other methods (sensitive):
-edge transport
-eigenstate preparation
-local estimate of density of states

- Due to trapping there is always one disk with atoms in topological state that give $\nu = \pm 1$
- Temperature is expected to change the length of $\langle \sigma \rangle$ and not its orientation of $\mathbf{S}$