Charge and Spin Dynamics

in 1D interacting electron systems

with modulated Rashba Spin-Orbit Coupling.

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WORKSHOP
Quantum Field Theory Aspect
of Condensed Matter Physics

INFN-Laboratori Nazionale di Frascati
September 6-th, 2011

OUTLINE

- **Part I:** *1D electrons with modulated Rashba coupling*

  M. Malard, I. Grusha, H. Johannesson and G.I. Japaridze,

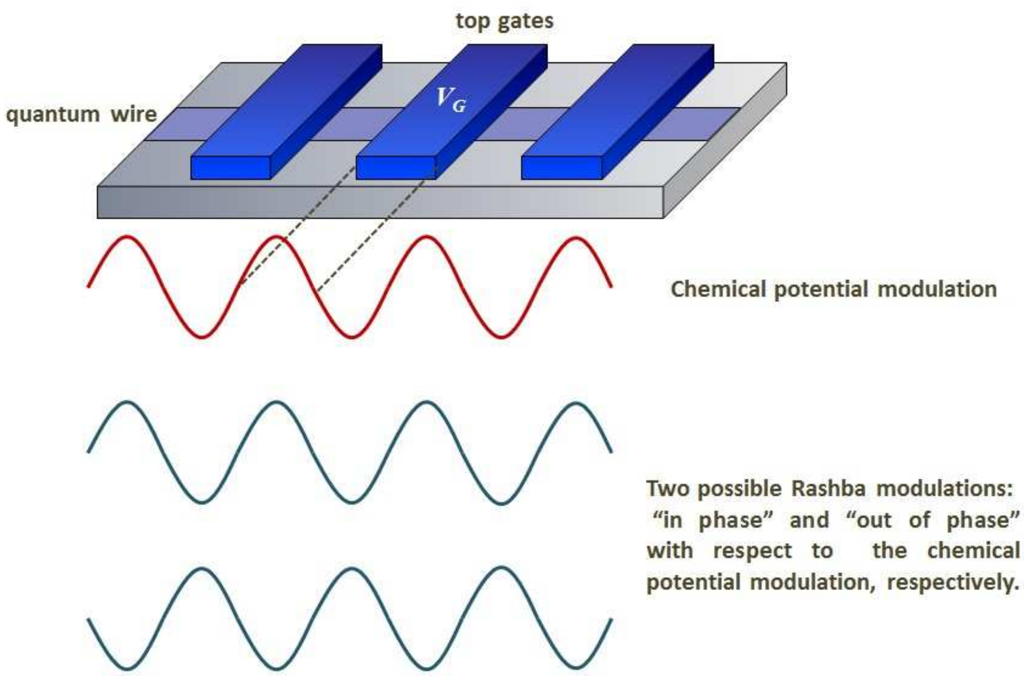
  Phys. Rev. B **84**, 075466 (2011).

- **Part. II** *Edge Dynamics in a Quantum Spin Hall State: Effects from Rashba Spin-Orbit Interaction*

  A. Ström and H. Johannesson, and G.I. Japaridze,

  Phys.Rev.Lett. **104** 256804 (2010).

Part I. 1D electrons with modulated Rashba SO coupling



We consider:

a. 1D quantum wire formed in a gated 2D quantum well supported by a semiconductor heterostructure;

b. we assume that the electrons are ballistic; $\Rightarrow l \simeq$ few microns;

c. the wire carries only one conduction channel;

The lattice Hamiltonian

$$H_0 = -t\sum_{n,\alpha}\left(c^\dagger_{n,\alpha}c_{n+1,\alpha}+\text{H.c.}\right)-\mu\sum_{n,\alpha}c^\dagger_{n,\alpha}c_{n,\alpha},$$

$$H_{\text{e-e}} = \frac{1}{2}\sum_{n,n',\alpha,\alpha'}V(n-n')c^\dagger_{n,\alpha}c^\dagger_{n',\alpha'}c_{n',\alpha'}c_{n,\alpha}.$$

The electrons in a 2D quantum well are subject to two types of spin-orbit interactions, the *Dresselhaus* and *Rashba* interactions, both originating from the inversion asymmetry of the potential

$$V(\boldsymbol{r}) = V_{\mathsf{cr}}(\boldsymbol{r}) + V_{\mathsf{ext}}(\boldsymbol{r}),$$

$$H_{SO} = \lambda_{\mathsf{cr}}\,(\boldsymbol{k} \times \nabla V_{\mathsf{ext}}(\boldsymbol{r})) \cdot \boldsymbol{\sigma} \,-\, \boldsymbol{b}(\boldsymbol{k}) \cdot \boldsymbol{\sigma}.$$

For 1D wire, when motion is restricted let say along $\hat{x}$ axis

$$H_{DR} = H_\alpha + H_\beta = k_x(\beta\sigma_x + \alpha\sigma_y) \Rightarrow$$

$$\Rightarrow -\, i\sum_{n,\alpha,\beta} c^\dagger_{n,\alpha}\left[\gamma_D\, \sigma^x_{\alpha\beta} + \gamma_R\, \sigma^y_{\alpha\beta}\right] c_{n+1,\beta} + \mathsf{H.c.}.$$

where $\gamma_D = \beta a^{-1}$, $\gamma_R = \alpha a^{-1}$, with a the lattice spacing.

Here the Dresselhaus coupling γ_D is a material-specific parameter, while the Rashba coupling
$gamma_R$ has complex dependence on

a) the ion distribution in the nearby doping layers

b) the relative asymmetry of the electron density at the two quantum well interfaces

c) and what is important for us, the applied gate electric field [Rashba 1960].

In the case of UNIFORM SO interaction the NON-interacting Hamiltonian $H = H_0 + H_{DR}$ can be diagonalized by two rotations in the spin space.

$$\begin{pmatrix} d_{n,+} \\ d_{n,-} \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\theta} c_{n,\uparrow} - i e^{i\theta} c_{n,\downarrow} \\ -i e^{-i\theta} c_{n,\uparrow} + e^{i\theta} c_{n,\downarrow} \end{pmatrix},$$

where

$$2\theta = \arctan(\gamma_D / \gamma_R)$$

.

With this, we write the transformed Hamiltonian as

$$\begin{aligned} H' &= H'_0 + H'_{\text{e-e}} + H'_{DR} \\ &= -t\sum_{n,\tau}\Big(d^\dagger_{n,\tau}d_{n+1,\tau}+\text{H.c.}\Big)-\mu\sum_{n,\tau}d^\dagger_{n,\tau}d_{n,\tau} \\ &- i\,\gamma_{\text{eff}}\sum_{n,\tau}\tau\, d^\dagger_{n,\tau}d_{n+1,\tau}+\text{H.c.}, \\ &+ \frac{1}{2}\sum_{n,n',\tau,\tau'}V(n-n')d^\dagger_{n,\tau}d^\dagger_{n',\tau'}d_{n',\tau'}d_{n,\tau} \end{aligned}$$

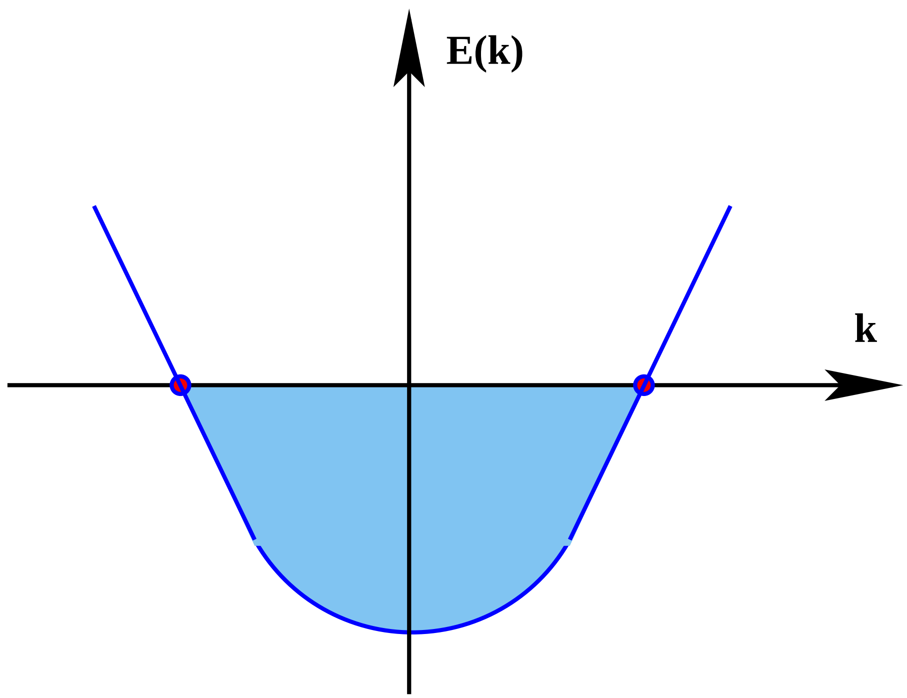
V. Gritsev, G. Japaridze, M. Pletyukhov, and D. Baeriswyl , Phys. Rev. Lett. **94**, 137207 (2005).

S. Gangadharaiah, J. Sun, and O. A. Starykh, Phys. Rev. B **78**, 054436 (2008).

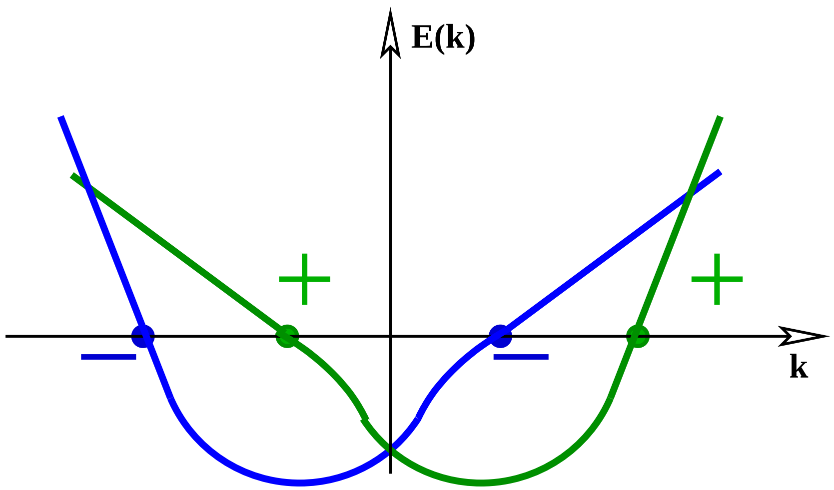
The Luttinger Liquid Phase:

Gapless Spin and Charge Excitations!

$$H_{lat} = -t \sum_{n,\sigma} (c^{\dagger}_{n,\sigma} c_{n+1,\sigma} + h.c.)$$



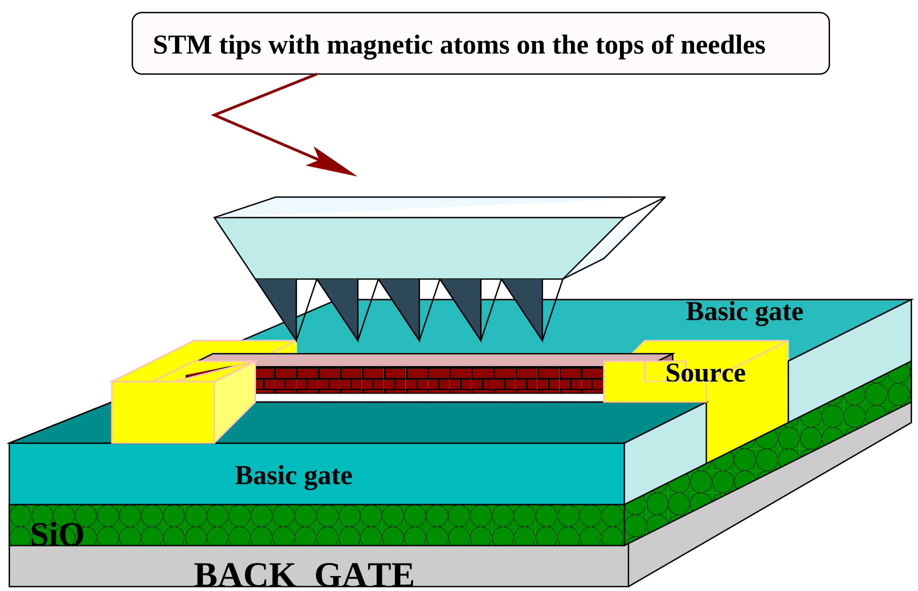
$$H_{lat} = -t \sum_{n,\sigma} (c^\dagger_{n,\sigma} c_{n+1,\sigma} + h.c.)$$
$$+\alpha_R \sum_n [\,(c^\dagger_{n,\uparrow} c_{n+1,\downarrow} - c^\dagger_{n,\downarrow} c_{n+1,\uparrow}) + h.c.\,]$$



- A: Diagonalization of the free Hamiltonian with Rashba term:

Rotation in the spin space $\Rightarrow$ shift of the spectrum

$$
\begin{aligned}
&-t\sum_{n,\alpha}\left(c^\dagger_{n,\alpha}c_{n+1,\alpha}+H.c.\right)+\sum_{n,\alpha}\mu(n)c^\dagger_{n,\alpha}c_{n,\alpha}\\
&-i\sum_{n,\alpha,\beta}\gamma_R(n)\left(c^\dagger_{n,\alpha}\sigma^y_{\alpha\beta}c_{n+1,\beta}+H.c.\right)\\
&+U\sum_{n}c^\dagger_{n,\uparrow}c_{n,\uparrow}\cdot c^\dagger_{n,\downarrow}c_{n,\downarrow}\,.
\end{aligned}
$$



In what follows we will restrict consideration by the simple harmonic modulation

$$\mu(n) = \mu_0 + \mu_1 \cos(Q_0 n a_0) \qquad \gamma_{SO}(n) = \gamma_R^0 + \gamma_R^1 \cos(Q_0 n a_0)$$

Under the same rotation as in the uniform case the Hamiltonian can be rewritten in the rotated $\tau = +, -$ spin basis as following

$$\begin{aligned} H &= -\sum_{n,\tau} \left[\left(t - i\tau\gamma_R^0\right) d_{n,\tau}^\dagger d_{n+1,\tau} \, + \, H.c. \right] \\ &+ \; i \sum_{n,\tau} \gamma_R^1(n) \sin(2\theta) \left(\tau d_{n,\tau}^\dagger d_{n+1,-\tau} \, - \, H.c. \right) \\ &- \; i \sum_{n,\tau} \gamma_R^1(n) \tau \cos(2\theta) \left(\tau d_{n,\tau}^\dagger d_{n+1,\tau} \, - \, H.c. \right) \\ &+ \; \sum_{n,\tau} [\mu_0 + \mu_1 \cos(Q_0 n a)] d_{n,\tau}^\dagger d_{n,\tau} \\ &+ \; \frac{U}{2} \sum_{n,\tau} d_{n,\tau}^\dagger d_{n,\tau} \cdot d_{n,-\tau}^\dagger d_{n,-\tau} \, . \end{aligned}$$

Non-interacting electrons $U = 0$ in absence of modulation $\gamma_1 = \mu_1 = 0$

At $\gamma_1 = \mu_1 = 0$, the Hamiltonian can be easily diagonalized using the Fourier transform to give

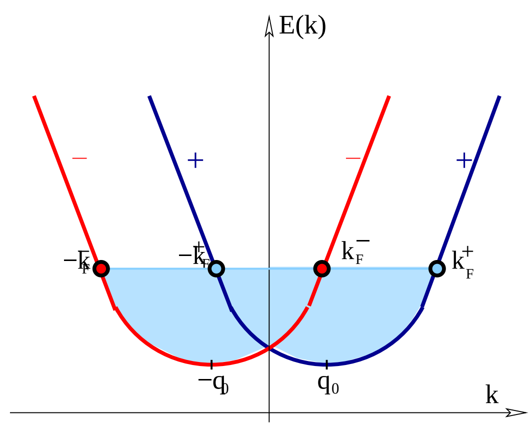
$$H^0 = \sum_{k,\tau} E^0_\tau(k) d^\dagger_{k,\tau} d_{k,\tau}$$

where

$$E^0_\tau(k) = -2\tilde{t}\cos[(k - \tau q_0)a] + \mu_0\,,$$

$$\tilde{t} = \sqrt{t^2 + \gamma^2_{eff}}$$

$$q_0 = \arctan(\gamma_{eff}/t)/a\,.$$



Four Fermi points

$$k^{\tau}_{F,R} = k^0_F + \tau\, q_0, \qquad\qquad k^{\tau}_{F,L} = -k^0_F + \tau q_0 \;\; (\tau = \pm)$$

where $k^0_F = \pi\nu/a_0$.

Decomposition into right- and left-moving fields $R_\tau(x)$ and $L_\tau(x)$,

$$d_{n,\tau} \longrightarrow \sqrt{a}\Big(\mathsf{e}^{i(k^0_F+\tau q_0)x} R_\tau(x) + \mathsf{e}^{-i(k^0_F-\tau q_0)x} L_\tau(x)\Big),$$

the Hamiltonian takes the form $H = \int dx\, (\mathcal{H}_+ + \mathcal{H}_-)$, with

$$\begin{aligned} H_\tau &= \int dx \Big\{ -iv_F\big(:R^\dagger_\tau(x)\partial_x R_\tau(x): - :L^\dagger_\tau(x)\partial_x L_\tau(x):\big) \\ &+ \Gamma_R \cos(Qx)\big(\mathsf{e}^{-2ik^0_F x} R^\dagger_\tau(x) L_\tau(x) + H.c.\big)\Big\}, \end{aligned}$$

where $v_F = 2a\sqrt{t^2+\gamma_0^2}$, and $\quad \Gamma_R = \mu_1 - 2\gamma_1 \sin(q_0 a)\mathsf{e}^{-i\pi\nu}$.

Bosonization.

Using the standard mapping

$$R_\tau(x) = \frac{\eta_\tau}{\sqrt{2\pi a_0}} e^{i\sqrt{\pi}[\varphi_\tau(x) - \vartheta_\tau(x)]},$$
$$L_\tau(x) = \frac{\bar{\eta}_\tau}{\sqrt{2\pi a_0}} e^{-i\sqrt{\pi}[\varphi_\tau(x) + \vartheta_\tau(x)]},$$

where $\varphi_\tau(x)$ and $\vartheta_\tau(x)$ are dual bosonic fields satisfying $\partial_t \varphi_\tau = v_F \partial_x \vartheta_\tau$,

$$H = \sum_\tau \int dx \left\{ \frac{v_F}{2} [\,(\partial_x \vartheta_\tau)^2 + (\partial_x \varphi_\tau)^2\,] + \frac{M_R}{\pi a} \cos[\,(Q_0 - 2k_F^0)x + \phi_0 + \sqrt{4\pi}\varphi_\tau\,] \right\}$$

where,

$$M_R = \sqrt{\Delta_R^2 - \mu_1 \Delta_R \cos(\pi\nu) + \mu_1^2/4}.$$

$$\phi_0 = \arctan\left(\frac{\mu_1 - 2\Delta_R \cos(\pi\nu)}{2\Delta_R \sin(\pi\nu)} \right).$$

At $Q_0 - 2k_F^0 \simeq \mathcal{O}(1/a_0)$, both cosine term $\sim M_R$ is rapidly oscillating and average to zero. Thus, in this limit the model describes free $+$ and $-$ bosons

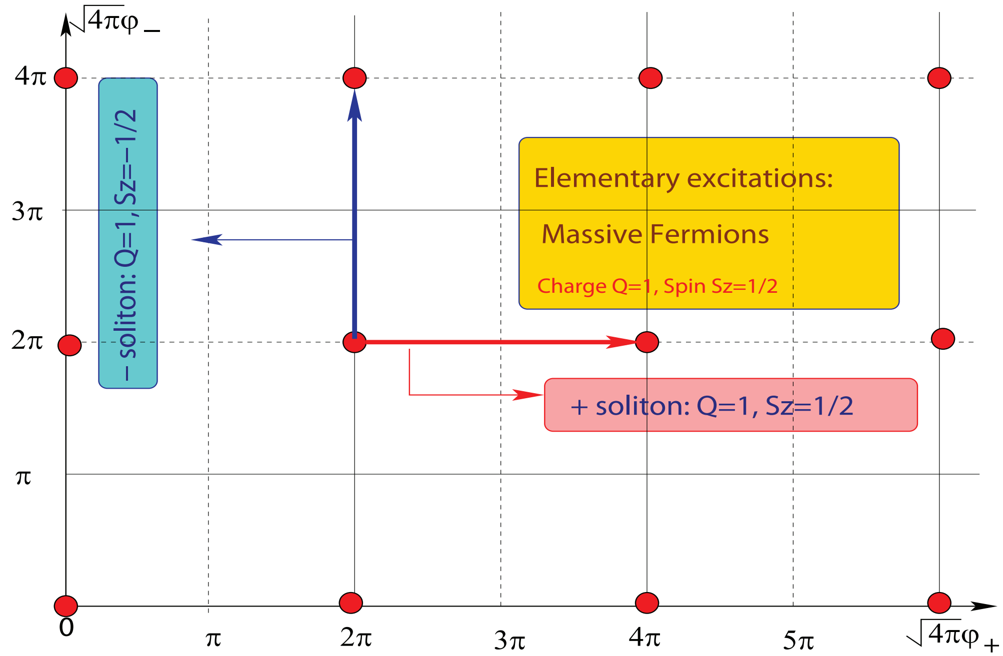
$$H = \sum_\tau \int dx \left\{ \frac{v_F}{2}[\,(\partial_x \vartheta_\tau)^2 + (\partial_x \varphi_\tau)^2\,] \right\}$$

At $|Q_0 - 2k_F^0| \ll \mathcal{O}(1/a_0)$ the component of the modulated Rashba coupling comes into play. In this case the Hamiltonian reads

$$H_\tau = \sum_\tau \int dx \left\{ \frac{v_F}{2}[\,(\partial_x \varphi_\tau)^2 + (\partial_x \vartheta_\tau)^2\,] - \frac{\mu_{\mathit{eff}}}{\sqrt{\pi}} \cdot \partial_x \varphi_\tau + \frac{M_R}{\pi a} \cos(\sqrt{4\pi}\varphi_\tau) \right\} \quad ,$$

where

$$\mu_{\mathit{eff}} = v_F(2k_F^0 - Q_0)/2.$$



$$N_\tau = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dx \partial_x \varphi_\tau(x)$$

The total charge and the total spin carried by **massive spinful fermions** are defined as

$$Q = N_\uparrow + N_\downarrow, \qquad S_z = \frac{1}{2}(N_\uparrow - N_\downarrow)$$

The MI transition becomes transparent if we introduce the **charge** and **spin** fields

$$\varphi_c = \frac{1}{\sqrt{2}}(\phi_+ + \phi_-), \qquad \vartheta_c = \frac{1}{\sqrt{2}}(\theta_+ - \theta_-)$$
$$\varphi_s = \frac{1}{\sqrt{2}}(\varphi_+ - \varphi_-), \qquad \vartheta_s = \frac{1}{\sqrt{2}}(\theta_+ - \theta_-)$$

HAMILTONIAN

$$\mathcal{H} = \mathcal{H}_s + \mathcal{H}_c + H_{cs} \, ,$$

$$\mathcal{H}_c = \int dx \Big\{ \frac{v_c}{2K_c}(\partial_x \varphi_c)^2 + \frac{v_c K_c}{2}(\partial_x \vartheta_c)^2 \Big\},$$
$$\mathcal{H}_s = \int dx \Big\{ \frac{v_s}{2K_s}(\partial_x \varphi_s)^2 + \frac{v_s K_s}{2}(\partial_x \vartheta_s)^2 \Big\},$$
$$\mathcal{H}_{cs} = \frac{2M_R}{\pi a_0} \int dx \cos([Q_0 - 2k_F^0]x + \sqrt{2\pi}\varphi_c) \cos(\sqrt{2\pi}\varphi_s)$$

$$\mathcal{H}_c = \int dx \Big\{ \frac{v_c}{2K_c} (\partial_x \varphi_c)^2 + \frac{v_c K_c}{2} (\partial_x \vartheta_c)^2 \Big\},$$

$$\mathcal{H}_s = \int dx \Big\{ \frac{v_s}{2K_s} (\partial_x \varphi_s)^2 + \frac{v_s K_s}{2} (\partial_x \vartheta_s)^2 \Big\},$$

$$\mathcal{H}_{cs} = \frac{2M_R}{\pi a_0} \int dx \cos([Q_0 - 2k_F^0]x + \sqrt{2\pi}\varphi_c)\cos(\sqrt{2\pi}\varphi_s)$$

The spin-charge coupling!

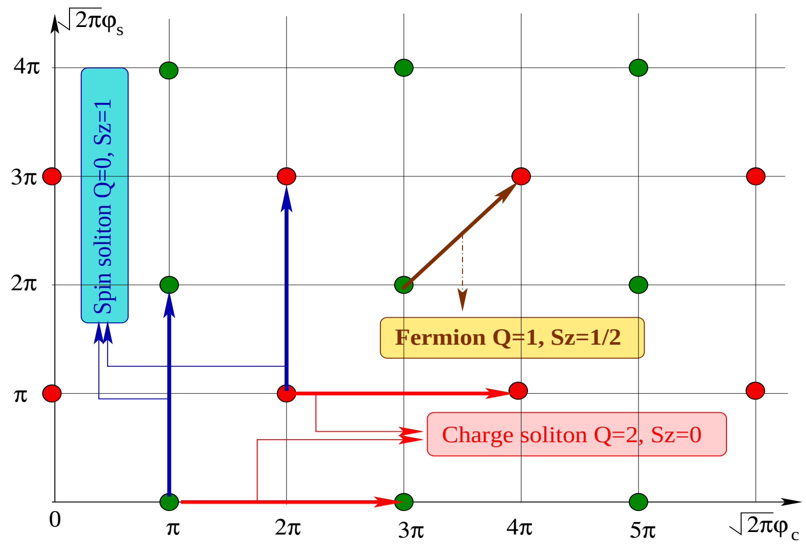
For $(q_0 - 2k_F)x \gg 1$ the Rashba term OSCILLATES and is irrelevant!!!

For $(q_0 - 2k_F)x \ll 1$ the Rashba term is relevant!!!

Simultaneous generation of the Charge and Spin Gaps

The Luttinger-liquid - Nonmagnetic insulator transition!

At commensurate band-filling ($Q_0 = 2k_F^0$) **the spectrum is fully gapped** and the system relays in one of the it's potential minima:



Set of minima of the effective electron-holon coupling potential

$$\mathcal{V}_{cs} = \frac{2M_R}{\pi a_0}\cos(\sqrt{2\pi}\varphi_c)\cos(\sqrt{2\pi}\varphi_s)$$

The Mean-Field bosonized Hamiltonian

$$
\begin{aligned}
\mathcal{H}_c &= \int dx \Big\{ \frac{v_c}{2}\Big[\frac{1}{2}(\partial_x \varphi_c)^2 + \frac{1}{2}(\partial_x \vartheta_c)^2\Big] - \mu_{eff}\sqrt{\frac{2K_c}{\pi}}\partial_x \varphi_c \\
&\quad - \frac{m_c^r}{\pi a_0}\cos(\sqrt{2\pi K_c}\phi_c)\Big\}, \\
\mathcal{H}_s &= \int dx \Big\{ \Big[\frac{1}{2}(\partial_x \varphi_s)^2 + \frac{1}{2}(\partial_x \vartheta_s)^2\Big] - \frac{m_s^r}{\pi a_0}\cos(\sqrt{2\pi K_s}\phi_s)\Big\}.
\end{aligned}
$$

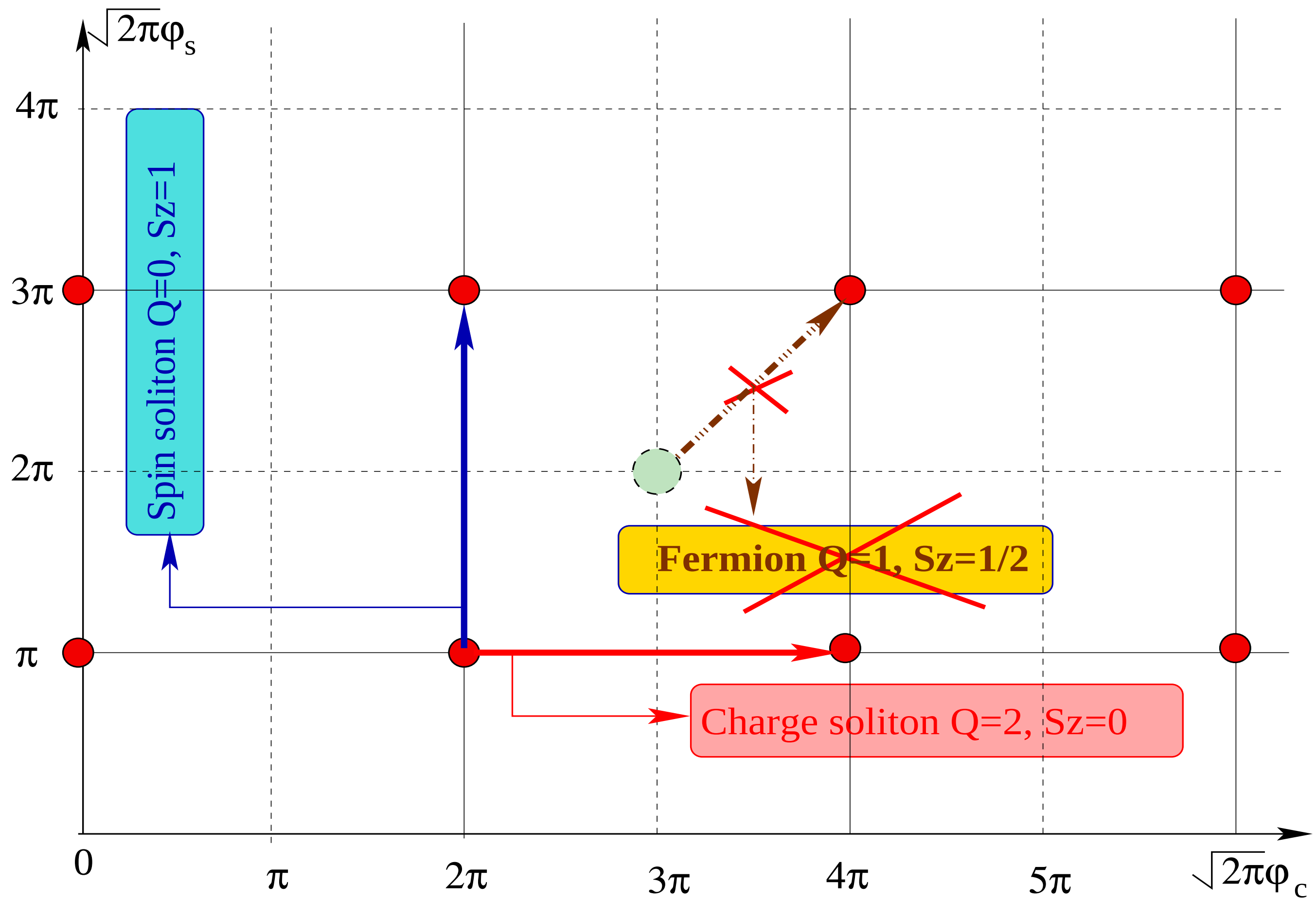
where

$$
m_c^r = 2M_R \cdot \langle \cos(\sqrt{2\pi K_s}\phi_s)\rangle \qquad m_s^r = 2M_R \cdot \langle \cos(\sqrt{2\pi K_c}\phi_c)\rangle \, ,
$$

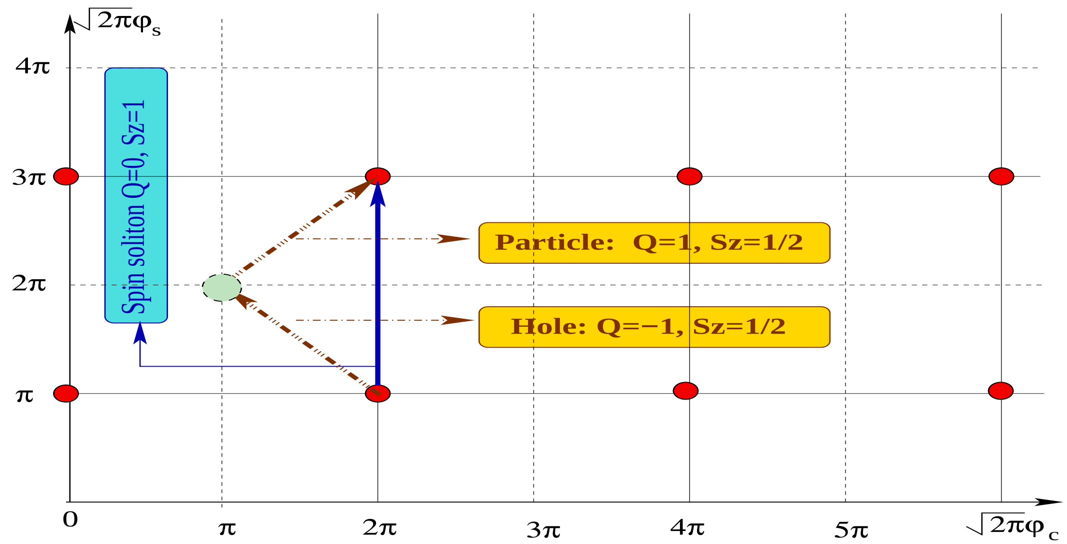
H_c is the Hamiltonian of the Commensurate-Incommensurate transition

Japaridze and Nersesyan, JETF Pis'ma **27**, 356 (1978);

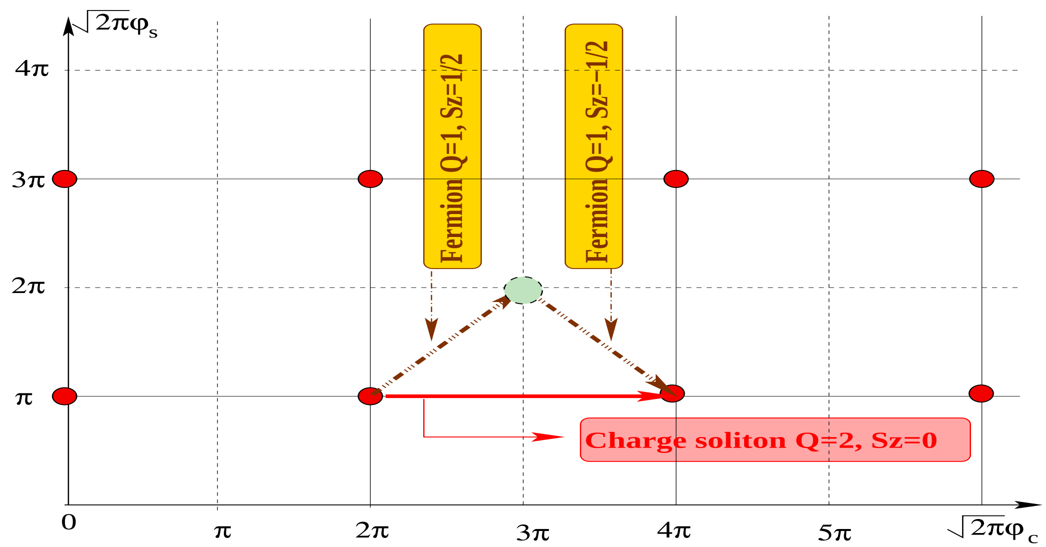
Pokrovsky and Talapov, PRL **42**, 65 (1979).



Set of minima of the mean-field decoupled Hamiltonian



$\sqrt{2\pi}\varphi_s$
4π
3π
2π
π
0
π
2π
3π
4π
5π
$\sqrt{2\pi}\varphi_c$
Spin soliton Q=0, Sz=1
Particle: Q=1, Sz=1/2
Hole: Q=−1, Sz=1/2



$\sqrt{2\pi}\varphi_s$
4π
3π
2π
π
0
π
2π
3π
4π
5π
$\sqrt{2\pi}\varphi_c$
Fermion Q=1, Sz=1/2
Fermion Q=1, Sz=−1/2
Charge soliton Q=2, Sz=0

Two relations by **two Zamolodchikovs**

In the case of the SG model, given by the Hamiltonian

$$H = \int dx \Big\{ \frac{v_F}{2} [(\partial_x \theta)^2 + (\partial_x \phi)^2] - \frac{m_c^0}{\pi a_0} \cos(\sqrt{2\pi K}\phi) \Big\},$$

1. The soliton mass M is related with its bare value m_0 as

$$M/\Lambda = C(K)\,(m_0/\Lambda)^{2/(4-K)}\,,$$

where

$$C(K) = \frac{2\Gamma(\frac{K}{8-2K})}{\sqrt{\pi}\Gamma(\frac{2}{4-K})} \cdot \left[\frac{\Gamma(1-K/4)}{2\Gamma(K/4)}\right]^{\frac{2}{4-K}}$$

and Λ is an energy cutoff.

Al. B. Zamolodchikov , Int. Jour. Mod. Phys. A **10**, 1125-1150 (1995).

2. The vacuum expectation value of the cosine field is related to the **soliton mass** M by

$$\langle\cos(\sqrt{2\pi K}\phi)\rangle = B(K)(M/\Lambda)^{K/2},$$

with

$$\begin{aligned} B(K) &= [\Gamma(1/2+\xi/2)\Gamma(1-\xi/2)]^{(K/2)-2} \\ &\cdot \left[\frac{2\sin(\pi\xi/2)}{4\sqrt{\pi}}\right]^{K/2}\left[\frac{(1+\xi)\pi^2\Gamma(1-K/4)}{\sin(\pi\xi)\Gamma(K/4)}\right], \end{aligned}$$

where $\xi = K/(4-K)$.

S. Lukyanov, A. Zamolodchikov, Nucl.Phys. B **493**, 571 (1997).

$$
\begin{aligned}
M_c &= \Lambda C(K_c)\left(\frac{2M_R}{\Lambda}\langle\cos(\sqrt{2\pi}\phi_s)\rangle\right)^{\frac{2}{4-K_c}} \\
M_s &= \Lambda C(K_s)\left(\frac{2M_R}{\Lambda}\langle\cos(\sqrt{2\pi}\phi_c)\rangle\right)^{\frac{2}{4-K_s}}
\end{aligned}
$$

The self-consistency equation reads

$$
\begin{aligned}
\frac{2M_R}{\Lambda} &= [C(K_c)]^{-\frac{4-K_c}{2}}[B(K_s)]^{-1}\left(\frac{M_c}{\Lambda}\right)^{\frac{4-K_c}{2}}\left(\frac{M_s}{\Lambda}\right)^{-\frac{K_s}{2}} \\
&= [C(K_s)]^{-\frac{4-K_s}{2}}[B(K_c)]^{-1}\left(\frac{M_s}{\Lambda}\right)^{\frac{4-K_s}{2}}\left(\frac{M_c}{\Lambda}\right)^{-\frac{K_c}{2}}
\end{aligned}
$$

gives

$$\lambda_c M_c \quad = \quad \lambda_s M_s.$$

where

$$\lambda_c \quad = \quad \frac{\sqrt{B(K_c)}}{[C(K_c)]^{1-K_c/4}}, \qquad \text{and} \qquad \lambda_s = \frac{\sqrt{B(K_s)}}{[C(K_s)]^{1-K_s/4}}.$$

Noninteracting particles:, $K_c = K_s = 1$

$$M_c \quad = \quad M_s.$$

The Single-particle gap

$$\bar{M}_{\text{mean}} = \kappa(K_c, K_s)\Lambda \left(2M_R/\Lambda\right)^{2/(4-K_c-K_s)},$$

with

$$\kappa(K_c, K_s) \equiv \frac{1}{4}\eta_c^{-1}(1,1)(\eta_c(K_c, K_s)+\eta_s(K_c, K_s)).$$

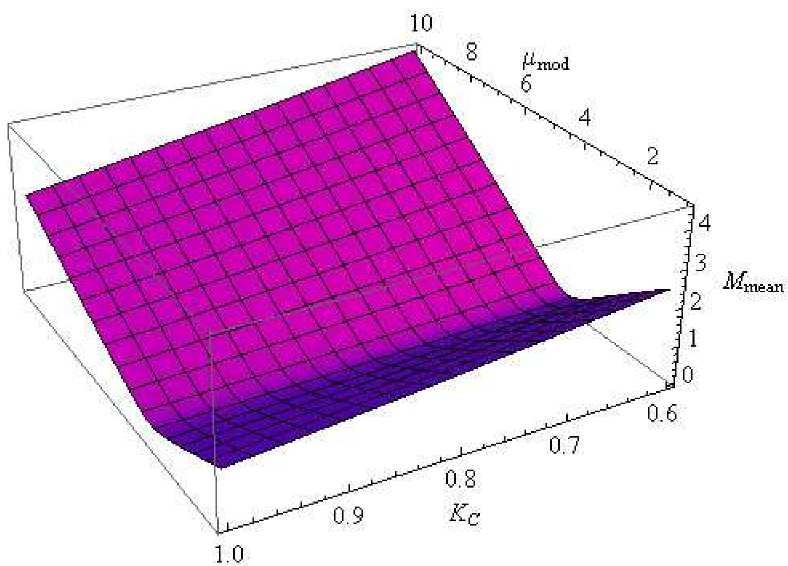


Figure 1: The mean-field value of $\bar{M}_{\text{mean}}$ [meV] of the single-particle gap as a function of the parameters K_c and μ_{mod} in the experimentally relevant parameter range $0.6 \leq K_c \leq 1$, $K_s = 1.1$, and with $\lambda_R = -2$ meV, $\nu = 0.04$, $\Lambda = 100$ meV, and 1 meV $\leq \mu_{\text{mod}} \leq 10$ meV.

EXPERIMENTAL CONSEQUENCES

A narrow **InAs quantum wire** with:

carrier density $n_e \approx 1 \times 10^{12}$ cm^{-2},

effective mass $m^* \approx 0.4 m_e$ and

$\alpha_R = (0.6 - 4) \times 10^{-11}$eVm,

We find that for realistic values of the Coulomb repulsion

$$M_c \simeq 4meV$$

the characteristic length scale

$$\xi \sim \hbar v_F / M_c \simeq 1 \mu m$$

This number does not fits easily the quantum ballistic regime of an

InAs quantum wire, with MF path $\gtrapprox 1.5\,\mu$m

CONCLUSIONS:

1. we have analyzed the spin- and charge dynamics in a ballistic single-channel quantum wire in the presence of a gate-controlled harmonically modulated Rashba spin-orbit interaction with a concurrent a harmonic modulation of the local chemical potential.

2. Depending on the relation between the common wave number q of the two modulations, the Fermi momentum k_F, and a parameter q_0 which encodes the strength of the Dresselhaus and the uniform part of the Rashba interaction, the electrons in the wire may form a metallic or an insulating state.

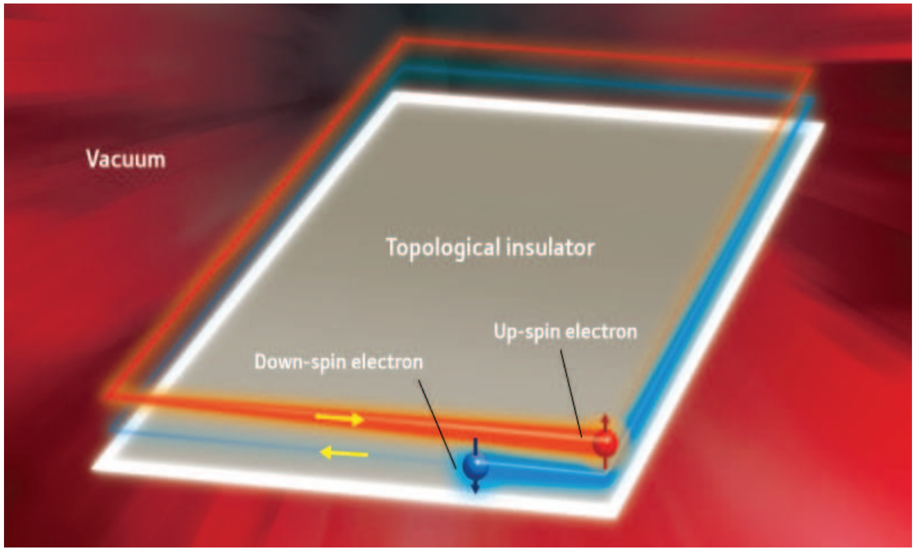
3. Specifically, and most interesting from the viewpoint of potential spintronics applications, when $|q - 2k_F| \ll \mathcal{O}(1/a)$ and $|q \pm 2q_0| \simeq \mathcal{O}(1/a)$ (with a the lattice spacing), a *nonmagnetic insulating state* is formed, with an effective band gap which depends on the amplitudes of the Rashba and chemical potential modulations as well as on the strengths of the uniform Dresselhaus and Rashba interactions.

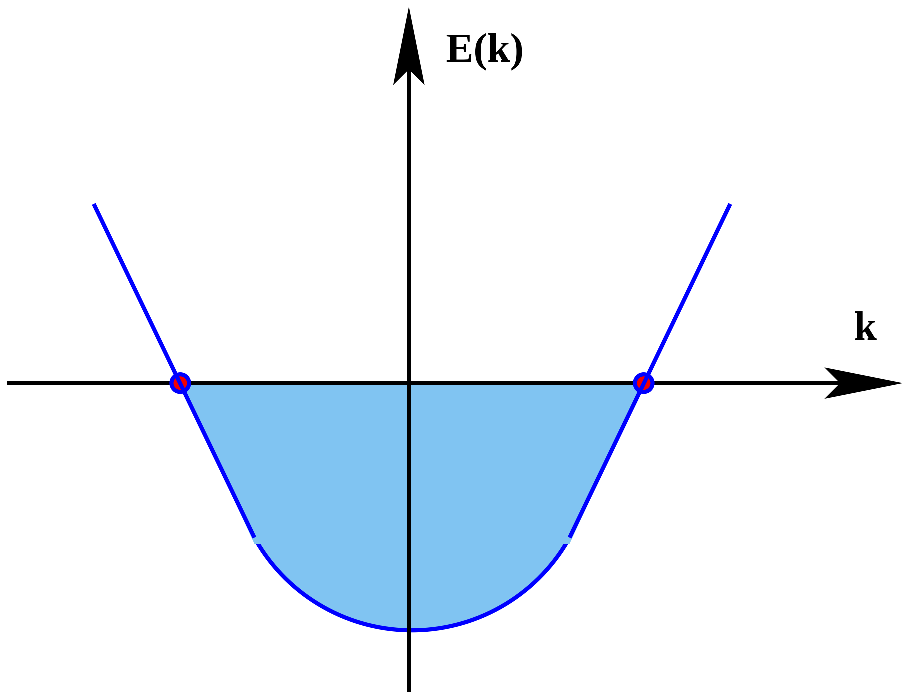
Part II. Edge Dynamics in a Quantum Spin Hall State:

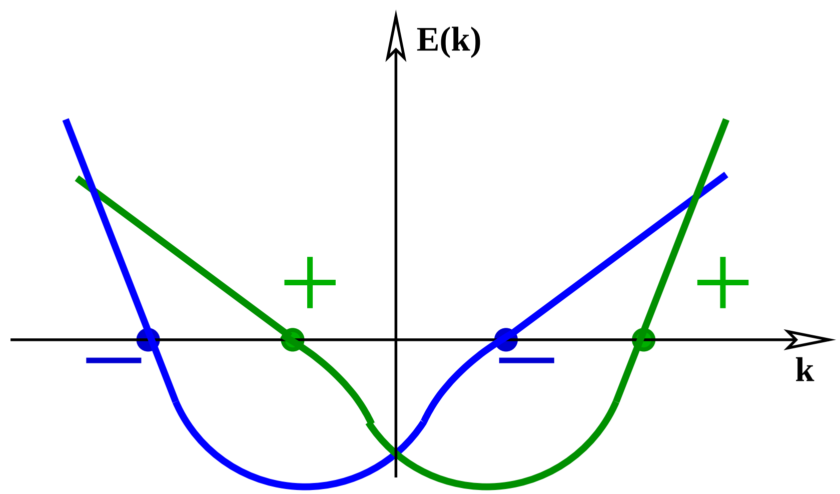
Effects from modulated Rashba Spin-Orbit Interaction

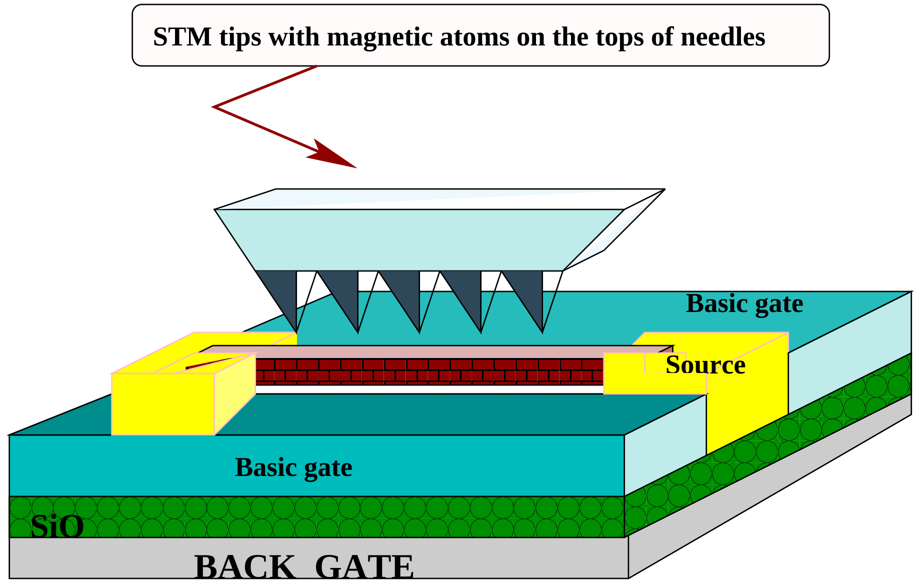
Anders Ström, Henrik Johannesson, and G. I. Japaridze,
Phys.Rev.Lett. **104** 256804 (2010).

Quantum Spin Hall Effect




E(k)
k


E(k)
k
+
−
−
+


STM tips with magnetic atoms on the tops of needles
Basic gate
Source
Basic gate
SiO
BACK GATE

In what follows we will restrict consideration by the simple harmonic modulation

$$\mu(n) = \mu_0 + \mu_1 \cos(Q_0 n a_0) \qquad \gamma_{SO}(n) = \gamma_R^0 + \gamma_R^1 \cos(Q_0 n a_0)$$

Under the rotation $d_{n,+} = \frac{1}{\sqrt{2}}(c_{n,\downarrow} - i c_{n,\uparrow})$ $d_{n,-} = \frac{1}{\sqrt{2}}(c_{n,\uparrow} - i c_{n,\downarrow})$ the Hamiltonian can be rewritten in the rotated $\tau = +, -$ spin basis as following

$$\begin{aligned} H &= -\sum_{n,\tau} \left[\left(t - i\tau\gamma_R^0 \right) d^\dagger_{n,\tau} d_{n+1,\tau} + H.c. \right] \\ &+ i\gamma_R^1 \sum_{n,\tau} \cos(Q_0 n a) \left(\tau d^\dagger_{n,\tau} d_{n+1,\tau} - H.c. \right) \\ &+ \sum_{n,\tau} [\mu_0 + \mu_1 \cos(Q_0 n a)] d^\dagger_{n,\tau} d_{n,\tau} \\ &+ \frac{U}{2} \sum_{n,\tau} d^\dagger_{n,\tau} d_{n,\tau} \cdot d^\dagger_{n,-\tau} d_{n,-\tau} \, . \end{aligned}$$

Decomposition into right- and left-moving fields $R_\uparrow(x)$ and $L_\downarrow(x)$,

$$d_{n,\tau} \to \sqrt{a}\big(\mathsf{e}^{i(k_F^0+q_0)x} R_+(x) + \mathsf{e}^{-i(k_F^0+q_0)x} L_-(x)\big),$$

the Hamiltonian takes the form

$$H_0 = -iv_F \int dx \left(\psi_\uparrow^\dagger \partial_x \psi_\uparrow - \psi_\downarrow^\dagger \partial_x \psi_\downarrow\right),$$

$$H_d = g_d \int dx\, \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow^\dagger \psi_\downarrow, \quad H_f = \frac{g_f}{2} \int dx\, \psi_\alpha^\dagger \psi_\alpha \psi_\alpha^\dagger \psi_\alpha,$$

We now add a Rashba SO interaction

$$H_R = \int dx\, \alpha(x) \Psi^\dagger_\alpha(x) \sigma^y_{\alpha\beta}\, k_x \Psi_\beta(x) + \text{h.c.},$$

$$H_R = \int dx\, \alpha(x) \Big((\partial_x \psi^\dagger_\uparrow) \psi_\downarrow - \psi^\dagger_\uparrow \partial_x \psi_\downarrow \Big)\, e^{-2ik_F x} + \text{h.c.}$$

Bosonization.

Using the standard mapping

$$\begin{aligned} R_\tau(x) &= \frac{\eta_\tau}{\sqrt{2\pi a_0}} e^{i\sqrt{4\pi}\varphi_{R,\tau}(x)}, \\ L_\tau(x) &= \frac{\eta_\tau}{\sqrt{2\pi a_0}} e^{-i\sqrt{4\pi}\varphi_{L,\tau}(x)} \end{aligned}$$

In terms of the charge and spin fields we have

$$\phi_{R\uparrow} + \phi_{L\downarrow} = \sqrt{2}(\phi_c - \theta_s)$$

and

$$\phi_{R\downarrow} + \phi_{L\uparrow} = \sqrt{2}(\phi_c + \theta_s).$$

$$\alpha(x) = \langle \alpha(x) \rangle + \sum_n \hat{\alpha}(k_n) e^{i k_n x}$$

Hence, to lowest order,

1. perturbation with a uniform Rashba interaction has no influence on the low-energy properties.

2. The lowest-order effect produced by H_R for *any* $\alpha(x)$ can at most be of $\mathcal{O}(\hat{\alpha}^2)$.

3. For noninteracting electrons, shows that an $\mathcal{O}(\hat{\alpha}^2)$ process is irrelevant (in renormalization-group (RG) sense) implying robustness of the edge states against perturbations with a Rashba interaction, even when spatially fluctuating.

BOSONIZATION

$$H_0 + H_d + H_f = v \int dx \left(\frac{1}{2K} (\partial_x \phi)^2 + \frac{K}{2} (\partial_x \theta)^2 \right),$$

$$H_R = \frac{1}{\sqrt{\pi} K} \int dx \left[\eta(x) (\partial_x \theta) e^{i\sqrt{4\pi}\phi} + \text{h.c.} \right],$$

with

$$\eta(x) \equiv \sum_n \hat{\alpha}(k_n - 2k_F) e^{ik_n x}$$

$$K = ((\pi v_F + g_f - g_d)/(\pi v_F + g_f + g_d))^{1/2}$$

To pass to a Lagrangian formalism, we use that $\Pi = \sqrt{K}\partial_x \theta$ serves as conjugate momentum to $\phi/\sqrt{K}$ and integrate out Π from the partition function Z to arrive at

$$Z \sim \int \mathcal{D}\varphi e^{-S[\varphi]},$$

with the Euclidean action

$$S[\varphi] = \frac{1}{2}\int dx d\tau \Big(\frac{1}{v}(\partial_\tau \varphi)^2 + v(\partial_x \varphi)^2\Big) - \frac{1}{2\pi\kappa}\int dx d\tau \left(\xi(x) e^{-i\lambda_K \varphi} + \text{h.c.}\right).$$

Here

$$\xi(x) \equiv 1/(4Kv\kappa)\sum_{n,n'} \hat{\alpha}(k_n - 2k_F)\hat{\alpha}(k_{n'} - 2k_F) \times e^{i(k_n + k_{n'})x}$$

$$\lambda_K \equiv \sqrt{16\pi K}$$

and

$$\varphi \equiv \phi/\sqrt{K}$$

By averaging over the randomness in $S[\varphi]$, using the Gaussian statistics

$$P[\xi(x)] = \mathsf{exp}\Big[- D_\xi^{-1} \int dx\, \xi^*(x)\xi(x) \Big]$$

so that

$$\langle \xi(x) \rangle = 0, \qquad \langle \xi^*(x)\xi(x') \rangle = D_\xi \delta(x - x')$$

the replica method yields the disorder-averaged action

$$\begin{aligned} S_n[\varphi] &= \frac{1}{2} \sum_a \int dx d\tau \Big(\frac{1}{v} (\partial_\tau \varphi_a)^2 + v (\partial_x \varphi_a)^2 \Big) \\ &- \frac{D_\xi}{(2\pi\kappa)^2} \sum_{a,b} \int dx d\tau d\tau' \cos[\lambda_K(\varphi_a(x,\tau) - \varphi_b(x,\tau'))] \end{aligned}$$

where $a, b = 1, ..., n$ are the replica indices and

$$D_\xi = n_i/(8\pi K^2 v^4) \left\langle \mathrm{Re}\, \hat{\alpha}^4(k) \right\rangle$$

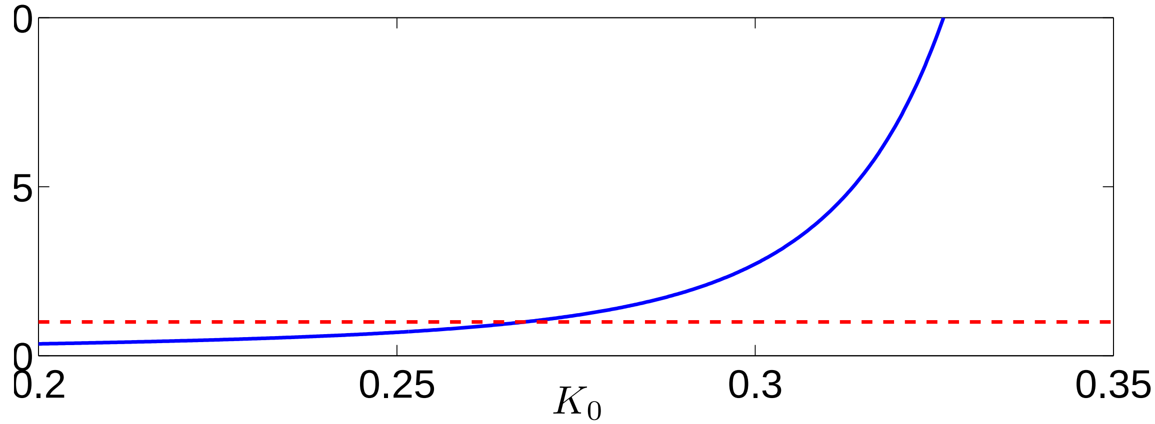
The second-order RG equations of D_ξ, v, and K, generated by the scaling $(\tau, x) \rightarrow (\tau, x)\exp(-\ell)$ $(\ell > 0)$, are given by

$$\begin{aligned} \partial D_\xi/\partial\ell &= (3-8K)D_\xi, \\ \partial v/\partial\ell &= -2vKD_\xi, \\ \partial K/\partial\ell &= -2K^2 D_\xi \end{aligned}$$

The Rashba coupling grows under renormalization when $K < K_c = 3/8$, driving a transition to an Anderson-type localized state. B. A. Bernevig *et al.*, Science **314**, 1757 (2006).

Importantly, to find out whether the edge electrons of a given experimental sample do become localized, one must test for the condition $\xi_{\text{loc}} < L$, with ξ_{loc} the localization length and L is the micron-sized length of the sample.

To make an estimate of $\xi_{\rm loc}$ for a HgTe QW we need to put numbers on our parameters.

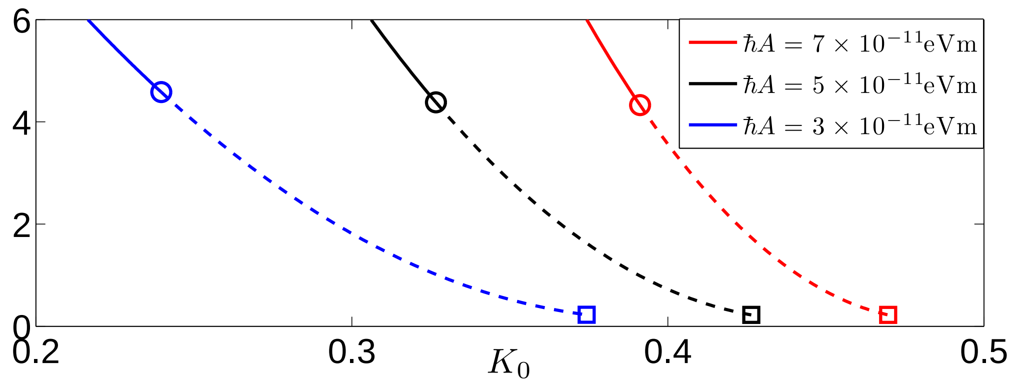


The edge localization length $\xi_{\rm loc}$ for different values of the interaction parameter K_0.
The dashed line marks the length of a micron-sized HgTe QW sample

In the case of periodically modulated Rashba coupling we take $\alpha(x) = A\cos(Qx)$ the action in becomes that of the sine-Gordon model.

$$S[\varphi] = \frac{1}{2}\int dx d\tau \Big(\frac{1}{v}(\partial_\tau \varphi)^2 + v(\partial_x \varphi) - 2g\cos(\lambda_K \varphi)\Big), \tag{1}$$

with $g \equiv A^2/(4\pi K \kappa^2 v)$.



The gap Δ for different Rashba amplitudes $\hbar A$ and values of K_0. The circles and squares mark the smallest gaps for HgTe QW samples of length $1\ \mu$m and $20\ \mu$m respectively.

One additional different and interesting Laboratory to understand

the character of the transition

from the Mott insulating phase caused by a periodic Rashba modulation

to the Anderson-type localization

Thank you for your attention!