Rings, boxes and spins with dissipative environments

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- Motivation
- Rings – particle + environment & conductance [1]
- Coulomb Boxes -- Relaxation resistance [1]

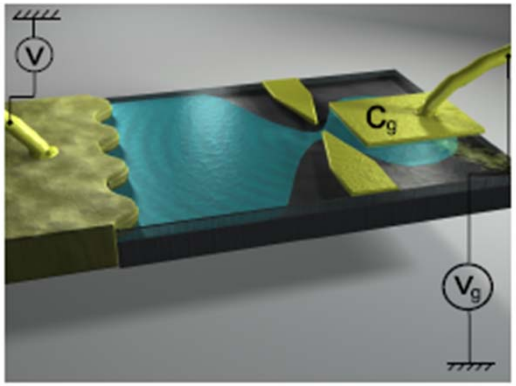
  → Non-equilibrium quantum critical point
- Spin dephasing on a ring – mapping to a spinless problem [2]

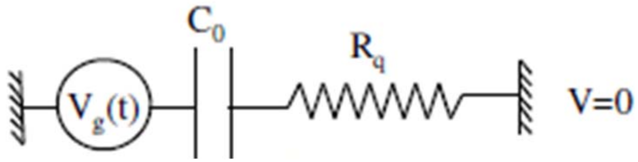
[1] Y. Etzioni, B. Horovitz and P. Le Doussal, Phys. Rev. Lett. **106**, 166803 (2011)

[2] B. Horovitz, P. Le Doussal and G. Zarand, Euro. Phys. Lett. **95**, 57004 (2011)

Workshop on Quantum Filed Theory aspects of Condensed Matter Physics, Frascati 9/2011

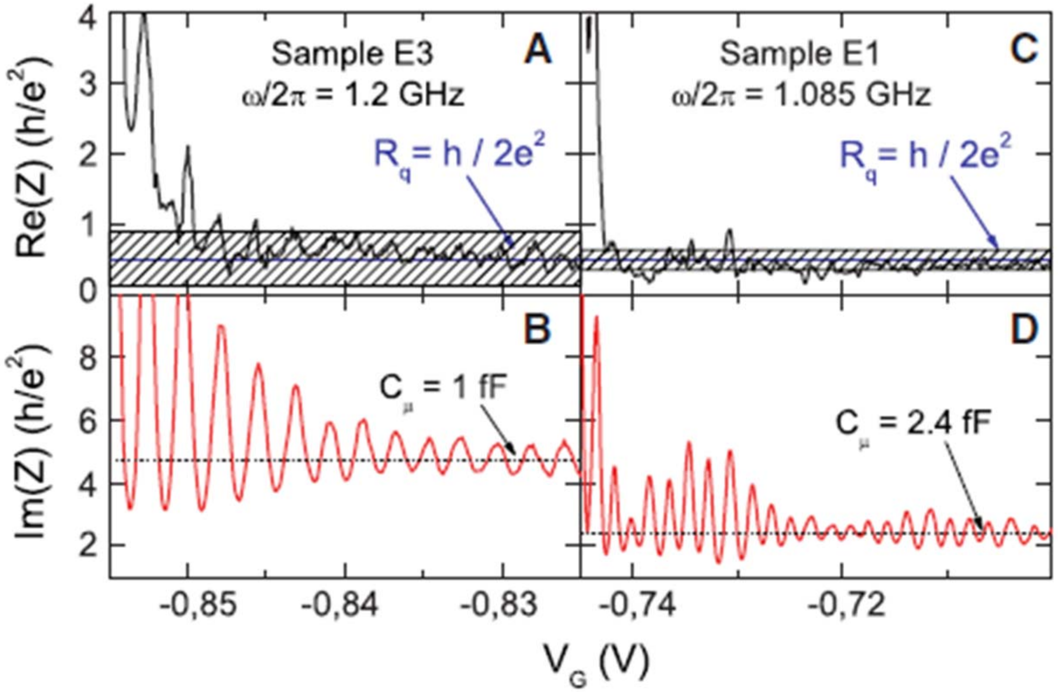
Coulomb box motivation





$$V_g = \frac{Q}{C_0} - i\omega Q R_q \quad \Rightarrow$$

$$\frac{\partial Q}{\partial V_g} = \frac{1}{(1/C_0) - i\omega R_q} = C_0(1 + i\omega C_0 R_q) + O(\omega^2)$$



J. Gabelli, G. Fève, J.-M. Berroir,
B. Plaçais, *A. Cavanna,*
B. Etienne, Y. Jin, D. C. Glattli,
Science **313**, 499 (2006)

Büttiker, Prêtre and Thomas (1993)

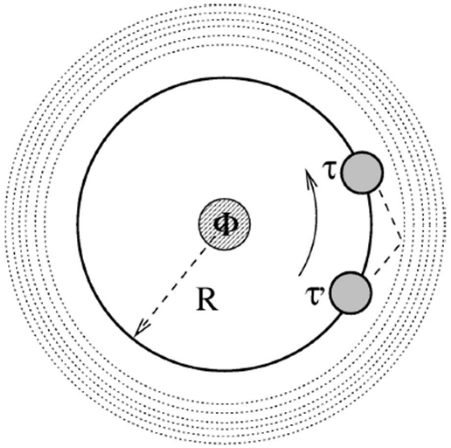
non-interacting: $R_q = h/(2N_c e^2)$

Mora & Le Hur (2010) -- interacting

N_c=1: small dots $R_q = h/2e^2$

Large dots $R_q = h/e^2$

Caldeira-Legget environment



Expand in particle coordinate $\vec{\mathrm{R}}(\tau)$

$$\tilde{S} = \int d\tau \{ \frac{1}{2} M \left(\frac{d\vec{R}}{d\tau} \right)^2 + \sum_i \vec{R}(\tau) \lambda_i Q_i(\tau) + L_{bath}[Q_i(\tau)] \}$$

Integrate bath coordinates, dissipation is obtained if

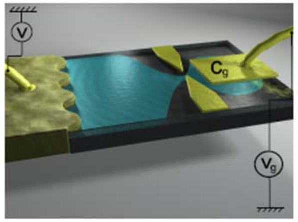
$$S_{\text{int}} = \gamma \int d\omega |\omega| \vec{R}(\omega) \vec{R}(-\omega) = \gamma \int d\tau \int d\tau' \frac{[\vec{R}(\tau) - \vec{R}(\tau')]^2}{(\tau - \tau')^2}$$

Since $\vec{R}(\tau) = R[\cos\theta(\tau), \sin\theta(\tau)]$

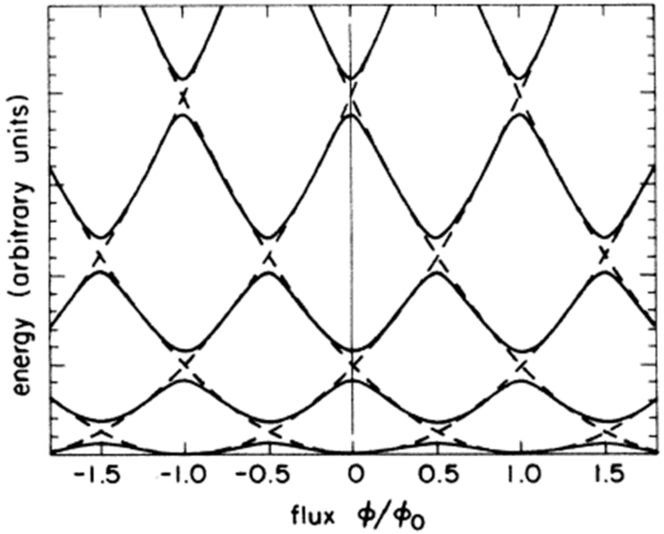
$$S = \int d\tau \frac{1}{2} MR^2 \left(\frac{d\theta}{d\tau} \right)^2 + \pi\eta \int d\tau \int d\tau' \frac{1 - \cos[\theta(\tau) - \theta(\tau')]}{(\tau - \tau')^2} - \phi_x \int d\tau \dot{\theta}$$

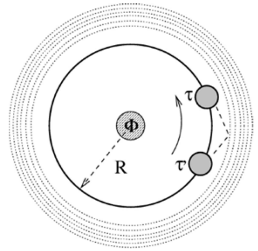
ϕ_x is an external flux (in units of $\phi_0 = hc/e$)

$\Rightarrow$ Long range interaction

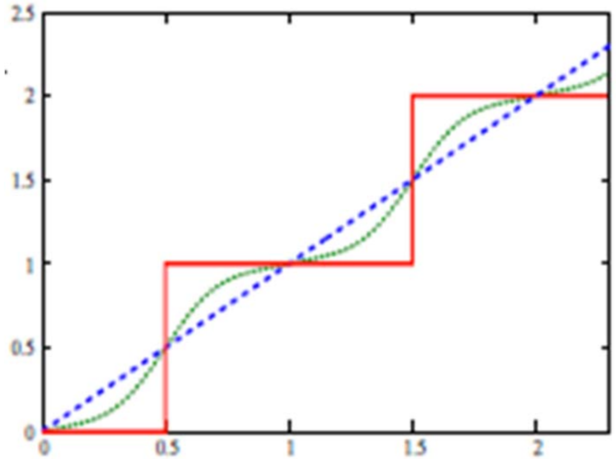


Mapping





$\langle \hat{N} \rangle$



N_0

$E_c = e^2 / 2C_g \qquad V_g = 2E_c N_0$

single particle energies:

charging energy: $E_c[\hat{N} - N_0]^2$

$(\text{m-}\phi_x)^2 / 2M$ m is winding

Ambegaokar, Eckern & Schön (82) $\eta = |\mathrm{t}|^2 N_c \rho_{dot}(0) \rho_{lead}(0), \;\; t \to 0, \;\; N_c \to \infty$

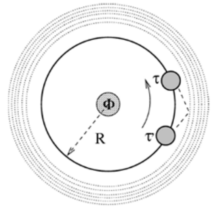
$$\langle \dot{\theta}_t \rangle = 2E_c[\langle \hat{N} \rangle - N_0] \qquad M = 1/2E_c \qquad \phi_x = N_0$$

$K_{t,t'} = i\theta(t-t')\langle [\hat{N}_t, \hat{N}_{t'}] \rangle$

$\tilde{K}_{t,t'} = i\theta(t-t')\langle [\dot{\theta}_t, \dot{\theta}_{t'}] \rangle$

$$\tilde{K}(\omega) + 2E_c = 4E_c^2 K(\omega) = (4E_c^2/e^2)\mathrm{C}_0(1 + i\omega C_0 R_q)$$

Langevin dynamics - nonequilibrium



$$M\ddot{\mathbf{x}} + \eta\dot{\mathbf{x}} = \boldsymbol{\xi}(t) \qquad E = \dot{\phi}_x$$

$$< \xi_i(t)\xi_j(0) >= B(t)\delta_{ij}, \qquad B(\omega) = \hbar\eta\,|\,\omega\,| \quad (T=0)$$

$$M\ddot{\theta}(t) + \eta\dot{\theta}(t) = \xi_x \cos\theta(t) + \xi_y \sin\theta(t) + E \quad (R=1)$$

$$\omega \to 0 \;\; <\theta_t >= -E/(i\omega\eta_R), \qquad \text{without noise } \eta_R = \eta$$

$$\frac{1}{\eta_R} = \frac{1}{\eta} - \frac{1}{\pi\eta^2}\ln\frac{\mathrm{v}_0}{\omega_c} + \frac{1}{\pi^2\eta^3}[\ln^2\frac{\mathrm{v}_0}{\omega_c} + b_0\ln\frac{\mathrm{v}_0}{\omega_c}] + \ldots$$

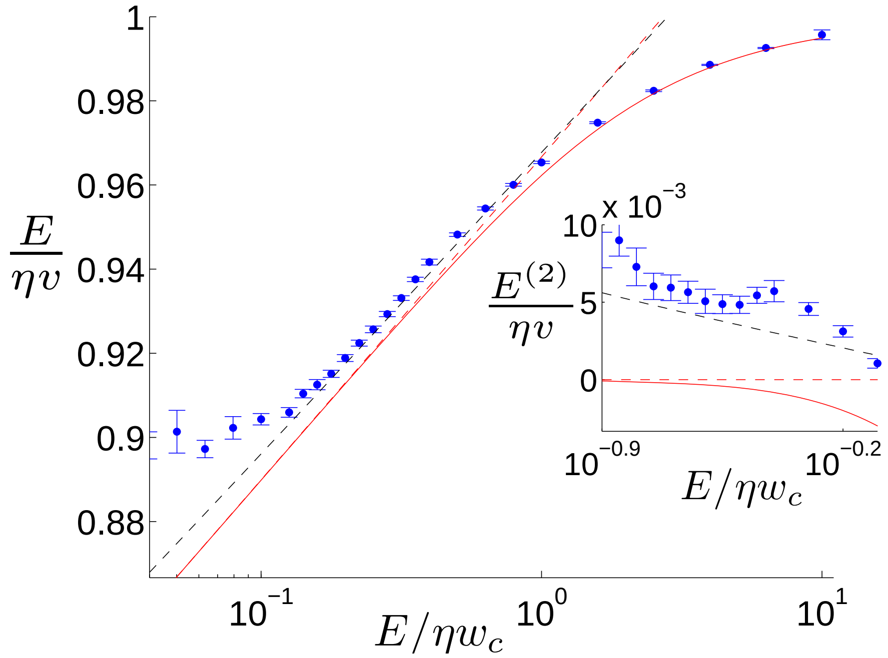
$$\mathrm{v}_0 = E/\eta, \quad \omega_c = \eta/M$$

Equilibrium: $\lim_{\omega\to 0}\lim_{E\to 0} \quad \Rightarrow \mathrm{b}_0 = -1$

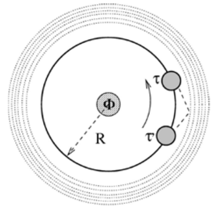
[Hofstetter & Zwerger 1997.]

Non-equilibrium:

$$\lim_{E\to 0}\lim_{\omega\to 0} \qquad \Rightarrow \mathrm{b}_0 = 0$$



Equilibrium vs non-equilibrium



Linear response to E (coupling $E\theta$) is $R(\omega)=-1/i\omega\eta_R$

Linear response to ϕ_x (coupling $\phi_x\dot{\theta}$) is $\tilde{K}(\omega)=-K_0+i\omega K_1 \overset{?}{=} \omega^2 R(\omega)$

constant term $K_0(\phi_x)$ is missing?

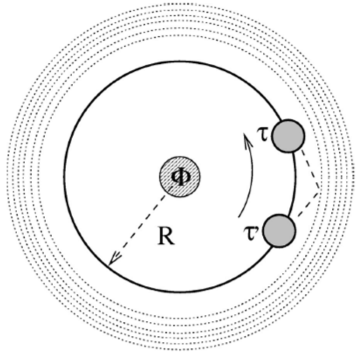
Claim: ϕ_x can be eliminated in total flux ϕ_x+Et, or

$K_0(\phi_x)=K_0(Et)$ is periodic, for dc response $\int_0^1 K_0(\phi_x)d\phi_x=0$

$$\Rightarrow \quad \lim_{E\to 0}\lim_{\omega\to 0}\tilde{K}(\omega)/i\omega=\int_0^1 K_1(\phi_x)d\phi_x=1/\eta_R \quad \Rightarrow \quad \text{Keldysh}$$

$$\frac{1}{\eta_R(E)}=\frac{1}{\eta}-\frac{2}{\pi\eta}\sin\frac{1}{2\eta}\ln\frac{v_0}{\omega_c}+\frac{4}{\pi^2}\sin^2\frac{1}{2\eta}\sin\frac{1}{\eta}[\ln^2\frac{v_0}{\omega_c}+b_0\ln\frac{v_0}{\omega_c}]$$

unexpected small parameter $\sin(1/2\eta)$ $\Rightarrow$ fixed point at $\eta_R=1/2\pi$



$\sin(1/2\eta) = 0$ has $\eta = 1/(2\pi n)$

but $\langle \cos\theta_t \cos\theta_0 \rangle \sim t^{-2n}$ with n>1 is consistent

with Spohn-Zwerger "theorem" $\langle \cos\theta_t \cos\theta_0 \rangle \sim t^{-2}$

$\Rightarrow$ for $\eta > \eta_R$ |-----•--------------- $\leftarrow \eta$

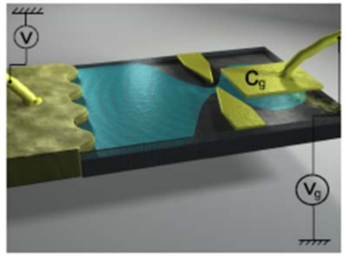
$\eta_R = 1/2\pi \quad \Rightarrow \quad G_{ring} = e^2/h$

Thouless charge pump

slow change of ϕ_x by 1 unit with $\dot{\phi}_x = \eta_R \langle \dot{\theta}_t \rangle = \langle \dot{\theta}_t \rangle / 2\pi$

$$\int dt\, \dot{\phi}_x = 1 = \int dt\, \langle \dot{\theta}_t \rangle / 2\pi$$

i.e. the particle comes back to the same position on the ring and a unit charge has been transported.



$$\int_0^1 \frac{C_0^2(N_0)}{C_g^2} R_q(N_0) dN_0 = \frac{h}{e^2}$$

For large η $C_0 \to C_g$, expect R_q independent of N_0

$$\Rightarrow \quad R_q = \frac{h}{e^2}[1 + \mathrm{O}(e^{-\pi\eta})]$$

Box Experiment

sample with many N_c (e.g. Al), $\hbar\omega_c \approx 1\text{meV}$

sweep gate voltage at a rate $E/\hbar \approx 10^8\,\text{Hz} << \hbar\omega_c$

need: level spacing << ω, T <<10^8Hz

charge fluctuations -- quantized noise

$$S_Q(\omega) = e^2 \langle \hat{N}_t \hat{N}_{t'} \rangle_\omega = \frac{e^2}{4E_c^2}\frac{\omega}{\eta_R} = 2\pi\omega \frac{e^2}{4E_c^2}$$

Spins

$$H_0 = -\frac{\hbar^2}{2m}[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}] + V_0(r)$$

$$H' = \frac{p_\theta^2}{2mr^2} + \alpha_0(S_x p_y - S_y p_x) + \beta_0(S_x p_x - S_y p_y)$$

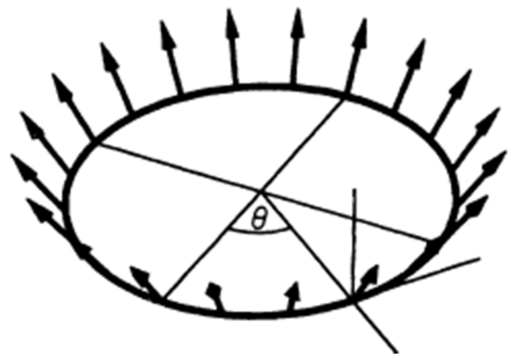
$$H_{ring} = \frac{1}{2mr^2}[p_\theta + \mathbf{h}(\theta)\cdot\mathbf{S}]^2$$

$$\mathbf{h}(\theta) = (\alpha\cos\theta - \beta\sin\theta, \alpha\sin\theta - \beta\cos\theta) \quad \alpha = mr\alpha_0, \quad \beta = \mathrm{mr}\beta_0$$

β=0: rotation invariance, $J_z = p_\theta + S_z$ is conserved

$$\Rightarrow H_{ring} = \frac{1}{2mr^2}[J_z - \mathbf{n}(\theta)\cdot\mathbf{S}\sqrt{1+\alpha^2}]^2 \quad \mathbf{n}(\theta) = [-h_x(\theta), -h_y(\theta), 1]/\sqrt{1+\alpha^2}$$

For $\alpha < \sqrt{3}$ the ground state is a spin coherent state $|\mathbf{n}(\theta)\rangle$



Adding environment:

$H = H_{ring} + V(\theta, \xi)$ ξ are coordinates of a dissipative environment.

$$\dot{\theta} = \frac{p_\theta + \mathbf{h}(\theta)\cdot\mathbf{S}}{mr^2} \qquad \ddot{\theta} = -\frac{1}{Mr^2}\partial_\theta V(\theta,\xi)$$

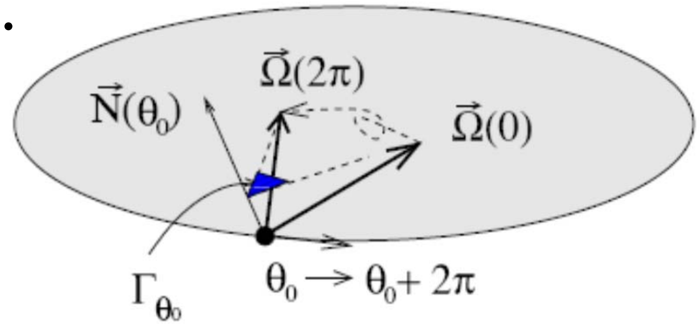
Dynamics of θ are independent of the spin-orbit coupling.

Spin dynamics: $\frac{d\mathbf{S}}{dt} = \dot{\theta}\mathbf{h}(\theta)\times\mathbf{S} \Rightarrow \qquad \frac{d\mathbf{S}}{d\theta} = \mathbf{h}(\theta)\times\mathbf{S}$

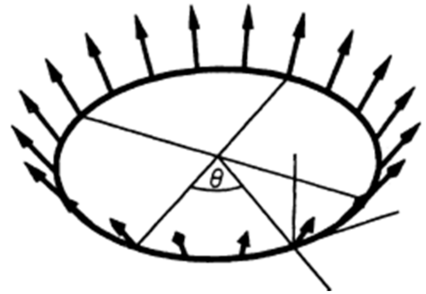
The solution is a linear mapping $S_i(\theta) = R_{ij}(\theta, \theta_0)$

In particular for $\theta = \theta_0 + 2\pi$ the rotation has a unit vector $\mathbf{N}(\theta_0)$ as axis of rotation and Γ the rotation angle.

$\Gamma = 2\pi(\sqrt{1+\alpha^2} - 1)$ incommensurate



Spin 1/2



$$U_{spin}(\theta,\theta_0)\psi_\pm(\theta_0) = \mathrm{e}^{\pm iG(\theta_0-\theta)}\psi_\pm(\theta)$$

$$G = \frac{1}{2}(1-\sqrt{1+\alpha^2})$$ incommensurate

spin correlations involve $P_{a,\Phi}(t) = \langle \mathrm{e}^{-ia\theta_t}\mathrm{e}^{ia\theta_0}\rangle_\Phi$ of spinless problem

$$\langle \mathrm{S}_x(t)S_x(0)\rangle = \frac{1}{4}\sin^2\bar{\alpha}[P_{1,G}(t)+P_{-1,G}(t)] + \cos^4\frac{\bar{\alpha}}{2}P_{-2G,G}(t) + \sin^4\frac{\bar{\alpha}}{2}P_{2-2G,G}(t)$$

$$\langle \mathrm{S}_z(t)S_z(0)\rangle = \cos\bar{\alpha} + P_{-1-2G,G}(t)\sin^2\bar{\alpha}$$

S_z does not dephase

Large η expect $P_{a,\Phi}(t) \sim t^{-a^2/\pi\eta} \to t^{-2a^2}$?

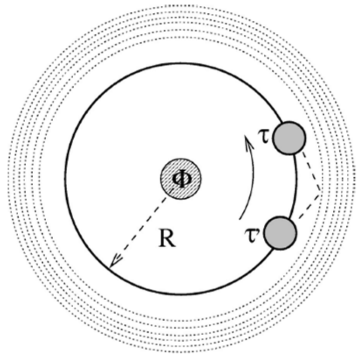
small η perturbation $\cos^4\frac{\bar{\alpha}}{2}$ has a finite correction, no dephasing.

Conclusions & messages

1. Non-equilibrium limit

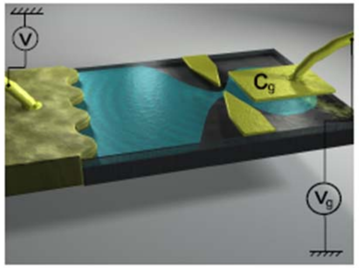
$$\lim_{E\to 0} \lim_{\omega\to 0} \tilde{K}(\omega)/i\omega = \int_0^1 K_1(\phi_x)d\phi_x = 1/\eta_R$$

$$G_{ring} = e^2/h$$



2. Quantized noise experiment

$$S_Q(\omega) = 2\pi\omega \frac{e^2}{4E_c^2}$$



3. Spin dephasing via $\langle \mathrm{e}^{-ia\theta_t} \mathrm{e}^{ia\theta_0} \rangle_\Phi$