The electron-electron interaction in graphene

Outline

- Interactions and screening in single layer graphene
- Renormalization, experiments and theory
- Interactions in bilayer graphene and other allotropes
- Edge states and the topology of the Brillouin Zone
- Magnetism at edges and defects

Quantum Fiend Theory
aspects of Condensed Matter Theory
Frascati, Sept. 6-9, 2011
The Dirac equation

\[ \hbar c \mathbf{F} = \epsilon \omega D(\omega) \]

Density of states

\[ D(\omega) \propto \frac{\omega}{\Lambda^2} \]

Graphene is a semimetal

No metallic screening.

Logarithmic divergences, as in QED
Electron-electron interactions

\[ H = H_{\text{kin}} + H_{\text{int}} = \hbar v_F \int \overline{\psi} \sigma_i \partial_i \psi + \frac{e^2}{\varepsilon} \int \overline{\psi}(\vec{r})\psi(\vec{r}) \frac{1}{|\vec{r} - \vec{r}'|} \overline{\psi}(\vec{r}')\psi(\vec{r}') \]

\[ E_{\text{kin}} \propto \hbar v_F n^{3/2} \]

\[ E_{\text{Coulomb}} \propto \frac{e^2}{\varepsilon} n^{3/2} \]

\[ \alpha = \frac{e^2}{\varepsilon \hbar v_F} \approx 2.3 - 2.5 \quad (\varepsilon = 1) \]
The coupling constant in graphene.

The lattice constant of a solid is determined by the balance between the kinetic and potential energies.

\[
E_{kin} = \frac{\hbar^2 k_F^2}{2m} \approx \frac{\hbar^2}{ma^2}
\]

\[
E_{Coul} \approx -\frac{e^2}{a}
\]

The “fine structure constant” in solids is always of order unity.
Screening in graphene

$\varepsilon_{subs} = \frac{1 + \varepsilon_{diel}}{2} \approx \begin{cases} 2.5 & \varepsilon_{SiO_2} \approx 3.9 \\ 5.4 & \varepsilon_{SiC} \approx 9.7 \\ 2.3 & \varepsilon_{BN} \approx 4.5 \end{cases}$

$\varepsilon_{graphene}^{RPA} = 1 + \frac{\pi e^2}{2 \hbar v_F} \approx 4.6$

$\varepsilon_{graphene}^{RPA+vertex} \approx 5.5$

M. M. Fogler, M. I. Katsnelson, M. Polini, A. Principi, F. G., unpublished
Measurements of $\alpha$

Observation of Plasmarons in Quasi-Freestanding Doped Graphene

Aaron Bostwick, Florian Speck, Thomas Seyller, Karsten Horn, Marco Polini, Reza Asgari, Allan H. MacDonald, Eli Rotenberg

$\alpha_G$ were extracted (Fig. 3I). Comparing to our measurements, we conclude that the best fit is for $\alpha_G \sim 0.5$. From this value, we determine the average screening $\epsilon \sim 4.4$, corresponding to substrate screening contribution $\epsilon_b \sim 7.8$ for graphene on H-SiC in vacuum.

$\alpha_G \geq 2$
Renormalization
Marginal Fermi liquid behavior in graphene

The Fermi velocity increases at low energies
Graphene becomes more insulator-like

\[
\Sigma_{HF}(k, \omega) = \int_{\mathbb{R}^2} \frac{d^2 q}{(2\pi)^2} G(q) \delta(q + k) \delta(\omega + \omega_q)
\]

\[
\Sigma_{RPA}(k, \omega) = \int_{\mathbb{R}^2} \frac{d^2 q}{(2\pi)^2} G(q) \delta(q + k) \delta(\omega + \omega_q)
\]

\[
\text{Im} \Sigma(k, \varepsilon_k) = \frac{\pi}{6} \frac{e^2}{\hbar v_F} |\varepsilon_k|
\]

The lifetime of quasiparticles increases is proportional to the energy

\[
\frac{\Lambda}{v_F} \frac{\partial v_F}{\partial \Lambda} = -\frac{e^2}{4\hbar v_F} \left[ 1 - \frac{8\pi v_F \cos^{-1}\left(\frac{\pi N e^2}{8\hbar v_F}\right)}{Ne^2} + \frac{2}{\pi^2} + \frac{1}{\sqrt{1 - \left(\frac{\pi N e^2}{8\hbar v_F}\right)^2}} \right]
\]

\[
\frac{e^2}{\hbar v_F} \ll 1
\]

\[
\frac{\Lambda}{v_F} \frac{\partial v_F}{\partial \Lambda} = -\frac{2}{\pi^2} + \frac{8\pi v_F \cos^{-1}\left(\frac{\pi N e^2}{8\hbar v_F}\right)}{Ne^2} + \frac{1}{N} \ll 1
\]

Logarithmic scaling:
See also M. S. Foster, I. L. Aleiner, Phys. Rev. B 77, 195413 (2008),
V. N. Kotov, B. Uchoa, V. M. Pereira, A. H. Castro Neto, F. G.
Excitonic transition?

Stoner criterium

\[ U \times N(E_F) \geq 1 \iff \frac{e^2}{\varepsilon |k_F|} \times \frac{|k_F|}{v_F} \approx \frac{e^2}{N v_F} \geq 1 \]

\[ |\Psi_g\rangle \equiv (\alpha_k + \beta_k c_{e,k}^+ c_{h,k}) \Psi_0 \]
Some early experiments

The dependence of gaps on nanotube radii can be written in terms of $1/R$ and $\log(R)$ contributions.


The quasiparticle lifetime increases linearly with energy.

Fabry–Perot interference in a nanotube electron waveguide

Wenjie Liang†, Marc Bockrath†, Dolores Bozovic†, Jason H. Hafner*, M. Tinkham; & Hongkun Park†

$v_F = 8.1 \times 10^5$ ms$^{-1}$
Recent experiments

Dirac charge dynamics in graphene by infrared spectroscopy

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Published online: 8 June 2008 | doi:10.1038/nphys989
Measurements of the effective mass

Suspended samples. Very high mobility

\[ \mu \approx 10^6 \text{cm}^2\text{V}^{-1}\text{s}^{-1} \]

\[ n = 1.4 \times 10^{10} \text{cm}^{-2} \]

\[ n = -7 \times 10^{10} \text{cm}^{-2} \]

 Dirac cones reshaped by interaction effects in suspended graphene

D. C. Elias\textsuperscript{1}, R. V. Gorbachev\textsuperscript{1}, A. S. Mayorov\textsuperscript{1}, S. V. Morozov\textsuperscript{2}, A. A. Zhukov\textsuperscript{2}, P. Blake\textsuperscript{3}, L. A. Ponomarenko\textsuperscript{1}, I. V. Grigorieva\textsuperscript{1}, K. S. Novoselov\textsuperscript{1}, F. Guinea\textsuperscript{4,*} and A. K. Geim\textsuperscript{1,3}

Fits to Renormalization Group calculations
Other recent measurements

Making ARPES Measurements on Corrugated Monolayer Crystals: Suspended Exfoliated Single-Crystal Graphene

Kevin R. Knox,1,2 Andrea Locatelli,3 Mehmet B. Yilmaz,4 Dean Cvetko,5,6 Tevfi G Onur Mentes,3 Miguel Angel Niño,3,7 Philip Kim,2 Alberto Morgante,5,8 and Richard M. Ogush, Jr.2

Broken-Symmetry States in Doubly Gated Suspended Bilayer Graphene


Magnetic ground state

Local Compressibility Measurements of Correlated States in Suspended Bilayer Graphene

J. Martin, B. E. Feldman, R. T. Weitz, M. T. Allen, and A. Yacoby

Nematic ground state

Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the “Parity Anomaly”

F. D. M. Haldane

Broken time reversal symmetry.
Ground state similar to the Integer Quantum Hall Effect

Divergent susceptibilities. Couplings become energy dependent.

F. G., Physics 3, 1 (2010)
Interaction-Driven Spectrum Reconstruction in Bilayer Graphene


Fig. 2. Temperature dependence of the width and the amplitude of the conductivity minimum for mono- and bilayer free-standing graphene samples. (A) Width of the conductivity minimum for bilayer graphene (open circles and open squares, experiments for two different samples; red line, theory for bilayer graphene with low-energy spectrum reconstructed due to nematic phase transition; green line, theory for bilayer with nonreconstructed spectrum). Fitting parameters: for the reconstructed spectrum (red line): \( m = 0.0280 \text{ meV}, \nu_y = 1.41 \times 10^6 \text{ m/s}, \nu = -6.32 \text{ meV} \); for the nonreconstructed spectrum (green line): \( m = 0.0280 \text{ meV}, \nu_y = 1.41 \times 10^6 \text{ m/s}, \nu = 0 \). (Inset) Amplitude of the conductivity minimum of bilayer graphene (yellow crossed circles, experiment; yellow solid line, a guide to the eye). Note the deviation from the straight line below 10 K (marked by arrow). (B) The broadening of the conductivity minimum for bilayer samples (circles, squares, and red and green lines are the same as in (A) and for monolayer graphene (blue line, theory)). (Inset) Left: Low-energy electronic spectrum as expected in the single-electron approximation; right: bilayer graphene low-energy electronic spectrum, reconstructed due to nematic phase transition. (C) The broadening of the conductivity minimum for monolayer graphene [blue and green triangles: experimental points for two different samples; blue line: theory, same as in (B)]. (Inset) Low-energy electronic spectrum for monolayer graphene.
Interactions and disorder
Strains induce resonances

Density of states

$E_F$

Science 329, 544 (2010)
Boundary conditions at edges


See also:

Edge states, topological aspects

When electron kiss. L. Balents, Physics (2011)

2D analog, D. P. Arovas, F. G., unpublished

\[
H(\phi) = \begin{pmatrix}
0 & v_F e^{i\phi} + \lambda k^2 e^{-2i\phi} \\
v_F e^{-i\phi} + \lambda k^2 e^{2i\phi} & 0
\end{pmatrix}
\]
Edge states, examples

Kagomé lattice

σ bands of graphene
Graphite has surface bands near the Fermi energy.
Edge states in graphene

Robustness of edge states in graphene quantum dots

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Fig. 1. (Color online) A graphene quantum dot. The excess density of states due to edge states is shown in a color plot. (a) and (b) display the smooth and rough edge, respectively. In general, edge states are present both near a smooth boundary (a) and a boundary with short-range disorder (b).
Magnetization at edges

Beyond mean field theory
Missing Atom as a Source of Carbon Magnetism

Vacancies

Disorder Induced Localized States in Graphene

Experiments

Theory
Proton irradiated graphite

M. Sepioni, R. R. Nair, S. Rablen, J. Narayanan, F. Tuna, R. Winpenny, A. K. Geim,
Local moments near vacancies: beyond the mean field approximation


Physical features not captured by mean field analyses:
- Magnetic fluctuations
- Kondo effect

The width of the resonance decreases as the interaction increases

Dynamical Mean Field Theory
Hubbard interactions

Curie like susceptibility
A local moment is formed near a vacancy.

- The moment is not quenched at low temperatures.
- The coupling between the moment and the conduction electrons is possibly ferromagnetic.
- Non local interactions enhance the formation of a moment.
- Interactions are not negligible in graphene

- Single layer graphene is a marginal Fermi liquid.

- Interactions in multilayered graphene can lead to novel broken symmetries

- The existence of edge states can be derived from topological arguments

- Magnetic moments are likely to be formed near edges and vacancies.

- Unusual Kondo effect, ferromagnetic and multichannel.