# The electron-electron interaction in graphene

A. Castro-Neto (Boston U.), N. M. R. Peres (U. Minho, Portugal), E. V. Castro, J. dos Santos (Porto), J. Nilsson (Boston U, Göteborg.), A. Morpurgo (Delft), M. I. Katsnelson (Nijmegen), D. Huertas-Hernando (Trondheim, Norway), D. P. Arovas, M. M. Fogler (U. C. San Diego), J. González, F. G., G. León, M. P. López-Sancho, T. Stauber, J. A. Vergés, M. A. H. Vozmediano, B Wunsch (CSIC, Madrid), A. K. Geim, K. S. Novoselov (U. Manchester), A. Lanzara (U. C. Berkeley), M. Hentschel (Dresden), E. Prada, P. San-José (Karlsruhe, Lancaster), J. L. Mañes (U. País Vasco, Spain), F. Sols (U. Complutense, Madrid), E. Louis (U. Alicante, Spain), A. L. Vázquez de Parga, R. Miranda, M. M. Ugeda, I. Brihuega, J. M. Gómez-Rodríguez (U. Autónoma, Madrid), B. Horovitz (Beersheva), P. Le Doussal (ENS, Paris), A. K. Savchenko (Exeter), F. von Oppen (Berlin), A. Akhmerov (Leyden), M. Wimmer (Regensburg, Leyden), T. Low (Purdue), V. Parente, A. Tagliacozzo (Naples), D. Rainis, F. Taddei, M. Polini (Pisa), V. I. Fal'ko (Lancaster), M. F. Crommie (UC Berkeley), P. Haase, Th Pruschke,..S. Fuchs (Göttingen), Hj. Gao (Beijing), R. Asgari (Tehran)

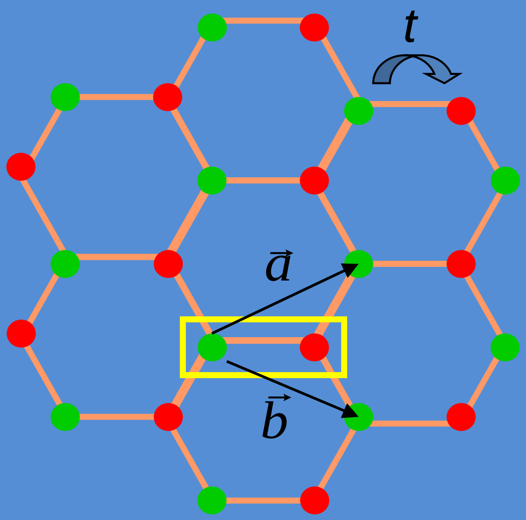
Quantum Fiend Theory aspects of Condensed Matter Theory
Frascati, Sept. 6-9, 2011

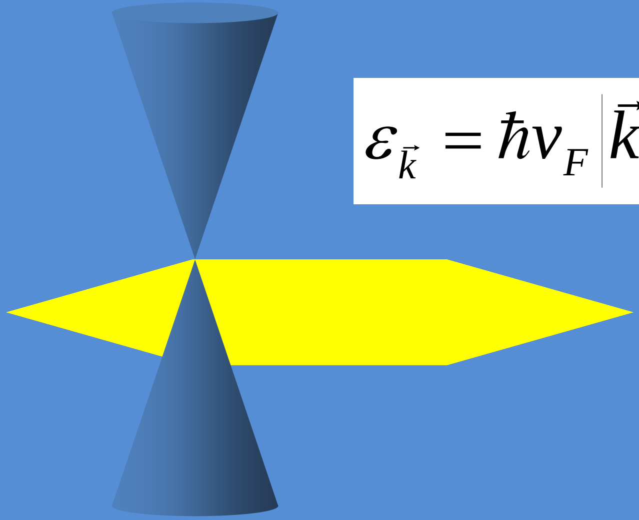
## Outline

- Interactions and screening in single layer graphene
- Renormalization, experiments and theory
- Interactions in bilayer graphene and other allotropes
- Edge states and the topology of the Brillouin Zone
- Magnetism at edges and defects

D. C. Elias, R. V. Gorbachev, A. S.. Mayorov, S. V. Mozorov, A. A. Zhukov, P. Blake, K. S. Novoselov, A. K. Geim, F. G., arXiv1104.1396, Nature Phys., in press (expts+theory)
M. M. Ugeda, I. Brihuega, F. G., J. M. Gómez-Rodríguez, Phys. Rev. Lett. **104**, 096804 (2010) (expts+theory)
P. Haase, S. Fuchs, Th. Pruschke, H. Ochoa, F. G, Phys. Rev. B **83**, 241408 (2011) (theory),
V. Kotov, B. Uchoa, V. M. Pereira, A. H. Castro Neto, F. G., arXiv:1012.3484, Rev. Mod. Phys., in press

# The Dirac equation



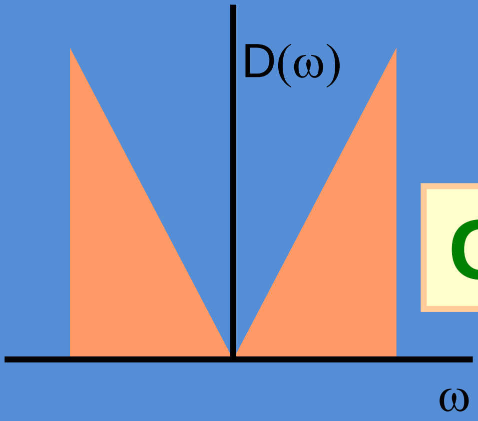


$$\varepsilon_{\vec k} = \hbar v_F \left|\vec k\right|$$

$$H \cong \frac{3ta}{2}\begin{pmatrix} 0 & k_x + i k_y \\ k_x - i k_y & 0 \end{pmatrix}$$

Density of states

$$D(\omega) \propto \frac{|\omega|}{\Lambda^2}$$

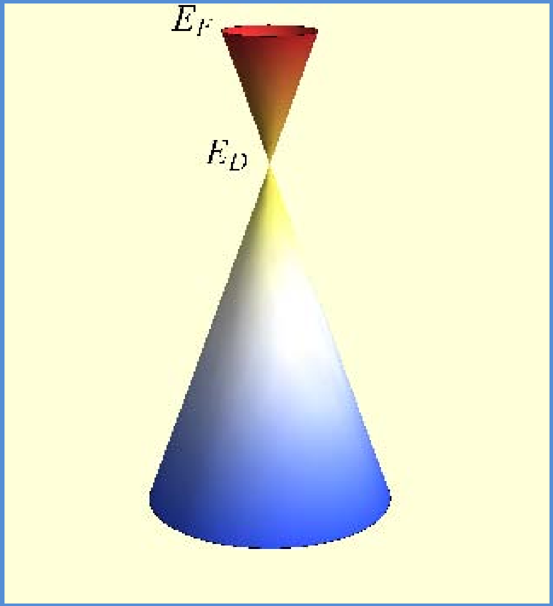


## Graphene is a semimetal

No metallic screening.
Logarithmic divergences, as in QED

# Electron-electron interactions

$$H = H_{kin} + H_{\text{int}} = \hbar v_F \int \overline{\psi}\sigma_i \partial_i \psi + \frac{e^2}{\varepsilon}\int \overline{\psi}(\vec{r})\psi(\vec{r})\frac{1}{|\vec{r}-\vec{r}'|}\overline{\psi}(\vec{r}')\psi(\vec{r}')$$

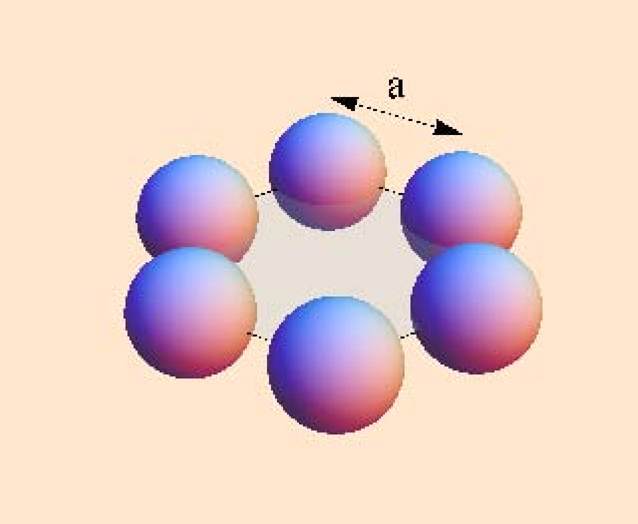


$$E_{kin} \propto \hbar v_F n^{3/2}$$

$$E_{Coulomb} \propto \frac{e^2}{\varepsilon} n^{3/2}$$

$$\alpha = \frac{e^2}{\varepsilon \hbar v_F} \approx 2.3 - 2.5 \quad (\varepsilon = 1)$$

# The coupling constant in graphene.



$$E_{kin} = \frac{\hbar^2 k_F^2}{2m} \approx \frac{\hbar^2}{ma^2}$$

$$E_{Coul} \approx -\frac{e^2}{a}$$

The lattice constant of a solid is determined by the balance between the kinetic and potential energies

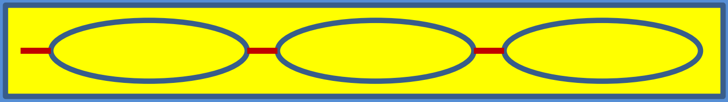
$$\left.\begin{array}{l} E_{kin} = \hbar v_F k_F \Rightarrow \hbar v_F \approx \dfrac{\hbar^2}{ma} \\ E_{kin} \approx E_{Coul} \Rightarrow \dfrac{\hbar^2}{ma^2} \approx \dfrac{e^2}{a} \end{array}\right\} \Rightarrow \frac{e^2}{\hbar v_F} \approx 1$$

The “fine estructure constant” in solids is always of order unity.

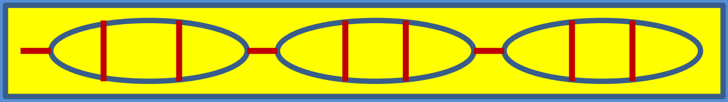
# Screening in graphene

$$\varepsilon_{subs} = \frac{1+\varepsilon_{diel}}{2} \approx \begin{cases} 2.5 & \varepsilon_{SiO_2} \approx 3.9 \\ 5.4 & \varepsilon_{SiC} \approx 9.7 \\ 2.3 & \varepsilon_{BN} \approx 4.5 \end{cases}$$

$$\varepsilon^{RPA}_{graphene} = 1 + \frac{\pi e^2}{2\hbar v_F} \approx 4.6$$

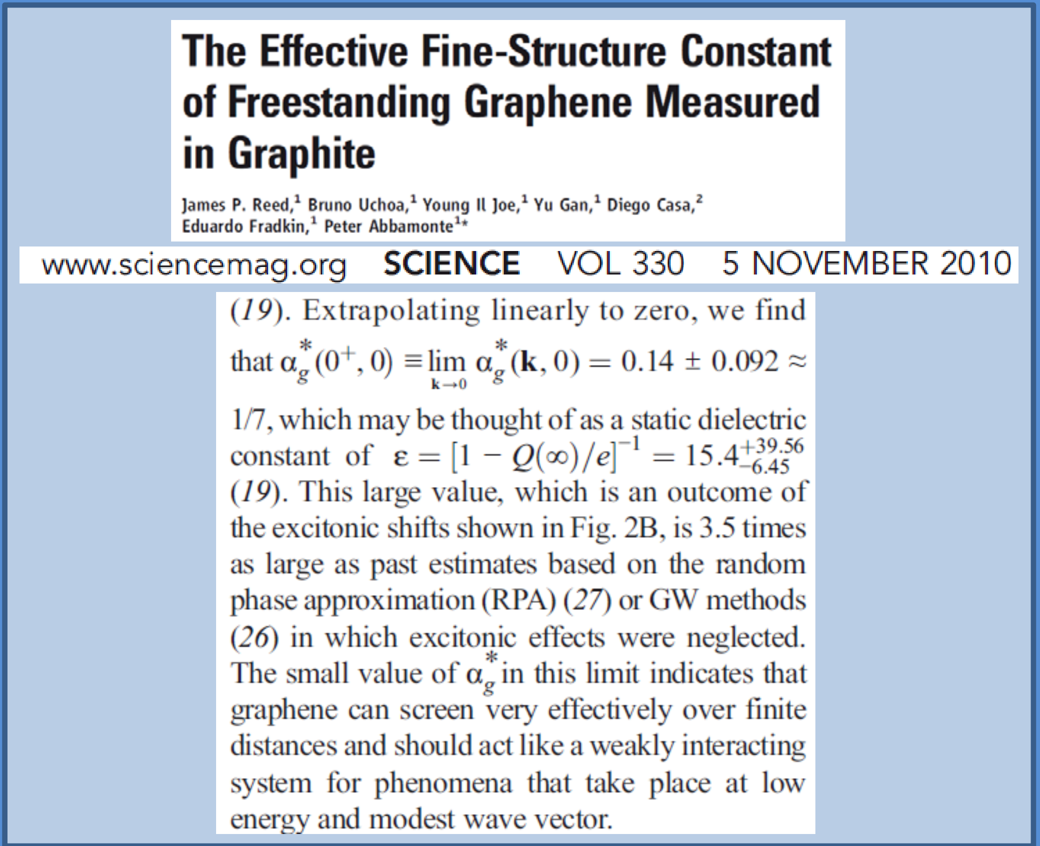


$$\varepsilon^{RPA+vertex}_{graphene} \approx 5.5$$



V. N. Kotov, B. Uchoa, A. H. Castro Neto, Phys. Rev. B 80, 165424 (2009)

M. M. Fogler, M. I. Katsnelson, M. Polini, A. Principi, F. G., unpublished



### The Effective Fine-Structure Constant of Freestanding Graphene Measured in Graphite

James P. Reed,[1] Bruno Uchoa,[1] Young Il Joe,[1] Yu Gan,[1] Diego Casa,[2] Eduardo Fradkin,[1] Peter Abbamonte[1]*

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(*19*). Extrapolating linearly to zero, we find that $\alpha_g^*(0^+,0) \equiv \lim_{\mathbf{k}\to 0} \alpha_g^*(\mathbf{k},0) = 0.14 \pm 0.092 \approx$ 1/7, which may be thought of as a static dielectric constant of $\varepsilon = [1 - Q(\infty)/e]^{-1} = 15.4^{+39.56}_{-6.45}$ (*19*). This large value, which is an outcome of the excitonic shifts shown in Fig. 2B, is 3.5 times as large as past estimates based on the random phase approximation (RPA) (*27*) or GW methods (*26*) in which excitonic effects were neglected. The small value of $\alpha_g^*$ in this limit indicates that graphene can screen very effectively over finite distances and should act like a weakly interacting system for phenomena that take place at low energy and modest wave vector.

# Measurements of $\alpha$

**Observation of Plasmarons in Quasi-Freestanding Doped Graphene**

Aaron Bostwick,[1] Florian Speck,[2] Thomas Seyller,[2] Karsten Horn,[3] Marco Polini,[4*] Reza Asgari,[5*] Allan H. MacDonald,[6] Eli Rotenberg[1†]

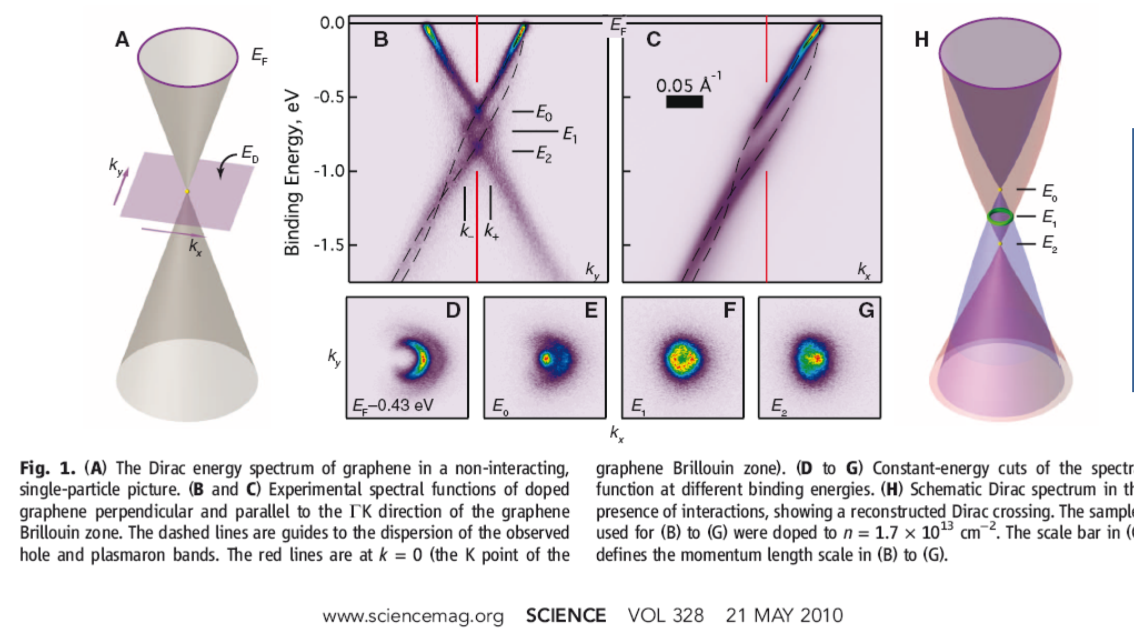

**Fig. 1.** (**A**) The Dirac energy spectrum of graphene in a non-interacting, single-particle picture. (**B** and **C**) Experimental spectral functions of doped graphene perpendicular and parallel to the ΓK direction of the graphene Brillouin zone. The dashed lines are guides to the dispersion of the observed hole and plasmaron bands. The red lines are at $k = 0$ (the K point of the graphene Brillouin zone). (**D** to **G**) Constant-energy cuts of the spectral function at different binding energies. (**H**) Schematic Dirac spectrum in the presence of interactions, showing a reconstructed Dirac crossing. The samples used for (B) to (G) were doped to $n = 1.7 \times 10^{13}$ cm$^{-2}$. The scale bar in (C) defines the momentum length scale in (B) to (G).

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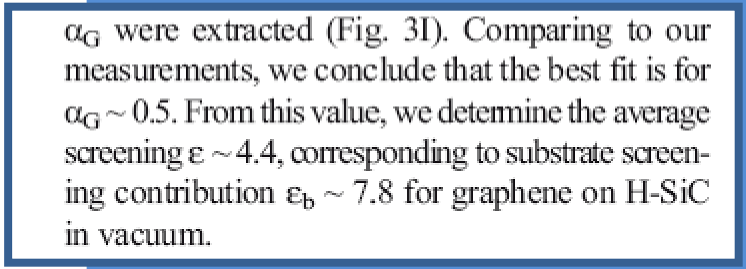


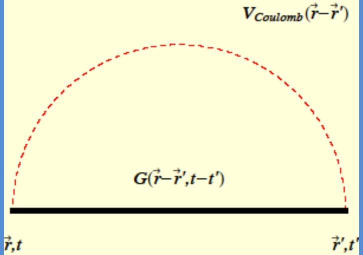
$\alpha_G$ were extracted (Fig. 3I). Comparing to our measurements, we conclude that the best fit is for $\alpha_G \sim 0.5$. From this value, we determine the average screening $\varepsilon \sim 4.4$, corresponding to substrate screening contribution $\varepsilon_b \sim 7.8$ for graphene on H-SiC in vacuum.

$$\alpha_G \geq 2$$

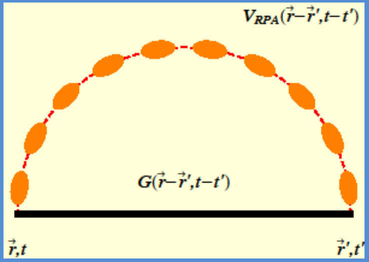
# Renormalization
# Marginal Fermi liquid behavior in graphene

The Fermi velocity increases at low energies
Graphene becomes more insulator-like

$$\Sigma_{HF}(k,\omega)=$$



$$\Sigma_{RPA}(k,\omega)=$$



$$\frac{\Lambda}{v_F}\frac{\partial v_F}{\partial\Lambda}=-\frac{e^2}{4\hbar v_F} \qquad \frac{e^2}{\hbar v_F}<<1$$

$$\frac{\Lambda}{v_F}\frac{\partial v_F}{\partial\Lambda}=-\frac{2}{\pi^2}\left[1-\frac{4\hbar v_F}{Ne^2}+\frac{8\pi v_F\cos^{-1}\left(\frac{\pi Ne^2}{8\hbar v_F}\right)}{\sqrt{1-\left(\frac{\pi Ne^2}{8\hbar v_F}\right)^2}}\right] \qquad \frac{1}{N}<<1$$

$$\operatorname{Im}\Sigma\left(\vec k,\varepsilon_{\bar k}\right)=\frac{\pi}{6}\frac{e^2}{\hbar v_F}\left|\varepsilon_{\bar k}\right|$$

The lifetime of quasiparticles increases is proportional to the energy

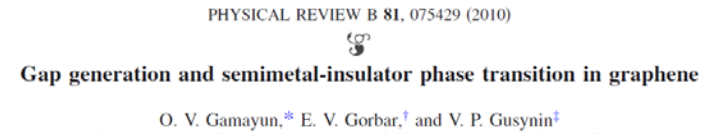
Logarithmic scaling:
A. A. Abrikosov, and D. Benelavski, Soviet, Physics, JETP **32**. 699 (1970).
RG and 1/N expansion: J. González, F. G., M. A. H. Vozmediano, Nucl. Phys. B **424**, 595 (1994), Phys. Rev. B **59**, R2974 (1999),
Quasiparticle lifetime: J. González, F. G., M. A. H. Vozmediano, Phys. Rev. Lett. **77**, 3586 (1996).
See also M. S. Foster, I. L. Aleiner, Phys. Rev. B **77**, 195413 (2008),
V. N. Kotov, B. Uchoa, V. M. Pereira, A. H. Castro Neto, F. G. arXiv:1012.3484, Rev. Mod. Phys., in press.

# Excitonic transition?


VOLUME 87, NUMBER 24 PHYSICAL REVIEW LETTERS 10 DECEMBER 2001

**Ghost Excitonic Insulator Transition in Layered Graphite**

D. V. Khveshchenko

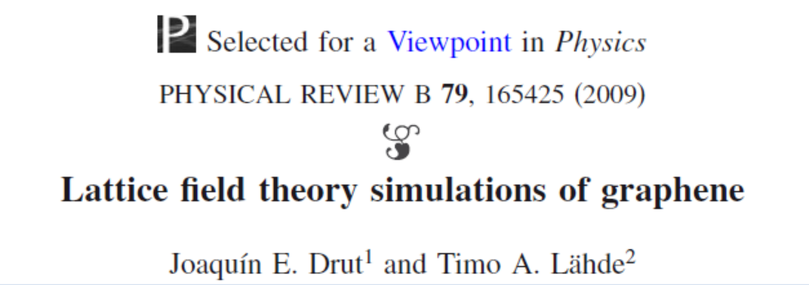

PHYSICAL REVIEW B **81**, 075429 (2010)

**Gap generation and semimetal-insulator phase transition in graphene**

O. V. Gamayun,* E. V. Gorbar,† and V. P. Gusynin‡

## Stoner criterium

$$U\times N(E_F)\geq 1 \Leftrightarrow \frac{e^2}{\varepsilon|k_F|}\times\frac{|k_F|}{v_F}\approx\frac{e^2}{Nv_F}\geq 1$$

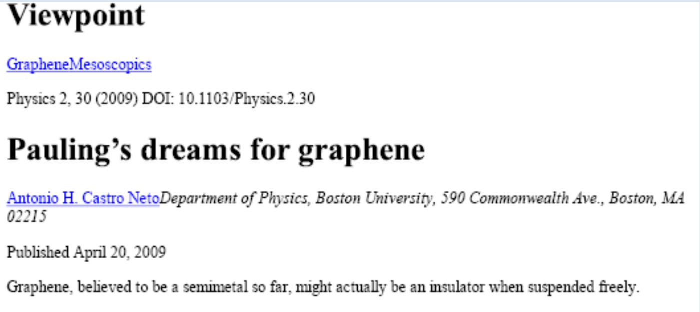

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW B **79**, 165425 (2009)

**Lattice field theory simulations of graphene**

Joaquín E. Drut[1] and Timo A. Lähde[2]

$$\alpha_c = 1.1 \pm 0.06\,?$$


**Viewpoint**

GrapheneMesoscopics

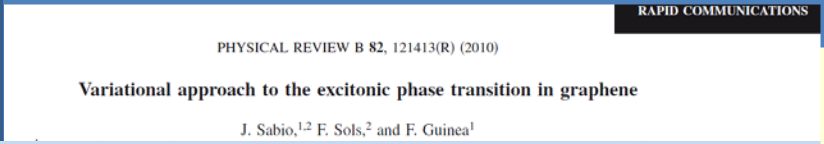
Physics 2, 30 (2009) DOI: 10.1103/Physics.2.30

**Pauling's dreams for graphene**

Antonio H. Castro Neto*Department of Physics, Boston University, 590 Commonwealth Ave., Boston, MA 02215*

Published April 20, 2009

Graphene, believed to be a semimetal so far, might actually be an insulator when suspended freely.

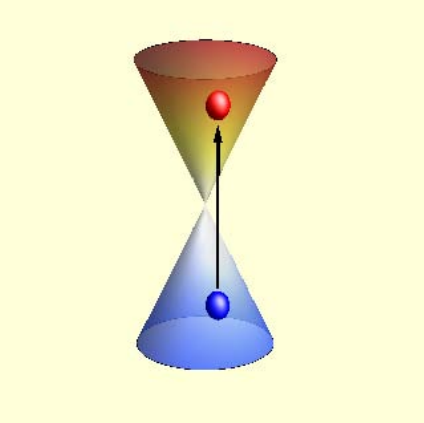

RAPID COMMUNICATIONS

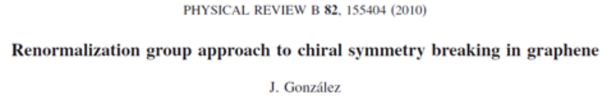
PHYSICAL REVIEW B **82**, 121413(R) (2010)

**Variational approach to the excitonic phase transition in graphene**

J. Sabio,[1,2] F. Sols,[2] and F. Guinea[1]

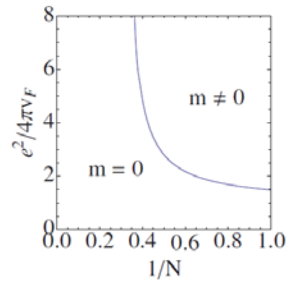
$$|\Psi_g\rangle \equiv \left(\alpha_k + \beta_k c^+_{e,k} c_{h,k}\right)|\Psi_0\rangle$$



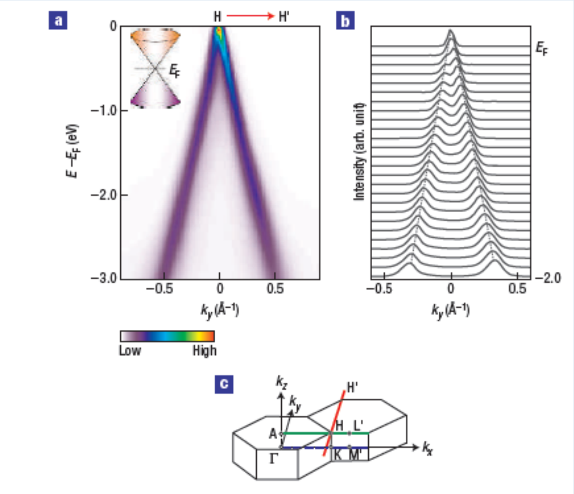

PHYSICAL REVIEW B **82**, 155404 (2010)

**Renormalization group approach to chiral symmetry breaking in graphene**

J. González

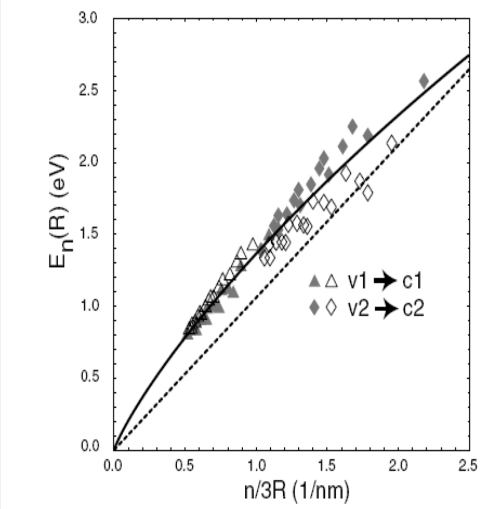


# Some early experiments



S. Y. Zhou, G.-H. Gweon, J. Graf, A. V. Fedorov, C. D. Spataru, R. D. Diehl, Y. Kopelevich, D.-H. Lee, Steven G. Louie and A. Lanzara, Nature Phys. **2**, 595 (2006)

The quasiparticle lifetime increases linearly with energy.



C. L. Kane and E. J. Mele, Phys. Rev. Lett. **93**, 197402 (2004)

The dependence of gaps on nanotube radii can be written in terms of 1/R and log(R) contributions.

## Fabry–Perot interference in a nanotube electron waveguide

Wenjie Liang*†, Marc Bockrath†‡, Dolores Bozovic‡, Jason H. Hafner*, M. Tinkham‡ & Hongkun Park*

$v_F = 8.1 \times 10^5$ m s$^{-1}$

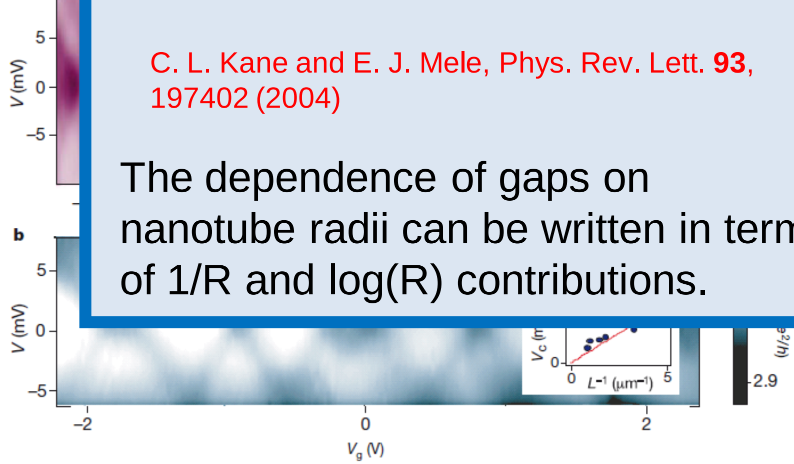
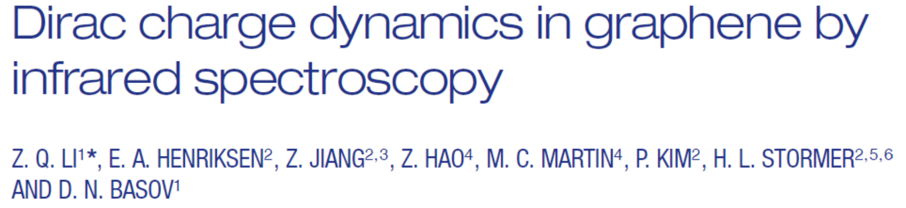


**Figure 2** Two-dimensional $\partial I/\partial V$ plots as a function of $V$ and $V_g$ measured at $T = 4$K. **a**, Data from a 530-nm SWNT device; **b**, data from a 220-nm SWNT device. Both plots show a quasi-periodic pattern of crisscrossing dark lines that correspond to the $\partial I/\partial V$ dips as $V$ and $V_g$ are varied. The bias voltage values ($V_c$) at which adjacent positively and negatively sloped lines intersect (white arrows) quantify the energy scales for $\partial I/\partial V$ oscillations. In **a**, $V_c$ is ~3.5 meV; in **b**, $V_c$ is ~6.5 meV. Inset, values of $V_c$ from seven devices plotted against the inverse nanotube length ($L^{-1}$). The solid curve is a line with a slope equal to $hv_F/2 = 1{,}670$ meV nm$^{-1}$, where $v_F = 8.1 \times 10^5$ m s$^{-1}$ is the Fermi velocity in the nanotube.

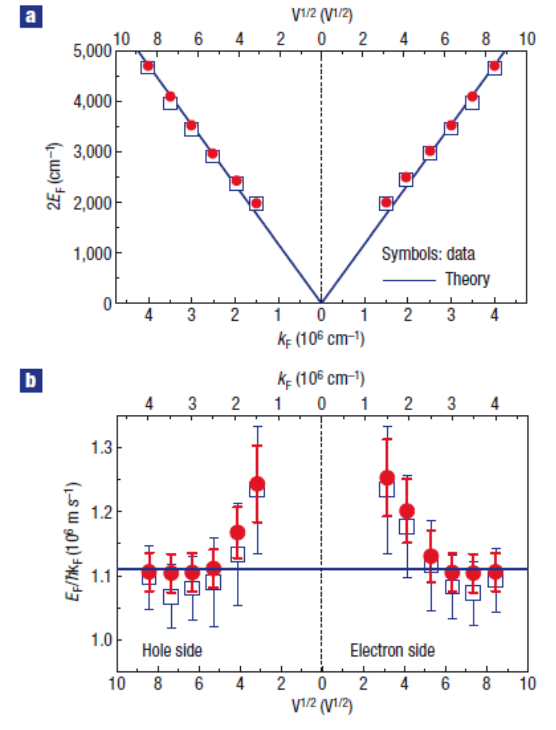
# Recent experiments



## Dirac charge dynamics in graphene by infrared spectroscopy

Z. Q. LI[1]*, E. A. HENRIKSEN[2], Z. JIANG[2,3], Z. HAO[4], M. C. MARTIN[4], P. KIM[2], H. L. STORMER[2,5,6] AND D. N. BASOV[1]

Nature Physics 4, 532 - 535 (2008)
Published online: 8 June 2008 | doi:10.1038/nphys989

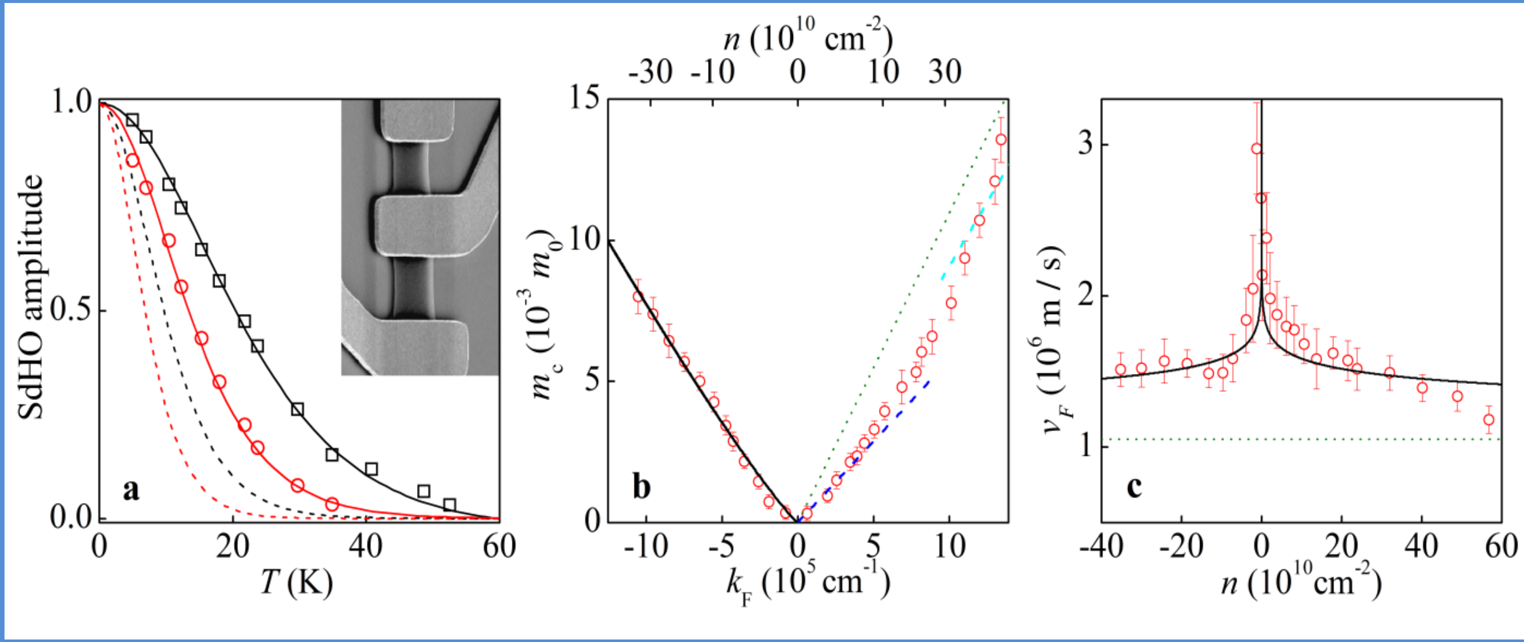


# Measurements of the effective mass

Suspended samples.
Very high mobility

$$\mu \approx 10^{6}\,\mathrm{cm^{2}V^{-1}s^{-1}}$$

$$n = 1.4\times10^{10}\,\mathrm{cm^{-2}}$$
$$n = -7\times10^{10}\,\mathrm{cm^{-2}}$$



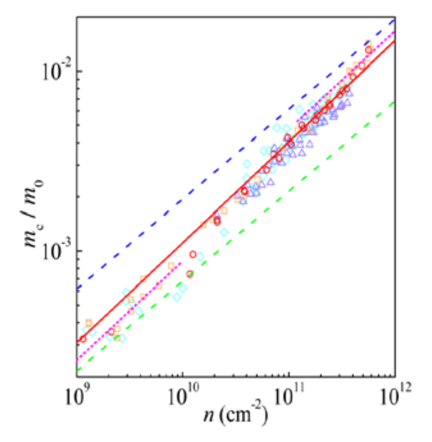
nature physics

LETTERS

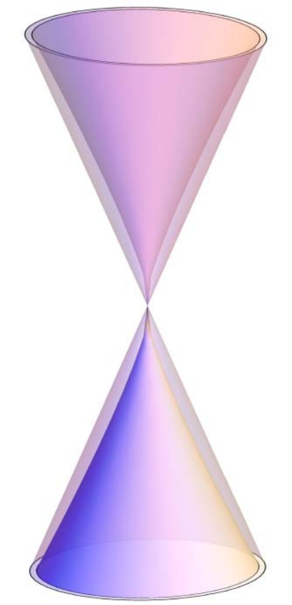
PUBLISHED ONLINE: 24 JULY 2011 | DOI: 10.1038/NPHYS2049

**Dirac cones reshaped by interaction effects in suspended graphene**

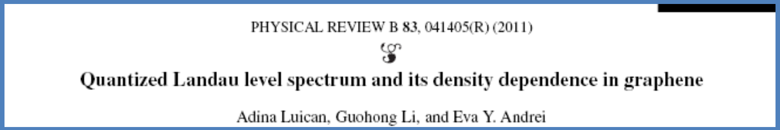
D. C. Elias[1], R. V. Gorbachev[1], A. S. Mayorov[1], S. V. Morozov[2], A. A. Zhukov[3], P. Blake[3], L. A. Ponomarenko[1], I. V. Grigorieva[1], K. S. Novoselov[1], F. Guinea[4]★ and A. K. Geim[1,3]



Fits to Renormalization Group calculations



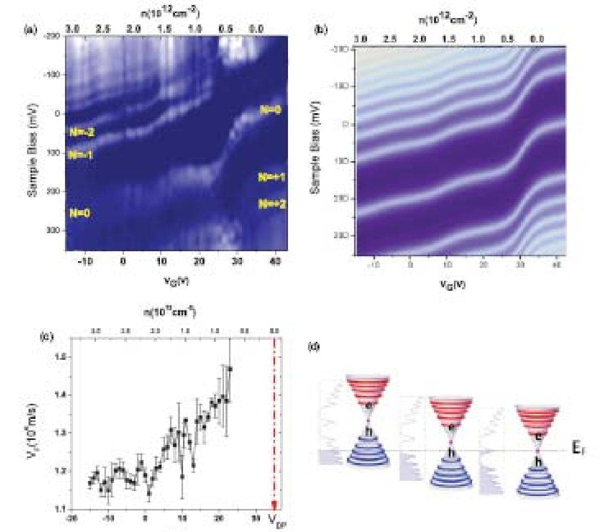
# Other recent measurements

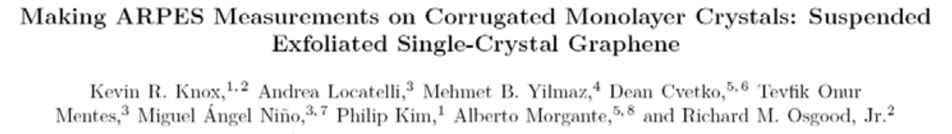


PHYSICAL REVIEW B 83, 041405(R) (2011)

**Quantized Landau level spectrum and its density dependence in graphene**

Adina Luican, Guohong Li, and Eva Y. Andrei

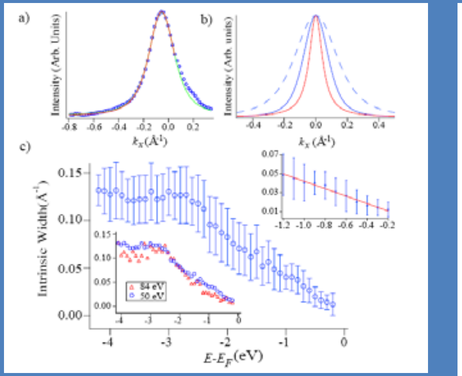


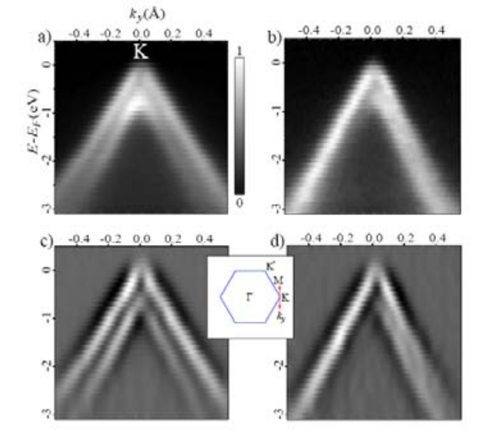


Making ARPES Measurements on Corrugated Monolayer Crystals: Suspended Exfoliated Single-Crystal Graphene

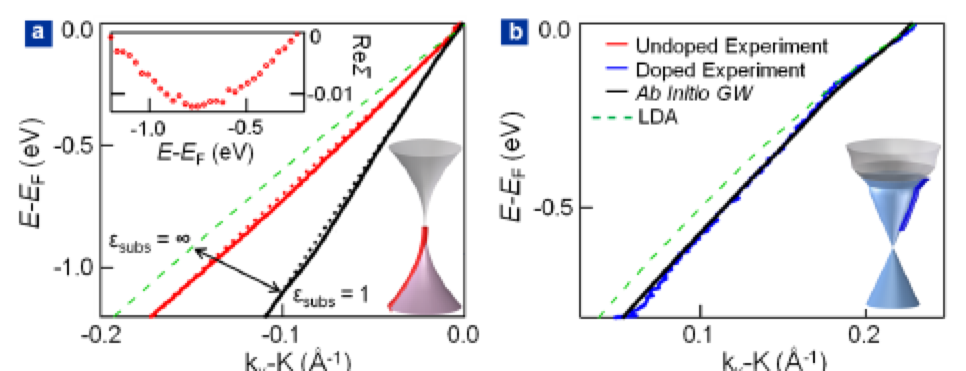
Kevin R. Knox,[1,2] Andrea Locatelli,[3] Mehmet B. Yilmaz,[4] Dean Cvetko,[5,6] Tevfik Onur Mentes,[3] Miguel Ángel Niño,[3,7] Philip Kim,[1] Alberto Morgante,[5,8] and Richard M. Osgood, Jr.[2]

arXiv:1104.2551

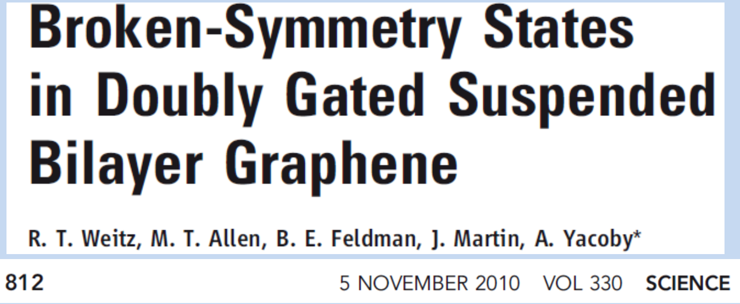




David A. Siegel, Cheol-Hwan Park, Choongyu Hwang, Jack Deslippe, Alexei V. Fedorov, Steven G. Louie, and Alessandra Lanzara, PNAS **108**, 11365 (2011)



# Bilayer graphene



## Broken-Symmetry States in Doubly Gated Suspended Bilayer Graphene

R. T. Weitz, M. T. Allen, B. E. Feldman, J. Martin, A. Yacoby*

812 5 NOVEMBER 2010 VOL 330 SCIENCE

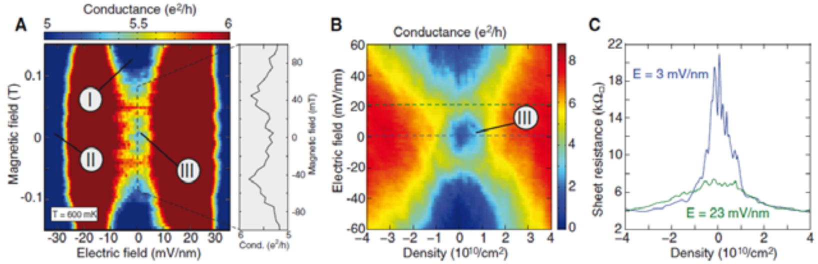
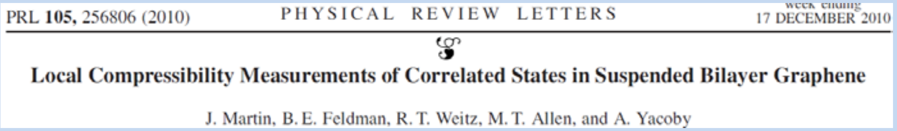


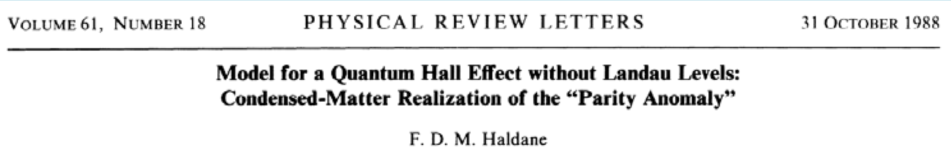
**Fig. 4.** Experimental evidence of a spontaneous gap in suspended bilayer graphene. (**A**) Detailed view of the conductivity at small electric and magnetic fields and zero average carrier density. The color scale has been restricted to between 5 and 6 $e^2/h$ to highlight the observed effect. (**B**) Conductivity as a function of electric field and density at zero magnetic field. (**C**) Two linecuts showing the resistivity at $E = 0$ and $E_{\rm off}$ are also shown. The scans in (B) and (C) were taken after thermal cycling of the sample, hence the difference in the minimal conductance at zero magnetic and electric field with respect to (A).



PRL **105,** 256806 (2010) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2010

### Local Compressibility Measurements of Correlated States in Suspended Bilayer Graphene

J. Martin, B. E. Feldman, R. T. Weitz, M. T. Allen, and A. Yacoby



VOLUME 61, NUMBER 18 PHYSICAL REVIEW LETTERS 31 OCTOBER 1988

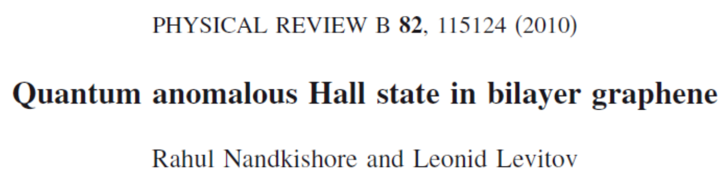
### Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane

PHYSICAL REVIEW B **77**, 041407(R) (2008)

### Pseudospin magnetism in graphene

Hongki Min,[1,*] Giovanni Borghi,[2] Marco Polini,[2] and A. H. MacDonald[1]

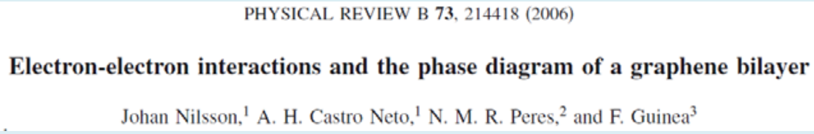


PHYSICAL REVIEW B **82**, 115124 (2010)

### Quantum anomalous Hall state in bilayer graphene

Rahul Nandkishore and Leonid Levitov

Broken time reversal symmetry.
Ground state similar to the Integer Quantum Hall Effect

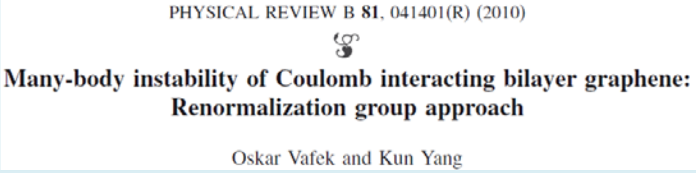


PHYSICAL REVIEW B **73**, 214418 (2006)

### Electron-electron interactions and the phase diagram of a graphene bilayer

Johan Nilsson,[1] A. H. Castro Neto,[1] N. M. R. Peres,[2] and F. Guinea[3]

Magnetic ground state



PHYSICAL REVIEW B **81**, 041401(R) (2010)

### Many-body instability of Coulomb interacting bilayer graphene: Renormalization group approach

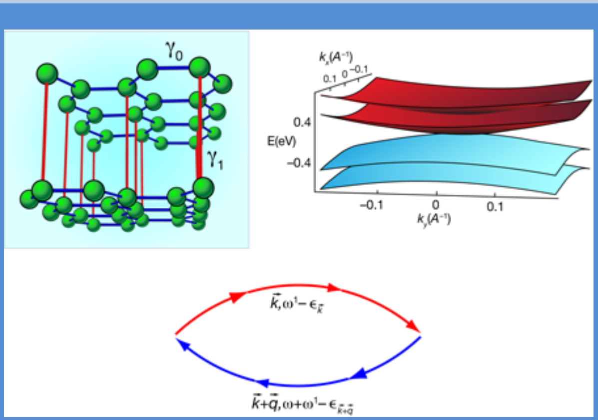
Oskar Vafek and Kun Yang

PHYSICAL REVIEW B **82**, 201408(R) (2010)

### Spontaneous symmetry breaking and Lifshitz transition in bilayer graphene

Y. Lemonik,[1] I. L. Aleiner,[1,2] C. Toke,[3] and V. I. Fal'ko[2,3]

Nematic ground state



Divergent susceptibilities. Couplings become energy dependent .
F. G., Physics **3**, 1 (2010)

# Interaction-Driven Spectrum Reconstruction in Bilayer Graphene

A. S. Mayorov,[1] D. C. Elias,[1] M. Mucha-Kruczynski,[2] R. V. Gorbachev,[3] T. Tudorovskiy,[4] A. Zhukov,[3] S. V. Morozov,[5] M. I. Katsnelson,[4] V. I. Fal'ko,[2] A. K. Geim,[3] K. S. Novoselov[1*]

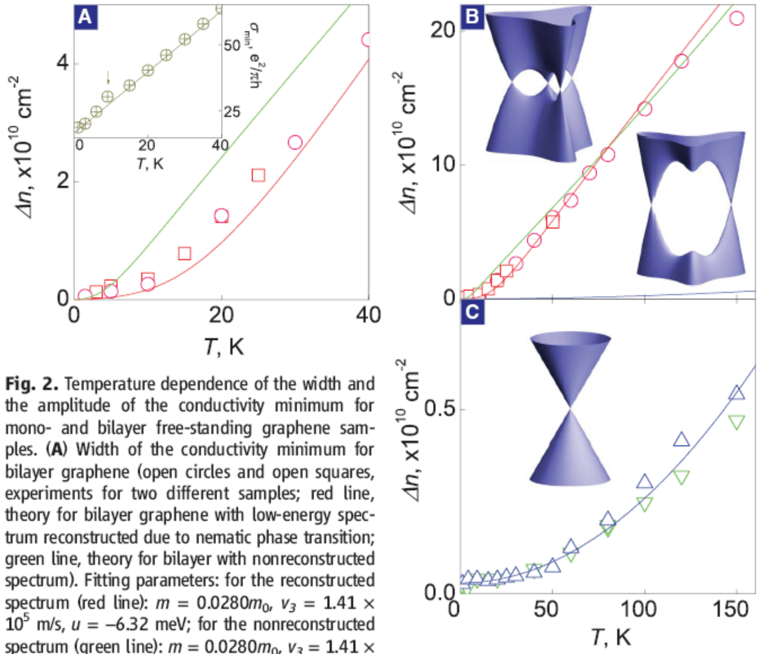


**Fig. 2.** Temperature dependence of the width and the amplitude of the conductivity minimum for mono- and bilayer free-standing graphene samples. (**A**) Width of the conductivity minimum for bilayer graphene (open circles and open squares, experiments for two different samples; red line, theory for bilayer graphene with low-energy spectrum reconstructed due to nematic phase transition; green line, theory for bilayer with nonreconstructed spectrum). Fitting parameters: for the reconstructed spectrum (red line): $m = 0.0280m_0$, $v_3 = 1.41 \times 10^5$ m/s, $u = -6.32$ meV; for the nonreconstructed spectrum (green line): $m = 0.0280m_0$, $v_3 = 1.41 \times 10^5$ m/s, $u = 0$. (Inset) Amplitude of the conductivity minimum of bilayer graphene (yellow crossed circles, experiment; yellow solid line, a guide to the eye). Note the deviation from the straight line below 10 K (marked by arrow). (**B**) The broadening of the conductivity minimum for bilayer samples [circles, squares, and red and green lines are the same as in (A) and for monolayer graphene (blue line, theory)]. (Insets) Left: Low-energy electronic spectrum as expected in the single-electron approximation; right: bilayer graphene low-energy electronic spectrum, reconstructed due to nematic phase transition. (**C**) The broadening of the conductivity minimum for monolayer graphene [blue and green triangles: experimental points for two different samples; blue line: theory, same as in (B)]. (Inset) Low-energy electronic spectrum for monolayer graphene.

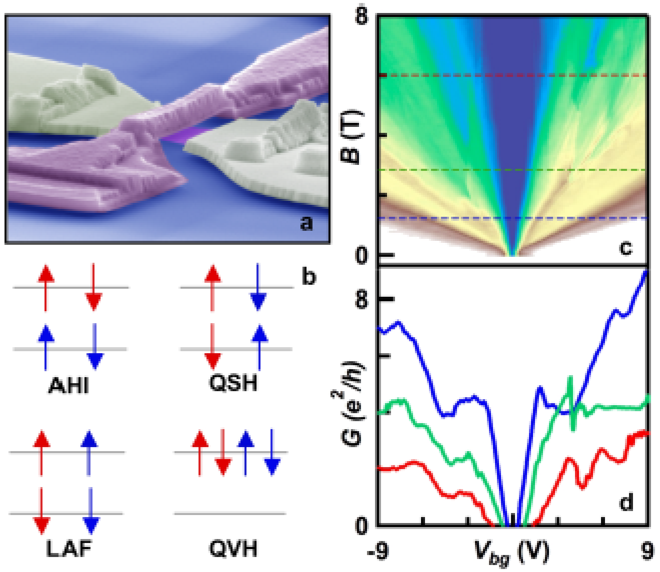
---

# Transport Spectroscopy of Symmetry-Broken Insulating States in Bilayer Graphene

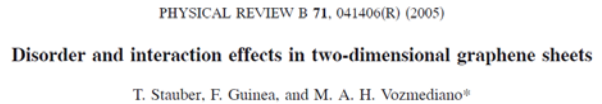
J. Velasco Jr., L. Jing, W. Bao, Y. Lee, P. Kratz, V. Aji, M. Bockrath, C.N. Lau[*] and C. Varma
*Department of Physics and Astronomy, University of California, Riverside, CA 92521*

R. Stillwell and D. Smirnov
*National High Magnetic Field Laboratory, Tallahassee, FL 32310*

Fan Zhang, J. Jung and A.H. MacDonald
*Department of Physic, University of Texas at Austin, Austin, TX 78712*



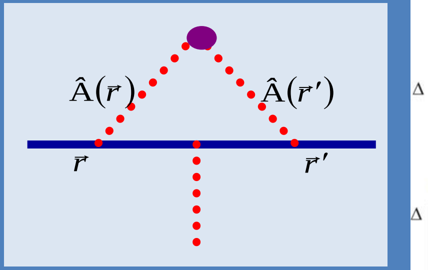
# Interactions and disorder

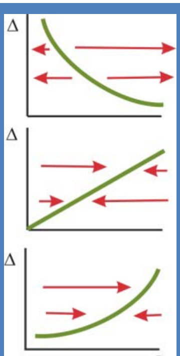


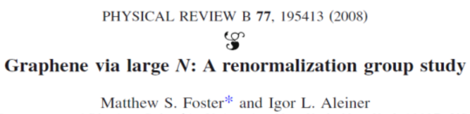
PHYSICAL REVIEW B **71**, 041406(R) (2005)

**Disorder and interaction effects in two-dimensional graphene sheets**

T. Stauber, F. Guinea, and M. A. H. Vozmediano*



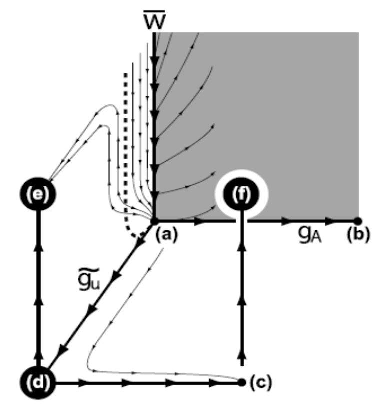


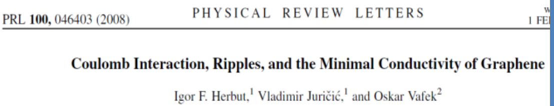


PHYSICAL REVIEW B **77**, 195413 (2008)

**Graphene via large *N*: A renormalization group study**

Matthew S. Foster* and Igor L. Aleiner

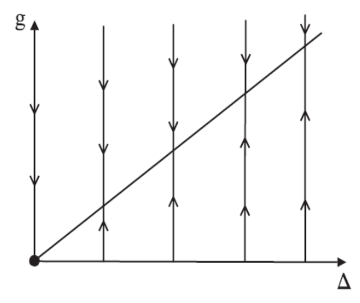


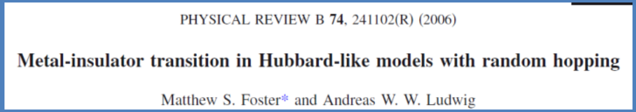


PRL **100**, 046403 (2008) PHYSICAL REVIEW LETTERS

**Coulomb Interaction, Ripples, and the Minimal Conductivity of Graphene**

Igor F. Herbut,[1] Vladimir Juričić,[1] and Oskar Vafek[2]

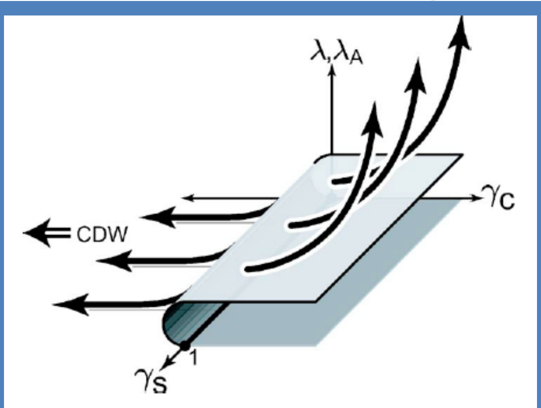


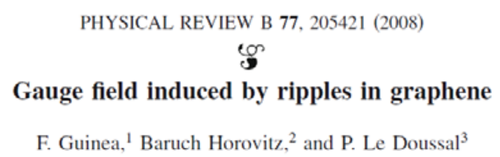


PHYSICAL REVIEW B **74**, 241102(R) (2006)

**Metal-insulator transition in Hubbard-like models with random hopping**

Matthew S. Foster* and Andreas W. W. Ludwig

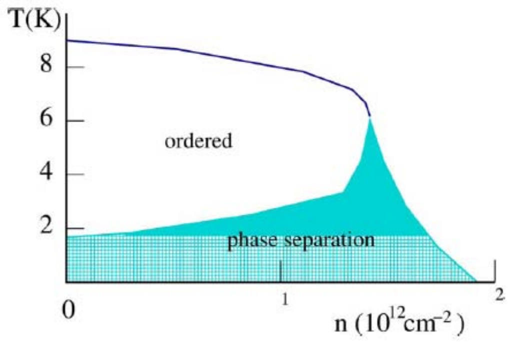




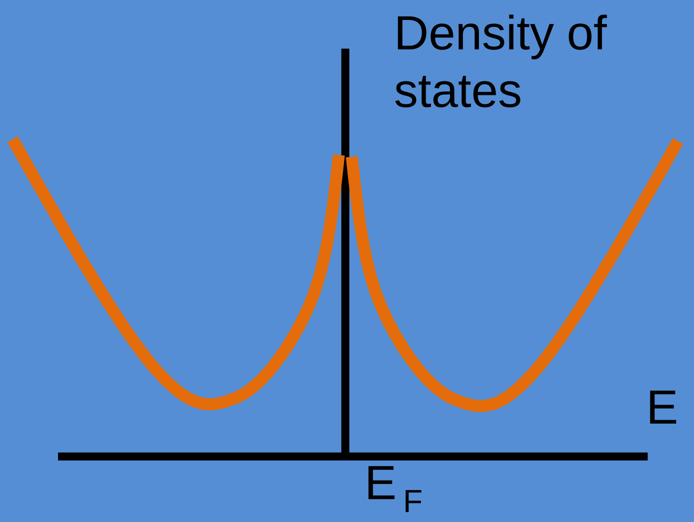
PHYSICAL REVIEW B **77**, 205421 (2008)

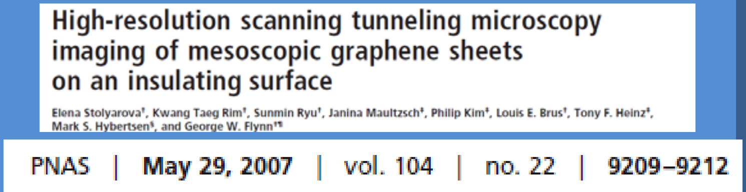
**Gauge field induced by ripples in graphene**

F. Guinea,[1] Baruch Horovitz,[2] and P. Le Doussal[3]



# Strains induce resonances





High-resolution scanning tunneling microscopy imaging of mesoscopic graphene sheets on an insulating surface

Elena Stolyarova†, Kwang Taeg Rim†, Sunmin Ryu†, Janina Maultzsch‡, Philip Kim‡, Louis E. Brus†, Tony F. Heinz‡, Mark S. Hybertsen§, and George W. Flynn†¶

PNAS | May 29, 2007 | vol. 104 | no. 22 | 9209–9212

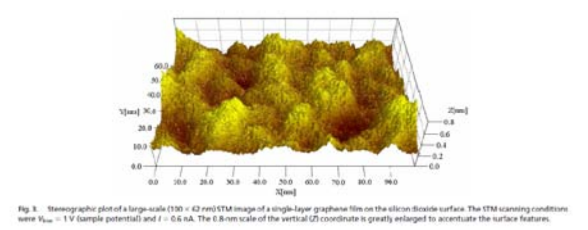
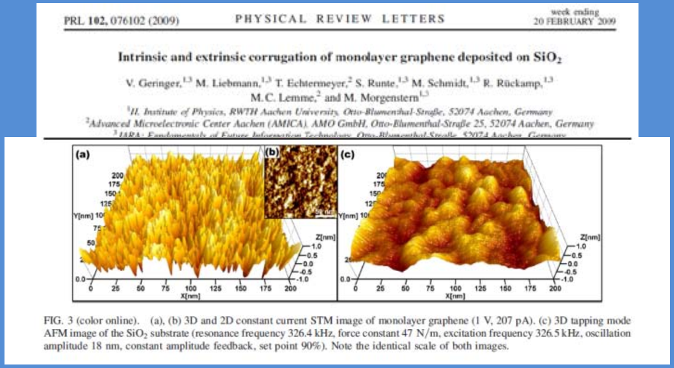


Fig. 3. Stereographic plot of a large-scale (100 × 42 nm) STM image of a single-layer graphene film on the silicon dioxide surface. The STM scanning conditions were $V_{bias}$ = 1 V (sample potential) and $I$ = 0.6 nA. The 0.8-nm scale of the vertical (Z) coordinate is greatly enlarged to accentuate the surface features.



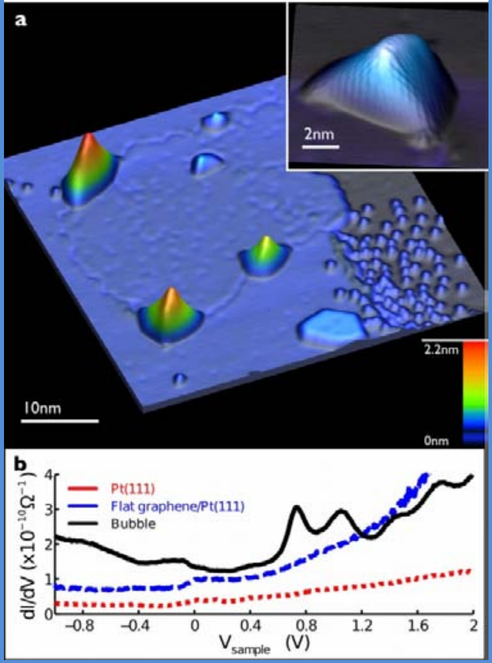
PRL 102, 076102 (2009) PHYSICAL REVIEW LETTERS week ending 20 FEBRUARY 2009

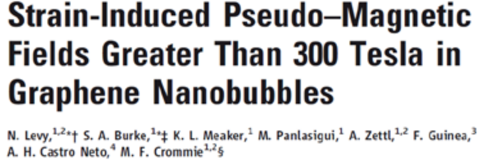
Intrinsic and extrinsic corrugation of monolayer graphene deposited on $SiO_2$

V. Geringer,[1,3] M. Liebmann,[1,3] T. Echtermeyer,[2] S. Runte,[1,3] M. Schmidt,[1,3] R. Rückamp,[1,3] M.C. Lemme,[2] and M. Morgenstern[1,3]

[1]II. Institute of Physics, RWTH Aachen University, Otto-Blumenthal-Straße, 52074 Aachen, Germany
[2]Advanced Microelectronic Center Aachen (AMICA), AMO GmbH, Otto-Blumenthal-Straße 25, 52074 Aachen, Germany
[3]JARA: Fundamentals of Future Information Technology, Otto-Blumenthal-Straße, 52074 Aachen, Germany

FIG. 3 (color online). (a), (b) 3D and 2D constant current STM image of monolayer graphene (1 V, 207 pA). (c) 3D tapping mode AFM image of the $SiO_2$ substrate (resonance frequency 326.4 kHz, force constant 47 N/m, excitation frequency 326.5 kHz, oscillation amplitude 18 nm, constant amplitude feedback, set point 90%). Note the identical scale of both images.



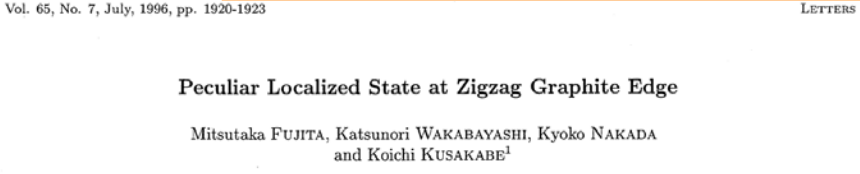


Strain-Induced Pseudo–Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles

N. Levy,[1,2]*† S. A. Burke,[1]*‡ K. L. Meaker,[1] M. Panlasigui,[1] A. Zettl,[1,2] F. Guinea,[3] A. H. Castro Neto,[4] M. F. Crommie[1,2]§

Science **329**, 544 (2010)

# Edge states in graphene



Journal of the Physical Society of
Vol. 65, No. 7, July, 1996, pp. 1920-1923

LETTERS

### Peculiar Localized State at Zigzag Graphite Edge

Mitsutaka FUJITA, Katsunori WAKABAYASHI, Kyoko NAKADA
and Koichi KUSAKABE[1]

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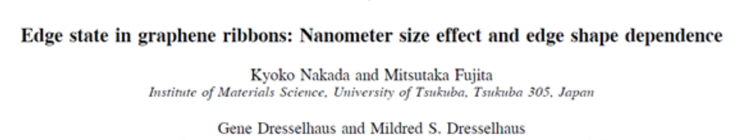

PHYSICAL REVIEW B VOLUME 54, NUMBER 24 15 DECEMBER 1996-II

### Edge state in graphene ribbons: Nanometer size effect and edge shape dependence

Kyoko Nakada and Mitsutaka Fujita
*Institute of Materials Science, University of Tsukuba, Tsukuba 305, Japan*

Gene Dresselhaus and Mildred S. Dresselhaus
*Massachusetts Institute of Technology, Cambridge, Massachusetts 02139-4307*

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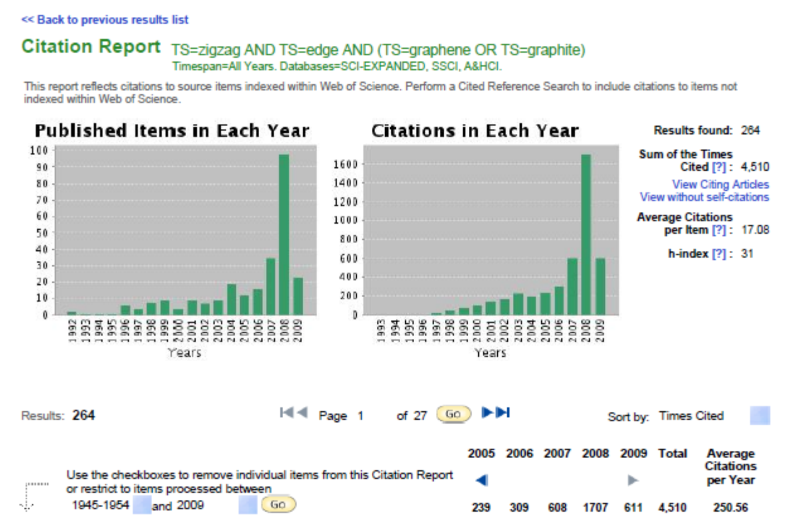

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LETTERS

| DOI: 10.1038/NPHYS1991

nature physics

## Spatially resolving edge states of chiral graphene nanoribbons

Chenggang Tao[1,2†], Liying Jiao[3†], Oleg V. Yazyev[1,2†], Yen-Chia Chen[1,2], Juanjuan Feng[1,4], Xiaowei Zhang[1,2], Rodrigo B. Capaz[1,5], James M. Tour[6], Alex Zettl[1,2], Steven G. Louie[1,2], Hongjie Dai[3] and Michael F. Crommie[1,2]*

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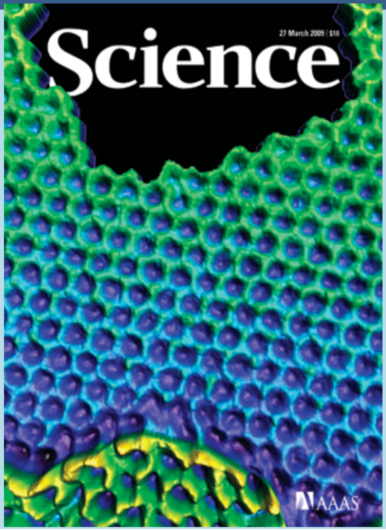

## Controlled Formation of Sharp Zigzag and Armchair Edges in Graphitic Nanoribbons

Xiaoting Jia,[1] Mario Hofmann,[2] Vincent Meunier,[3] Bobby G. Sumpter,[3] Jessica Campos-Delgado,[4] José Manuel Romo-Herrera,[4] Hyungbin Son,[2] Ya-Ping Hsieh,[2] Alfonso Reina,[1] Jing Kong,[2] Mauricio Terrones,[4] Mildred S. Dresselhaus[2,5]*

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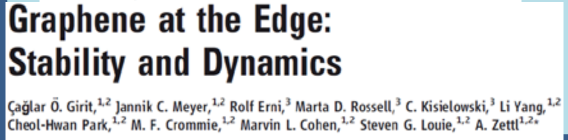

## Graphene at the Edge: Stability and Dynamics

Çağlar Ö. Girit,[1,2] Jannik C. Meyer,[1,2] Rolf Erni,[3] Marta D. Rossell,[3] C. Kisielowski,[3] Li Yang,[1,2] Cheol-Hwan Park,[1,2] M. F. Crommie,[1,2] Marvin L. Cohen,[1,2] Steven G. Louie,[1,2] A. Zettl[1,2]*

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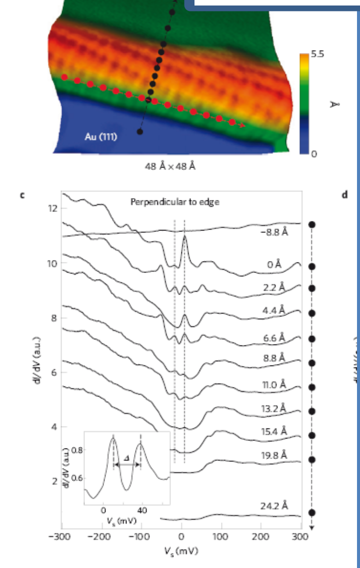


# Boundary conditions at edges

A. R. Akhmerov and C. W. J. Beenakker, Phys. Rev. B **77**, 085423 (2008)

See also:
M. V. Berry and R. J. Mondragon, Proc. R. Soc. London Ser. A **412**, 53 (1987),
E. McCann and V. I. Fal'ko, J. Phys.: Condens. Matter **16**, 2371, (2004),
L. Brey and H. A. Fertig Phys. Rev. B **73**, 235411 (2006)

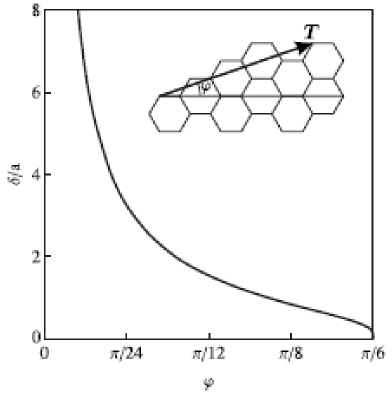


FIG. 2. Dependence on the orientation $\varphi$ of the distance $\delta$ from the boundary within which the zigzag-type boundary condition breaks down. The curve is calculated from formula (3.14) valid in the limit $|T| \gg a$ of large periods. The boundary condition becomes precise upon approaching the zigzag orientation $\varphi = \pi/6$.

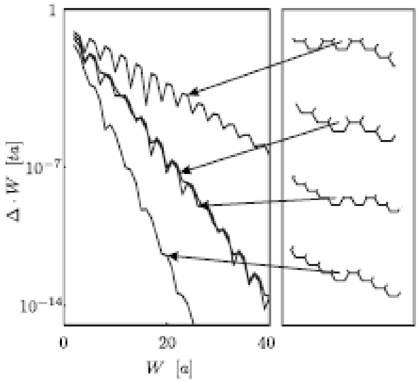
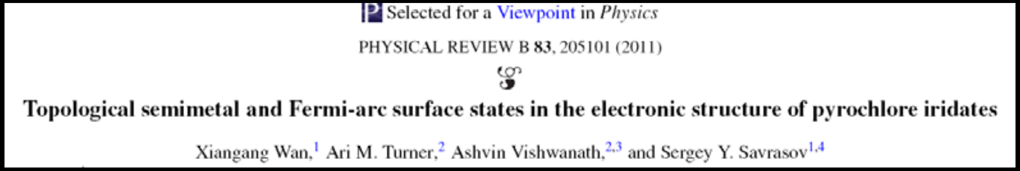


FIG. 7. Dependence of the band gap $\Delta$ of zigzaglike nanoribbons on the width $W$. The curves in the left panel are calculated numerically from the tight-binding equations. The right panel shows the structure of the boundary, repeated periodically along both edges.

From A. R. Akhmerov and C. W. J. Beenakker, Phys. Rev. B **77**, 085423 (2008)

# Edge states, topological aspects

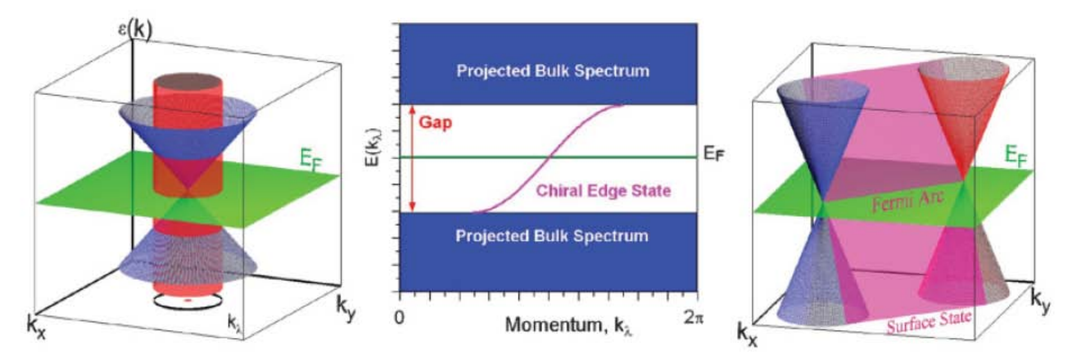


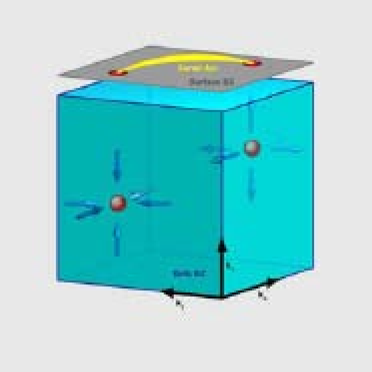
Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW B **83**, 205101 (2011)

**Topological semimetal and Fermi-arc surface states in the electronic structure of pyrochlore iridates**

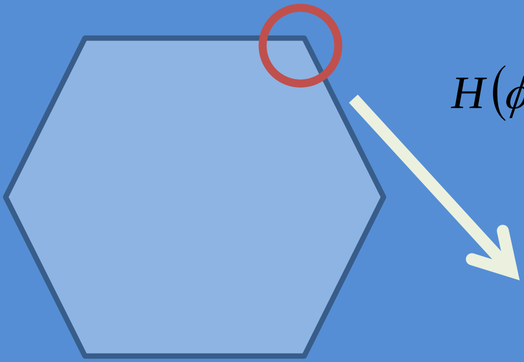
Xiangang Wan,[1] Ari M. Turner,[2] Ashvin Vishwanath,[2,3] and Sergey Y. Savrasov[1,4]



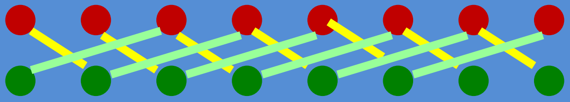


When electron kiss. L. Balents, Physics (2011)

2D analog, D. P. Arovas, F. G., unpublished

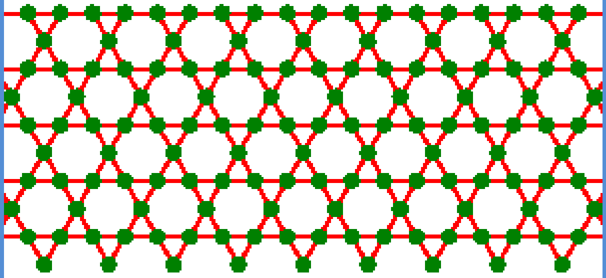


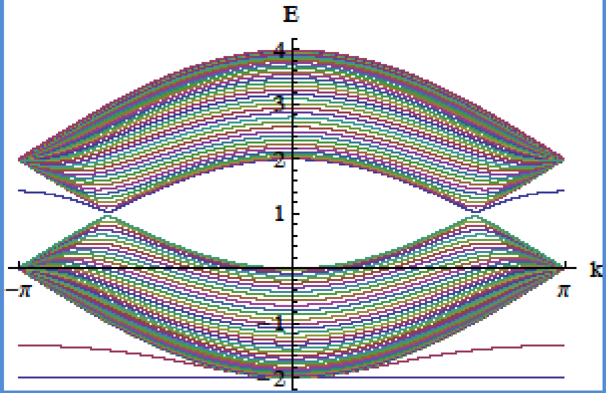
$$H(\phi) \equiv \begin{pmatrix} 0 & v_F k e^{i\phi} + \lambda k^2 e^{-2i\phi} \\ v_F k e^{-i\phi} + \lambda k^2 e^{2i\phi} & 0 \end{pmatrix}$$



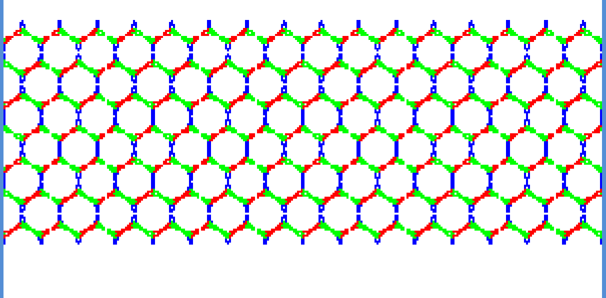
# Edge states, examples

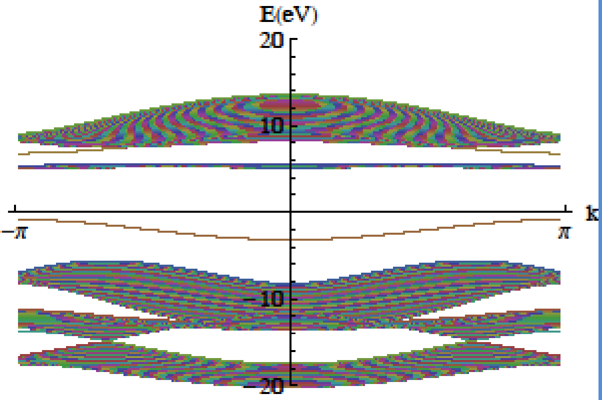
## Kagomé lattice

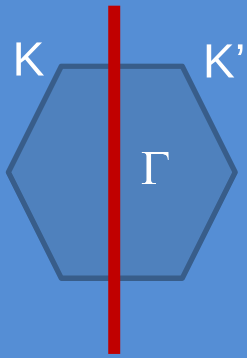




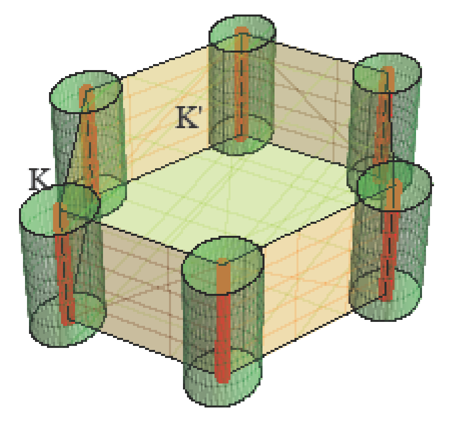
## σ bands of graphene



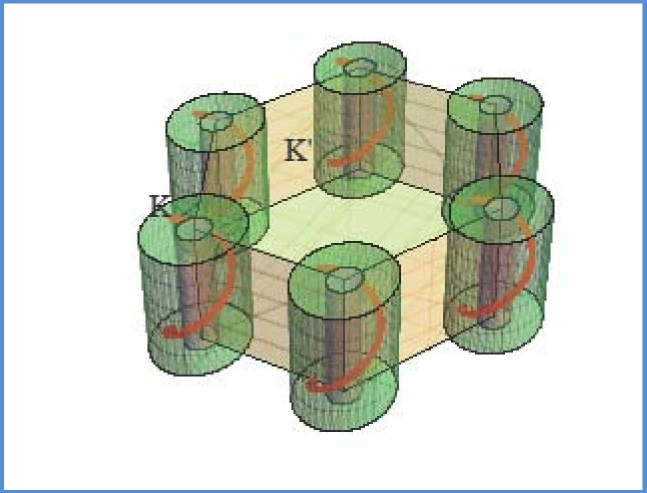




# Surface states, graphite



Bernal graphite



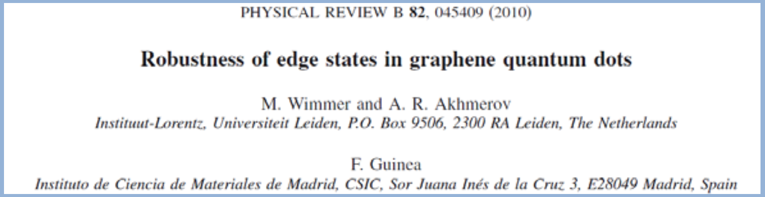
Rhombohedral graphite

A
B
A
B

A
B
C
A
B
C

Graphite has surface bands near the Fermi energy

# Edge states in graphene

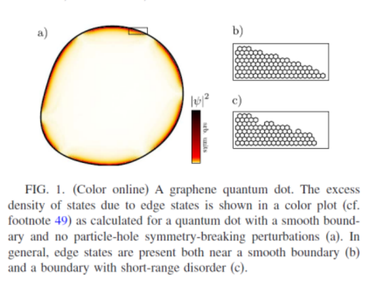


PHYSICAL REVIEW B **82**, 045409 (2010)

**Robustness of edge states in graphene quantum dots**

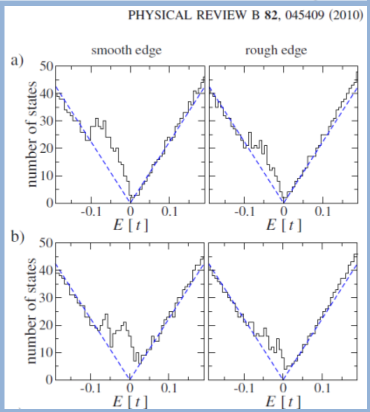
M. Wimmer and A. R. Akhmerov
*Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands*

F. Guinea
*Instituto de Ciencia de Materiales de Madrid, CSIC, Sor Juana Inés de la Cruz 3, E28049 Madrid, Spain*

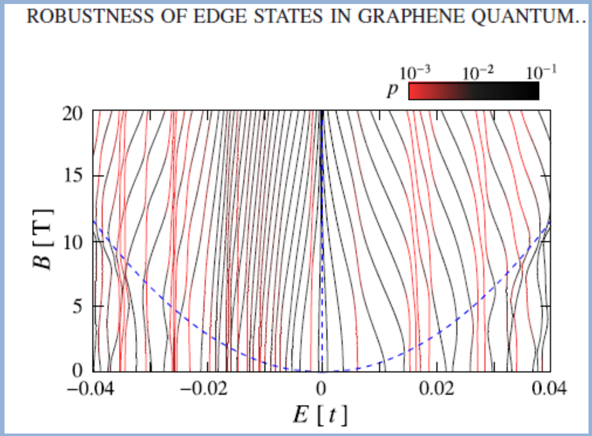


WIMMER, AKHMEROV, AND GUINEA

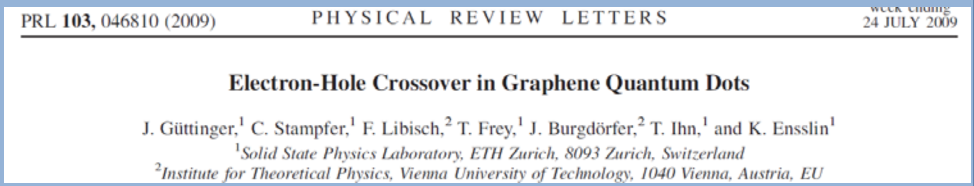
FIG. 1. (Color online) A graphene quantum dot. The excess density of states due to edge states is shown in a color plot (cf. footnote 49) as calculated for a quantum dot with a smooth boundary and no particle-hole symmetry-breaking perturbations (a). In general, edge states are present both near a smooth boundary (b) and a boundary with short-range disorder (c).



PHYSICAL REVIEW B **82**, 045409 (2010)



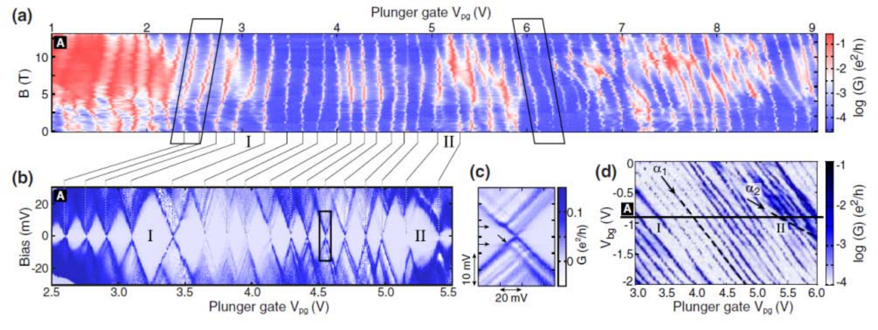
ROBUSTNESS OF EDGE STATES IN GRAPHENE QUANTUM...

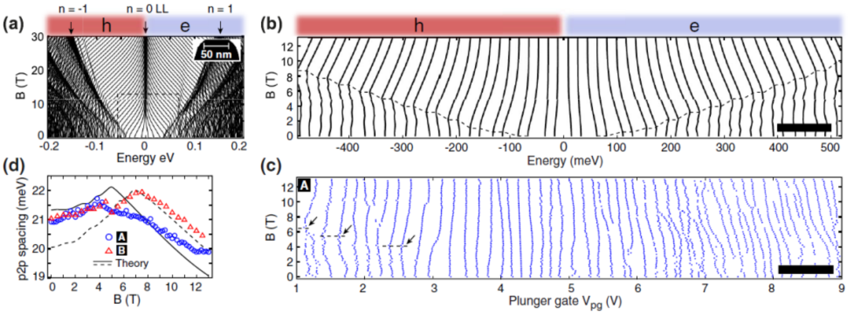


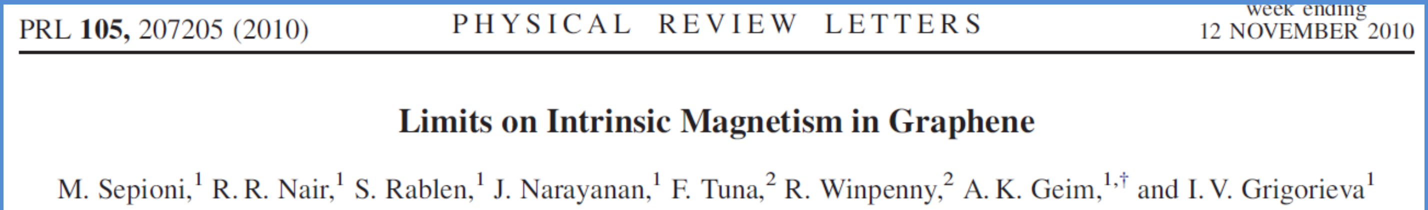
PRL **103**, 046810 (2009) PHYSICAL REVIEW LETTERS week ending 24 JULY 2009

**Electron-Hole Crossover in Graphene Quantum Dots**

J. Güttinger,[1] C. Stampfer,[1] F. Libisch,[2] T. Frey,[1] J. Burgdörfer,[2] T. Ihn,[1] and K. Ensslin[1]
[1]*Solid State Physics Laboratory, ETH Zurich, 8093 Zurich, Switzerland*
[2]*Institute for Theoretical Physics, Vienna University of Technology, 1040 Vienna, Austria, EU*





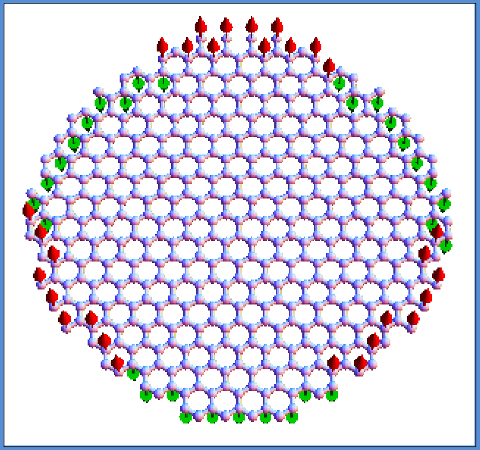


PRL **105**, 207205 (2010) PHYSICAL REVIEW LETTERS week ending 12 NOVEMBER 2010

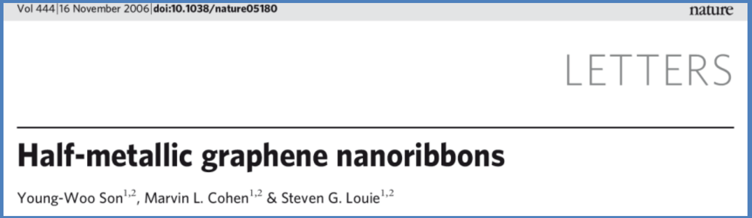
**Limits on Intrinsic Magnetism in Graphene**

M. Sepioni,[1] R. R. Nair,[1] S. Rablen,[1] J. Narayanan,[1] F. Tuna,[2] R. Winpenny,[2] A. K. Geim,[1,†] and I. V. Grigorieva[1]

# Magnetization at edges



Mean field



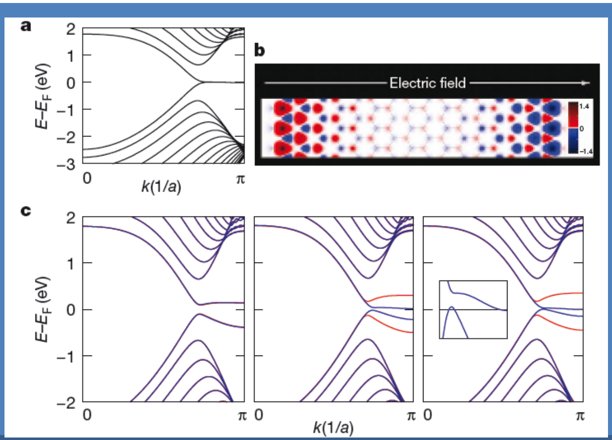
Vol 444 | 16 November 2006 | doi:10.1038/nature05180

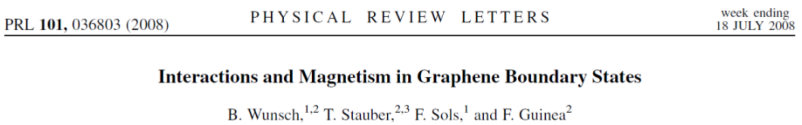
nature

LETTERS

## Half-metallic graphene nanoribbons

Young-Woo Son[1,2], Marvin L. Cohen[1,2] & Steven G. Louie[1,2]

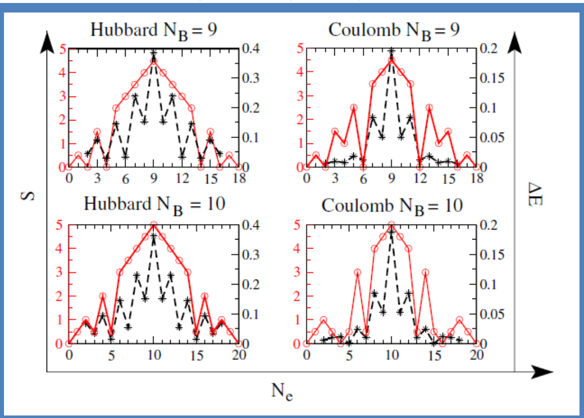




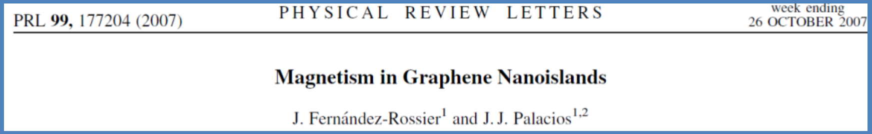
PRL 101, 036803 (2008) PHYSICAL REVIEW LETTERS week ending 18 JULY 2008

## Interactions and Magnetism in Graphene Boundary States

B. Wunsch,[1,2] T. Stauber,[2,3] F. Sols,[1] and F. Guinea[2]



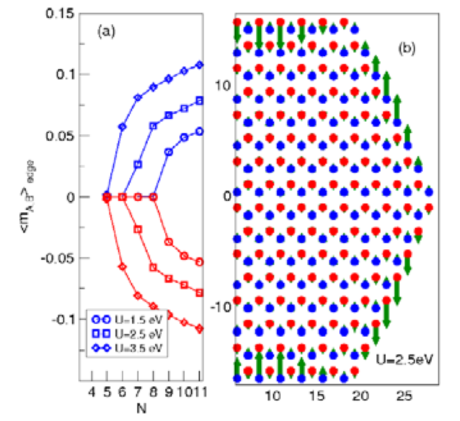
Beyond mean field theory



PRL 99, 177204 (2007) PHYSICAL REVIEW LETTERS week ending 26 OCTOBER 2007

## Magnetism in Graphene Nanoislands

J. Fernández-Rossier[1] and J. J. Palacios[1,2]

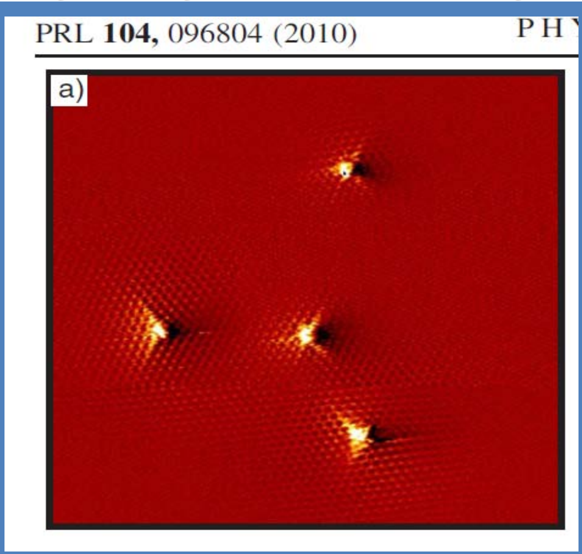


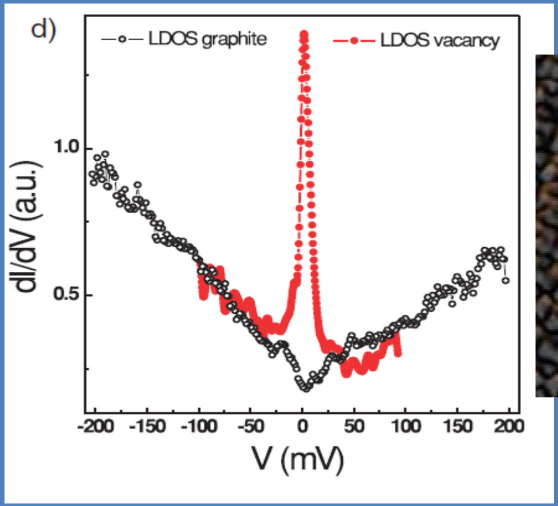
# Vacancies

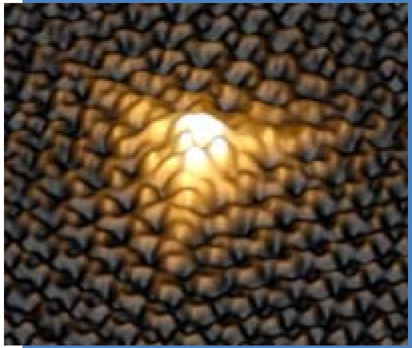
PRL **104,** 096804 (2010) PHYSICAL REVIEW LETTERS week ending 5 MARCH 2010

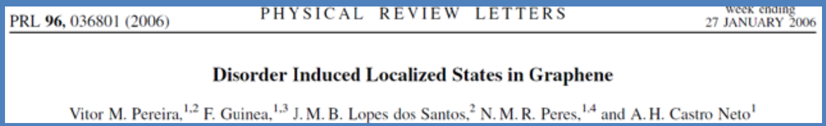
**Missing Atom as a Source of Carbon Magnetism**

M. M. Ugeda,[1] I. Brihuega,[1,*] F. Guinea,[2] and J. M. Gómez-Rodríguez[1]





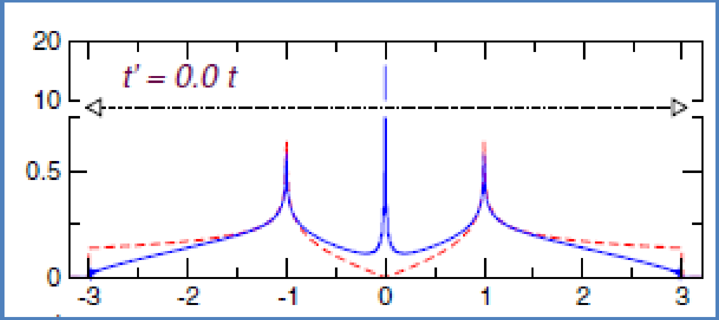


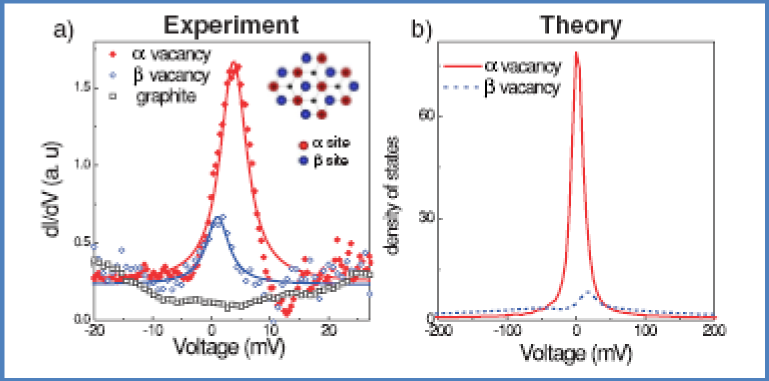


PRL **96,** 036801 (2006) PHYSICAL REVIEW LETTERS week ending 27 JANUARY 2006

**Disorder Induced Localized States in Graphene**

Vitor M. Pereira,[1,2] F. Guinea,[1,3] J. M. B. Lopes dos Santos,[2] N. M. R. Peres,[1,4] and A. H. Castro Neto[1]

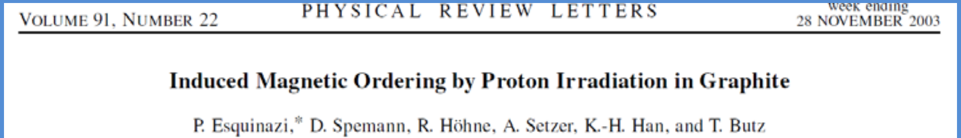




Experiments

Theory

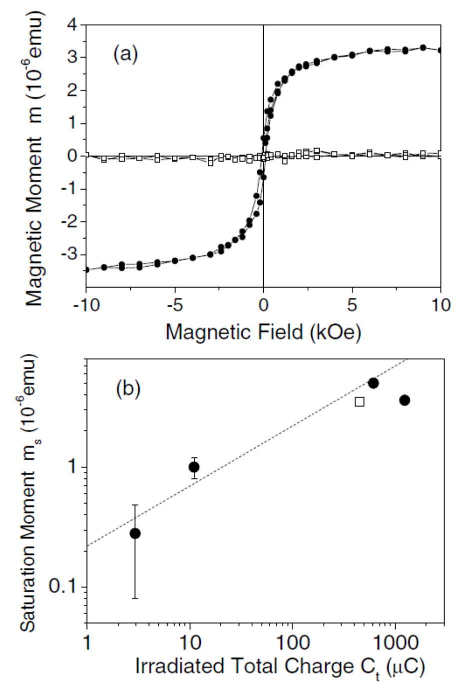
# Proton irradiated graphite

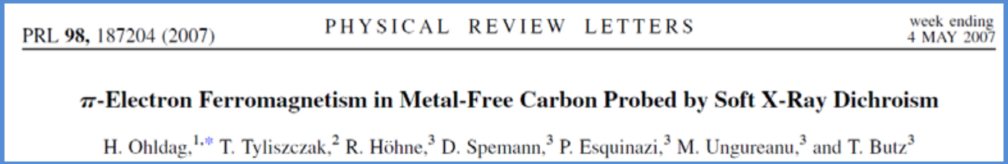


VOLUME 91, NUMBER 22 PHYSICAL REVIEW LETTERS week ending 28 NOVEMBER 2003

**Induced Magnetic Ordering by Proton Irradiation in Graphite**

P. Esquinazi,* D. Spemann, R. Höhne, A. Setzer, K.-H. Han, and T. Butz

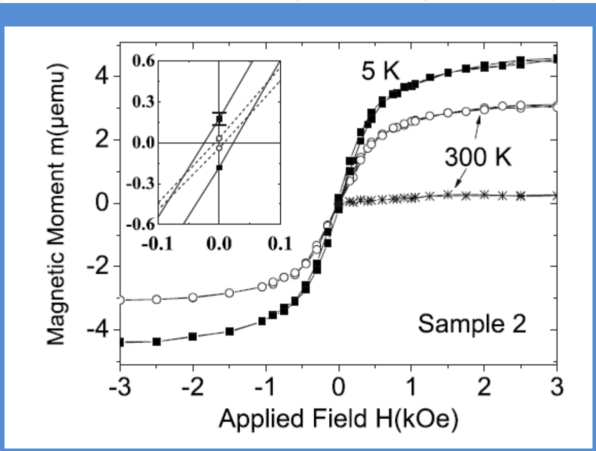




PRL **98,** 187204 (2007) PHYSICAL REVIEW LETTERS week ending 4 MAY 2007

**π-Electron Ferromagnetism in Metal-Free Carbon Probed by Soft X-Ray Dichroism**

H. Ohldag,[1,*] T. Tyliszczak,[2] R. Höhne,[3] D. Spemann,[3] P. Esquinazi,[3] M. Ungureanu,[3] and T. Butz[3]



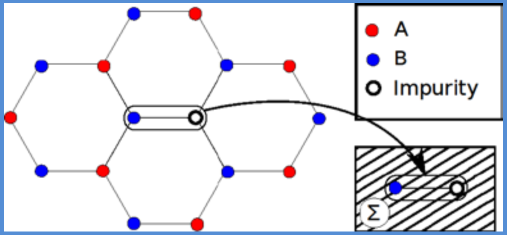
See also J. Cervenka, M. I. Katsnelson, C. F.. J. Flipse, Nature Phys. **5**, 840 (2009)
M. Sepioni, R. R. Nair, S. Rablen, J. Narayanan, F. Tuna, R. Winpenny, A. K. Geim, and I. V. Grigorieva, Phys. Rev. Lett. **105**, 207205 (2010)

# Local moments near vacancies: beyond the mean field approximation

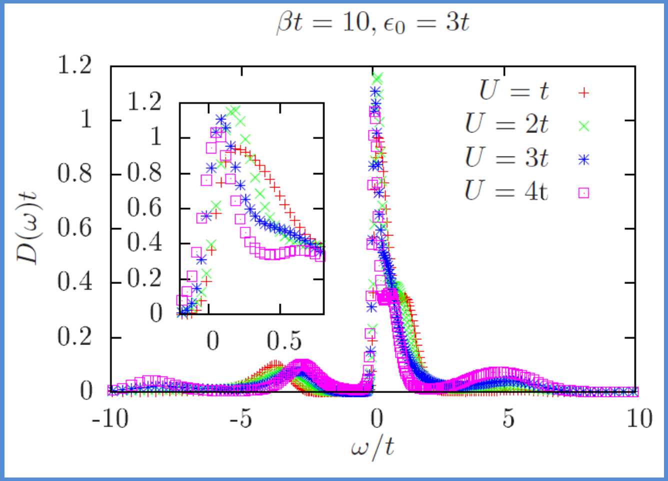
P. Haase, S. Fuchs, Th. Pruschke, H. Ochoa, F. G., Phys. Rev. B **83**, 241408 (2011)

Physical features not captured by mean field analyses:

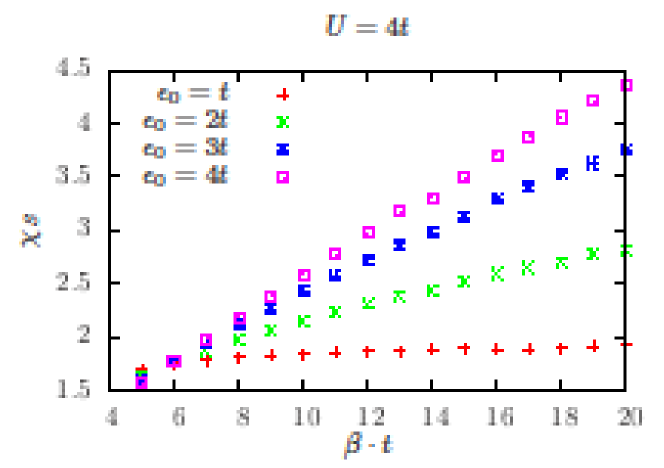
- Magnetic fluctuations
- Kondo effect



Dynamical Mean Field Theory
Hubbard interactions

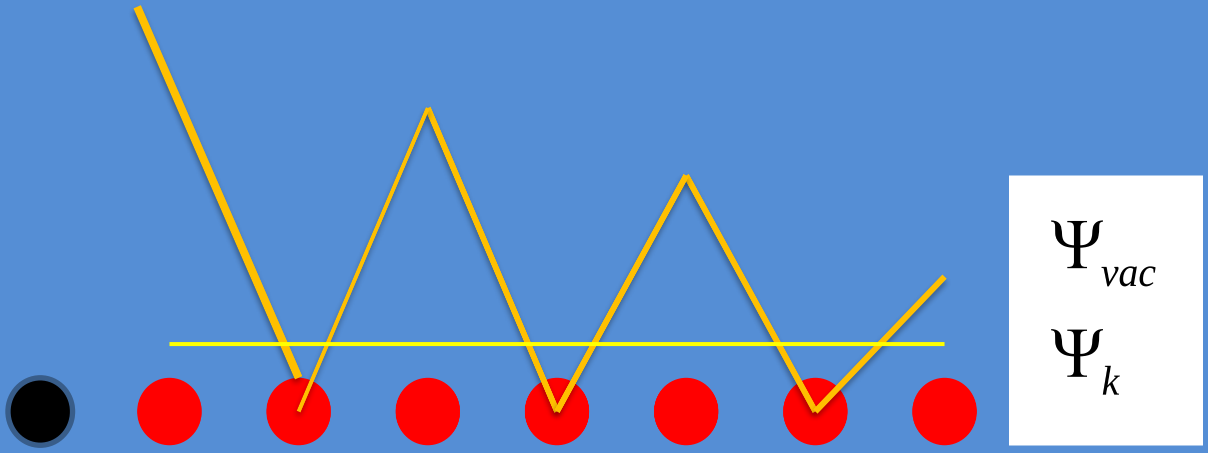


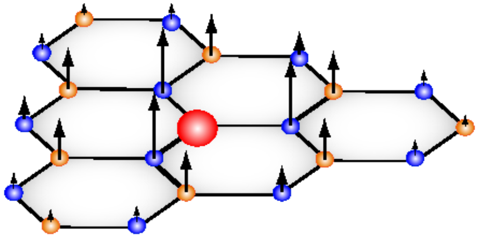
The width of the resonance decreasses as the interaction increases



Curie like susceptibility

# Local moments near vacancies: beyond the mean field approximation





- A local moment is formed near a vacancy
- The moment is not quenched at low temperatures
- The coupling between the moment and the conduction electrons is possibly ferromagnetic
- Non local interactions enhance the formation of a moment

- Interactions are not negligible in graphene

- Single layer graphene is a marginal Fermi liquid.

- Interactions in multilayered graphene can lead to novel broken symmetries

- The existence of edge states can be derived from topological arguments

- Magnetic moments are likely to be formed near edges and vacancies.

- Unusual Kondo effect, ferromagnetic and multichannel.